Computational Continuum Mechanics

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Lecture – 37

Finite Element formulation of Ductile Fracture in Coupled-Thermo-Elastoplastic Dynamic Contact Problems

Welcome to this, last section of this course. And, in this particular lecture and in the coming 2 to 3 lectures, what we are going to do is we are going to see the Finite Element formulation of Ductile Fracture in Coupled-Thermo-Elastoplastic Dynamic Contact Problem ok.

So, in our previous discussion the constitutive model that we had taken was for a compressible hyper elastic material ok, which was neo Hookean material ok. So, now, that is a very special kind of material, which is where you can derive the stresses from the stored strain energy density potential ok.

So, there was no dissipation involved and as such it led to a very convenient and easily implementable finite element form ok. So, we had derived till Newton Raphson procedure in our previous lecture.

So, now, to give you an idea and glimpse of the application of continuum mechanics in the computational setting, I have taken this complete formulation of coupled thermoelastoplastic dynamic contact problem, where you have to simulate ductile fracture ok. So, it will be a very concise presentation in a sense that I am not going into deep into each and every aspect, but the idea of this whole presentation ok.

In the coming lectures is to show you that, if you can identify the physics of the problem you can actually simulate a lot of complicated problems ok. Involving dynamic effects thermal effect contact ok, plasticity and on top of that the behavior of the material degradation ok, because of the cracks that originate inside the body, because of the loading that it is subjected to ok. So, the title of our presentation is the finite element formulation of ductile fracture in coupled thermo elastoplastic dynamic contact problems.

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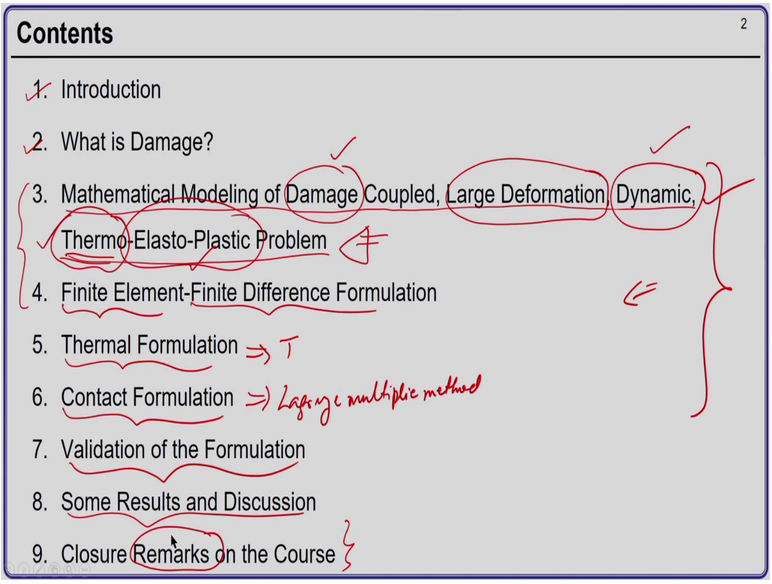

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So, our plan for the next few lectures is as follows ok. So, we will first have a brief introduction followed by in the present lecture, we will discuss what is actually meant by damage ok. So, after that in the next two lectures, we will see the mathematical modeling of a damage coupled, large deformation, dynamic thermo-elastoplastic contact problem ok.

So, here it will be a mathematical model where we will have the effect of the damage ok. So, the degradation of the material performance because of the voids, the cracks, that form inside the body. The body will be allowed to undergo large deformation, which means the strains will be very high ok. Which is generally found when the body undergoes severe loading, weather loading takes it beyond the yield limit.

And, in this course we will deal with ductile material only I mean in this particular discussion that we are going to do. We will also consider the effect of time ok. Till now we did not consider the effect of time, but now we will bring in the effect of time also, where the inertia terms mass into acceleration become important ok. Then, we will also bring in thermal effects.

So, you can read here, it is called thermo. Thermal effect means, when the body is getting deformed. So, the plastic work that is being dissipated ok. So, body is getting plastically deform. So, the plastic work associated with this plastic strain ok, gets released in the form of heat inside the body.

So, that heat also effects the deformation. And, also when the two bodies contact each other because this is a contact problem. So, because of the friction which happens between the two surfaces of the contacting body there will be frictional heat which is generated.

So, part of the heat goes inside one body and part of the heat goes inside another body. So, this heat also ok, effects the deformation at least at the local level where at the near to the contact surface ok. And, then our material will be elastoplastic, which means the material is allowed to deform beyond the elastic limit ok, it can deform plastically.

So, you see we have one physics which is damage ok, we have another physics which is time, we have another physics which is thermal effect thermo and then we have elastoplastic ok. So, in all we have four different physics which come into this particular problem ok.

So, there is this multi physics problem. So, once we have developed the mathematical model for this damage coupled large deformation dynamic thermal elastoplastic contact problem, then what we will get is a equation, which is similar to the newtons second law ok.

So, now, to solve that ok. So, that will be a finally, we will derive the weak form and then that weak form has to be converted to the finite element form ok. So, there will be time involved. So, we have to do what is called the finite difference formulation in time ok.

So, in space we will do finite element formulation and in time we will do finite difference formulation. So, we will use new marks time integration scheme for finite difference formulation. So, we will also do the thermal formulation ok, for so, we have to take into account the plastic work and the frictional heat which is generated and top of that we will also consider whether some outside heat is also coming inside the body.

So, with taking all this into account we will develop our thermal formulation ok, to find out the temperature T ok. And, then because it is a contact problem we have to carry out the contact formulation. So, that we can ensure that one body, which is impacting the another body, should not go inside the another body ok.

So, for this we will use what is called the Lagrange multiplier method ok. We will use Lagrange multiplier method ok. And, once when we develop a formulation ok, from step 3 to step 6 we have our formulation ok.

We have developed our formulation, we have developed the mathematical formulation, we have discretize in space and time here. And, then once we have described everything and we have developed our finite element formulation we go and implement that finite element formulation inside a computer code and we develop a computer code, which first needs to be validated. So, we will see how do we do the validation Ok.

So, the purpose of this is to show the steps when you yourself will write any formulation ok. Develop any code and based on some formulation that you have developed in that case you will first have to do the validation of your problem. Validation means, you take some examples from the literature, which are already available.

Which can be done using the formulation that you have developed in your code and then you run those problems using your own code and you match your results with the results that are reported in the literature for that particular problem.

It can be an analytical problem like for example, Howard Stone contact for which a analytical solution is available or it might be some experimental results which people have already done, or it might be the numerical results, or the finite element results of somebody else ok.

So, you take those problems you solve using your own code and you see how good your code matches results of your code matches with the results of the literature ok. Once, you have done this and you are satisfied with your results that is you have completed your validation, then we will see some of the results that we have obtained using our formulation that we have developed from step 3 to 6 ok.

And, finally, I will conclude this course with some concluding remarks ok, what we have done in this course and what we have left in this course? And, if you want to go into detail about various aspects about this course, then we have to look for suitable literature ok.

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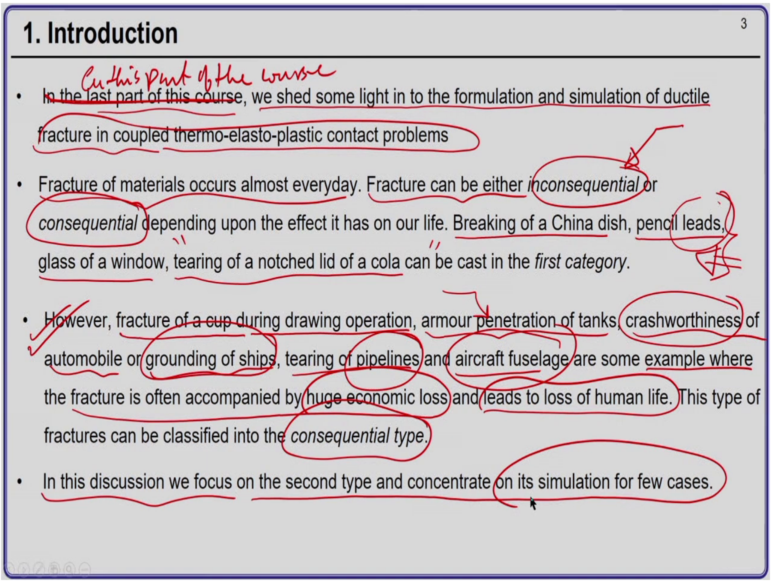

1. Introduction

In this part of the course

- In the last part of this course, we shed some light in to the formulation and simulation of ductile fracture in coupled thermo-elasto-plastic contact problems
- Fracture of materials occurs almost everyday. Fracture can be either *inconsequential* or *consequential* depending upon the effect it has on our life. Breaking of a China dish, pencil leads, glass of a window, tearing of a notched lid of a cola can be cast in the *first category*.
- However, fracture of a cup during drawing operation, armour penetration of tanks, crashworthiness of automobile or grounding of ships, tearing of pipelines and aircraft fuselage are some example where the fracture is often accompanied by huge economic loss and leads to loss of human life. This type of fractures can be classified into the *consequential type*.
- In this discussion we focus on the second type and concentrate on its simulation for few cases.

So, with this we first see, that in the last part of this course, we had shed some light into the formulation and simulation of fracture in coupled and thermo- elasto-plastic coupled problem. So, I will say that in this part of the course ok.

So, this part of the course, we will shed some light into the formulation and simulation of ductile fracture in coupled thermo elastoplastic contact problem ok. Now, fracture as you all would know ok. So, we come across fracture in almost our day today life very often ok. So, some of the fracture types are in consequential type and the some can be consequential in nature.

So, inconsequential fracture are for example, breaking of a china dish or pencil leads glass of a window ok, tearing a notched lid of a cola can ok, all this kind of problem where you have a fracture. So, the dish brakes and there are cracks which is generated and your dish is broken into multiple pieces, or a pencil lead when you apply a lot of pressure beyond a certain limit the lead breaks. So, all these kind of fracture do not have any consequent severe consequence on your day to day life ok.

So, if your dish is broken you will buy a new dish a pencil lead is broken you will sharpen again, if a glass of a window is broken you will replace with a new glass ok. And, of course, the cola can once you have completed the drinking the cola can you will throw that in the dustbin ok. So, these are some of the day today inconsequential fracture problem that we come across ok.

So, we do not often study that of course, I mean designing a China dish which will not break or a pencil lead, which will not break till a certain limit ok, or glass window which is much more resistant to certain kind of impact. I mean these are can be studied, but from our point of view these are not that much important and we classify them into what is called the inconsequential type of fracture ok.

