Fundamentals of Compressible Flow
Prof. Niranjan Sahoo
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module – 03
Quasi-One Dimensional Isentropic Flow
Lecture - 07
Quasi-One Dimensional Isentropic Flow - I

Welcome you again for this course that is Fundamentals of Compressible Flow. We are in the module 3, it is a new module, title of this module is Quasi One Dimensional Isentropic Flow.

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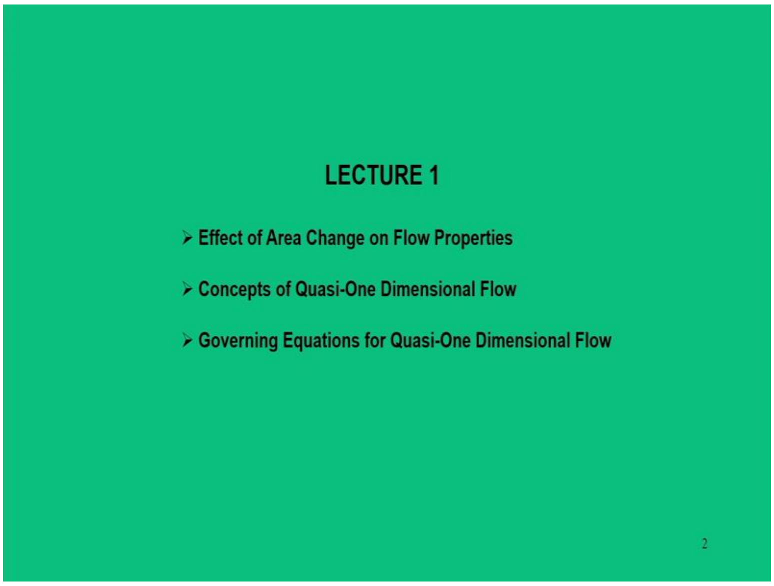

So, the content of this lecture are as follows; that is effect of area changes on flow properties, concepts of quasi one dimensional flow, governing equations for quasi one dimensional flow.

Now, let us see that we have brought about a new word, which is quasi one dimensional. So, in all previous sections, we essentially emphasize the flow to be one dimensional in nature; in a sense that it is the flow is entering in a constant area duct, that duct in particular we assume to be a stream tube. In fact, the cross sectional area at the inlet and the exit are essentially same.

Now, while analyzing the flow field, we really do not bother about how the flow is changing within the duct; rather we are essentially bothered about the inlet and exit condition of the duct. And while analyzing all those things it is assumed that, the flow is isentropic in nature. So, that is what we have analyzed till this point of time.

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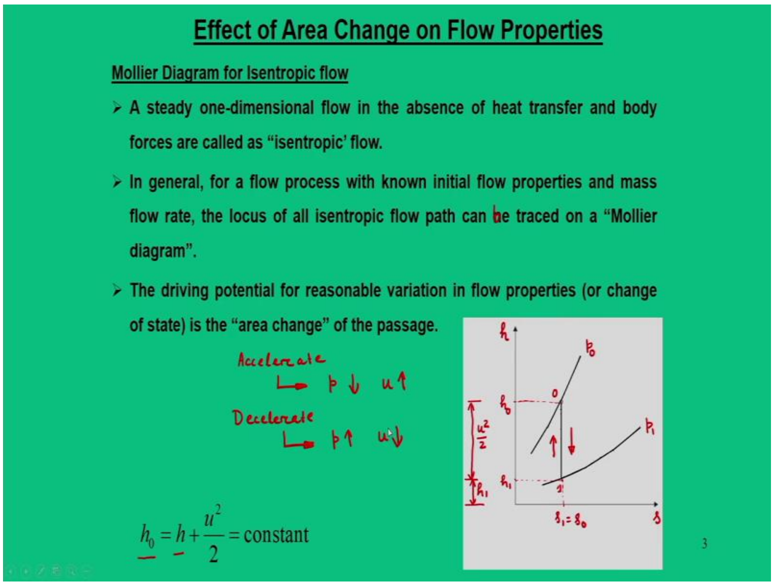

Now, moving further, we are talking about the quasi one dimensional flow. So, in this approach I will explain that, how it is different from the conventional one dimensional flow and why the word quasi is comes into picture. So, before you move further, we will bring into effect of area change on the flow properties; rather we will say that, we have to do the relaxation of the one dimensional nature. So, one simplest thing that we can do is that, we can the change the area; but still the flow can be treated to be isentropic.

So, there is some thermodynamical physical insight into it. So, for which we will revisit the Mollier diagrams just we explain that how we can do area change; while doing so the properties are going to increase or decrease or there is a change in the flow properties. So, let us revisit this Mollier diagram.

So, when you draw the Mollier diagram, we normally refer it is as in enthalpy entropy plane as shown in this figure. So, on this enthalpy entropy plane, we are defining a state which is any arbitrary state 1 and we will also talk about a stagnation state 0.

So, when I say this stagnation state, though I can define a stagnation enthalpy $h_0 = h + \frac{u^2}{2}$. So, for this case for the state 1, any arbitrary state 1; if I can define this as enthalpy as h_1 in this diagram and corresponding to this stagnation state, I can define h_0 as its state. Now when I am at point 1, we are at pressure p_1 which is the any arbitrary pressure and when we are at point 0, we are at corresponding stagnation pressure. So, this stagnation pressure p_0 corresponds to the static pressure p_1.

