Fundamentals of Compressible Flow
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Module – 02
Wave Propagation in Compressible Medium
Lecture – 06
Wave Propagation in Compressible Medium - III

I welcome you again for this course that is Fundamentals of Compressible Flow. We are in module 2 that is Wave Propagation in the Compressible Medium.

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So, prior to this we have finished 2 lectures on this module. Just to brief about what we have completed that is we just introduced to what a compressible flow is, how it is different from the incompressible flow and we discussed about wave propagation in one dimensional medium. Here, we specifically talked about when a medium starts to become compressible, it generates a sound waves.

How that sound wave is generated and we also derived the speed of sound from this method. Then we also discussed about one dimensional flow equations that is pertaining to speed of sound. Then next we move to isentropic flow where the one fundamental thermodynamic properties was defined which is specific for the compressible fluid that is stagnation properties; that means, for a given thermodynamic states we can have a stagnation properties and for to define those properties the flow needs to be isentropic throughout.

Now, then moving to further there is some other properties known as characteristics properties of the compressible fluid. And this was also derived for a situation when a flow is hypothetically moved to a sonic state; that means, at that state where the Mach number becomes 1. So, corresponding to those states we say it is a characteristics properties.

Now, moving further in this lecture we will try to emphasize two important topics that is again with respect to isentropic flow. We will talk about the energy equations in more exhaustive manner; that means, in one dimensional flow equations earlier energy equation was one of the fundamental equations and we are trying to express in different forms.

So, that is what it is a alternative form of energy equations. In fact, these equations are derived based on those properties we defined earlier like stagnation properties, characteristics properties, how you can correlate those informations. Then on other the segment what we are going to discuss is again with respect to pressure disturbance in the compressible medium, but here the philosophy will be something different I will come back to as and when it comes.

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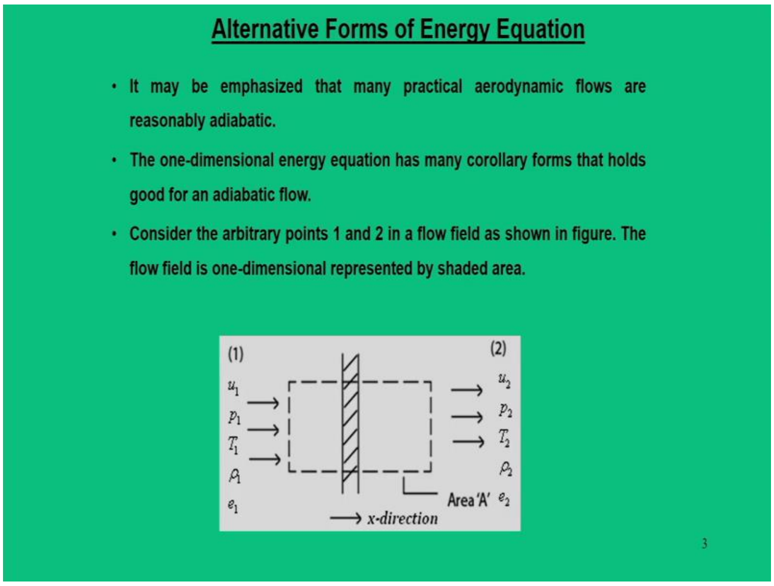

So, moving to the first topic that is alternative form of energy equations, the very basic idea is that in a given flow situations or flow along a streamlines, the property change with respect to point to point or with respect to space. But here what we are trying to say is that we are looking at two different locations in a medium or that locations may be treated as a arbitrary locations.

So, we are talking about 2 locations 1 and 2 and for that 1 and 2 in earlier discussions we have derived the mass continuity equations, we have derived momentum equations also we have derived energy equations. So, that means, the properties at state 1 and 2 are related in some fashion. So, if that is the situations and in fact, we says these 2 are arbitrary situations.

Now, what we are trying to look at here we are very specifically looking at this energy equation and with respect to this energy equations we are trying to see that if I move these arbitrary states to a very specific state or any arbitrary state to a very specific state. This means for example condition 1 or state 1 at these conditions velocity u_1 and pressure $p_{1,}$ temperature $T_{1,}$ density ρ_1 and total energy e_1.

Had this condition be a stagnation state what would have been the relation between 1 and 2 or in other situations also if state 1 is stagnation state, state 2 is a characteristic state. So, what will be the relations between these two and in fact, all the energy equation is valid because we are looking at the domain in a one dimensional medium and the flow is occurring along this constant area duct.

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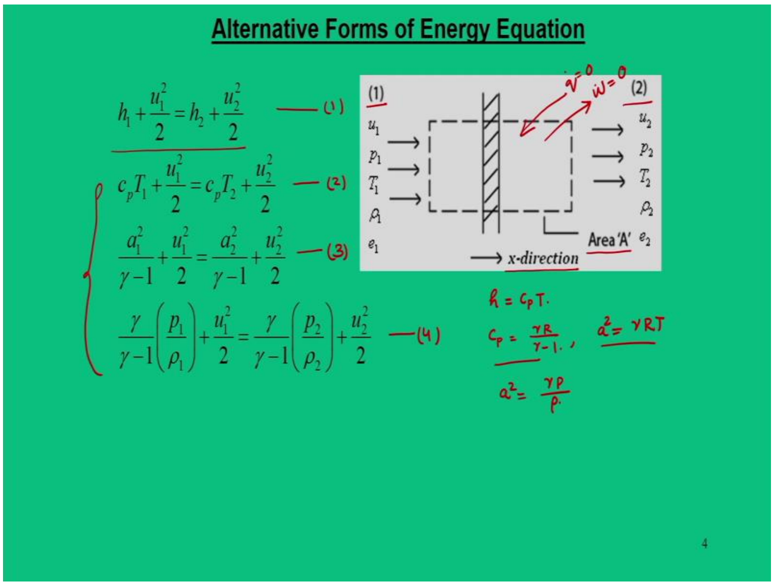

So, this is how the philosophy is all about. Again here to start with the first form of a energy equations for the situations as shown in this case where we say q or work that they are 0 mean there is no work transfer or there is no heat transfer.

So, if this is the case and its a constant area duct of the area A and the medium is one dimensional; so, it is flow is only in one directions. The first form of a energy equation is $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$. This is how the state 1 and 2 are related.