Now; however, if you have fracture of a cup during the drawing operation ok, or you have a armour penetration of tanks, or crashworthiness of automobile, or the grounding of ships, tearing of pipelines or the rigidity or the tearing of aircraft fuselage, because of the cracks that might appear ok.

In the fuselage are some of the examples, where fracture is often accompanied by huge economic loss and it also may lead to loss of the human life. So, there are these kind of problem ok. For example, armour penetration of tanks. So, if you are not designed the armour of a tank very properly, then the shell the anti armour shell may penetrate the armour of the tank, and may explode inside the tank causing severe casualties for the tank room ok.

Similarly, a car for which the crashworthiness has not been carried out properly may lead to the severe injury of the occupants of the car, if there is an impact head on impact or side on impact ok, it may lead to casualty ok.

And, then this grounding of ship so, the hull of the ship. If the ship is grounded it touches the sea floor and it might happen, then the cracks appear in the hull and if the ship is carrying some nasty kind of chemical like, petroleum, it may spill and cause a lot of problem ok..

And, then the petroleum pipelines ok, the tearing of pipelines are used all over the industry, they carry a lot of harmful combustible substances like, gases or petroleum, and under the action of certain kind of loading, if there is an earthquake or if there is a thermal load or something like this. And, there is a tearing of these pipelines it also leads to a lot of economic losses and sometime huge loss of human life ok.

So, also aircraft fuselage aircraft goes so, an airplane goes up in the air ok. So, it goes from atmospheric pressure, where it is the cabin is depressurized it goes up where it is pressurized it comes down lands it is again depressurized. And, the same cycle goes on and on for many times and then this kind of loading that is this kind of fatigue loading, which is there may cause problem in the fuselage.

And, nowadays what happens with a lot of companies focusing on thinner and thinner material, trying to reduce the weight of the aircraft ok. So, the fuselage is very critical for the safe performance of the aircraft ok.

So, if fuselage breaks mid air, then there is certainly loss of human life is involved. So, these type of fracture is what we called the consequential types of fracture, there is certain economic consequences or human life loss as a consequence of these kind of fracture. So, these kind of fracture is what we see in this last part of this course ok. So, we in this discussion we focus on the second type and concentrate on it is simulation for few test cases.

So, we are not going to discuss how to simulate the armour penetration or aircraft fuselage or something like this? But, the class of problem that we will do, we will have a similar kind of fracture which occurs in these kind of problem. Of course, if you want to write a code for these kind of problem it is going to be a very messy job. And, already there are commercial packages which are available which can do it for you ok. So, with this we move to the question of damage.

So, in this particular formulation that we are going to discuss the fracture is incorporated through what is called the damage mechanics approach ok. So, there is material which is said to be damaged, because of the loading which occurs on the material ok. So, material is acted upon by different kind of load, dynamic load, external load, thermal load.

And, because of that the material loses it is load carrying capacity. So, this loss of load carrying capacity is what we called as damage. So, the we say the material is damaged that is it cannot bear as much load ok, as it could before the loads were applied ok.

So, before it the loads were applied suppose the material could take 100 Newton, but after the loads are applied there are cracks and all that form inside the body and the overall strength carrying capacity of the material goes down from say 100 Newton, so to 80 Newton. So, you cannot go to 100 now, you have to go till 80 only ok.

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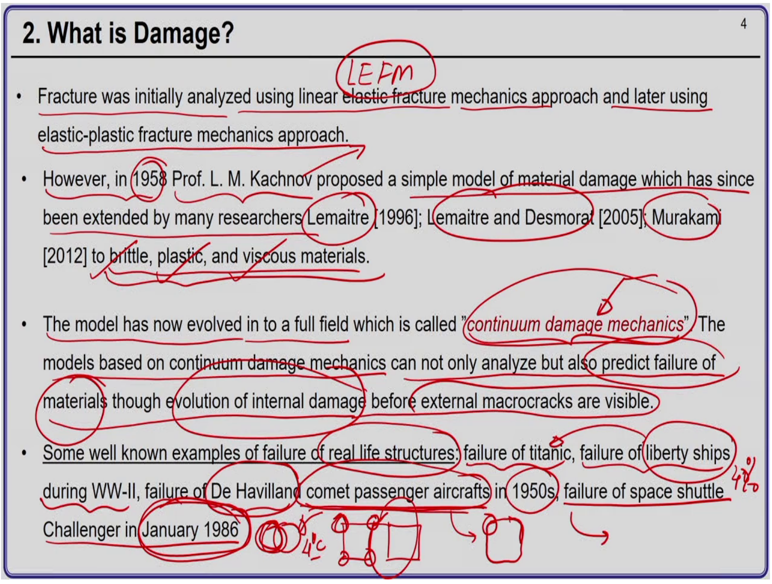

2. What is Damage?

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- Fracture was initially analyzed using linear elastic fracture mechanics approach and later using elastic-plastic fracture mechanics approach.
- However, in 1958 Prof. L. M. Kachnov proposed a simple model of material damage which has since been extended by many researchers Lemaitre [1996]; Lemaitre and Desmorat [2005]; Murakami [2012] to brittle, plastic, and viscous materials.
- The model has now evolved in to a full field which is called "*continuum damage mechanics*". The models based on continuum damage mechanics can not only analyze but also predict failure of materials though evolution of internal damage before external macrocracks are visible.
- Some well known examples of failure of real life structures: failure of titanic, failure of liberty ships during WW-II, failure of De Havilland comet passenger aircrafts in 1950s, failure of space shuttle Challenger in January 1986

So, so, facture was initially analyzed using linear elastic fracture mechanics approach ok. And later on using the elasto-plastic fracture mechanics approach ok. So, the linear elastic fracture mechanics is called LEFM, this is still very popular where we find the stress intensity factor J integral and all these kind of concepts are there ok.

So; however, in 1958 ok. So, Professor L. M. Kachnov proposed a simple model of material damage which has since been extended by many researchers like, Lemaitre and this Desmorat Murakami to brittle, plastic, and visco elastic or visco plastic material.

Now, Professor Kachnov suggested that there is something called damage. And, with this damage you can analyze brittle, plastic or viscous kind of material fracture in these kind of materials ok. So, this particular model has now evolved into a new field which is called the continuum damage mechanics field ok.

So, there was a whole journal of international of journal international journal or damage mechanics which caters to this field of continuum damage mechanics ok. So, the models based on continuum damage mechanics can not only analyze, but it also predict failure of materials through evolution of internal damage, before external microcracks are visible.

So, in linear elastic fracture mechanics and all ok, there has to be a crack already present inside the body and then you analyze when can the crack grow and in which direction it can grow whether at all it can grow. But, here in continuum damage mechanics there is no such restriction that there has to be a crack ok.

So, it can not only lead to the prediction of failure of material, but it can also predict the evolution of the damage. So, from no damage inside the material to complete damage, the whole path of fracture can be done using the field of continuum damage mechanics ok. Now, there are some well known examples of failure of real life structures.

So, it is interesting to see how the failure to understand the correct response of the material to different cases ok, has led to severe failure of real life structures in past ok. So, one of the most famous example is the failure of the titanic ship ok.

As you have know in 1911 titanic ship was going from UK to America. And, over the Atlantic Ocean it collided with an iceberg which cause the tearing of the underside of the ship, through which water entered inside the ship, and then it eventually sank titanic ok.

Now, what caused is fracture so; obviously, you see there was this iceberg which tor through the steel plates ok, which formed the outer covering of the ship. Now, later on it was found out that the steel which was used ok. So, under test condition the steel was very ductile ok, which means that it would deform, but to cause a failure it took a lot of effort.

But, then when it was floating in a Ocean where the temperature was very low then the behavior of the steel changed from ductile to say a lower level of ductile closer to the brittle nature.

So, when there was a huge force which was applied through that iceberg, that steel plate instead of undergoing large plastic deformation it immediately cracked ok, which cause the failure of the outer hull and then through which water the entered leading to the eventual sinking of the ship ok. Also United States launched a lot of ships called the liberty ships during World War II.

So, close to 2500 ships were made and these ships were rapidly made. And, these ships were used to carry armaments and all those kind of stuff during World War II. And, because of the share pace at which they were built, there was no time to study the behavior of the metal which was being used to make those ship under very cold condition ok.

So, close to like forty percent of these ships sank because of the similar kind of failure which was seen in titanic ok. So, when they entered very cold water, the outer metal of the ship became very brittle and then it led to the failure of the ship. The complete ship would crack into half and then sink ok.

Another, very famous example is the failure of the De Havilland comet passenger aircraft in 1950s ok. So, this failure is led to the rise of American companies in the passenger aircraft field for example, Boeing and which continues to this date ok. So, De Havilland Aircrafts were the first Jet Engine Aircrafts and they would flow they would manufactured in UK. And, they will actually fly from one place to another and then what happened was in the fuselage all the windows were of this particular shape. So, they had sharp corners ok.

So, it of few of the aircrafts when they were operating they cracked into half in mid flight leading to the crash of the air craft and all the passenger lives were lost. Later on when the problem was studied they were found that due to the continuous pressurization and De pressurization of the cabin. There was this fatigue loading which was being exerted on the fuselage.

And, also the windows and these sharp corners, where a crack would originate ok. Because, there was the sharp corner and then it because of the sharp corner there was a stress concentration and the cracks would originate. And, those sharp corners and then after a certain number of loading cycles there was a catastrophic propagation of this crack from one window to another and also over the fuselage leading to breaking of the fuselage into 2 parts and eventual crash.

So, with when this was found out till that time Boeing had started making windows, which were like of this shape ok. So, there was no sharp corner there was a blunt corner ok. Which led to stress relaxation and stress would not concentrate and leading to much better performance of the aircraft ok.

Also another famous example is the failure of space shuttle challenger in January 1986, because of the failure of the rubber of the O rings which are there in the rocket ok. So, the O rings are which made of rubber and they are there two there are two sets of O rings, which are there to protect the leakage of fuel from the booster of the rocket.

But, in January 1986 when the shuttle was launched that day the temperature was very low, something like 4 degree centigrade and at this temperature the rubber rings ok, that is the O rings which are made of rubber they become very brittle ok.

So, there was a transition of material which was ductile from ductile to brittle, which cause the catastrophic failure of this O rings, which led to the escape of hot gases and fuels which makes eventually leading to the blast of the challenger and all 7 lives were lost.

