Fundamentals of Compressible Flow
Prof. Niranjan Sahoo
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 08
Lecture - 22
Measurement Diagnostics and Experimental Facilities for Compressible Flow

Welcome to this course Fundamentals of Compressible Flow. We are in the last module that is module 8. The title of this module is Measurement Diagnostics and Experimental Facilities for Compressible Flow. So, this particular module is designed to have some overview on flow measurement methods that are particularly used for compressible flow, this is first part.

Second part is most of the compressible flow phenomena takes place in aerodynamic testing facilities and we will try to give some glimpses about one of the experimental facilities that is shock tube and shock tunnel. Since this course is dealt with the compressible flow, we will focus on those aspects that are mostly relevant for compressible flow category.

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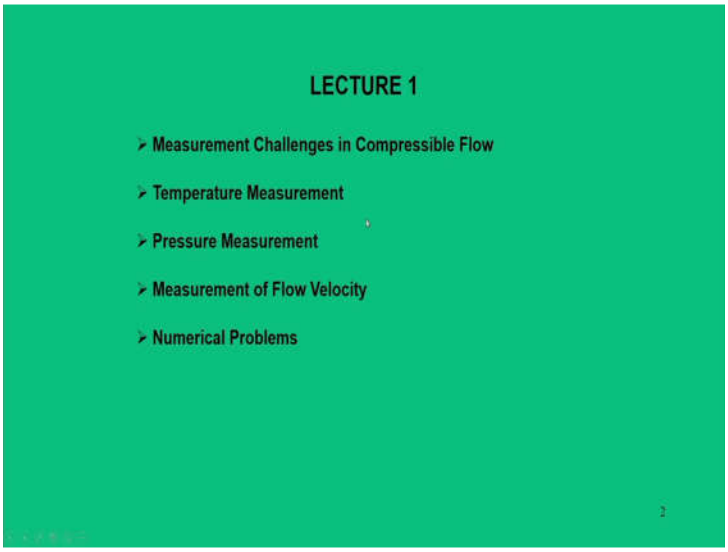

So, in the first lecture of this module, our intention is to do some classical measurement techniques. The first is temperature measurement, pressure measurement then flow velocity. So, these three measurements are very critical to know the flow phenomena, because since the fluid is already compressible and it is ensured that it is moving at very high velocity. In fact, this high velocity flow gives many challenges with respect to measurements.

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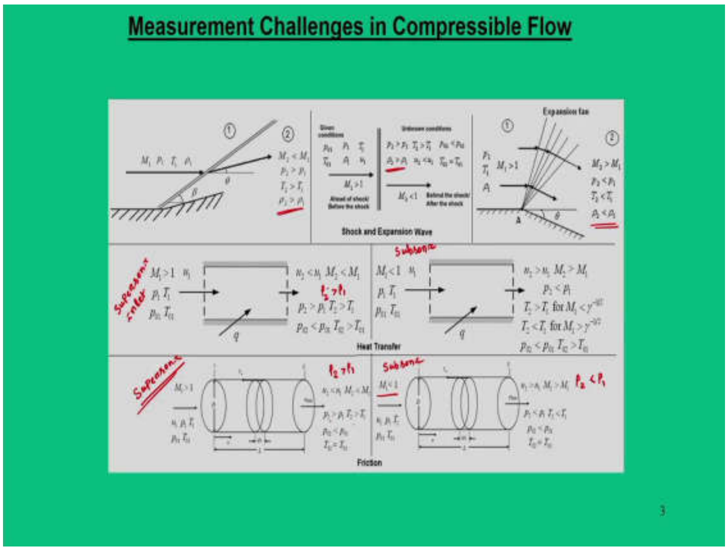

So, the first topic of this today's lecture is, that we will discuss about some measurement challenges in the compressible flow. Since we are already in the last module and prior to this module, we have analyzed various aspects of the compressible flow. We discussed about the shock wave and expansion wave formations. In this category we have oblique shocks, we have expansion waves, we have normal shocks. This is one category the flow property changes across a shock or expansion waves.