So, here h is enthalpy and u as you know this flow velocity and u_2 is the flow velocity at state 2. Now, from in the equation 1 if you write a calorically perfect gas with $h = C_p T$, we get the equation 2. $C_p T_1 + \frac{u_1^2}{2} = C_p T_2 + \frac{u_2^2}{2}$

Now, when we write $C_p = \frac{\gamma R}{\gamma - 1}$. This is how it is related *Cp* and R and then we also can write $a = \sqrt{\gamma RT}$ or $a^2 = \gamma RT$. When these two equations are put here, we get equation 3.

$$\frac{a_1^2}{\gamma - 1} + \frac{u_1^2}{2} = \frac{a_2^2}{\gamma - 1} + \frac{u_2^2}{2}$$

Now, moving further there is another form of the speed of sound that is $a^2 = \frac{\gamma p}{\rho}$. If you put it for state 1 and 2, these expression we get equation 4.

$$\frac{\gamma}{\gamma-1}\left(\frac{p_1}{\rho_1}\right)+\frac{u_1^2}{2}=\frac{\gamma}{\gamma-1}\left(\frac{p_2}{\rho_2}\right)+\frac{u_2^2}{2}$$

So, just to say that although we started the equation is the basic equation that is 1, but we have 3 other important forms of equations that are of importance. So, this is how one can use this equations in a one dimensional medium as and when the parameters are given.

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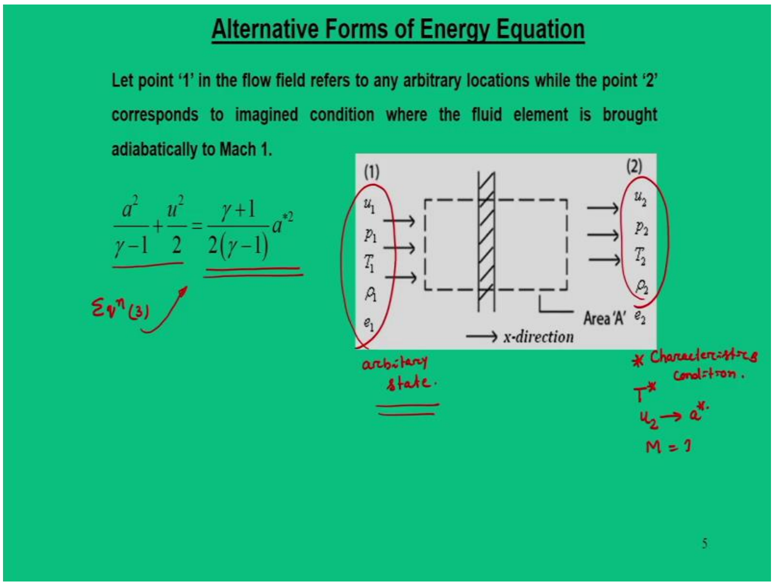

So, moving further here what we are trying to say that in a flow field domain, if the condition 1 is any arbitrary state. Arbitrary state I mean that is instead of defining as 1, I can write u, p, temperature T, density ρ and energy e, but these conditions that is condition 2 we will called it as a * conditions or characteristics condition.

Why is a star condition? Because, normally the properties are defined as T^*. So, hypothetically what we are trying to say that this condition is reached in an adiabatic manner for which u_2 becomes a* and M becomes 1. Let say we reaching the sonic state.

So, what we can do is that one form of equations that is in the last slide in equation 3 what we can write, if we put this condition 2 as *; the right hand side of the equation will turn about this expression that is $\frac{\gamma+1}{2(\gamma-1)}a*^2$ whereas, the left hand side of the equations is a any arbitrary state.

$$\frac{a^2}{\gamma-1}+\frac{u^2}{2}=\frac{\gamma+1}{2(\gamma-1)}a*^2$$

So, equation 3 from the last slide will turn into this particular expressions. So, we can say this is also another form of equations where one of the condition is a * condition or characteristics condition.

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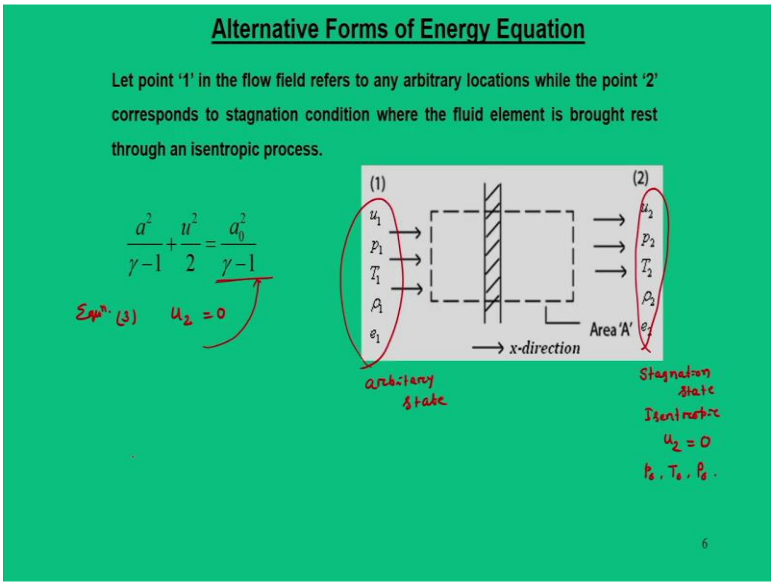

The next expression that we can do in a similar way; we also can say that this condition 1 is again as it is. It is a arbitrary state and these condition we will say it is a stagnation state which means this condition is reached in an isotropic manner for which the velocity u_2 is equal to 0 and all the property parameters becomes p_0, T_0, ρ_0 etc.

So, in same equation 3 when you say u_2 is equal to 0, this the expression turns out to be in this form.

$$\frac{a^2}{\gamma-1}+\frac{u^2}{2}=\frac{a_0^2}{\gamma-1}$$

So, that means, the left hand side of the equation becomes $\frac{a_0^2}{\gamma - 1}$ because that point of time we defined this as a_0 as a stagnation speed of sound.

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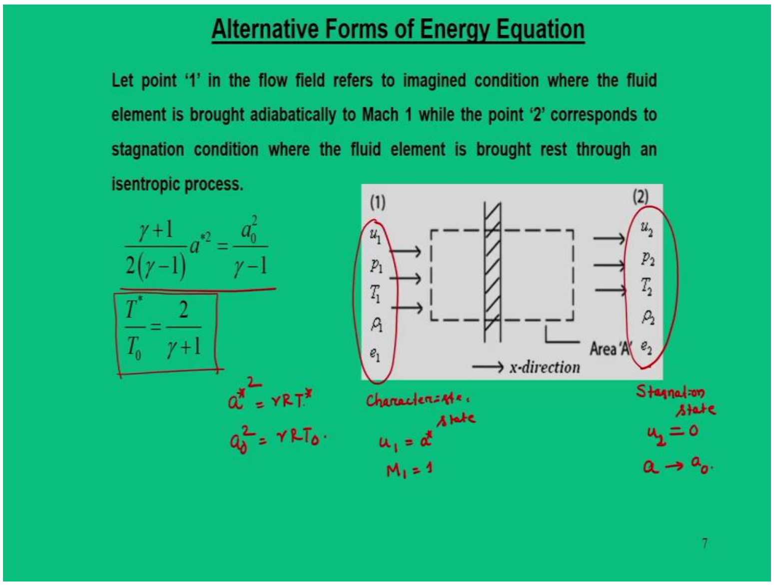

Now, moving further here what we are trying to say is that when we say like in previous 2 situations, we say one as arbitrary other as any of the possible conditions either stagnation state or the characteristic states.