Now, looking at these equations what we can say is that, the total stagnation enthalpy is divided into two parts; one is the static enthalpy h_1, other is the dynamic enthalpy that comes by virtue of its velocity, that is $\frac{u^2}{2}$. So, in other words what I can say, I can go from state 1 to 0. So, when I go from state 1 to 0; that means, I am moving isentropically. So, for this process 1 to 0 or 0 to 1, essentially the entropy s_1 is equal to s_0.

So, when we are moving from 1 to 0; that means, we are increasing the pressure. So, relative weightage of u comes down. When I am moving from 0 to 1; the relative weightage of u increases. So, in one case I am accelerating the flow, in other case I am decelerating the flow. So, this is the very important basics that one can accelerate the flow. When I say I have to accelerate the flow then we have to go from state point 0 to 1, so that pressure must decrease. Pressure decreases, velocity increases.

When you say decelerate the flow; this means pressure increases, velocity decreases. So, this is the very basic fundamental things that how the flow processes change isentropically, when we are moving from stagnation state to a static state or from static state to stagnation state in isentropic manner.

So, this is how the Mollier diagram is framed of, in which the flow process with known initial properties and mass flow rate; the locus of all isentropic path can be traced in a Mollier diagram. And in this case while accelerating or to accelerate or decelerate the flow; the driving potential for this variation in the flow properties comes through area change of the passage.

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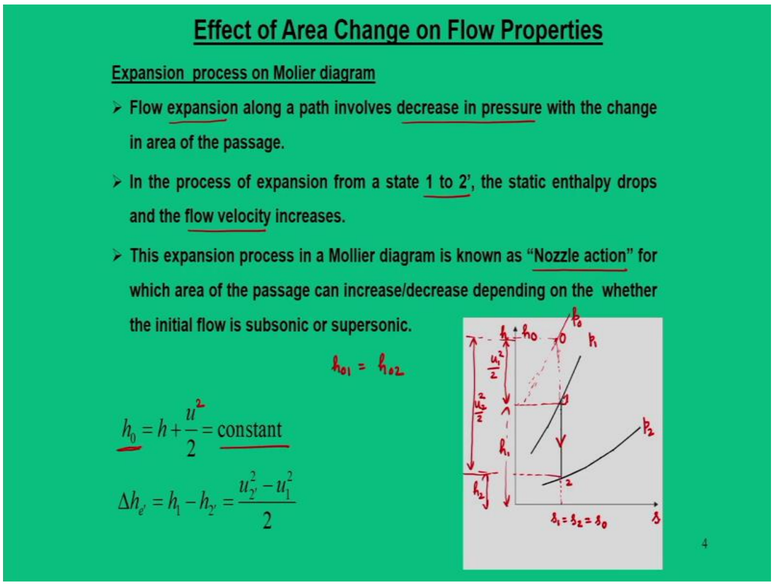

In fact, while decreasing the pressure and increasing the pressure, we come across the change in the velocity; now to achieve this, we must take into account the area change of the passage.

Now, let us see that, we will go by one by one; the first thing what we are going to see is that on a Mollier diagram, we have to see that how an expansion process is taken place on a Mollier diagram. So, to study the meaning of flow expansion; it means that the flow path involves decrease in pressure with change in the area, so the pressure must decrease in the direction of the flow with change in the area.

So, that is what the flow expansion means. So, like emphasize this word; flow expansion means decrease in the pressure. And this through this process, the area can increase or decrease. So, those things will come in the later part; but the basic understanding goes that when we say flow expansion it must decrease the pressure.

Now, to do this, so let us take a state which is going from 1 to 2; in the previous case we say we are moving from stagnation to static or static to stagnation, now you are saying that it is any arbitrary process we are moving from state 1 to 2 and this process is treated to be an expansion process.

So, what I drew is that, in the same Mollier diagram that is h-s plot; I am putting a point 1 that is coordinate 1 and point 2. So, we are moving in the direction of 1 to 2. So, in a manner such that, we say s_1 is equal to s_2. So, the process is isentropic. So, when I am moving these things; now on a Mollier diagram these two lines represent the constant pressure lines. So, at point 1, the constant pressure lines can be represent in this manner, and for point 2 one can draw the constant pressure line in this manner. So, this is how it is shown in the figure.

So, next important point is that the flow process is isentropic. So, when I say flow process is isentropic. So, for state 1 and state 2; we will have same stagnation enthalpy that is h_0, which is $h+\frac{u^2}{2}$ is equal to constant. So, this needs to be constant.

So, what I can do is that, I can draw another constant pressure line which can go in this manner such that I can define for this point 1 and 2; I can then define a point o for which the total pressure line is defined in this manner and corresponding stagnation enthalpy is defined as h_0. So, what I can say, your $h_{01}= h_{02}$ and this is nothing, but s_0.

So there is no change in the entropy. So, let us define the state point 1 and the relative weightage of h and u. So, for point 1, I can draw a line as h_1 which is this and corresponding $\frac{u_1^2}{2}$ we can write as shown in this figure. Now, for point 2 we say, this is your h_2 and corresponding $\frac{u_2^2}{2}$ is defined by this line.

So, in this process what we can say by maintaining total enthalpy constant, one can play with the relative decrease in the enthalpy or increase in the enthalpy. So, let us say if you are moving from point 1 to 2; what I am trying to say, in this process and the process goes in an isentropic manner, in this process I am decreasing the enthalpy, so that means it drops from h_1 to h_2.