All the astronauts were lost along with a young teacher who was also going into the space for the first time ok. So, that were some real life example where failure to appropriately study the nature of fracture has led to catastrophic fail ok.

So, that is why studying the response of the material under different loading conditions is very important.

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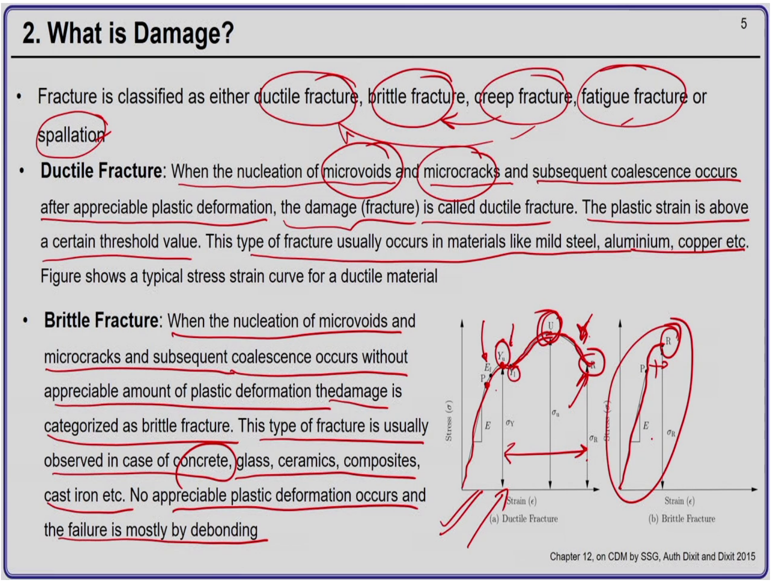

2. What is Damage?

- Fracture is classified as either ductile fracture, brittle fracture, creep fracture, fatigue fracture or spallation
- **Ductile Fracture**: When the nucleation of microvoids and microcracks and subsequent coalescence occurs after appreciable plastic deformation, the damage (fracture) is called ductile fracture. The plastic strain is above a certain threshold value. This type of fracture usually occurs in materials like mild steel, aluminium, copper etc. Figure shows a typical stress strain curve for a ductile material
- **Brittle Fracture**: When the nucleation of microvoids and microcracks and subsequent coalescence occurs without appreciable amount of plastic deformation thedamage is categorized as brittle fracture. This type of fracture is usually observed in case of concrete, glass, ceramics, composites, cast iron etc. No appreciable plastic deformation occurs and the failure is mostly by debonding

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Now, fracture can be classified into what is called the ductile fracture, the brittle fracture, creep fracture, fatigue fracture and spallation ok. So, although these three can be categorized into either of these two, we have kept in separate and we just call them 5 different cases ok. So, ductile fracture is when the nucleation of microvoids and microcracks and subsequent coalescence occurs after appreciable plastic deformation.

So, the material undergoes a lot of plastic deformation before the body generates a microvoids or microcracks inside it ok, which leads to the ultimate fracture. So, in this the damage or the fracture is called the ductile fracture and then the plastic strain in ductile fracture goes above a threshold value ok. So, these kind of fracture usually occurs in materials likes, mild steel, aluminum, copper etcetera. So, this figure over here shows the graph of a stress strain graph of a typical ductile material ok.

So, when you load a ductile material it deforms elastically till a certain point called the proportional limit after which it reaches the elastic limit and then at this point the material yields ok. So, this is called the upper yield limit, and there is a lower yield limit, and after and if you keep on applying the load the stress again increases, when it reaches the maximum point called the ultimate stress. And, finally, there is stress concentration and a lot of cracks and all coalesce together ok.

So, the voids and cracks start forming after yielding, but after ultimate point they start coalescing together with each other resulting in the failure at point R. So, there is a you can see after the yielding has started still there is a considerable deformation which happens, before the eventual fracture takes place ok.

Now, then second type is called the brittle fracture where the nucleation of microvoids and microcracks and subsequent coalescence occurs without appreciable amount of plastic deformation ok, and the damage is categorized as brittle fracture. So, this type of fracture is usually observed in concrete, glass, ceramics, composites, cast iron etcetera ok. So, there is no appreciable plastic deformation which occurs and the failure is mostly by debonding ok.

So, you see the plot of the stress strain curve of a brittle fracture. So, there is elastic deformation and immediately after plastic deformation the material a small amount of plastic deformation the material fails. So, if there is no considerable plastic deformation in brittle fracture.

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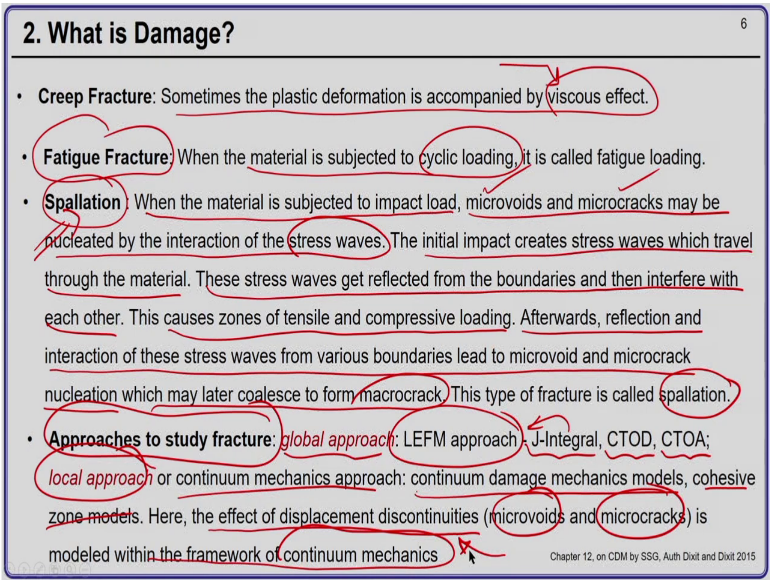

2. What is Damage?

- **Creep Fracture**: Sometimes the plastic deformation is accompanied by viscous effect.
- **Fatigue Fracture**: When the material is subjected to cyclic loading, it is called fatigue loading.
- **Spallation**: When the material is subjected to impact load, microvoids and microcracks may be nucleated by the interaction of the stress waves. The initial impact creates stress waves which travel through the material. These stress waves get reflected from the boundaries and then interfere with each other. This causes zones of tensile and compressive loading. Afterwards, reflection and interaction of these stress waves from various boundaries lead to microvoid and microcrack nucleation which may later coalesce to form macrocrack. This type of fracture is called spallation.
- **Approaches to study fracture**: *global approach*: LEFM approach, J-Integral, CTOD, CTOA; *local approach* or continuum mechanics approach: continuum damage mechanics models, cohesive zone models. Here, the effect of displacement discontinuities (microvoids and microcracks) is modeled within the framework of continuum mechanics

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Now, we quickly look in to what is meant by creep fracture Ok. So, creep fracture is sometimes the plastic deformation is accompanied by viscous effect ok. So, in creep fracture you have to take into account viscous effect and in fatigue fracture the material is subjected to cyclic loading ok.

So, just like an aircraft which is pressurized, when it is up in the air and depressurized, when it lands this kind of loading is called the cyclic loading and then the failure associated with such kind of loading is called the fatigue failure.

Finally, spallation is when the material is subjected to impact load, microvoids and microcracks maybe nucleated by the interaction of stress wave ok. So, in spallation the material is subjected to dynamic loading.

And, because of the dynamic loading there is stress waves which are generated inside the body and these stress waves go back and forth, inside the body they are reflected from the free surfaces and the surfaces where it is held as tensile or compressive wave. And, these waves when they interact with each other they lead to the creation of microvoids and microcracks ok.

So, the initial impact creates stress waves, which travel through the material. And, these stress wave get reflected from the boundaries and then they interfere with each other. This causes zones of tensile and compressive loading and afterwards the reflection and interaction of these waves from various boundaries lead to microvoid and microcrack nucleation which may later coalesce from microcrack.

So, this kind of fracture is called a spallation kind of fracture. So, this is when you hit a body is hit by a load, which is changing rapidly in time, then there will be stress waves and these stress waves will interact with each other creating to fracture ok. So, there are two approaches to study fracture problem, the first approach is a global approach.

So, in this approach we have what is called the linear elastic fracture mechanics approach, where the concepts of J-Integral, Crack Tip Opening Displacement, Cracking opening angle etcetera are considered ok. So, these all fall into the area of global approach. And, also we have what is called the local approach or the continuum mechanics approach ok.

So, in the continuum mechanics approach, we have continuum damage mechanics based models and cohesive zone models ok. So, in local approach the effect of displacement discontinuities, that is the microvoids and microcracks is modeled within the framework of continuum mechanics.

So, we have already discussed in this course some part of continuum mechanics though we did not discussed the thermodynamic part in the constitutive relations part that extensively. But in a pure course on continuum mechanics you would actually go through the those parts, which are needed to model the continuum damage mechanics or the cohesive zone models ok.

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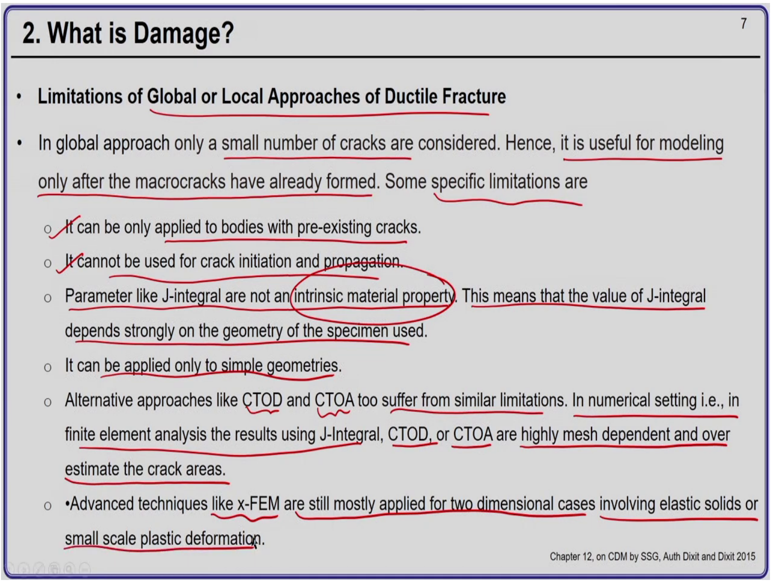

2. What is Damage?

- **Limitations of Global or Local Approaches of Ductile Fracture**
- In global approach only a small number of cracks are considered. Hence, it is useful for modeling only after the macrocracks have already formed. Some specific limitations are
    - It can be only applied to bodies with pre-existing cracks.
    - It cannot be used for crack initiation and propagation.
    - Parameter like J-integral are not an intrinsic material property. This means that the value of J-integral depends strongly on the geometry of the specimen used.
    - It can be applied only to simple geometries.
    - Alternative approaches like CTOD and CTOA too suffer from similar limitations. In numerical setting i.e., in finite element analysis the results using J-Integral, CTOD, or CTOA are highly mesh dependent and over estimate the crack areas.
    - •Advanced techniques like x-FEM are still mostly applied for two dimensional cases involving elastic solids or small scale plastic deformation.