Here, we will say that your condition 1 is a characteristic state which means u_1 is equal to a* and at this stage M is equal to 1 and this condition is a stagnation state where u_2 becomes is equal to 0 and a speed of sound becomes a_0.

So, in that aspect that means, we say the condition 1 and 2 can be one of the energy equations we will take the form of this shape.

$$\frac{\gamma + 1}{2(\gamma - 1)} a^{*2} = \frac{a_0^2}{\gamma - 1}$$

That means, in the previous way we derived those equations. Now, if we can correlate them, then it will be becomes like $\frac{\gamma + 1}{2(\gamma - 1)} a^{*2} = \frac{a_0^2}{\gamma - 1}$.

So, here we can further simplify this relations that is $a^{*2} = \gamma RT^*$ and $a_0^{\ 2} = \gamma RT_0$. When you put these equations and simplify, we will get a very fundamental equation that is $\frac{T^*}{T_0}$.

So, this is a very important expressions that we will be frequently using in the subsequent analysis of this compressible flow classes that when a fluid goes from stagnation state having temperature T_0 to a state where it is Mach number becomes 1 that is at that point of time your temperature become T^*.

$$\frac{T^*}{T_0} = \frac{2}{\gamma + 1}$$

It is the relation between T^* and T_0 is a function of gamma. It is only a function of gamma. So, it is no other parameter comes into picture. So, this is how the first important expression is derived.

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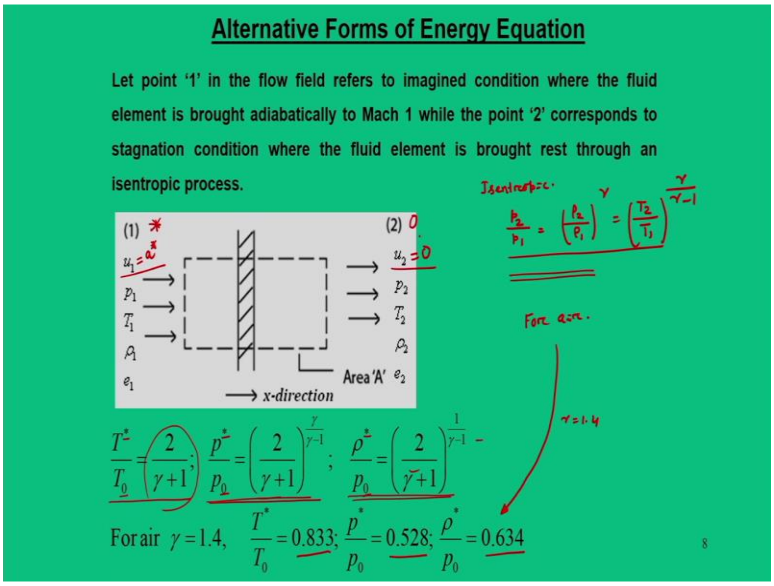

Now, moving for the next similar expressions; now, here this is also same situation that condition 1 become * and condition 2 becomes stagnation. Stagnation means * is used as a superscript, 0 is used as a subscript in the parameters that is $\frac{T^*}{T_0}$, $\frac{p^*}{p_0}$, $\frac{\rho^*}{\rho_0}$.

So, in the last expression we derive this first relation that is $\frac{T^*}{T_0} = \frac{2}{\gamma + 1}$. Now, we can recall our isentropic relation like $\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$.

Now, when I use this equations and for which between pressure and temperature, then we get this equations because temperature ratio we already know.

$$\frac{p^*}{p_0} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}}$$

Now, when I use between pressure and density, then I can get this expression because I already know the pressure.

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma-1}}$$

So, what we see is that we get a relation between a state where u_1 is equal to a* and u_2 is equal to 0 that is stagnation. So, in such situation the property relations are derived in such a manner that it turns out to be a function of gamma.

Now, for air; so, these expressions can be derived or found out which is a fixed number that is $\frac{T^*}{T_0} = 0.833$ $\frac{p^*}{p_0} = 0.528$ $\frac{\rho^*}{\rho_0} = 0.634$ and this is for gamma is equal to 1.4. So, this is a very important numbers that gives lot of design calculations in the compressible flow analysis.

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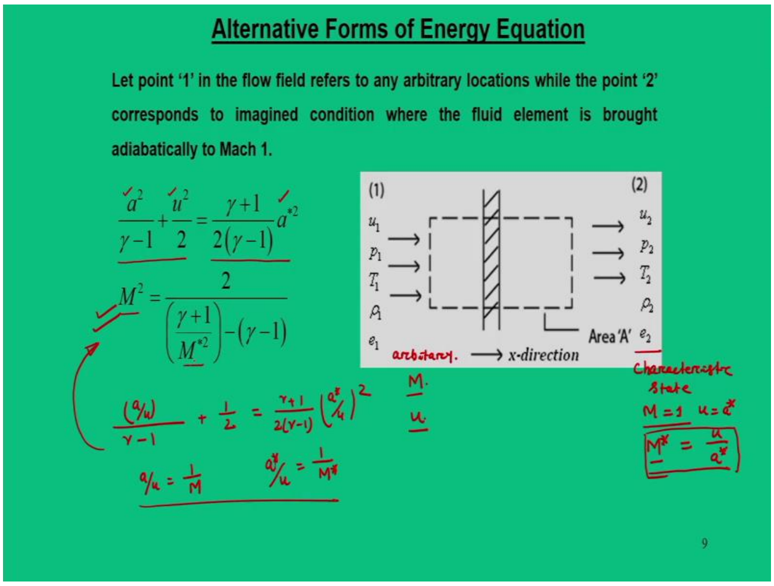

Now, having said this now we will talk about another relations where we will bring Mach number into account. So, what does this mean that any arbitrary state that is condition 1 if it is any arbitrary state for which your Mach number is M and this condition 2 is a characteristic state for which your Mach number is 1.

And when you say Mach number is 1, but we can define as $M^* = \frac{u_2}{a*}$. So, you can also define this M^* for this Mach number. So, we reach here u=a*. So, Mach number for this we can write, if in this state your speed is u. So, you can say $\frac{u}{a*}$ which is Mach number M^*.