But at the same time I am increasing its flow speed; that is from $\frac{u_1^2}{2}$ to $\frac{u_2^2}{2}$ or in terms of kinetic energy is increased. So, the increase in the velocity happens at the cost of decrease in the static enthalpy and in this process I am going from 1 to 2, so pressure must decrease. So, this means that I am expanding the flow that involves decrease in the pressure and in this process I am also increasing the flow velocity at the cost of static enthalpy.

So, this is a very fundamental aspect and we call this expansion process on a Mollier diagram, we call this as a nozzle action. So, when I say nozzle action; that means, I must expand the flow, the pressure must decrease. Now, this is a mechanism in which one can increase or decrease the area, whether your inflow is subsonic or supersonic; which means if at the inlet state your velocity is subsonic, we can increase the velocity or for a Nozzle action the velocity must increase and pressure must decrease, this is the fundamental aspects.

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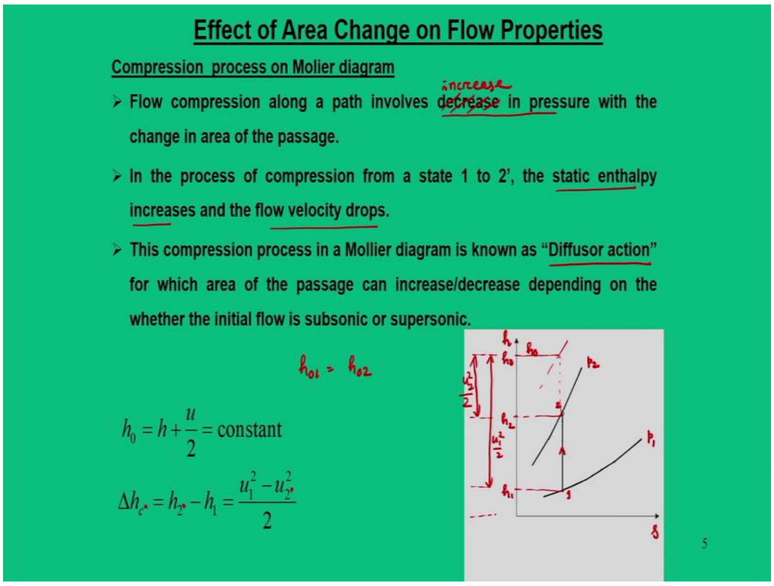

Now, moving further we will just take a reverse situation. So, there is nothing just to explain here we are just saying that, it is a compression process on a Mollier diagrams. So, we redraw the same figure. So, I will just put the reverse symbol like we are now at point 1 and moving to point 2 that is pressure must increase; but still your stagnation pressure remains same.

So, direction of this is given as 1 to 2. So, there is a increase in the pressure. So, you have your h_1, you have h_2 and this is h_0. So, in this process we are saying this as $h_1 + \frac{u_1^2}{2}$, $h_2 + \frac{u_2^2}{2}$.

So, here 2 and 2` are same. So, in this case also your $h_{01}= h_{02}$. So, the function is just exactly opposite. So, this process we call as a compression process and it is called as a diffuser action.

So, in this process what is going to happen; your static enthalpy increases, but flow velocity drops, and the path involves the area change. So, there is the flow compression. it’s a increase in the area. So, the flow compression involves a path that involves increase in the pressure with the change in the flow passage.

So, now we are able to say that process is isentropic and we can define the compression process and expansion process on a Mollier diagrams. So, accordingly we define diffuser action or nozzle actions. Till this point of time we do not say anything about the area; whether it should increase or decrease, that part, that aspect will come later. But for a nozzle action pressure must decrease and for diffuser action pressure must increase.

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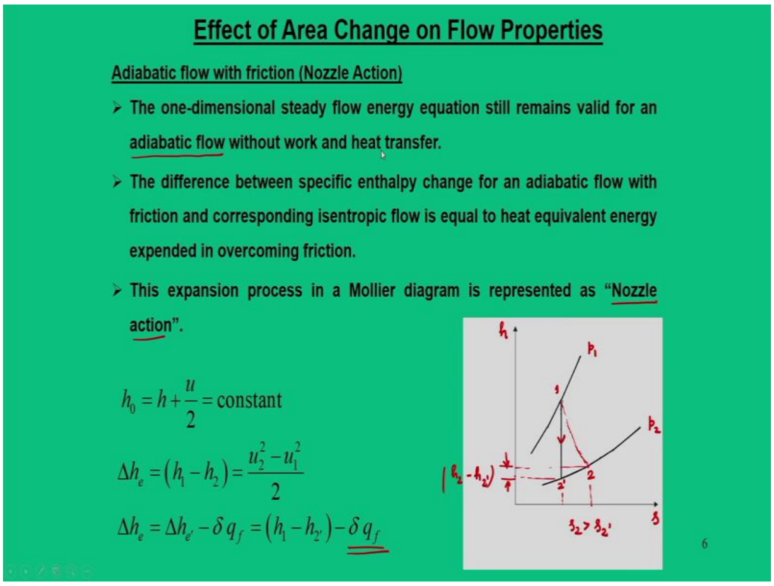

Now, let us do a kind of a relaxation saying that, instead of talking the flow to be isentropic; we can say it is a adiabatic. So, when I say adiabatic; so I bring friction into account. So, in the same nozzle action, we can bring into friction; so essentially flow is not isentropic, rather flow can still can be thought of to be a adiabatic.

So, what does this mean? So, again we will revisit this enthalpy entropy diagram h-s; now since it is a nozzle actions, so we define this point to be situated and the top. So, it is a expansion process; so one is moving from 1 to 2. So I will define this state as 2`, so which is isentropic.

