Fundamentals of Compressible Flow
Prof. Niranjan Sahoo
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module – 01
Review Concepts of Fluid Mechanics and Thermodynamics
Lecture – 03
Review Concepts of Fluid Mechanics and Thermodynamics - III

We are back to again this module 1 this last part of this module that is Review Concept of Fluid Mechanics and Thermodynamics. So, I think in the previous two lectures I have clarified the some of the important topics of fluid mechanics and thermodynamics.

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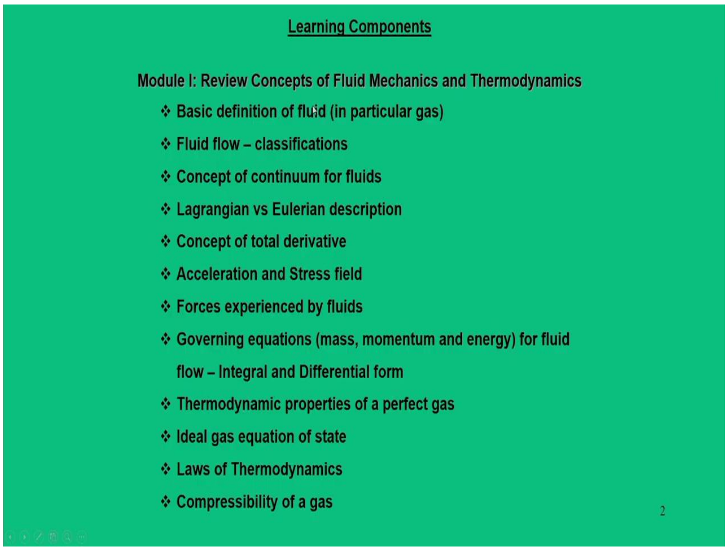

And in fact, these two topics that is fluid mechanics and thermodynamics is very vast, but to the best possible extent, I just tried to give some brief introductions that are essential for this course that is modern compressible flow. And I have talked about those topics of interest that are related to some sense to the compressible flow.

So, if you have to recall the learning components from this module then you must ask yourself that whether you know this or not on these broad topics. That is you should by this time you should be able to define the definition of a fluid and in particular if it is a gas; because the shear stress is absent here. Also we talked about fluid flow its classifications we introduced the concept of continuum into the fluid motion and with that relations there are Lagrangian and Eulerian descriptions.

Then we introduced the concept of total derivatives, and acceleration and stress; these are the two important topics and for this fluid particles if you want to find out its acceleration and its stress and if we want to express it in terms of space as well as time then the field concept is introduced. So, we call this as a acceleration and stress field.

Then we introduced the concept of how the forces are experienced by the fluid. And in particular the forces are expressed as a normalized component and tangential components. Then after having discussing all these things we try to give some insight to the governing equations that is; mass, momentum, energy equations for a fluid flow. And in particular we did it for an inviscid analysis and these equations can be represented as an integral as well as differential form.

In fact, we tried to find out that how this integral and differential form of equations for an inviscid compressible flow will be beneficial for our application. Now these are mostly on fluid part and coming back to thermodynamic part because why I am saying thermodynamic part. Because this is equally important because since the properties many thermodynamic properties of the gas many thermodynamic properties of the gas and fluid they are related in many sense.

In fact, to have this more clear we can say the first law of energy equations and first law of thermodynamics and the energy equation of the fluids they are almost same. Then we have this these thermodynamic properties, we also talked about ideal gas equations, laws of thermodynamics. And then important part is the compressibility in particular for a gas.

This is one of the vital part we are concluding this in this thing; so, that in the next module we will be able to start from this as a beginning in the sense that this is the essentially the most important part of this course.

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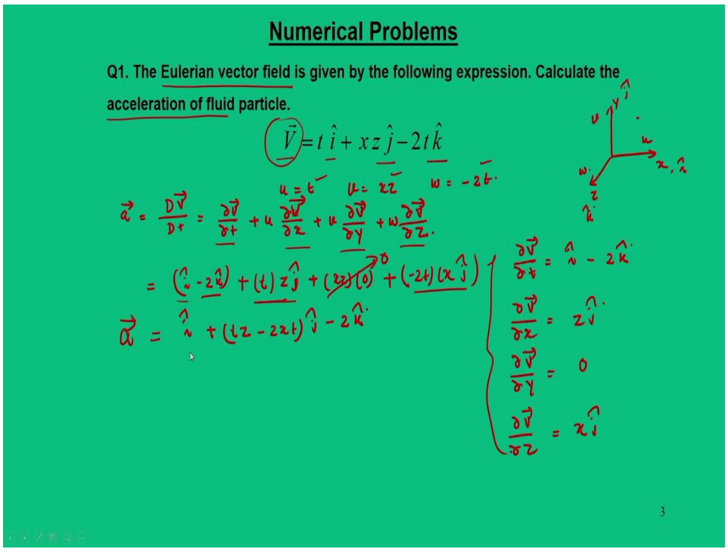