If this is the situation, then we are trying to see what is the relation between the M and M star. So, for that what you are writing one form of the energy equations; one for the arbitrary state, other for the characteristic state.

$$\frac{a^2}{\gamma-1} + \frac{u^2}{2} = \frac{\gamma+1}{2(\gamma-1)} a*^2$$

So, here we have brought the parameters into account that is speed of sound and velocity of the gas or body. We also define a* that is corresponding to the speed of sound at the characteristic state. So, in this equations we can simplify a or divide that equation by u that is on both sides.

So, we can write $\frac{\left(a/u\right)^2}{\gamma-1}+\frac{1}{2}=\frac{\gamma+1}{2(\gamma-1)}\left(a^*/u\right)^2$. So, once you simplify this we can write $a/u=\frac{1}{M}$ and $a^*/u=\frac{1}{M^*}$. So, after putting this expression we can simplify this equation in this manner where you talks about Mach number relation with the corresponding Mach number at the characteristic state.

$$M^2=\frac{2}{\left(\frac{\gamma+1}{M^{*2}}\right)-(\gamma-1)}$$

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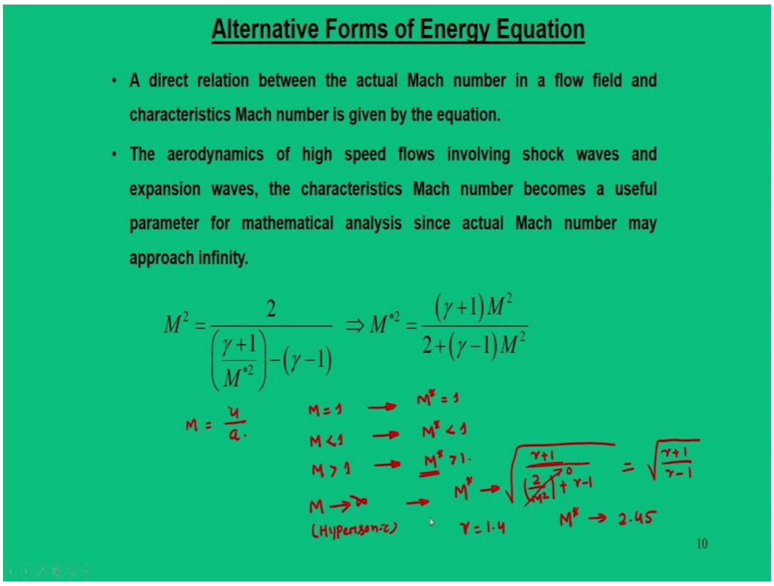

So, just to give you brief about this particular parameter. So, what we can say the direct relation between the actual Mach number in a flow field and characteristics Mach number is related by this equation. And the aerodynamics of high speed flow involving shock waves, expansion waves which further we will be discussing further, these characteristics Mach number becomes a useful and significant parameters. How I will just try to explain that what is going to happen. So, on normal circumstances when I say define Mach number we say $\frac{u}{a}$. So, when your M is equal to 1, that is sonic state your M^* goes to 1 that is you can check from this equation.

Now, when M is less than 1 that is flow is subsonic your M^* will be also less than 1. When M is greater than 1, these M^* also will be greater than 1. So, by this one can say that the way the M changes similar way also M^* changes.

Then question arises then why should I define M^*. So, the main reason of this definition of M^* comes when M is much much higher or M goes to infinity. So, when M goes to infinity, this is a situation when the flow becomes hypersonic that is Mach number is predominantly high. So, under that circumstances if I simplify this equations, what turns out to be M^*.

So, what we do is you rewrite this equation in this manner like divide M on numerator and denominator.

$$M^{*2} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$$

So, what we will get $M^* = \sqrt{\frac{\gamma+1}{\left(\frac{2}{M^2}\right)+(\gamma-1)}}$. So, when M^*goes to infinity, this part goes to 0. So, this turns out to be $\sqrt{\frac{\gamma+1}{\gamma-1}}$. So, putting gamma is equal to 1.4, your M^* happens to be a fixed number that is 2.45.

So, this is a very interesting analysis for this expression that as long as you are in the domain of subsonic, sonic and supersonic, your M and M^* they are related in same fashion. But, when you go to hypersonic situation, the M^*becomes a fixed number although M goes to infinity.

So, this analysis makes a very simplified analysis in terms of mathematical expressions because instead of using M as infinity people tried to express M in terms of $M^*$where there is no question of infinity comes, it turns out to be a fixed number. So, this is not much covered details in this course. Just I tried to explain what is the significance of M^*in this situation.

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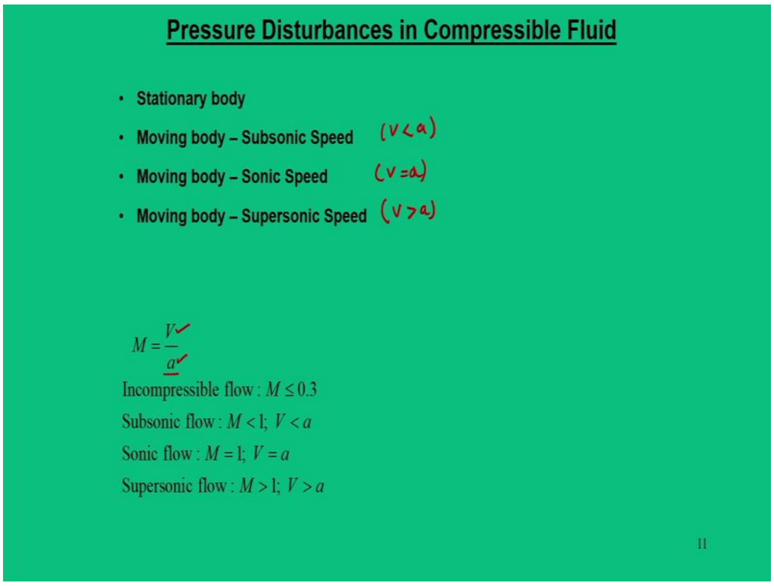

So, now we are going to move to a different topic that is pressure disturbances in the compressible fluid. So, in the previously situations we talked about wave propagation in a compressible medium in which we said that how a disturbance gets generated in a medium and when this disturbance is generated, it moves at a speed of sound. If this disturbance has very small magnitude, then that we called as a speed of sound.

So, now you imagine that in any medium this disturbance always moves at a speed of sound. So, that speed of sound is always remains in the medium as long as there is a disturbance. At the same time, we also have a moving body. So, a body also is moving at the same time it is creating disturbance. So, there is a velocity associated to it. So, there are 2 velocities now. One is the speed at which the body is moving and other velocity or other speed is that speed at which the disturbance is moving.