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Now, there are certain limitations which are associated with the global and local approaches of ductile fracture ok.

So, in the global approach you can only model a small number of cracks ok. And, hence it is useful for modeling only after the macro cracks have been already formed ok. Now, there are some specific limitations of the global approach, which are that it can only be applied to bodies with pre existing cracks.

It cannot be used for crack initiation and propagation, although now you can combine it with x fem kind of approaches where you can have the crack propagation, but mostly it is limited to 2 D and very difficult to do in 3 D.

So, parameters like J-integral are not an intrinsic material property this means that the value of the J-integral depends strongly on the geometry of the specimen which is used ok. So, that is the problem as we change the geometry of the problem the J-integral also changes ok.

So, it is not a property of the intrinsic property of the material. So, this global approach can be applied only to simple geometries. Now, alternative approaches like crack tip opening displacement or crack tip opening angle too suffer from similar limitations ok.

So, in the numerical setting in finite element analysis the results using J-integral CTOD, CTOA are highly mesh dependent and overestimate the crack areas. Now, there are advanced technique like x-FEM that are still mostly used for two dimensional cases involving elastic solids or small scale plastic deformation.

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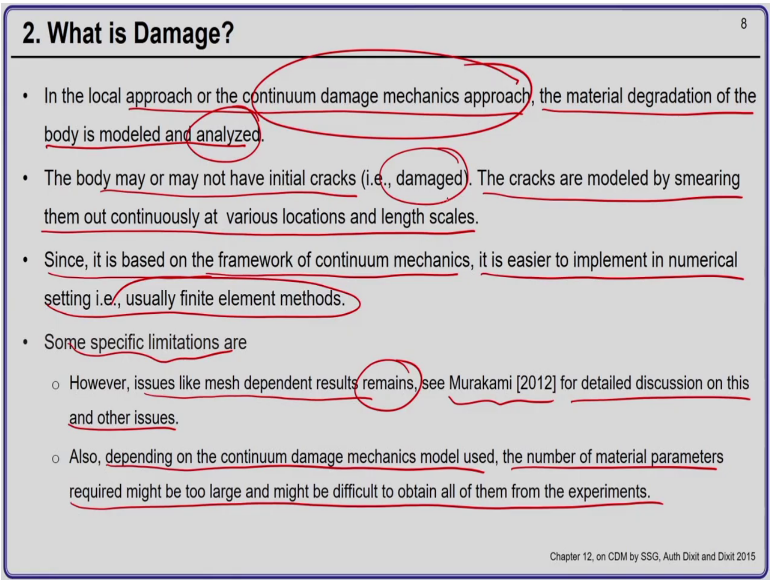

2. What is Damage?

- In the local approach or the continuum damage mechanics approach, the material degradation of the body is modeled and analyzed.
- The body may or may not have initial cracks (i.e., damaged). The cracks are modeled by smearing them out continuously at various locations and length scales.
- Since, it is based on the framework of continuum mechanics, it is easier to implement in numerical setting i.e., usually finite element methods.
- Some specific limitations are
  - However, issues like mesh dependent results remains, see Murakami [2012] for detailed discussion on this and other issues.
  - Also, depending on the continuum damage mechanics model used, the number of material parameters required might be too large and might be difficult to obtain all of them from the experiments.

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Now, in the local approach or the continuum damage mechanics approach which is what we will use in the formulation that will present the material degradation or the body is modeled ok.

So, you will explicitly model the material degradation, which is not considered inside the global approach and then we will analyze the behavior of the material ok. So, the body may or may not have initial crack that is it may or may not be damaged from the start. So, the cracks are modeled by smearing them out continuously at various location and at various length scale.

So, as the deformation progresses you will have certain criteria which will originate new cracks where there was no crack initially present. Now, since local approach is based on the framework of continuum mechanics it is easier to implement inside numerical setting for example, in usual finite element code.

So, if you have already an existing non-linear fem code, which can model dynamic elastoplastic behavior than it is not very difficult to incorporate the continuum damage mechanics approach, rather it is very simple to do it ok. Now, some of these specific limitations of local approach are as follows the issue of mesh dependent result still remains ok.

So, you can see the book by Murakami for a detailed discussion on this and other issues associated with the local approach ok. Also depending on the continuum damage mechanics model use the number of material parameters, required might be too large and might be too difficult to obtain all of them from the experiment ok.

So, now, to model this damage you have to have a damage growth law ok. We will see next how to get this damage growth law for a material and there are certain constants and these constants have to be found out using experiments ok.

Now, in many cases the number of constants are so large that it is very difficult to do experiments on them, and then you may have to drop some of the constants and you have to go for very simple model, which may not then truly reflect the behavior of the material under the different loading condition.

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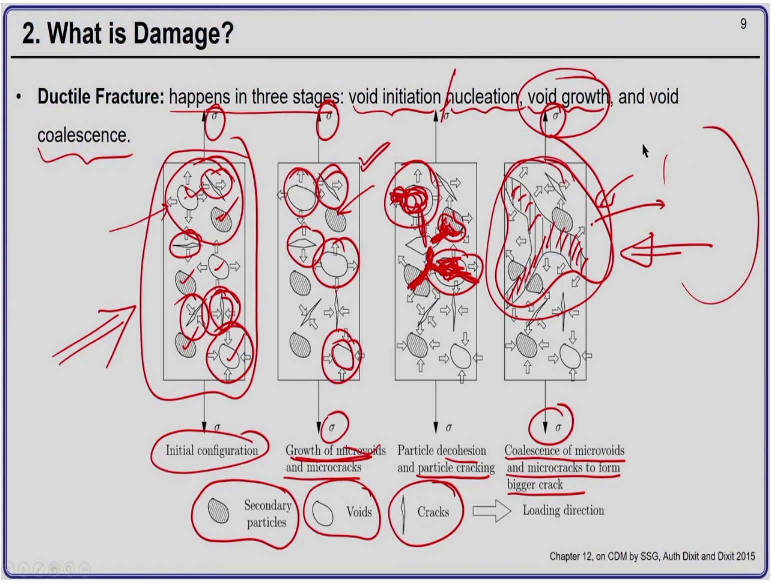

2. What is Damage?

- **Ductile Fracture:** happens in three stages: void initiation/nucleation, void growth, and void coalescence.

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So, now what is ductile fracture, because in this particular 2 3 lectures we are discussing ductile fracture.

So, now ductile fracture is what kind of fracture where we have considerable elastoplastic deformation before the actual body fractures. So, now, ductile fracture happens in three stages, there is one what is called the void nucleation, initiation, there is void nucleation or initiation, there is void growth and there is void coalescence ok.

So, what is meant by these? You can see in this particular image. So, let us say this shows you the secondary particle. So, inside a material there might be some secondary impurities which might be present, these are particles which are of different material than the actual material. So, the bulk material may be it you may say it is iron and the secondary particle might be say some impurities.

So, this represent the voids and the cracks are represented by following figure ok. So, when you have a body in initial configuration ok. So, let us say this is your body or part of body and this is the initial configuration and when you apply loads on the body, then so you have voids ok.

So, you have for example, voids you have these secondary particles and you may have some cracks ok. So, cracks particle they all may be oriented in different direction they may have different shapes. Although, I have shown here with the same shape, they may have different shapes and different orientation ok.

And, we have we are saying for the simplicity in 2 D, but this can well extend in 3 D. Now, as you apply the loads the voids the cracks and the particles will be subjected to different loads. Some of them may be tensile, some of them may be in compression. So, like this here is compression this might here be intention ok.

Now, what happens? Once, you apply the load and you increase the load, what will happen is these voids which are there ok, or this cracks which are there they will grow ok. So, they will the size of the crack although the ends will not grow, but the size of the crack or the voids will grow. So, this is called the void growth face ok.

So, also at this point you will not have any new voids right now coming into picture. So, as you keep on increasing the load, what will happen? Your size of the voids will grow to a large amount and also new voids ok.

As you can see here, the there is a delamination between the bulk material and the secondary particle which has created this kind of new surface or the secondary particle, because the under of the action of the force may break into two parts and it may create a new set of void ok.

So, you may have void which may form because of de cohesion or you may have void which which can form because of particle cracking. So, now, you have new voids ok. So, earlier the voids were growing. Now, you have new voids which have come up ok. And, under sustain loading say your load you are keep on applying the load ok. Then, what can happen is these cracks ok.

So, the this crack may coalesce with this de cohesion this crack may coalesce with this particle cracked it may coalesce with this particular void it may coalesce with this particular void here ok. And, this may grow on this side so, leading to a big micro crack.

So, the coalescence of the micro voids and micro cracks lead to the formation of this bigger crack over here and this is called the micro crack ok. So, that shows you how the ductile fracture actually happens? Ok.

So, from small voids and cracks a bigger crack may appear ok. And, these bigger cracks may coalesce together. So, this might be one crack in one part of the body this may coalesce with another micro crack micro crack in another part of the body to form a much bigger crack.

And, these much bigger cracks may reflect themselves upon the visual eyes of the person ok. So, it might actually be visible at the body itself ok. Then, which can be detected using other visual inspection or using some techniques like ultrasonic testing.