Now in this course although there are many numerous problems that are available, but I will try to give some insight of some standard problems how just to clear certain concepts. So, this particular problem that is first problem; in which we are talking about a Eulerian vector field.

So, normally when you say a vector field it is represented by three components i, j and k; that is in orthogonal coordinates, if I write x y z and this unit vectors are $\hat{i}, \hat{j}$ and $\hat{k}$ and we can say the velocity components u, v and w. And here for any point in the space is defined by the vector field then we can say your u = t, v = xz and w = -2t.

Now, we wanted to have this acceleration of this fluid particle. So, acceleration for acceleration we can recall this as derivative of this velocity vector $\frac{D\vec{V}}{Dt}$ and this is nothing, but $\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \left(u\frac{\partial \vec{V}}{\partial x} + \upsilon\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} \right)$

Now, from this equation one can directly calculate we know 'u' we know 'v' and we know 'w' and we want to find out these components. So, from this equation we can rewrite $\frac{\partial \vec{V}}{\partial t}$; that means, partial derivative with respect to t both. So, when you do this the only where the t term is present we have to retain this that is $\frac{\partial \vec{V}}{\partial t} = \hat{i} - 2\hat{k}$.

And $\frac{\partial \vec{V}}{\partial x}$, so, we can write so the first component, there is no x, only there is a second component there is x. So, $\frac{\partial \vec{V}}{\partial x} = z\hat{j}$. $\frac{\partial \vec{V}}{\partial y}$, so there is no y in any of the term so we can put it 0. $\frac{\partial \vec{V}}{\partial z}$ so if you look at z here, only middle term has z component. So, we can write that is $x\hat{j}$. So, this is how we can write.

So, after finding these terms we can simplify this as

$$\vec{a} = \left(\hat{i} - 2\hat{k}\right) + (t)z\hat{j} + (zx)(0) + (-2t)x\hat{j}$$

Now from here we can simply write we can separate it out i, j and k.

$$\vec{a} = \hat{i} + (tz - 2tx)\hat{j} - 2\hat{k}$$

So, you can write this acceleration vector as this so $\hat{i} + (tz - 2tx)\hat{j} - 2\hat{k}$. So, from this velocity vector by using this derivative one can find out this acceleration vector.

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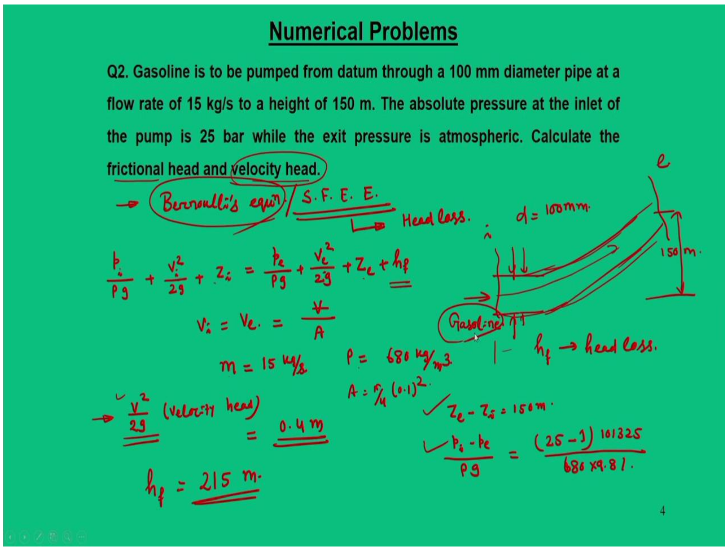

The next problem talks about the use of what I can say Bernoulli's equation or in other words we can rewrite as steady flow energy equation that involves head loss. So, when there is no head loss this steady flow equation turns out to be Bernoulli's equation.