So, this is related by a number which you called as a Mach number. So, one is your speed of the body, other is the speed of sound and we made this ratio. So, the relative difference between these two speed will essentially talk about the Mach number whether it is higher or lower, other is medium is compressible or incompressible that we discussed in the previous lecture.

What we are trying to say is that in this slide that we are talking about pressure disturbance in a compressible medium when there is a relative importance between the speed of the body and the speed of sound.

So, what does this mean? That is what there are 4 possible cases we have discussed that first is even your body is stationary; that means, a standing body it is creating a disturbance or a source that is creating a disturbance it is moving in the different directions.

When it moves a different direction, it creates a speed of sound; that speed of sound at a certain time it encloses area defined by a_t. So, it is a spherical domain a_t that is what till that time the disturbance should have travelled. Second case what we are going to see is that when you at a subsonic speed.

So, apart from this, instead of velocity of the body is 0, if the body is also moving and creating disturbance; that means, disturbance moves at the same speed, but your body is moving relatively higher. So, if you progressively increase this velocity when it is a subsonic situation that means V less than a; when it is a sonic when V becomes equal to a. Now, when at supersonic speed V greater than a, then what happens in the medium. So, that is the basic philosophy of this case which we are going to analyze.

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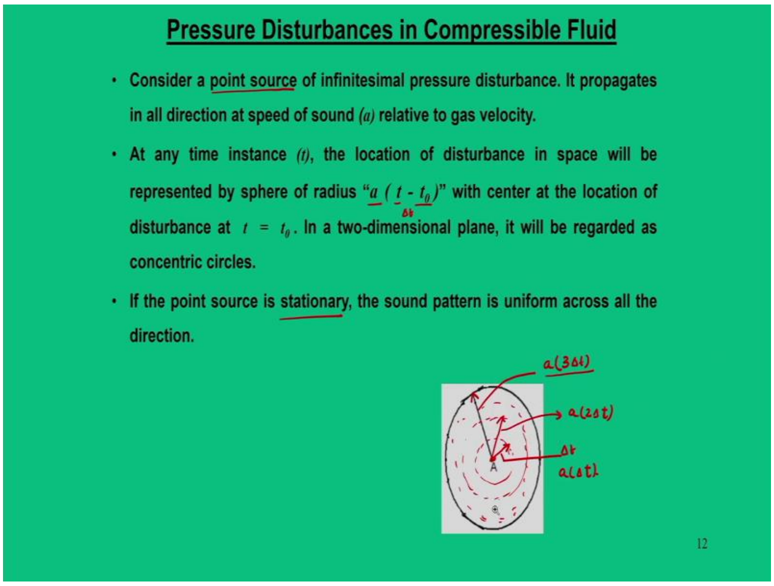

And in fact it is very important as far as our understanding is concerned. So, the first situation what we are trying to say just to explain this phenomena, we are saying there is a point source which is located at point A and this point source is stationary; that means, point source is not moving, but it creates disturbance around the medium.

That one instance and this disturbance moves at a speed ‘a’. Now, at certain time δt; that means, if your $t=t_0$ is initial time at time t. So, this $t- t_0$ becomes δt. So, at different time steps it creates a disturbance that moves in a spherical fashion.

So, one can say the by time δt, this disturbance would have moved aδt; at time 2δt, the disturbance would have moved a2δt and at time 3δt, the disturbance would have moved a3δt. So, this is a very simple situation wave is moving and since body is not moving, the disturbance moves at this location or at this distances. So, by staying this what do we physically mean that, if you say at time a =3δt, even though the body is at point A, but entire domain outside the point A is aware about the presence of the body at point A. That means, entire flow information like pressure, temperature, velocity or the density for this point A is essentially shared in the medium.

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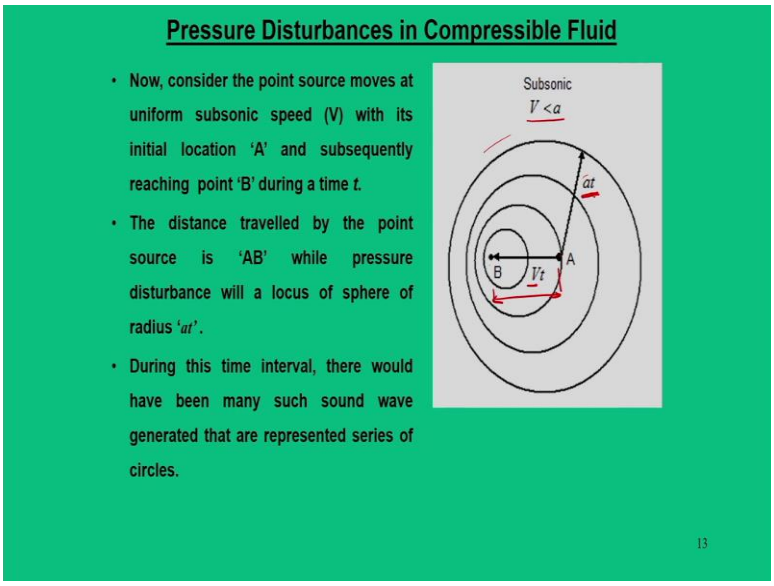

So, now moving further if I say instead of sitting at point A, the body has moved at certain velocity V. So, at time t the body would have travelled from point A to point B. So, the distance travelled by the body will be $V \times t$. So, at same time the disturbance would have moved a distance $a \times t$ and that is represented as a sphere in the space. So, the most important point here is that within the space entire domain is aware of the body’s movement from point A to B.

So, whatever changes in the flow field that is pressure, temperature, density, etc are already shared by the medium. How? Because, this has been shared through the disturbance that moves at a speed of sound. How long these disturbance or the informations are known; till the distance $a \times t$ enclosed in a spherical domain. So, but one important point is that at any time instant, the point B as long as your V is less than a, the point B will be always remain within the bigger circle that is $a \times t$.

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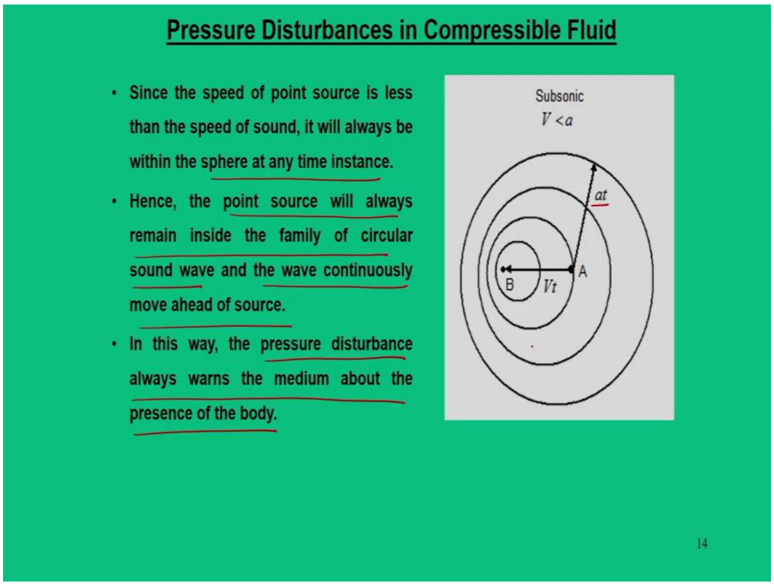

So, because V is less than a and we are looking at the distance at same time t. So, this is what it means that since the speed of point source is less than the speed of sound, it will always be remain within the sphere at any time instance. So, the point source will always remain inside the family of circular sound waves and the wave continuously move ahead of the source.