In one case, the flow Mach number decreases that is in the shock wave and in expansion wave category, the flow Mach number increases, but the very basic bottom line of the fact, since we are talking about a compressible flow, we say that as and when passes through the shock waves or in the expansion waves, the important factor that changes is the flow density.

In fact, this is very vital with respect to measurement point of view. So, this is how we get from the shock waves. And the other methods of changing the density in the flow field is due to heat transfer; the pressure and temperature increases; so obviously, your density will also increase or this is what we do for supersonic flow inlet.

For subsonic inlet, we have similar phenomenon where pressure drops, temperature may increase or may decrease. So, it all depends on the relative increase or decrease of pressure and temperatures. And in terms of friction, this also leads to increase in the pressure and temperature. So, density will also increase or decrease.

So, if the flow is in this case is subsonic inlet, in this case you have supersonic inlet. So, this is what the summary what we have known so far. And in fact, in all the phenomena, the parameter that changes is the density. This imposes some measurement problem in doing certain measurements in the compressible flow.

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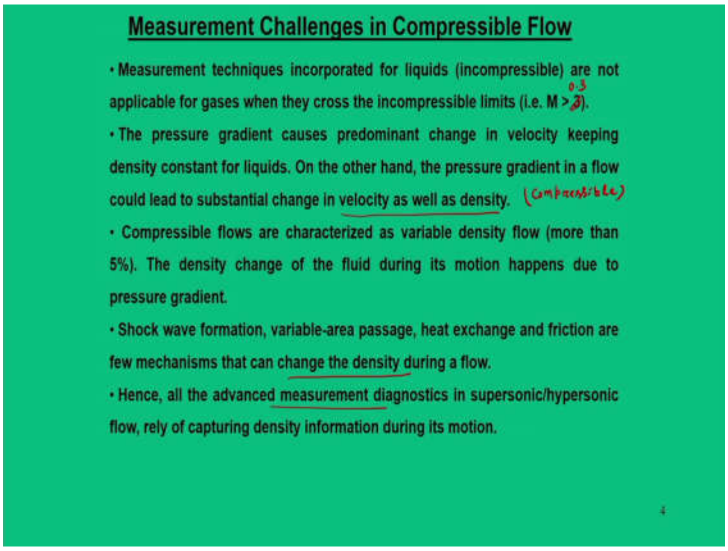

So, just to give some glimpses that, how the measurements in the compressible flow is different from the incompressible flow? So, in case of incompressible flow, which is mainly for liquids, whatever classical techniques that are employed, they are not applicable, because for gases they pose a dual nature that density can change in the flow field. And when the gases change the incompressible flow limit that is Mach number is greater than 0.3, then it gives a indication that flow is approaching towards the compressible nature.

Now, due to this compressible nature, what happens is the pressure gradient change. And when you would say for liquids, this pressure gradient change can only do the change in the velocity by keeping density constant for liquids, but these pressure gradient has dual role in changing the velocity and density for compressible flow. So, when the flow becomes compressible or flow velocity increases, then the pressure gradient cause the change in the velocity as well as the density.

Now, we may say that the compressible flow can be thought of a variable density flow and when we say this density is more than 5 percent, then the flow is approaching towards the compressibility limit. And in fact, when we are approaching the compressibility limit, what are the mechanisms? As I mentioned in the last slide that shockwave formation, variable area passage, heat exchange, friction; these are the mechanisms that can change the density during a flow.

So, looking at this change in the density, many advanced measurement techniques or flow diagnostics in supersonic and hypersonic facilities, they rely on capturing density informations. So, rather in a high speed flow situation, they try to capture the picture with a principle that what are the methods that can change the density in a flow field through this advanced measurement techniques.

So, although you pose its one of the challenges at the same time, this is also one of the beneficial factor that by somehow if one can get the information of density change in a flow field, mainly using some kind of electric discharge methods or some kind of optical techniques. So, all these methods if you can incorporate, then we can take the advantage of capturing the density informations. In fact, all measurement diagnostics in advanced flow phenomena, they rely on this issues.