So, what the problem talks about that; we want to pump a gasoline in a pipe of diameter 100 mm and the distance I want to go in an elevation of 150 meter. So, the fluid is gasoline and what needs to be calculated? We have to calculate the frictional head and velocity head. So, this frictional head why there is a frictional head? because this is being and you know this gasoline is being carried in a long pipe of this diameter and as and when it flows that will be loss in the pipe and that loss is accounted as a head loss h_f.

Now, to calculate this one simplest expression of steady flow energy equation can be used which is; that means, we can put as 'i' as inlet 'e' as exit. So, $\frac{p_i}{\rho g}+\frac{V_i^2}{2g}+z_i=\frac{p_e}{\rho g}+\frac{V_e^2}{2g}+z_e+h_f$. This h_f will be added in this part because this is at the exit, so as when flow ends here, so we will have this head loss. So this has to be calculated.

So, one thing we can say is, since it is a constant diameter pipe and it is a gasoline, flow is incompressible and when you say flow is incompressible, an area does not change. So, we can say $V_i=V_e=\frac{\dot{V}}{A}$

So, volume flow rate we can find out because we know the mass. So, since we know mass and we know that is 15 kg per second. And for density of gasoline that we can find out from the any book that is about 680 kg/m^3.

So, once we know mass and density we can find out volume. So, once we know volume, we know area = $\frac{\pi}{4}d^2$, d is your 100 mm that is 0.1 meter. So, after putting it we can find out this term $\frac{V^2}{2g}$ that is nothing, but velocity here. So, this term after putting all these values we can calculate this $\frac{V^2}{2g}$ to be about 0.4 meter. That is what we require the velocity head.

Now, to calculate this friction head we have to use this equation. And in fact, in this equation the velocity head remains the same for the both sides. So, what we can do is here and you can have $z_e-z_i=150m$. And this $\frac{p_i-p_e}{\rho g}=\frac{(25-1)101325}{680\times 9.81}$.

So, this we know; this we know velocity head remains same $z_e - z_i$ also we know. So, from this equation we can calculate this h_f to be about 215 meter. So, head loss would be about 215 meter.

So, this example gives you that how to use this steady flow energy equation in a given problem. And in this problem the fluid is gasoline and in general it is incompressible.

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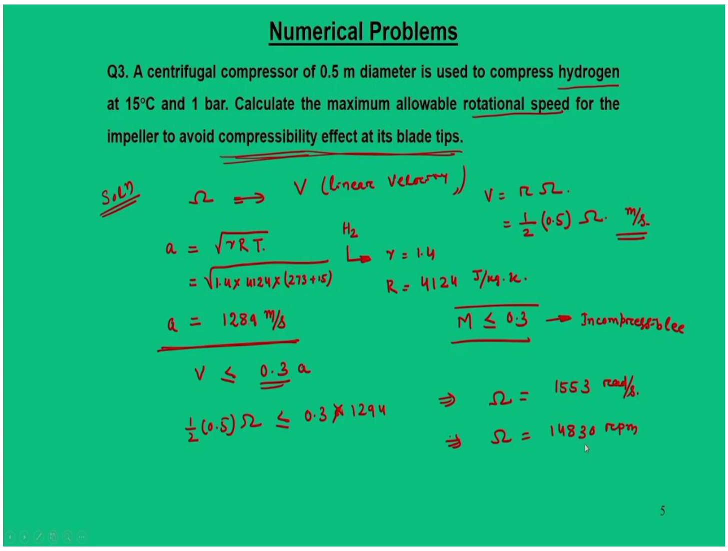

And next important problem we are going to talk about a centrifugal compressor of 0.5 meter diameter is used to compress hydrogen at 15 degrees centigrade and 1 bar. Calculate the maximum allowable rotational speed to avoid the compressibility effect.

So, here the solution we should start from the fact that we are asked to calculate the rotational speed Ω. So, the Ω and this velocity can be related as omega and V that is linear velocity, can be related by the fact that $V = r\Omega$.

So, this will be $V = 0.5(0.5)\Omega$ so this will be in meter per second. But what it is asking that to avoid the compressibility effect at the blade tip, then we have to calculate what is the sonic velocity. Sonic velocity is defined by $\sqrt{\gamma RT}$.

Now, here your medium is hydrogen so for H_2 we say gamma to be again 1.4, but R is something different that from data sheet we can find out that is 4124 J/kg-K. So, by putting this we know γRT and temperature is about 15°C.