These are the important phenomena that from our analysis we can understand. So, because of this reason, the pressure disturbance always warns the medium about the presence of the body. So, within that circle $a \times t$ or spherical space $a \times t$, the pressure disturbance warns the medium. Medium here I mean apart from the body enclosing space is your medium. It warns the medium about the presence of the body.

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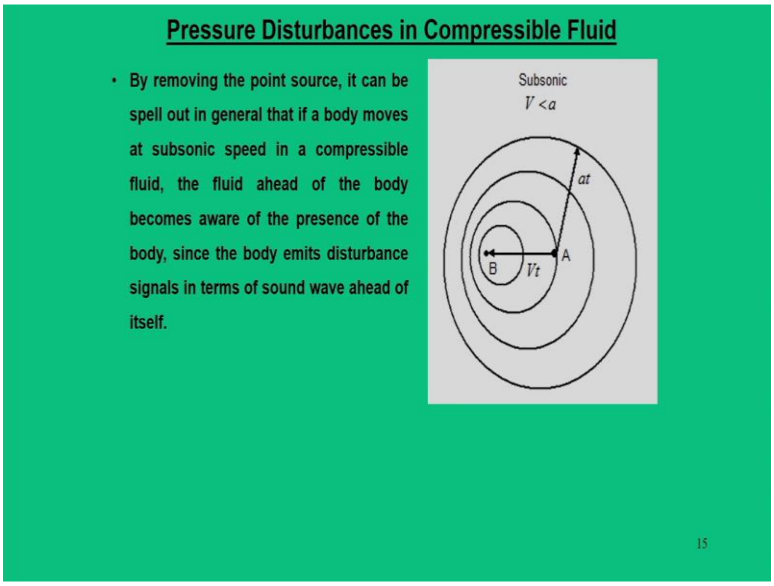

So, in a very technical term or from with respect to our analysis term what we can say by removing the point source it can be spell out in general that if a body moves at a subsonic speed in a compressible fluid, the fluid ahead of the body becomes aware of the presence of the body. How? Because the body emits disturbance signal in terms of sound wave ahead of the itself. So, by the time body reaches there, the sound wave would have traveled and it has warn the medium. This is the one of the important conclusion from this analysis.

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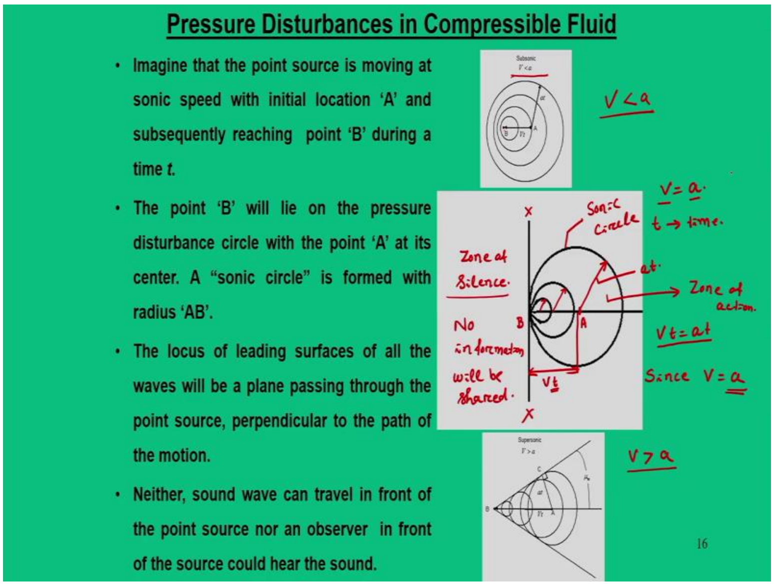

So, we discussed about what happens when your V is less than a which is subsonic. Now, let us see what will happen if V becomes equal to a; that means, your speed of the body is equal to the speed of sound means the point source is moving at a speed which is equal to speed of sound. So, for that case referring to this figure we can say that the relative difference of point B will be more and more as and when its velocity is increased.

So, at any time instant t if you can say that your initial point or position of the body is A and by that time t if you are looking at this pressure disturbance space of travel, we can say it is $a \times t$. And at same time the point B would have moved to a location B and that distance should be AB that is equal to $V \times t$, But, here interesting part is that all the circles which were emerged out of this will try to merge at a point B only.

So, these are like essentially different sound waves that would have generated at time t. And all this sound waves will try to merge at this point B and obviously, when V=a, this circle will have same radius that is $V \times t = a \times t$ or the sphere or circle whatever we say will be $V \times t$ is will be equal to $a \times t$. So, what does this physically mean to us? So, we can draw a line X-X and we can clearly distinguish two different zones.

So, what are the zones? We say this, the space which is left to this X-X we say as zone of silence and within this spherical circle we can say these are zone of action means what? Since V=a, we can say the speed of sound or disturbance cannot move faster than the velocity of the body. So, beyond this line X-X no information will be shared. So, what does this mean?

That whatever flow information pressure, temperature, density, it is limited within this spherical domain of $a \times t$ radius and the and the region outside the line of line X-X has no information about the presence of body, but that is not the case when it is a subsonic. Now what will happen if you just move further that is when V becomes greater than a?

So, what we are trying to see that this line X-X will try to gets bent in a fashion that as shown in this figure; that means, since because your velocity has becomes progressively higher than the speed of sound. So, the point B we will try to shift towards the left again and again and as and when B is more than A, you will have number of circles and these circles will try to move towards the left. And entire information which was about the sharing of the medium is limited within this distance limited by the radius $a \times t$.

So, this is how what happens when the body moves at sonic speed and this circle is known as sonic circle. That is the maximum radius what we can have when it is at that point of time.