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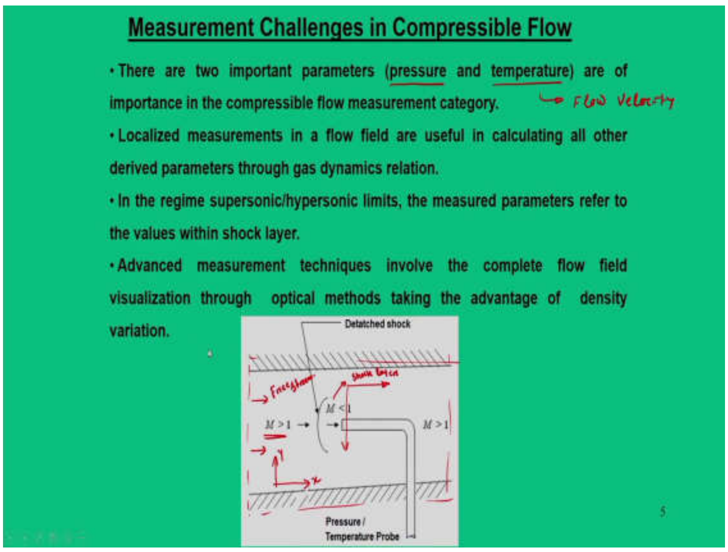

Now, in this lecture, we will talk about the two important parameters mostly pressure and temperature measurements and these pressure and temperature measurement can combinely gives the flow velocity. So, our lecture is mainly rely on the flow velocity measurement, pressure and temperature measurement in a compressible flow situation.

In fact, when you do the pressure and temperature measurements, they are localized measurements. Localized measurement means that in a given flow field as shown in this figure, we can introduce some kind of probe and this probe try to measure either pressure or temperature. And the location of the probe can vary in the lateral direction or in the longitudinal directions, rather I can move this probe in these directions or in these directions.

So, lateral and longitudinal directions movement will give you localized measurements in this flow domain. So, if your domain of the flow is this test section; so, one can play with locating this probe or placing this probe at any locations in the axial or normal direction.

So, this is how the flow information is captured in a localized manner, but one basic problem that arises here that, when we have a subsonic flow; obviously, there will be no shockwave, but when we have a supersonic flow, the measurement device itself will have a shock wave sitting on this surface.

So, this means that whatever we are capturing the information from the flow either pressure or temperature, this number should be the value which is supposed to be at after the shock not before the shock, ok. So, these particular things we call this as a freestream, this is we say within shock layer.

So, this free stream and shock layer, these are the important categories in supersonic and hypersonic limits. But however, the advanced flow measurement techniques involves complete flow field visualization through optical methods by taking advantage of density variations. Here we will not discuss about these methods, so we will talk about only localized measurements.

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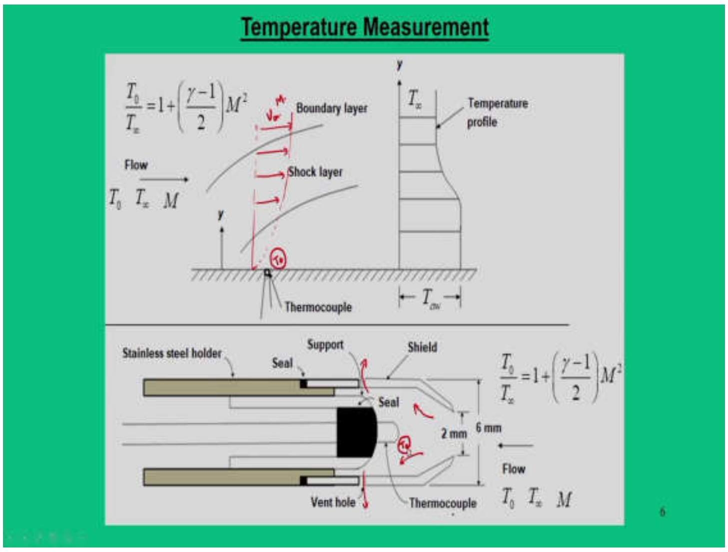

Now, first measurement that we are talking about is the temperature measurements. Now, when you deal with the temperature measurements, there are two ways one can deal like, if you have a free stream flow having free stream temperature T_∞, total temperature T_0 and Mach number M and we are trying to know that, what is the static temperature and total temperature of this flow.