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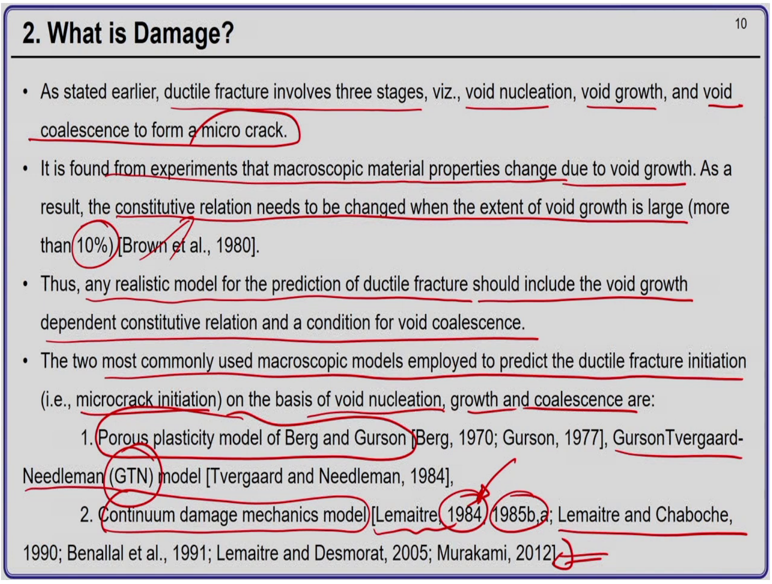

2. What is Damage?

- As stated earlier, ductile fracture involves three stages, viz., void nucleation, void growth, and void coalescence to form a micro crack.
- It is found from experiments that macroscopic material properties change due to void growth. As a result, the constitutive relation needs to be changed when the extent of void growth is large (more than 10%) [Brown et al., 1980].
- Thus, any realistic model for the prediction of ductile fracture should include the void growth dependent constitutive relation and a condition for void coalescence.
- The two most commonly used macroscopic models employed to predict the ductile fracture initiation (i.e., microcrack initiation) on the basis of void nucleation, growth and coalescence are:

1. Porous plasticity model of Berg and Gurson [Berg, 1970; Gurson, 1977], GursonTvergaard-Needleman (GTN) model [Tvergaard and Needleman, 1984],
2. Continuum damage mechanics model [Lemaitre, 1984, 1985b,a; Lemaitre and Chaboche, 1990; Benallal et al., 1991; Lemaitre and Desmorat, 2005; Murakami, 2012]

Now, as stated earlier ductile fracture involves three stages void nucleation, void growth, and void coalescence to form a micro crack. And, it is found from a experiments that macroscopic material properties change due to void growth ok.

So, as a result the constitutive relation needs to be changed when the extent of void growth is large. For example, if you have the void growth the void growth is more than 10 percent, then you need to account the effect of void growth on the constitutive relation ok.

Thus, any realistic model for the prediction of ductile fracture should include the void growth dependent constitutive relation and a condition for void coalescence ok. So, two most commonly used macroscopic models employed to predict the ductile fracture initiation, that is micro crack initiation on the basis of void nucleation, growth and coalescence are the porous plasticity model of berg and Gurson ok.

Its called the Gursons model or the Gursons Tvergaard Needleman model which is in short is very popularly known as gtn model ok. And, there is continuum damage mechanics model this is from the original work of lemaitre or lemaitre and chaboche and you can see some of the references, where it had been discussed.

So, it first came in 1984 within the framework of continuum mechanics and it is called the continuum damage mechanics model ok. So, there are two macroscopic models which are employed to predict ductile fracture. Porous plasticity model and the continuum damage mechanics model. And, in our discussion we will concentrate on the continuum damage mechanics models.

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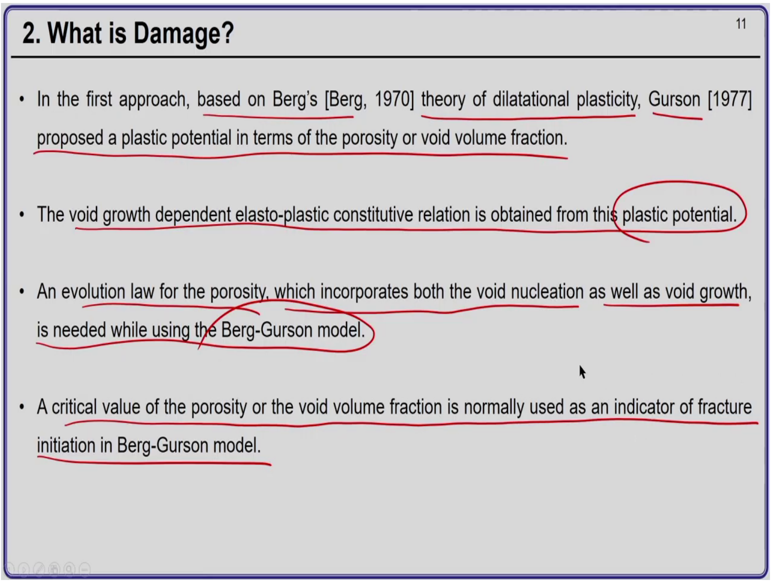

2. What is Damage?

- In the first approach, based on Berg's [Berg, 1970] theory of dilatational plasticity, Gurson [1977] proposed a plastic potential in terms of the porosity or void volume fraction.
- The void growth dependent elasto-plastic constitutive relation is obtained from this plastic potential.
- An evolution law for the porosity, which incorporates both the void nucleation as well as void growth, is needed while using the Berg-Gurson model.
- A critical value of the porosity or the void volume fraction is normally used as an indicator of fracture initiation in Berg-Gurson model.

So, in the first approach which is based on the bergs theory of dilatational plasticity, Gurson proposed a plastic potential in terms of porosity or void volume fraction. And, the void growth dependent elastoplastic constitutive relation is obtained from this particular plastic potential ok. So, an evolution law for the porosity, which incorporates both the void nucleation as well as void growth is needed when you are using the Berg and Gursons model.

And, the critical value of the porosity or the void volume fraction is normally used as an indicator to fracture ok. Indicator for fracture initiation in the berg and Gurson model.

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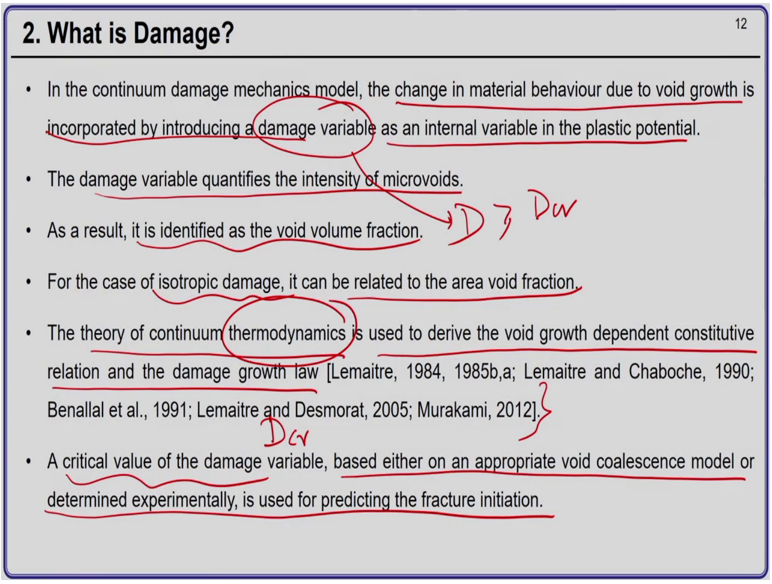

2. What is Damage?

- In the continuum damage mechanics model, the change in material behaviour due to void growth is incorporated by introducing a damage variable as an internal variable in the plastic potential.
- The damage variable quantifies the intensity of microvoids.
- As a result, it is identified as the void volume fraction. $D \geq D_{cr}$
- For the case of isotropic damage, it can be related to the area void fraction.
- The theory of continuum thermodynamics is used to derive the void growth dependent constitutive relation and the damage growth law [Lemaitre, 1984, 1985b,a; Lemaitre and Chaboche, 1990; Benallal et al., 1991; Lemaitre and Desmorat, 2005; Murakami, 2012].
- D_{cr} A critical value of the damage variable, based either on an appropriate void coalescence model or determined experimentally, is used for predicting the fracture initiation.

Now, in continuum damage mechanics model, the change in the material behavior is due to the void growth and it is incorporated by introducing a new variable which is called the damage variable. In our coming slides we will see what is meant by this damage variable, which is an internal variable in the plastic potential.

So, this damage variable quantifies the intensity of the microcracks and as a result it is identified as the void volume fraction. Now, for isotropic damage, this can be treated to be related to the area void fraction.

So, the theory of continuum thermodynamics is used to derive the void growth dependent constitutive relation and the damage growth law. Now, in our course we have not discussed this continuum thermodynamics, but this concept of continuum thermodynamics is needed to derive a thermodynamically consistent void growth dependent constitutive relation and the damage group law. So, you can see these references over here for your reference.

And, now a critical value of the damage variable which is called D c r D critical based either on an appropriate void coalescence model or determined experimentally is used for predicting the fracture initiation. So, once the damage D.

So, this quantity damage variable which is denoted by D, when this quantity becomes greater than D critical, greater than equal to D critical then we say the body has failed at that particular point, there is a full micro crack which has originated ok.

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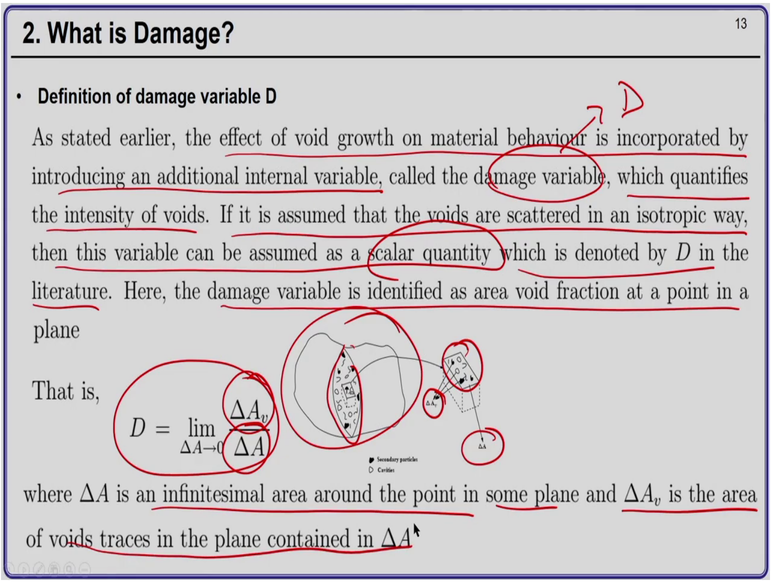

2. What is Damage?

- **Definition of damage variable D**

As stated earlier, the effect of void growth on material behaviour is incorporated by introducing an additional internal variable, called the damage variable, which quantifies the intensity of voids. If it is assumed that the voids are scattered in an isotropic way, then this variable can be assumed as a scalar quantity which is denoted by D in the literature. Here, the damage variable is identified as area void fraction at a point in a plane

That is,

$$D = \lim_{\Delta A \to 0} \frac{\Delta A_v}{\Delta A}$$

where ΔA is an infinitesimal area around the point in some plane and ΔA_v is the area of voids traces in the plane contained in ΔA

So, damage variable D is defined as follows. So, the effect of void growth on material behavior is incorporated by introducing an additional internal variable, called the damage variable, which is denoted as D which quantifies the intensity of the voids. If it is assumed that the voids are scattered in an isotropic way, then this variable can be assumed as a scalar quantity although people have used it as a tensor quantity also and this is denoted by D in the literature ok.