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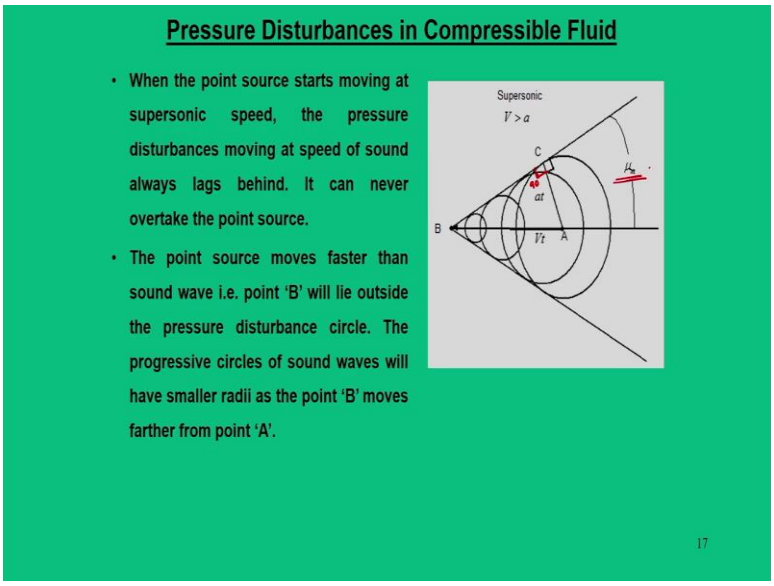

Now, as I explained earlier that when the body moves at supersonic speed, the point B will try to move towards left. So, in this process the figure turns out to be number of disturbance waves which is engulfed in a domain which is a cone.

So, in this cone what we will try to see if one interesting point that happens, if this velocity keeps on increasing and increasing, at one point of time it becomes a point. It can be imagined to be start at a point and this spherical circles of pressure disturbance will keep on increasing and increasing and with a maximum radius of $a \times t$.

And from that point, if you draw tangent to all this circles of pressure disturbance waves, then we will land of a very interesting geometrical figure and this will be represented by a right angle triangle ABC, where the $\angle ACB = 90^{\circ}$. And this angle $\angle ABC$ is represented as μ_m. What is that μ_m? We will explain now.

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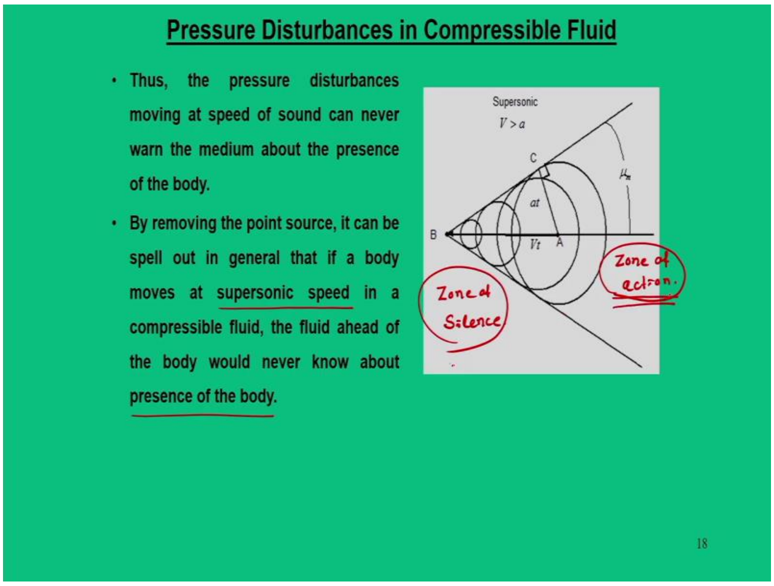

So, this is how I have explained that by removing the point source it can be spell out that in general that if a body moves at supersonic speed in a compressible fluid, the fluid ahead of the body would never know about the presence of the body.

So, this is another interesting or landmark remark when a body moves in this speed of sound. So, at this particular situation what we can say that as long as we are in this sonic circle we say this region will be zone of action. So, which means the entire domain within this cone is aware about the presence of body and outside this cone we call this as a zone of silence.

So, outside this cone we will have the zone of silence. So, if you are standing in this domain that is in the zone of silence, you will not be aware about the motion of the body or any physical presence of the body. Because you have not received the pressure disturbances in the form of speed of sound. So, the flow information has not been propagated to you. So, that is the reason this is called as a zone of silence and other thing that is within the cone we will define this as a zone of action.

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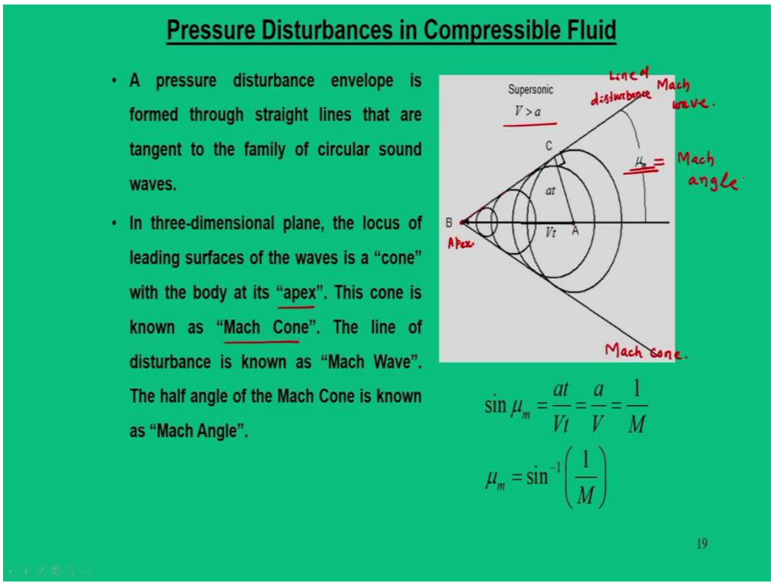

Now, moving about little bit of mathematical or geometrical understanding to this phenomenon that is when your V is less than a. So, what we say is that in a 3 dimensional plane we will try to find out some mathematical background or insight to this flow phenomena that is the pressure disturbance envelope is formed through a straight lines that are tangent to the family of circular waves.

So, that means, each point on this, it will be tangent when you would start from the point B. So, the point B is the final location of this P of your source, point A is your initial location of the source.

Now, in a 3 dimensional plane the locus of the leading surfaces of the waves is a cone with a body at its apex. So, this point B is known as the apex; that means, the location of the body is known to be its apex of the cone and this particular cone is known as Mach cone. So, we say it is a Mach cone. The line of disturbance is known as Mach wave. So, essentially the line of disturbance I mean the line that we draw the tangent; we call this as a line of disturbance that is known as a Mach wave. So, this is known as line of disturbance.

The half angle of the Mach cone is known as a Mach angle that is μ_m. Half angle means I think one can make it understand that along this line AB, this cone is symmetrically is nature with half angle μ_m that is known as Mach angle.