So, we can say sonic speed a $=\sqrt{1.4\times 4124\times(273+15)}=1289 m/s$. So, in order to have the velocity at the impeller tip that is $0.5(0.5)\Omega$. If this speed does not have to account the compressibility effect then we must satisfy these conditions that your V should be less than or equal to 0.3 of a.

That means when your Mach number is less than or equal to 0.3, we say that flow is treated to be incompressible; that means, hydrogen the medium the flow will be treated to be incompressible. So, that there will not be any compressibility in the blade tip.

So, the blade tip your velocity is V and your speed of sound is 1289 m/s. In order to have this compressibility effect to be neglected your V should be less than or equal to 0.3 times a. So, if we put it we can turn out to be $0.5(0.5)\Omega \le 0.3\times 1289$

So, this will land off Ω to be about 1553 rad/s. Once you put this number or you can say Ω is about 14830 rpm. So, this number is nothing but the rotational speed that is about 14830 rpm.

So, this example talks about that when a centrifugal compressor is operated in a compressible medium then you can see this rotational speed of this number about 14830 rpm. So, this number is essentially very high and normally all the aircrafts they operate at this rotational speeds. In fact, all conventional aircrafts the rotational speeds will be of this number in the range of 15,000 rpm.

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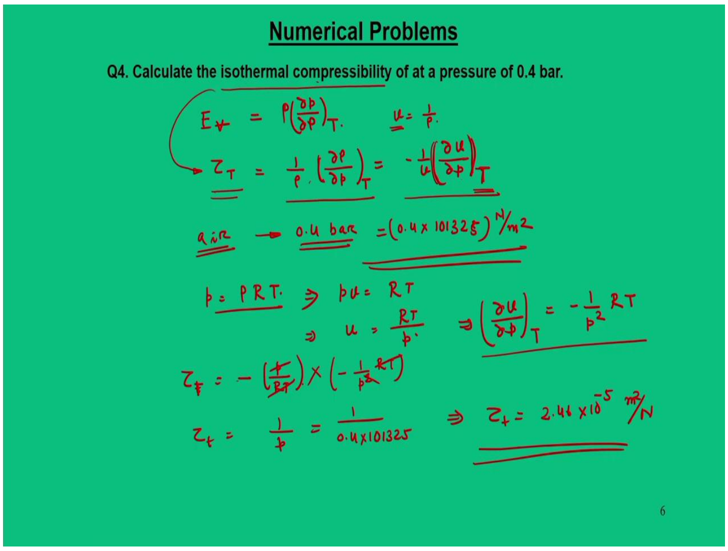

So, the next point again we bring here as the isothermal compressibility at a pressure of 0.4 bar. So, isothermal compressibility when you calculate, if you recall our analysis we define a parameter called as a bulk modulus that is $E_v = \rho\left(\frac{\partial p}{\partial \rho}\right)_T$.

And from this parameter if we put $\upsilon = 1/\rho$ we define a term τ_T that is $\tau_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)$. And when you put rho in terms of υ then we are going to get density in terms of specific volume that will be $-\frac{1}{\upsilon}\left(\frac{\partial \upsilon}{\partial p}\right)_T$.

So, this is the term which is used for isothermal compressibility and represented as a function of pressure and density or specific volume and pressure. So, here what you say $\frac{d\upsilon}{dp}$ in a ordinary differential form, but when I express them as $\left(\frac{\partial \upsilon}{\partial p}\right)_T$, then we are saying that we are defining this compressibility for a isothermal medium that is keeping temperature constant.

So, having said this we can say τ_T comes in this expressions. But what we can say is that if you say air which is at 0.4 bar pressure; that means, we can say 0.4 *101325 N/m^2; so this is the data given. So, we need to calculate the isothermal compressibility for an ideal gas.

So, for an ideal gas we can write $p = \rho RT$. So, when I say ρRT or we can simply write $p\upsilon = RT$. So, from this we can write $p = \frac{RT}{\upsilon}$. So, we require $\frac{\partial \upsilon}{\partial p}$. So, it is better that you would express v in terms of p, $\upsilon = \frac{RT}{p}$. So, this will be $\left(\frac{\partial \upsilon}{\partial p}\right)_T = -\frac{1}{p^2}RT$.

So, we can write $\tau_T = -\frac{1}{\upsilon}\left(\frac{\partial \upsilon}{\partial p}\right)_T = -\frac{p}{RT}\left(-\frac{1}{p^2}RT\right) = \frac{1}{p}$. So, by putting this we can write $\frac{1}{0.4 \times 101325}$. So, $\tau_T = 2.46 \times 10^{-5}$ m^2/N.