Now, if the flow is not isentropic, the actual process the flow can take place in a non isentropic manner. So, we say 2. So, the actual process we say it is a 1-2. So, we actually one can say that your s_2 is greater than $s_{2`}$; that means 1-2 process is non-isentropic process.

In this aspect we can say there is a enthalpy lost; so this is nothing but $h_2 - h_{2'}$. So, this particular concept; where it is lost; this is lost due to heat equivalent energy expended in overcoming this friction, that is δq_f. So, when I am performing a nozzle action while moving from 1 to 2` in isentropic manner, in reality it does not happen; we land of in another location 2, for which there is a heat loss in terms of friction, that is δq_f.

So, obviously, this is a negative quantity for a nozzle action. But, we can still say that flow can be assumed to be adiabatic without any work and heat transfer.

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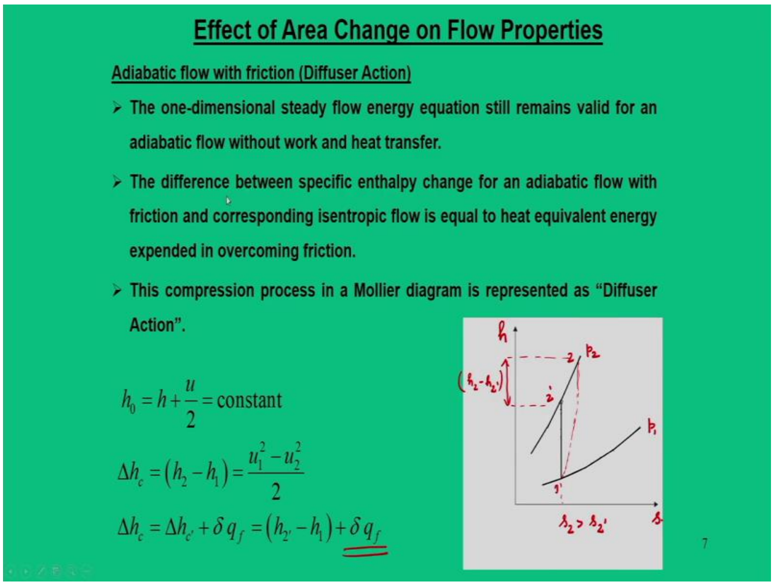

Now, in similar concept also apply here, but here the heat equivalent energy expended in overcoming friction for the situation of a diffuser action happens to be a positive quantity. So, in the same plot h-s, one can say that I am at p_1, I am moving to p_2. So, here an isentropic process would have gone to 1 to 2`; but a non-isentropic process will take to a real situation and your $h_2$ also greater than $h_{2`}$.

And in fact, there is a deviation in the enthalpy; that is $h_2 - h_{2'}$, that enthalpy deviation is taken into account as heat equivalent energy expended over to overcome the friction. But in any case the flow is still adiabatic and one dimensional.

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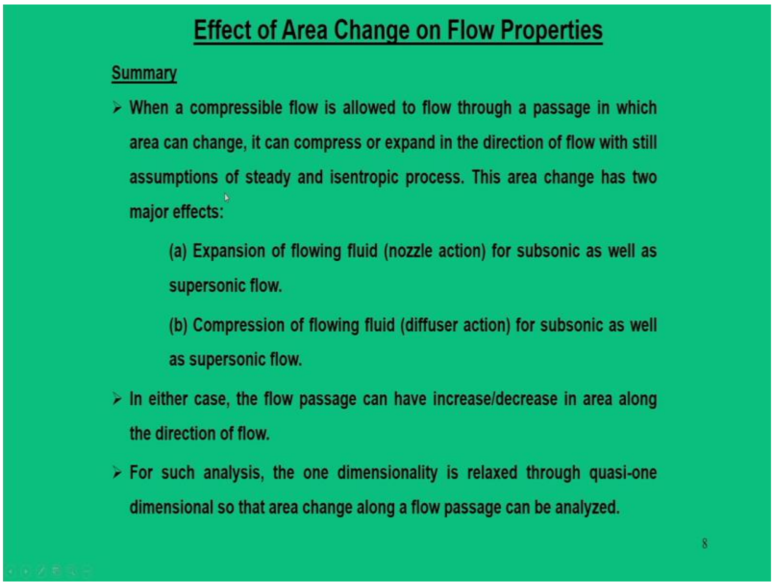

So while analyzing these two, if I can sum up what we have discussed so far; for a compressible flow, these are the summary what we are going to get. That is when a compressible flow is allowed to flow through a passage in which the area can change; that means we are changing the area, it can compress or expand, that means flow will compress and expand in the direction of the flow with still assumption of steady and isentropic process.

So, these area change has two major effects. So, first effect that we can expand the flowing fluid, so we call this as a nozzle action or we can compress the flowing fluid, we call this as a diffuser actions. Now, both the actions are possible or both the actions help us what to do; that means, if your inflow is subsonic, we can expand the fluid; if your inflow is supersonic, we can still expand the fluid, but what is going to happen whether we have to either increase or decrease the area.

So, it all depends, so we are bringing the condition of inlet condition into picture, whether the inlet condition is subsonic or supersonic. Now, that is what in either case or expansion or compression, the flow passage can have increase or decrease in area in the direction of the flow. So, whether you have to increase or decrease it all depends what condition your inflow is; means whether your inflow is subsonic or your inflow is supersonic.