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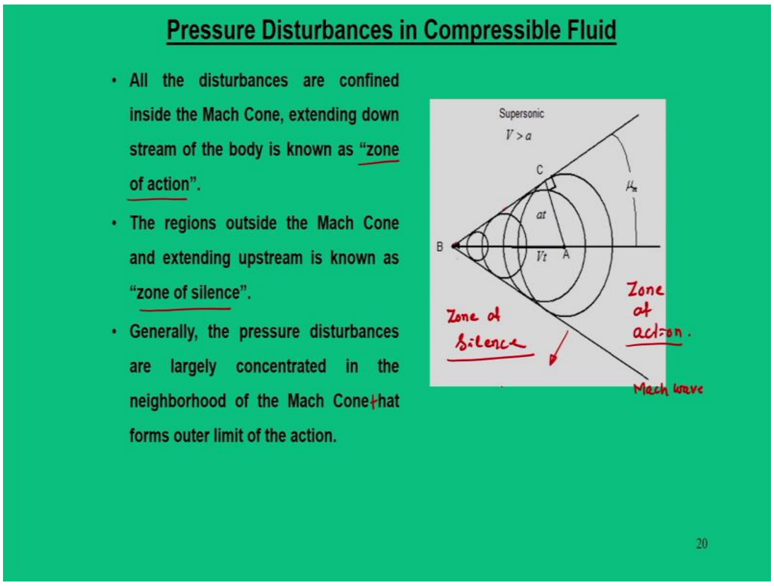

So, as I mentioned that for this case within the Mach cone we call this as a zone of action, outside the Mach cone we called as a zone of silence. So, this has been emphasized here. So, one can take a note of it. So, generally the pressure disturbances are largely concentrated in the neighborhood of the Mach cone as Mach cone that forms the outer limit of action. So, this is the limiting line of the Mach cone as a Mach wave that separate the zone of action and zone of silence.

So, it is essentially means if you are close to this Mach wave you will at least know the presence of the body. If you move away from this Mach wave, you are not aware of this presence of the body or you are not aware about the flow informations.

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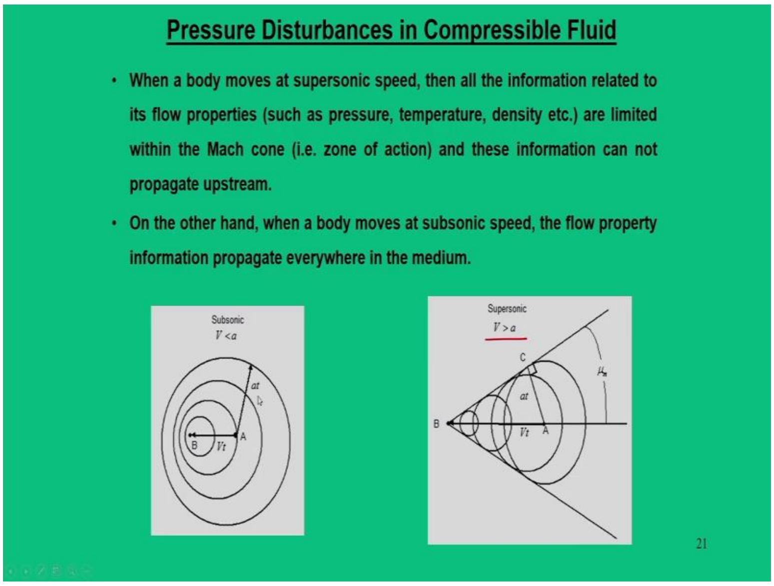

So, this is how I summarized with respect to compressible flow and in fact this concept has to be used as a basics of this particular course. That is when the body moves at a supersonic speed, then the information related to its flow properties such as pressure, temperature, densities are limited within the Mach cone that is in the second figure. That is what we call as a zone of actions and these information cannot propagate upstream.

On the other hand when the body is moves at subsonic speed the flow property information propagate everywhere in the medium. So, that means, in this case the flow properties are always moves everywhere in the flow field because the disturbance moves at higher speed than the body.

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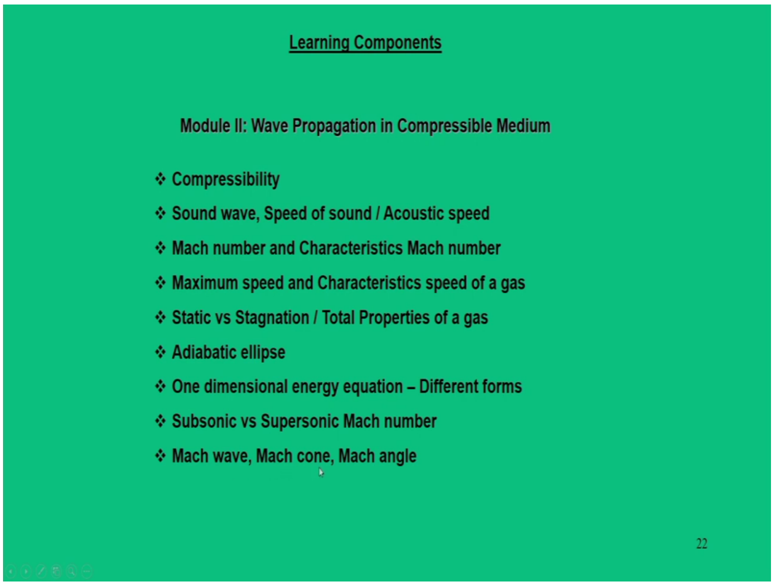

So, this is a like a closing module for this second module that is wave propagation in the compressible medium. With this we come to the end of this module 2. So, the highlights or learning components for this module we should understand that what is the compressibility.

We also know about sound waves, speed of sound or acoustic speed. We have to know about Mach number, characteristics Mach number. We derived the expressions for maximum speed of any gas and the characteristic speed of the gas. Then we discussed about static and stagnation properties of the any gas.

We also talked about a graphical representation of the importance of Mach number in terms of a ellipse which is known as a adiabatic ellipse. Then we also derived different forms of energy equations and we also discussed about their relative importance, their relative significance in a flow domain. In fact, we have now clear about the subsonic and supersonic Mach number, how they are different in terms of pressure disturbance.

Now, when we are in the supersonic Mach number, it starts with a Mach wave. Then we defined a Mach cone and also we also talked about Mach angles. So, with this I hope you should as a reader for this course at the end of this module you have to ask yourselves whether you are find the answers for all these terms or not.

So, this you can take it as a keywords for this module and try to dig into this definitions at appropriate slides. And, once you are once you find this answers, then you will get satisfied with your understanding. Now, with this I come to the end of this module 2. In the next subsequent discussions, we will start the module 3.

Thank you.