So, the damage variable is identified in as area void fraction at a point in a plane. So, you have a body and then you cut using a certain plane and on that particular plane, you count what is the area of the voids, and then you find out the area of that plane, and then the damage variable is nothing, but is the limiting value of the ratio of the voids, area of the voids divided by area of the infinitesimal area around that point in some plane ok. So, delta A v is the area of void traces in the plane contained in delta A.

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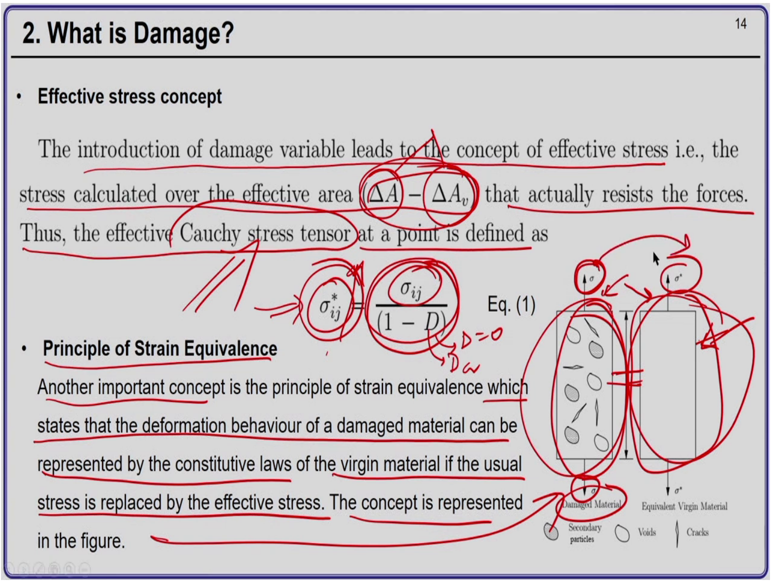

2. What is Damage?

- **Effective stress concept**

The introduction of damage variable leads to the concept of effective stress i.e., the stress calculated over the effective area $(\Delta A) - (\Delta A_v)$ that actually resists the forces. Thus, the effective Cauchy stress tensor at a point is defined as

$$\sigma_{ij}^{*} = \frac{\sigma_{ij}}{(1 - D)} \quad \text{Eq. (1)}$$

- **Principle of Strain Equivalence**

Another important concept is the principle of strain equivalence which states that the deformation behaviour of a damaged material can be represented by the constitutive laws of the virgin material if the usual stress is replaced by the effective stress. The concept is represented in the figure.

Now, the concepts of equivalence stress and principal is strain equivalence is usually used during the derivation of damage growth law. And, what is the effective stress concept? The introduction of the damage variable leads to the concept of effective stress, that is the stress calculated over the effective area ok.

So, when the body is damaged, the part where you have voids or the cracks those parts will not contribute to the load carrying capacity. So, the effective load will be carried by the remaining area. So, the total area is delta a and the area of the voids is delta v. So, the area which carries a load is nothing, but delta A minus delta A v ok.

So, this area is what actually resist the forces. Thus, the effective Cauchy stress tensor at a point is defined as sigma star equal to sigma divided by 1 minus D. So, you notice that when D is equal to 0, when the body does not have any damage it is free of any void, then the effective stress is equal to sigma itself ok. And, when D equal to one you will see a known value divided by 0, which means infinity. Which means that at the tip of the crack ok, when the crack occurs stress goes to a very high value. So, in general it will not go to infinity.

So, we will usually set D cannot cross a critical value. So, when D reaches a critical value what will do we will say that our body has failed ok. So, this means that the effective stress ok, total stress is actually carried out by this much area which is able to resist the load ok..

Now, the principle of strain equivalence is an another important concept and this states that the deformation behavior of a damaged material can be represented by the constitutive laws of the margin virgin material if the usual stress is replaced by the effective stress. And, this concept is represented in this figure over here ok.

So, if you have a real material which is damaged and this is the actual stress sigma, then the behavior of this material ok. And, the deformation behavior of this damage material can be represented by the constitutive rows of a virgin material, virgin means there are no voids, there are no cracks, there are no secondary particles. And, then the behavior of this virgin material is same considered same to be that of the damaged material, if I replace sigma by sigma star which is nothing, but this effective stress tensor ok.

So, then the behavior of this material damage material is same as behavior of this virgin material ok. So, I just have to replace sigma by sigma star and then I will have the similar behavior. So, this is called the principle of strain equivalence.

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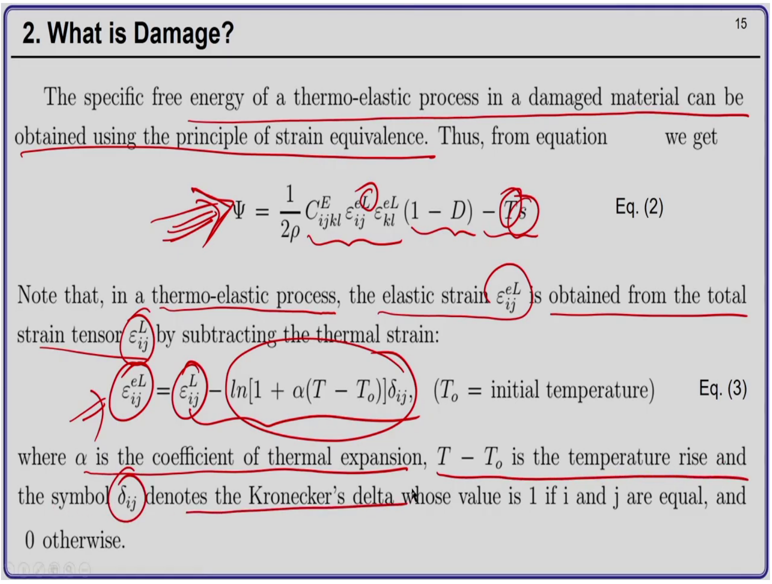

2. What is Damage?

The specific free energy of a thermo-elastic process in a damaged material can be obtained using the principle of strain equivalence. Thus, from equation we get

$$\Psi = \frac{1}{2\rho} C^{E}_{ijkl} \varepsilon^{eL}_{ij} \varepsilon^{eL}_{kl} (1 - D) - Ts \qquad \text{Eq. (2)}$$

Note that, in a thermo-elastic process, the elastic strain ε^{eL}_{ij} is obtained from the total strain tensor ε^{L}_{ij} by subtracting the thermal strain:

$$\varepsilon^{eL}_{ij} = \varepsilon^{L}_{ij} - ln[1 + \alpha(T - T_o)]\delta_{ij}, \quad (T_o = \text{initial temperature}) \qquad \text{Eq. (3)}$$

where α is the coefficient of thermal expansion, $T - T_o$ is the temperature rise and the symbol δ_{ij} denotes the Kronecker's delta whose value is 1 if i and j are equal, and 0 otherwise.

Now, the specific free energy of a thermo elastic process in a damaged material can be obtained using the principle of strain equivalence as follows, psi equal to 1 by 2 rho Cijkl epsilon ij epsilon kl and this L over here, denotes the logarithmic strain into 1 minus D minus T into s, where s is the specific entropy ok.

And, T is the temperature, because we have coupled thermo mechanical problem ok. Now, and this we get from continuum thermodynamics. So, we just straight away mention this. And, now in thermo elastic elastoplastic process the elastic strain can be obtained from the total strain ok. So, epsilon. So, elastic strain can be obtained from the total strain as following ok.

So, you have to subtract the thermal strain from the total strain to get the elastic strain ok. Here alpha is the coefficient of thermal expansion and T minus t 0 is the temperature rise and delta is the Kronecker delta.

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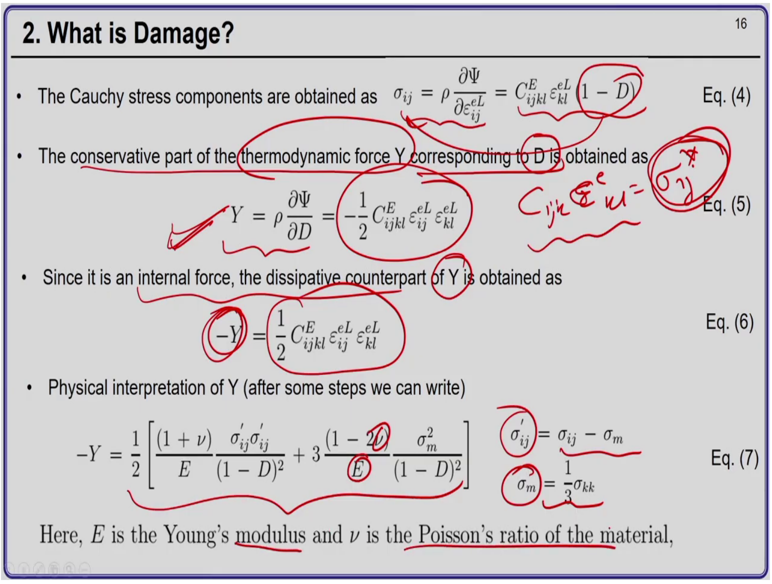

2. What is Damage?

- The Cauchy stress components are obtained as $\sigma_{ij} = \rho \dfrac{\partial \Psi}{\partial \varepsilon_{ij}^{eL}} = C_{ijkl}^{E} \varepsilon_{kl}^{eL} (1 - D)$ Eq. (4)
- The conservative part of the thermodynamic force Y corresponding to D is obtained as

$$Y = \rho \frac{\partial \Psi}{\partial D} = -\frac{1}{2} C_{ijkl}^{E} \varepsilon_{ij}^{eL} \varepsilon_{kl}^{eL} \qquad \text{Eq. (5)}$$

$$C_{ijkl} \varepsilon^{e}_{kl} = \sigma^{*}_{ij}$$

- Since it is an internal force, the dissipative counterpart of Y is obtained as

$$-Y = \frac{1}{2} C_{ijkl}^{E} \varepsilon_{ij}^{eL} \varepsilon_{kl}^{eL} \qquad \text{Eq. (6)}$$

- Physical interpretation of Y (after some steps we can write)

$$-Y = \frac{1}{2}\left[\frac{(1+\nu)}{E}\frac{\sigma'_{ij}\sigma'_{ij}}{(1-D)^2} + 3\frac{(1-2\nu)}{E}\frac{\sigma_m^2}{(1-D)^2}\right] \qquad \sigma'_{ij} = \sigma_{ij} - \sigma_m \qquad \sigma_m = \frac{1}{3}\sigma_{kk} \qquad \text{Eq. (7)}$$

Here, E is the Young's modulus and ν is the Poisson's ratio of the material,

Now, the Cauchy stress can then be obtained as rho into del psi by del epsilon E i j ok. And, this gives you sigma i j as C i j k l epsilon k l into 1 minus D ok. So, if you take 1 minus D on the other side yours C i j k l epsilon k l become sigma star i j ok.