So, accordingly we have to take a decision, whether to increase the area or decrease the area to perform the expansion action or compression action. So, now, when I say that area has to change, so we have to relax the assumption of constant area situations; that means there is no question of one dimensionality, so rather we are going to bring the concept what we call as quasi one dimensional approach.

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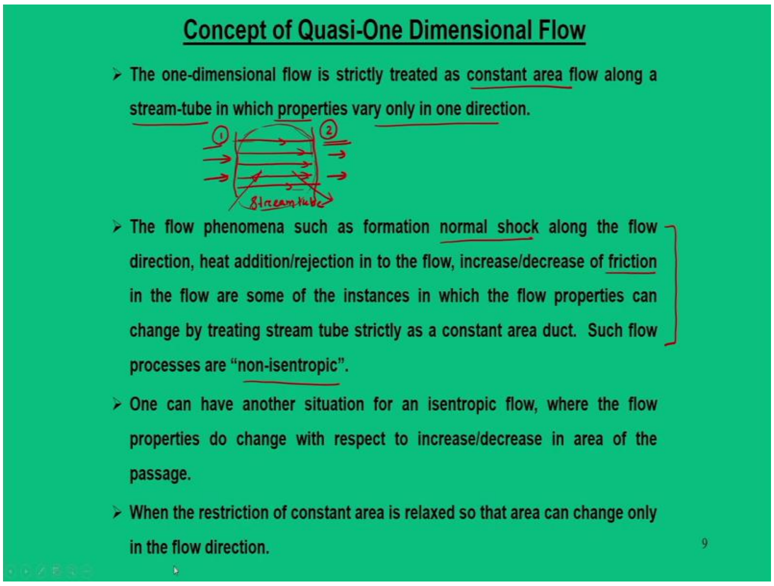

So, in a quasi one dimensional approach what really happens is that, in a normal situation we say the flow to be one dimensional contained infinite number of streamlines and we assume this to be a stream tube. And we are really bothered about the inlet and exit conditions, that is 1 and 2 and we say this process needs to be isentropic.

So, this was the concept we did for a constant area flow in a stream tube, where the properties vary in one directions. Now, when I say this is a stream tube, we really do not bother about what happens within the area; whether this flow properties change happens in any of the manner, whether the process is isentropic, non isentropic that does not matter, rather we are worried about two extreme conditions 1 and 2.

So, such a process either within the stream tube; this we can think of heat addition, we can think of heat rejections, we can think of any other phenomena such as formation of normal shock, we can think of any other phenomena such as friction.

So, there are variety of mechanisms in which the condition 1 and 2 can change and all these process are essentially non isentropic; in fact this particular part we will be discussing more details in all subsequent analysis. But the very basic point I need to emphasize here that, we really do not about the control surface of the stream tube, because it is a constant area and does not change.

And in fact, our all the governing equations which were treated, which were derived; it was essentially a following the assumption of constant area. But what we are trying to bring the concept here a approach, we call as quasi one dimensional approach.

So, what we are trying to say here; one can have another situation where the flow is isentropic, where the properties do change with respect to increase or decrease in the area of the passage. So, the restriction of constant area is relaxed; means area can change in the direction of the flow. So, how that is possible?

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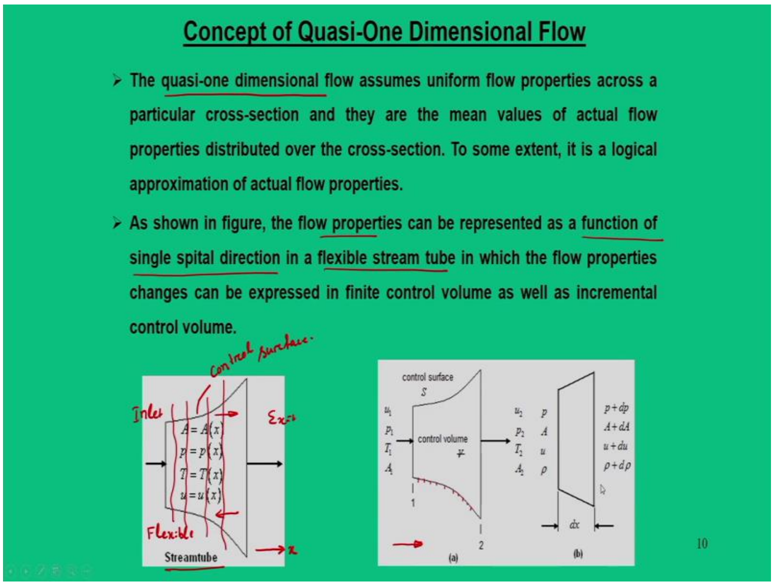

So, you bring this particular concept here is that, this is a kind of a situation where we can say the; it is a kind of a duct which has certain inlet area A_1 and also some exit area A_2 and in this case the area changing only in the direction of x. So, I can write A as a function of A(x). So, what you see is that, there is a flexible surface which is bounded from inlet to exit. So, this is nothing, but your control surface.

So, in this case it is also a stream tube stream tube, but it is a flexible. Flexible means, I can increase this area, also I can decrease in area; if I am coming from the other side, this is a situation of decrease in the area; if you are moving from inlet to exit, it is a situation is with increase in the area. So, the approach that we do here, like we are in all other cases, we are talking this entire volume to be a control volume approach; it is bounded by the control surface, the properties change in these directions.

So, the very basic point that needs to be emphasize that, although by relaxing these assumptions we say that; it is a quasi one dimensional approach, flow properties are uniform across a particular cross section.