So, there are two ways to measure this information. We can keep our probe or in this case let us say it is a thermocouple; keep fixing it at some locations in the wall. So, when I put this in the wall; so, it means this probe is going to capture the information of T_0 or T_∞.

Now, which temperature it will capture? To look at all those aspects; so, what we may have in the situations, we can say that since there is a very high speed flows or flows are in the compressible nature, we will think of a kind of a boundary layer that is popping up.

So, that boundary layer may grow with respect to wall. Now, when I say there is a boundary layer, we may have a velocity gradient, that is happening in this boundary layer. So, we will have a free stream velocity V_∞ or a Mach number. So, this velocity is 0 at this bottom of the wall and this value changes and it approaches to free stream.

So, this means that when this probe is experiencing a boundary layer which is velocity boundary layer in this case and we will also have a another boundary layer which is the temperature boundary layer, in which the temperature will also vary. Apart from this, since it is a corner; so, there may be shock layer and if the surface is inclined also, the shock layer will be more and more compressive.

So, in this process what we are trying to see is that, we must know that when we are inserting a thermocouple in this flow, our main intention is to capture the information of T_0 or T_∞, but we have to see that these informations are measured within the boundary layer or shock layer. So, this is one approach we do this.

And in other approach people have thought of designing a specific probe, where these thermocouple is a part of it. So, what you do is that, these particular probe consist of the stainless steel holder that holds this probe and we have a ceiling, we have a support system, we have a shielding, we have a vent holes.

So, all these have certain purposes. And what we do is that we try to collect a sample of the flow having T_0 temperature, T_∞ and M in a certain passage which is a diffuser passage and allow this thermocouple experiences this flow. And when it approaches then finally, through this vent holes this the flow gets released.

So, in this process without disturbing nature of the flow we capture the sample, thermocouple gives them indications and this thermocouple through this thermocouple wire and finally, the flow leaves out. So, entry of the flow comes in this way and finally, it leaves through this vent holes.

So, there are some purposes like shields that when you putting a shield, we do not allow any interference from the outer side. Then second thing we have to have certain ceiling. So, that whatever we are measuring from this thermocouple, it should be the measurement from the flow not from any other sources. So, these are the some kind of precautionary measures that needs to be taken.

So, in this process typically, we also experience the total temperature of the flow. In fact, here also, we measure the total temperature of the flow because on this wall your no-slip condition is satisfied. So, we say this measures also the total temperature of the flow.

So, true value is total temperature of the flow we are trying to capture, but in reality we have to see that really we are capturing this total temperature or something else. So, this is the typical design of a temperature probe.

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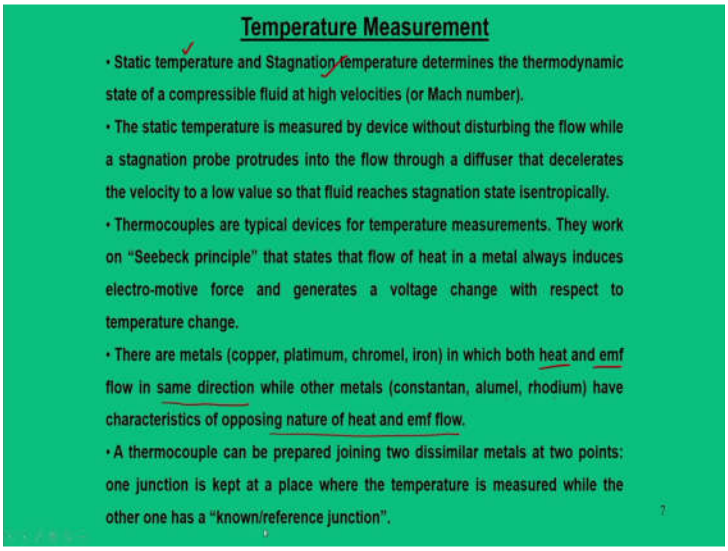