So, this is the so, this relation is for a virgin material you should remember. So, sigma star is your effective stress and that is what is your strain equivalence theorem ok. So, the conservative part of the thermodynamic force corresponding to the damage variable D ok.

So, the conservative part of the so, the force corresponding to strain is stress, similarly the conservative part of the thermodynamic force Y corresponding to damage is identified as Y equal to rho time del psi by del D, which gives me minus 1 by 2 C i j k l epsilon i j epsilon k l ok.

Now, because this force internal force is a dissipative, because damage causes dissipation then I take the counterpart of Y, which is minus Y as I dissipative counterpart of Y. And, this is equal to 1 by 2 Cijkl epsilon i j epsilon k l ok. Now, after some steps I can show that this dissipative part of Y can be written in following form ok, where sigma dash is the deviatoric part of the Cauchy stress.

And, sigma m is the mean stress 1 by 3 sigma kk. And, E and nu are the Youngs modulus and the poisons ratio of the material.

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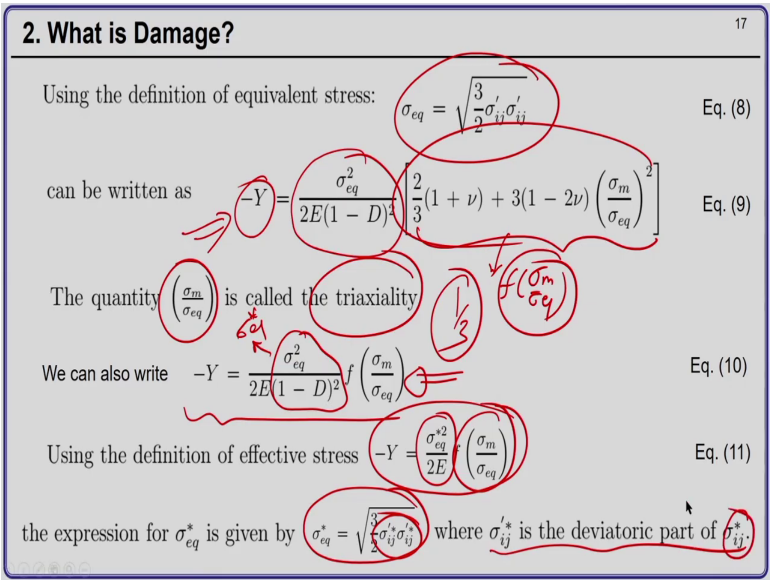

2. What is Damage?

Using the definition of equivalent stress: $\sigma_{eq} = \sqrt{\frac{3}{2}\sigma'_{ij}\sigma'_{ij}}$ Eq. (8)

can be written as $-Y = \frac{\sigma_{eq}^2}{2E(1-D)^2}\left[\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_m}{\sigma_{eq}}\right)^2\right]$ Eq. (9)

The quantity $\left(\frac{\sigma_m}{\sigma_{eq}}\right)$ is called the triaxiality

We can also write $-Y = \frac{\sigma_{eq}^2}{2E(1-D)^2} f\left(\frac{\sigma_m}{\sigma_{eq}}\right)$ Eq. (10)

Using the definition of effective stress $-Y = \frac{\sigma_{eq}^{*2}}{2E} f\left(\frac{\sigma_m}{\sigma_{eq}}\right)$ Eq. (11)

the expression for σ_{eq}^* is given by $\sigma_{eq}^* = \sqrt{\frac{3}{2}\sigma'^*_{ij}\sigma'^*_{ij}}$ where σ'^*_{ij} is the deviatoric part of σ^*_{ij}.

And, using the concept of equivalence stress ok, I can write minus Y ok. The dissipative part of Y as sigma equivalence square divided by 2 into 1 minus D square into this function. And, this function can be written as function of sigma m by sigma equivalent, where sigma ratio of sigma m the mean stress by equivalence stress is called triaxiality ok.

This is the very important parameter in ductile fracture, which is called the triaxiality and then and the value of this triaxiality is 1 by 3 for tension ok. For uniform simple tension this value is minus 1 by 3.

Now, we can also write if I write this as f then the dissipative part of Y is written as following equation that is equation 10. Now, I identify from the concept of effective stress this term over here I can write this as sigma star equivalent ok. This is the equivalent stress corresponding to the virgin material and then my dissipative part becomes sigma equivalent square divided by 2 E into function of triaxiality ok.

Here, the expression for equivalent stress of the virgin material is root over 3 by 2 sigma dash i j, sigma dash i j of the deviatoric of the effective stress ok. So, sigma dash i j star is nothing, but the deviatoric part of the effective stress sigma i j star ok.

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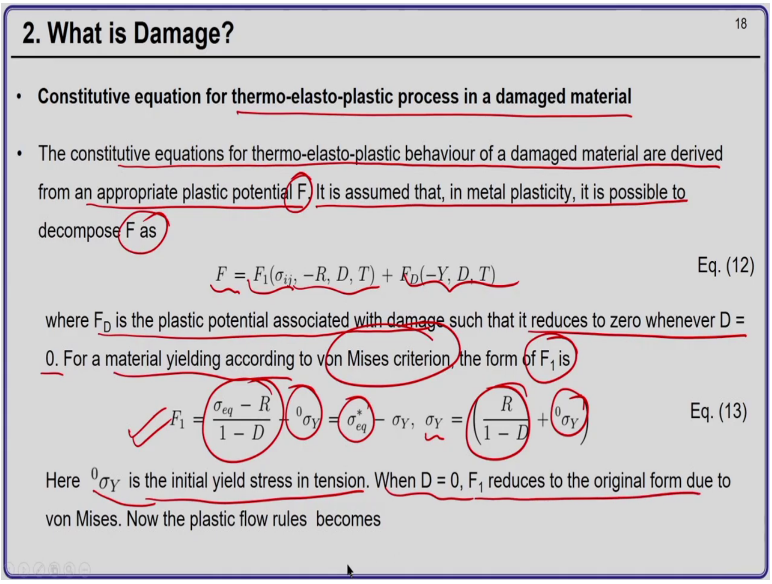

2. What is Damage?

- **Constitutive equation for thermo-elasto-plastic process in a damaged material**
- The constitutive equations for thermo-elasto-plastic behaviour of a damaged material are derived from an appropriate plastic potential F. It is assumed that, in metal plasticity, it is possible to decompose F as

$$F = F_1(\sigma_{ij}, -R, D, T) + F_D(-Y, D, T) \qquad \text{Eq. (12)}$$

where F_D is the plastic potential associated with damage such that it reduces to zero whenever D = 0. For a material yielding according to von Mises criterion, the form of F_1 is

$$F_1 = \frac{\sigma_{eq} - R}{1 - D} - {}^{0}\sigma_Y = \sigma^*_{eq} - \sigma_Y, \quad \sigma_Y = \left(\frac{R}{1 - D} + {}^{0}\sigma_Y\right) \qquad \text{Eq. (13)}$$

Here ${}^{0}\sigma_Y$ is the initial yield stress in tension. When D = 0, F_1 reduces to the original form due to von Mises. Now the plastic flow rules becomes

Now, the constitutive relation for a thermo elastoplastic process in a damaged material can be obtained as follows. So, the constitutive equation for the thermo elastoplastic behavior of a damaged material are derived from an appropriate plastic potential F. It is assumed that in metal plasticity it is possible to decompose F into 2 parts; one which is the plastic potential associated with the damage and one which is F 1 ok.

Now, F D is the plastic potential associated with damage such that it reduces to 0 whenever D is equal to 0. And, now for a material which is yielding according to the von Mises yield criteria, there are different yield criterias we are using von Mises yield criteria here the form of F 1 is given by sigma equivalent minus R upon 1 minus D minus sigma Y 0.

Now, I can write sigma equivalent upon 1 minus D as sigma star and I can write the yield stress as sigma Y 0 plus R upon 1 minus D. Here sigma 0 Y is the initial yield stress of the material in tension. And, when you have 0 damage F 1 reduces to it is original form due to Von Mises ok.