So, at a particular cross section means, I can define the cross section and these are essentially the cross section at which I can define. So, at these particular cross section, it may have a average property value and that remain same at all the points at this location; but if I move to different x, the properties are going to change.

So, to some extent it is a logical approximation of actual flow properties. So, this point also I have emphasized that the properties are represented as a function of spital directions in a flexible stream tube. This is how we say it is a control volume approach. In fact, while doing analysis one can say, we can think about this overall property change to happen in a very infinitesimal small manner through a small distance dx.

If I measuring x along these directions, so we can define very small elemental lengths of dx and that dx the property change is very marginal. And while doing so, we can make sure that isentropic process assumption still holds good.

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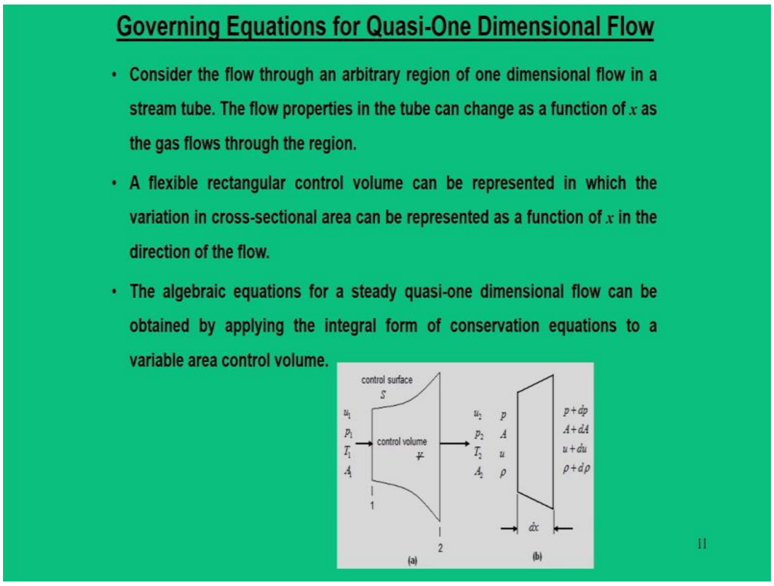

So, these are already I have explained, just to go through these, just I can just read out that; we are now moving for quasi one dimensional flow analysis for which we are going to derive the governing equations. In this case, we are we have to consider any arbitrary region in the one dimensional flow through a stream tube. The flow properties in the tube can change as a function of x.

So, that means area can change in the direction of the flow. And we can still need to have a flexible rectangular control volume, which is defined by this volume V and it is bounded by a control surface S; either you going from section 1 to 2 or 2 to 1, we can see their area varies, either increases, increases when you are moving from 1 to 2 and decreases when you are moving from 2 to 1.

So, the approach that we are flowing here that, we have to derive this entire volume as a very small elemental length dx, and across which we are going to fit the one dimensional flow equations.

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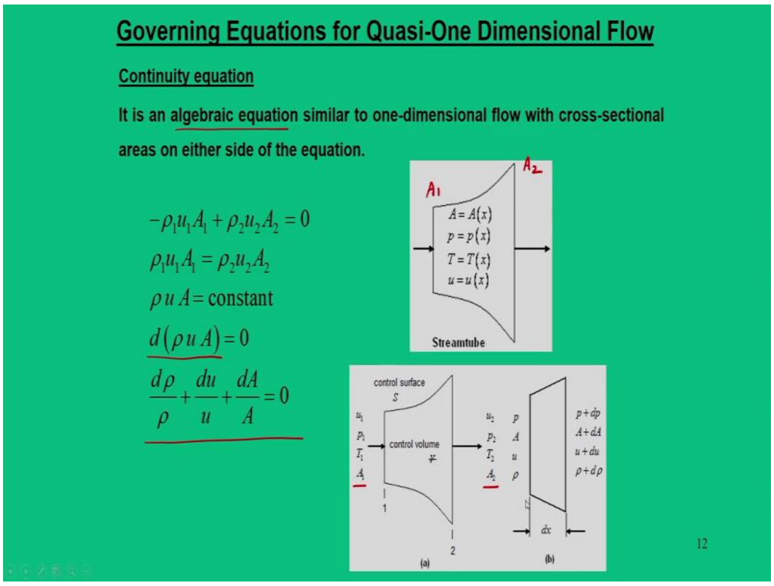

So, the first equation that comes out is the continuity equation; so obviously the continuity equation is a straight forward, its a algebraic equations, the very basic difference what we are bringing here. Whereas in previous equations, where the area was assumed to be constant; but here we are saying area as A_1 that is section 1 and area as A_2 here.

So, likewise these areas are defined. So, we get a straight forward continuity equations which is algebraic in nature, that is $\rho_1 u_1 A_1 = \rho_2 u_2 A_2$. So, otherwise we can say $\rho u A = const$; then we can differentiate this. So, we will take this one and moving further and doing simplify, we will use these particular expression $\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$. Now, question comes that, why you are using this approach?

Because for this analysis; we expect the property change to happen in a very small manner, so that the justification of isentropic process is ensured and that is possible only when we take a differential approach by dividing the entire control volume through elemental length dx.