So, this expression says that the isothermal compressibility for any gas at a pressure of 0.4 bar is each 2.46×10^{-5} m^2/N.

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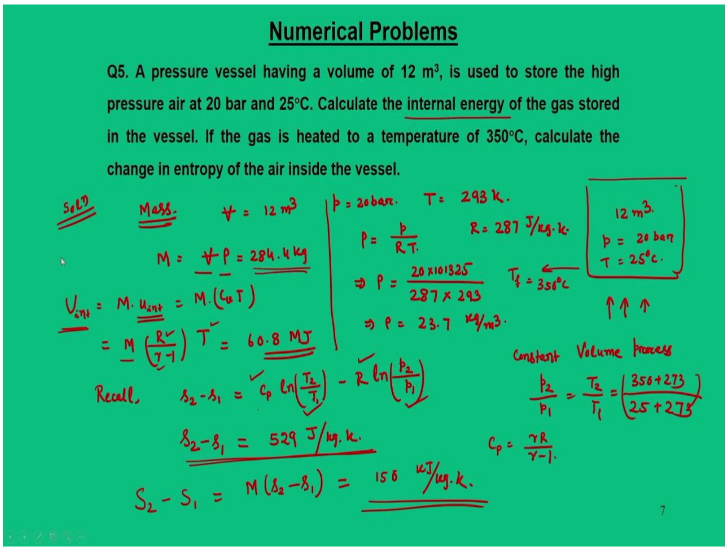

The next problem we are going to discuss about a situation where we have a pressure vessel that has a volume of 12 m^3 is used to store high pressure air at 20 bar and 25ºC. So, you want to calculate the internal energy of the gas that is stored in the vessel that is first part. Second part is that if the gas is to be heated at 350ºC then we have to find out entropy change.

So, this is a very classical thermodynamic problems; where we need to calculate the one of the properties that is internal energy. So, internal energy nothing, but it is we can say it is $C_v T$, total energy and we second part we are going to calculate about the entropy. So, let us solve the problem.

So, basically we have a container here that is that volume has 12 m^3. It has pressure 20 bar temperature is 25ºC. So, it is a closed container and in this container we are also going to add heat so that this final temperature becomes 350ºC. So, let us do part by part in the first part.

So, when I say this entire container so I must calculate what is the mass? Mass of the gas that is in the container. we have given the volume. So, we have given volume as 12 m^3, p as 20 bar and T as 25ºC that is 293K. So, mass we can calculate that is volume times density.

So, how do you calculate density? Density we can calculate $\rho = \frac{p}{RT}$. And since this is air, so we can we can say R is 287 J/kg-K. So, from this we can find out $\rho = \frac{20 \times 101325}{287 \times 293} = 23.7 kg/m^3$. So, we know volume, we know ρ, then you can find out mass. So, this will be 284.4 kg. So, once we know the mass we can find out internal energy.

So, total internal energy $U_{\text{int}} = mu_{\text{int}} = m(C_v T) = m\left(\frac{R}{\gamma - 1}\right)T$. So, when you put all the numbers which are known to us then; we know m, we know R, we know T, we know γ, then we can find out this total internal energy of the entire mass is about 60.8 MJ. So, this part it is now clear, internal energy.

Now if the gas is heated to 350ºC then what is the change in the entropy? So, you have to recall entropy equation that is $s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_2}{p_1}\right)$. Since it is a constant volume process, so we can say $\left(\frac{p_2}{p_1}\right) = \left(\frac{T_2}{T_1}\right)$.

So, we know $\frac{T_2}{T_1} = \frac{350 + 273}{25 + 273}$. So, we know temperature ratio, we know presser ratio, we know C_p, $C_p = \frac{\gamma R}{\gamma - 1}$. So, we know C_p, we know R, so we can find out $s_2 - s_1$ as 529 J/kg-K.

So, remember this is per unit mass, but we have total mass about 284. So, total entropy change that is $S_2 - S_1 = m(s_2 - s_1)$. So, this number would be about 150 kJ/kg-K. So, this is the total entropy change for the mass 284.4 kg. So, this entire problem talks about all types of thermodynamic properties how this is going to be calculate.

So, likewise there are many such problems that can arise in this particular module. So, it is suggested that the candidates should refer some standard problems to brush up the knowledge of fluid mechanics and thermodynamic fundamental aspects. So, with this I conclude this module 1.

Thank you for your attention.