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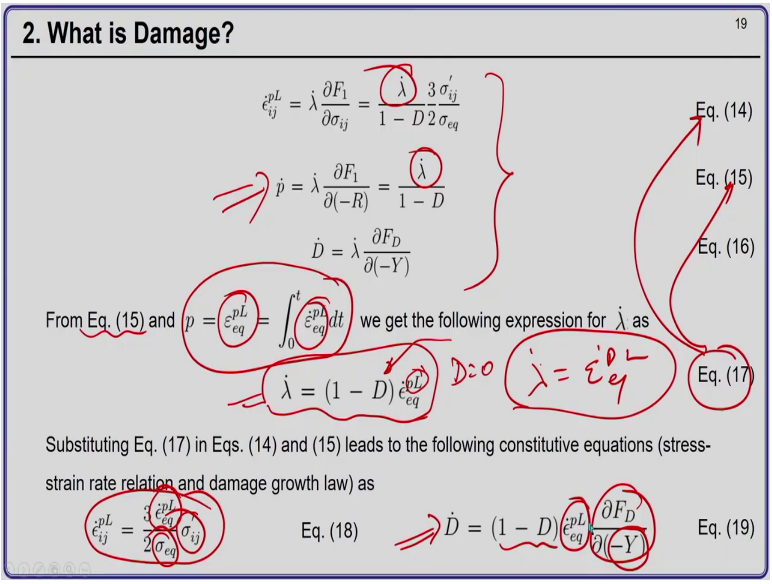

2. What is Damage?

$$\dot{\epsilon}_{ij}^{pL} = \dot{\lambda}\frac{\partial F_1}{\partial \sigma_{ij}} = \frac{\dot{\lambda}}{1-D}\frac{3}{2}\frac{\sigma'_{ij}}{\sigma_{eq}} \quad \text{Eq. (14)}$$

$$\dot{p} = \dot{\lambda}\frac{\partial F_1}{\partial(-R)} = \frac{\dot{\lambda}}{1-D} \quad \text{Eq. (15)}$$

$$\dot{D} = \dot{\lambda}\frac{\partial F_D}{\partial(-Y)} \quad \text{Eq. (16)}$$

From Eq. (15) and $p = \varepsilon_{eq}^{pL} = \int_0^t \dot{\varepsilon}_{eq}^{pL} dt$ we get the following expression for $\dot{\lambda}$ as

$$\dot{\lambda} = (1-D)\,\dot{\epsilon}_{eq}^{pL} \quad \text{Eq. (17)}$$

$D=0$ $\dot{\lambda} = \dot{\varepsilon}_{eq}^{pL}$

Substituting Eq. (17) in Eqs. (14) and (15) leads to the following constitutive equations (stress-strain rate relation and damage growth law) as

$$\dot{\epsilon}_{ij}^{pL} = \frac{3}{2}\frac{\dot{\epsilon}_{eq}^{pL}}{\sigma_{eq}}\sigma'_{ij} \quad \text{Eq. (18)} \qquad \dot{D} = (1-D)\,\dot{\epsilon}_{eq}^{pL}\frac{\partial F_D}{\partial(-Y)} \quad \text{Eq. (19)}$$

So, with this the plastic flow rule then can be written as following equation, where we have to find out this lambda star ok. So, here ok from equation number 15 which is here and this P, which can be identified with the equivalent plastic strain inside the body given by integral from 0 to 2, 0 to t integration of epsilon dot equivalent plastic into d t, we can obtain the value of lambda dot as 1 minus D into epsilon dot equivalent plastic remember.

When, D equal to 0 in plasticity lambda dot will come out. So, if damage is equal to 0 lambda dot will be epsilon dot P equivalent plastic ok. So, in your traditional plasticity course this is what you will get, but when a material is damaged that is it has some damage, lambda dot will come out to be one minus D time epsilon dot equivalent plastic ok.

Now, if we substitute equation number 17 in equation 14 and equation number 15 this lambda dots, then we can obtain the constitutive relations that is the stress strain relation and the damage growth law as follows ok. So, this is the constitutive relation, it is epsilon dot p L i j equal to 3 by 2 epsilon dot p L equivalent divided by sigma equivalent into sigma dash..

And, the damage growth D dot is 1 minus D into the rate of equivalent plastic strain into the differentiation of the dissipative part of the plastic potential with respect to the dissipative part of the thermo dynamic force corresponding to damage D ok.

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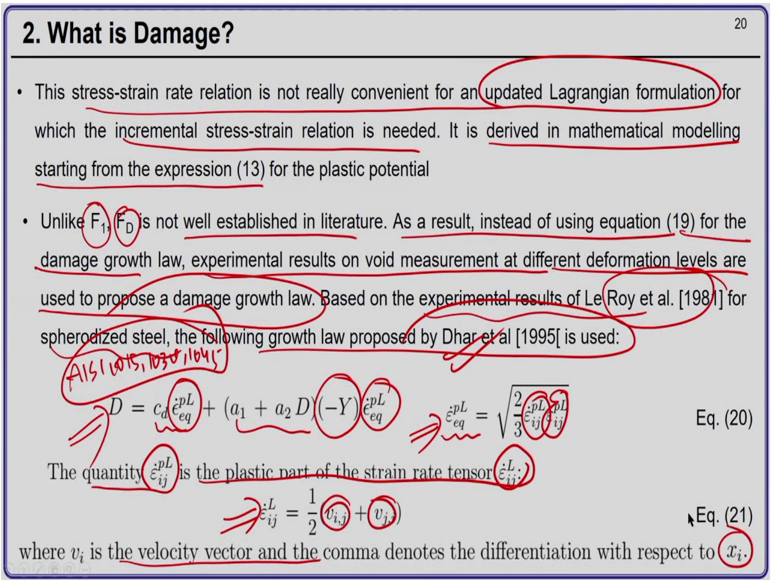

2. What is Damage?

- This stress-strain rate relation is not really convenient for an updated Lagrangian formulation for which the incremental stress-strain relation is needed. It is derived in mathematical modelling starting from the expression (13) for the plastic potential
- Unlike F1, FD is not well established in literature. As a result, instead of using equation (19) for the damage growth law, experimental results on void measurement at different deformation levels are used to propose a damage growth law. Based on the experimental results of Le Roy et al. [1981] for spherodized steel, the following growth law proposed by Dhar et al [1995[is used:

AISI 1015, 1030, 1045

$$D = c_d \dot{\varepsilon}^{pL}_{eq} + (a_1 + a_2 D)(-Y)\dot{\varepsilon}^{pL}_{eq} \qquad \dot{\varepsilon}^{pL}_{eq} = \sqrt{\frac{2}{3}\dot{\varepsilon}^{pL}_{ij}\dot{\varepsilon}^{pL}_{ij}} \qquad \text{Eq. (20)}$$

The quantity $\dot{\varepsilon}^{pL}_{ij}$ is the plastic part of the strain rate tensor $\dot{\varepsilon}^{L}_{ij}$:

$$\dot{\varepsilon}^{L}_{ij} = \frac{1}{2}(v_{i,j} + v_{j,i}) \qquad \text{Eq. (21)}$$

where v_i is the velocity vector and the comma denotes the differentiation with respect to x_i.

Now, this stress-strain relation is not convenient for the updated Lagrangian formulation and we will see what is meant by this for which the incremental stress strain relation is needed. So, it is derived in mathematical modeling starting with the expression 13 for the plastic potential.

Now, F 1 is well established in the literature; however, F D is not well established in the literature. So, some people has suggested the form of F D as a result instead of directly using equation 19.

So, what was equation 19, this was equation 19. So, because F D is not known so, what we usually do is instead of using equation 19 for the damage growth law experimental results for void growth measurement at different deformation levels are used to propose the damage growth law ok..

For example, in the discussion that we have we use the experimental results of Le Roy et al. for the spherodized steel ASI 1015, 1030 and 1045 ok. For these Le Roy has done experimental results for the void growth and we use that to propose a damage growth law which was originally proposed by Dhar et al in 1995 and this is given by equation number 20..

D dot equal to C d, which is a constant time rate of equivalent plastic strain plus a 1 plus a 2 D into the dissipative part of thermodynamics force corresponding to damage into epsilon dot equivalent plastic equivalent ok. Here, this equivalent rate of equivalent plastic strain rate is nothing, but root over 2 by 3 epsilon dot equivalent dot pi j epsilon dot p L i j.

So, this quantity epsilon dot p L i j is the plastic part of the strain rate tensor, this where this is given by 1 by 2 the gradient of the symmetric part of the velocity gradient tensor ok.

So, v i is the velocity and comma denotes the differentiation with respect to x.

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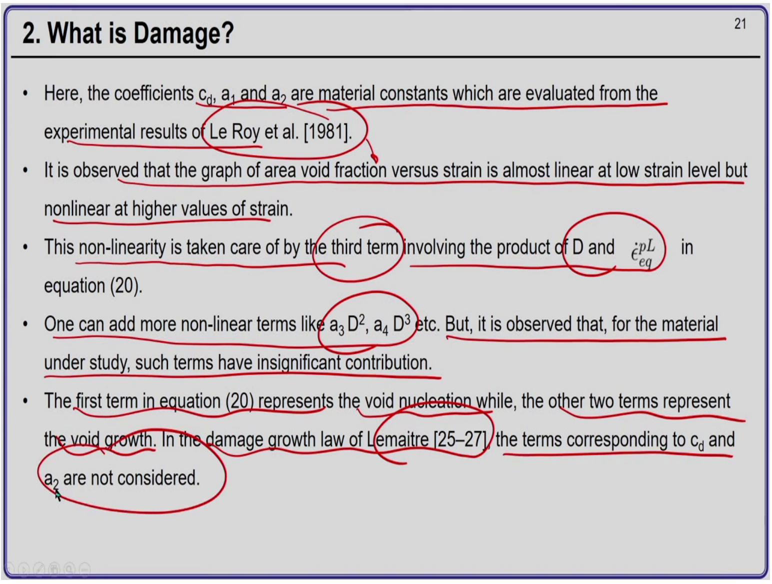

2. What is Damage?

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- Here, the coefficients c_d, a_1 and a_2 are material constants which are evaluated from the experimental results of Le Roy et al. [1981].
- It is observed that the graph of area void fraction versus strain is almost linear at low strain level but nonlinear at higher values of strain.
- This non-linearity is taken care of by the third term involving the product of D and $\dot{\epsilon}^{pL}_{eq}$ in equation (20).
- One can add more non-linear terms like $a_3 D^2$, $a_4 D^3$ etc. But, it is observed that, for the material under study, such terms have insignificant contribution.
- The first term in equation (20) represents the void nucleation while, the other two terms represent the void growth. In the damage growth law of Lemaitre [25–27] the terms corresponding to c_d and a_2 are not considered.

So, here the coefficients c d a 1 a 2 are material constant which are to be evaluated from the experimental results and for which we use the results of Le Roy and this we will discuss in the coming lectures. And, it is observed that the graph of area void fraction versus strain is almost linear at low strain level, but it is non-linearity higher values of strain ok.

So, this is what the observation in Le Roy's result. Therefore, to incorporate for this non-linearity by taking the third term involving the product of a and product of D with the rate of equivalent plastic strain. So, you see we have this product of D with strain.

So, one can add more non-linear terms like a 3 D square a 3 a 4 D cube, but it is observed in our case that the material under study, such terms have no significant impact ok. Now, in equation 20 the first term represents the void nucleation while the other two term represent the void growth. So, now, in the damage growth law which is proposed by Lemaitre the terms corresponding to c d and c 2 you will not find ok.

So, this term of C d and a 2 you will not find in the damage growth law of Lemaitre ok there you will only have a 1 times of minus Y into epsilon dot plastic ok. So, with this we end today's lecture and we have discussed what is meant by damage. In the next lecture we will discuss the mathematical modelling of this large deformation dynamic thermo elastoplastic problem ok.

Thank you.