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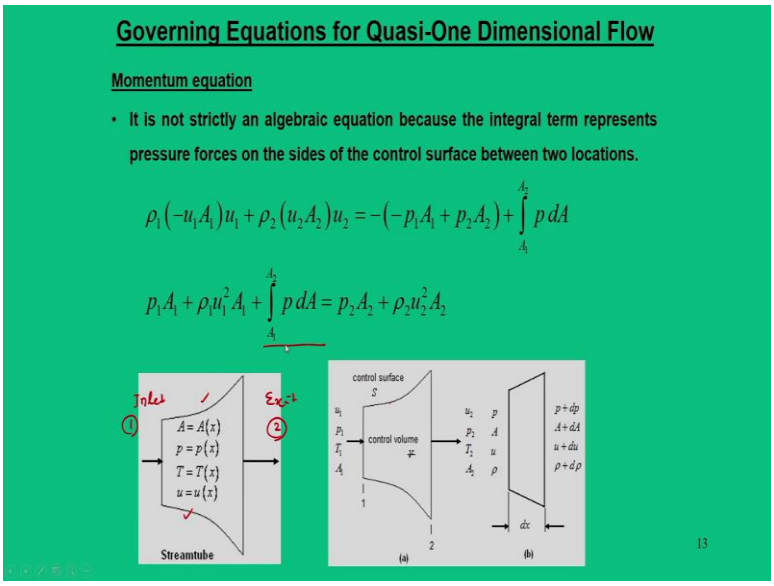

Now, moving to the momentum equations; momentum equations essentially it is a not strictly a algebraic equations, because the area change matters. So, what we see is that, from inlet to exit that is from 1 to 2 what we see is that the area that comes by virtue of pressure is p_1A_1 that is for this side and area that comes by virtue of exit is that p_2A_2. The press force that get exerted on this exit area is p_2A_2.

$$p_1A_1 + \rho_1 u_1^2 A_1 + \int_{A_1}^{A_2} p dA = p_2A_2 + \rho_2 u_2^2 A_2$$

In addition to that there is another pressure term that acts on this control surface and these changes as a integral. So, pressure changes over that area from A_1 to A_2 over this integrals.

So, that is what is not a strictly algebraic equations, but there is an integral into it. And while recalling the one dimensional equations; the only difference that comes here is this part, where your area is different and also the integral also has another parameter that gets added in the momentum equations.

Now, this is the equation we are trying to analyze in more details. So, this is how we get the first equation in the integral form.

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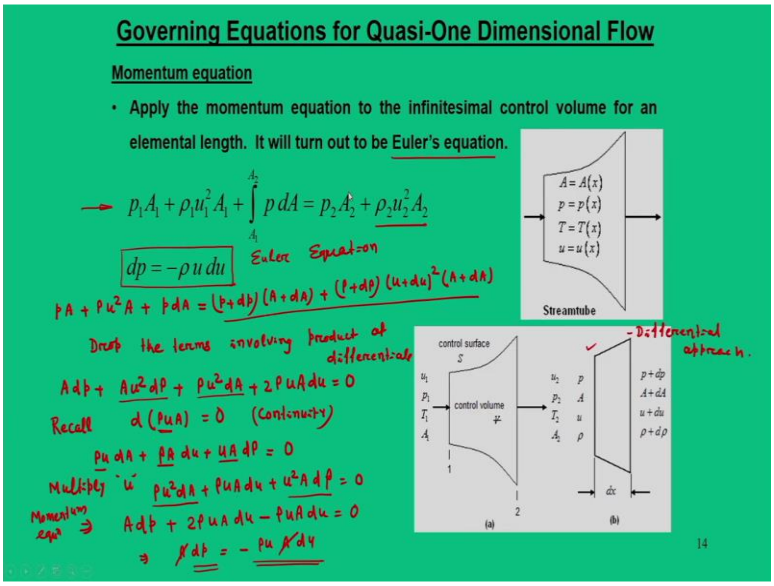

And what we are trying to look at here, we have to revisit the concept that we are dividing this entire control volume with small elemental length dx or elemental area dA and area changes from A to A+dA. So, accordingly when the area changes this; so the thermodynamic properties pressure, velocity and density they also change in a very small manner like p+dp, u+du and ρ+dρ.

So, difference between them is hardly a very small number dp. So, this approach we call this as a differential approach. So, for the time being we these particular equation what was derived earlier from integral calculations, we are going to apply for these two locations having this. So, if I write this equation for this before for the figure shown in b; so what we can write is

$$pA + \rho u^2 A + p dA = (p + dp)(A + dA) + (\rho + d\rho)(u + du)^2 (A + dA).$$

what you do now? You expand this right hand side term.

So, when I expand this right hand side drop, I must drop the terms involving product of differentials; by differential means, like ρdu^2, you will have here du^2, we will have dA^2, we will have $d\rho du$, these components wherever it is appearing we have to remove.

So, by doing so what we are going to get is that, $Adp + Au^2 d\rho + \rho u^2 dA + 2\rho uAdu = 0$. Now, we have to recall the continuity equation which says, $d\left(\rho ua\right) = 0$; this we call as from the continuity equation.

So, we can differentiate this in a manner, we can write that taking this parameter as one; we can say $\rho udA + \rho Adu + uAd\rho = 0$. So, you have to differentiate. So, every part these two terms kept constant keeping the differential of the other. So, this is how we do in the mathematics.

Then what we do? Multiply u on both sides. So, when I say multiply u. So, I will say $\rho u^2 dA + \rho uAdu + u^2 Ad\rho = 0$. So, when I say compare this equation; I have here $\rho u^2 dA$ is here this equation, we will have $u^2 Ad\rho$.

So, these two equations, so the term remaining here when I substitute here that equation; so I can write this momentum equation, it is $Adp + 2\rho uAdu - \rho uAdu = o$, ultimately we get $Adp = -\rho uAdu$

So, A gets cancelled; ultimately we come back $dp = -\rho udu$ which is nothing, but this particular equation. So, this is a very famous equations known as Euler equation. So, this Euler equation is a very famous equation in the compressible flow analysis. So, essentially we will be using Euler equation to study the quasi one dimensional flow approach.

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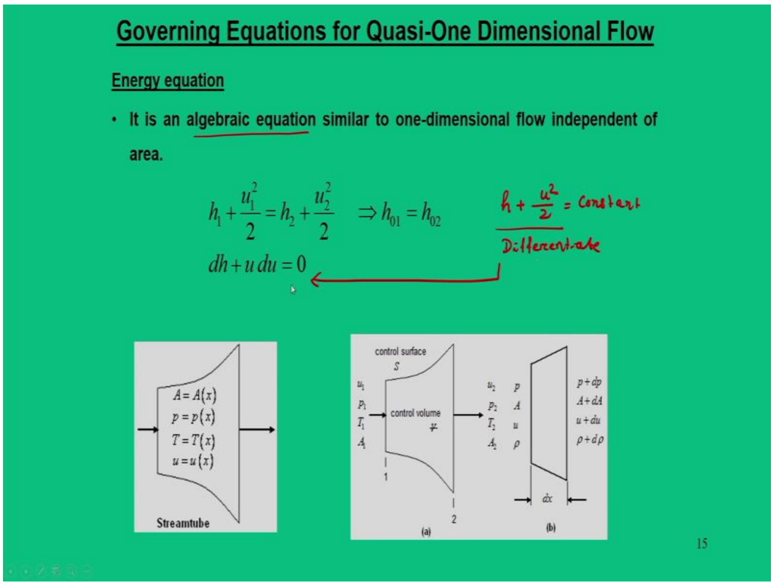

Now, moving to the energy equations; now as you can see the energy equation is still algebraic, because the flow is isentropic. So, total enthalpy remains constant, that is $h_{01} = h_{02}$. And, when I say total enthalpy is remains constant; that is if I write $h + \frac{u^2}{2} = const$ and differentiate this equation, then we move to the equation that is $dh + udu = 0$.

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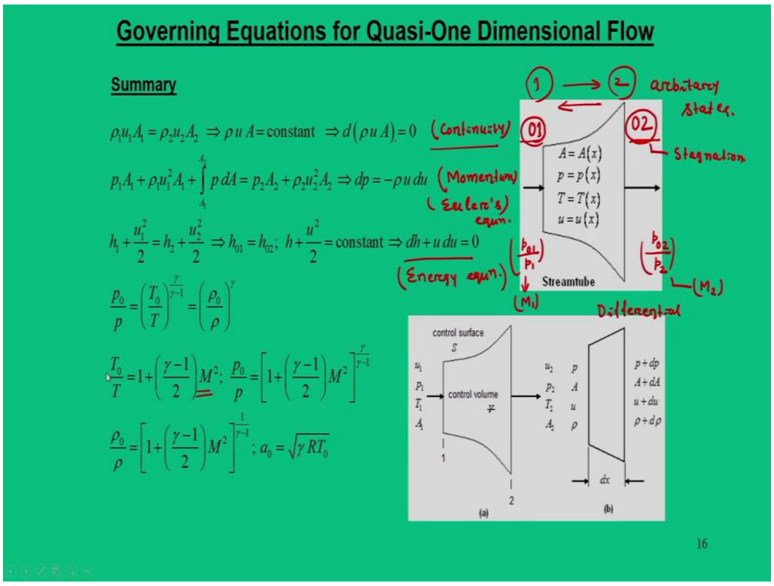

So, just to summarize whatever you have studied, I can make out that we are looking a quasi one dimensional flow approach where the area can change; but this area can change in a particular direction that is A as a function of x. It is first approach, first assumption of quasi one dimensional flow, essentially the flow is one dimensional; but it is still not one dimensional, but we are relaxing the dimensionality in some aspect. So, we say it is a quasi.

So, while doing so, we derived this governing equations; but our approach would be instead of not having a control volume approach, our approach would be a differential approach. While, deriving the conservation of momentum equation that is we will say it is a continuity equation; $d(\rho ua) = 0$. We will have Euler equation that is momentum equation or in the other words we say let us say Euler's equation and third one is energy equation.

So, this all these things form the very basic bottom line equations to be studied for quasi one dimensional approach. Now, while doing so, since we have already talked about the situations that the flows now here can move from 1 to 2 or it can go from also from 2 to 1, and this 1 and 2 are the arbitrary state,.

And when I say arbitrary states, these arbitrary states have their own stagnation state. So, that stagnation state we say 02 state and 01 state. So, these are stagnation states. Now when I say these two states, so; one can define their own stagnation states. So, to do that, we have to use these particular isentropic relations $\frac{p_0}{p}$. So that means for the state 1, I can write this as $\frac{p_{01}}{p_1}$, for this state I can write $\frac{p_{02}}{p_2}$.

Now, to define all this things we are going to use these isentropic relations, where it is a function of Mach number. So, this is a function of M_1 this is a function of M_2. And in fact, this is true for all other properties relations like temperature ratio and density ratio. We also have stagnation speed of sound $a_0 = \sqrt{\gamma RT_0}$. So, this is the very basic form of equations that needs to be used for the analysis which is quasi one dimensional in nature. So, with this I will conclude my talk.

So, thank you for your attention.