So, now before we go further let us see that what is the most important device that can do our temperature measurements? So, in fact, I told there is two types of temperature we are going to see; static temperature and also we have stagnation temperature and to measure those temperatures we are looking at thermocouples.

So, these are very robust type of devices that are used for temperature measurements. In fact, these are very classical or robust that has been existence from many years and they work on the principle what we call as Seebeck principle that states that flow of heat in metal always induces an electro-motive force and also generates a voltage change with respect to temperature change.

So, through this electro-motive force, a current gets generated and through this current we are going to capture this voltage. I will show how you are going to do that. So, this is one aspect. The next question is that, how do you make this thermocouple?

So, these thermocouple are made of with metals. There are two category of metals such as copper, platinum, chromel, iron these in these metals both heat and emf they flow in the same directions, whereas, in the other metals constantan, alumel and rhodium; they have characteristics of opposing nature of heat and emf flow.

Now, when you do this or prepare a thermocouple, you take the advantage of both the metals, take two dissimilar metals and join them together to form a junction. So, one junction is kept at the temperature where it is to be measured, while the other junction is known as reference junctions.

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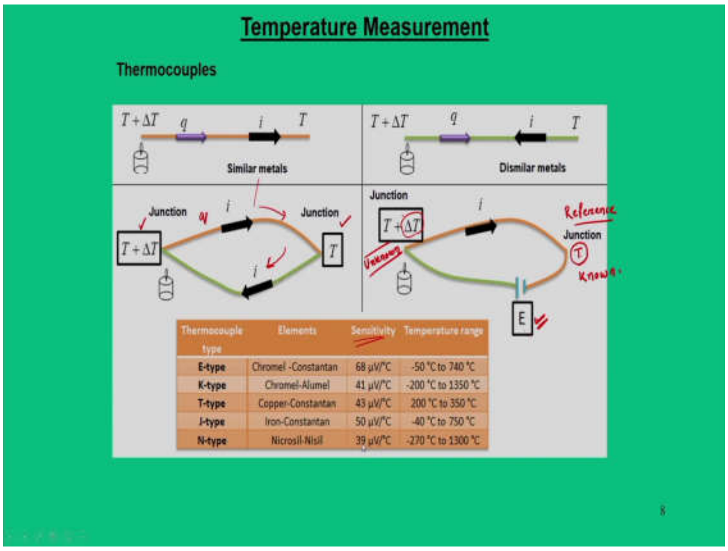

So, this is the typical picture whatever I have explained. First this is for similar metals, whenever there is a temperature difference T + ΔT that appears, a heat is getting generated. And because of this heat flow also there is a current flow in this metal and this is what happens in a similar metals, but same thing when you do in a dissimilar metals.

So, in a dissimilar metals both q and i the current direction they are opposite in nature. So, this is one characteristics, but both of them have potential to measure the temperature change. Then what do you do? Take these two metal materials or metals and join them together. Now, when you join them together, for same value of heat flow q in the circuit; for temperature T + ΔT and T, a current tries to flow in this circuit.

Since, they are in one way, the current flow in the same direction, in other way you have in a opposite directions. So, this is what we sees for similar metal, this is what we see for dissimilar metals for the other thing. Then what you do? Now, how to take the magnitude of this temperature change? Then what you do?

So, whenever there is a current flow in the circuit, try to gather the information about what voltage it will be induced by this current i? So, we say it is a value is E. So, in other words, for a two junctions at different temperature T + ΔT, when they are joined together. So, we say this junction as reference junction, for which the temperature is typically known and for this junction temperature is unknown, but what we get is in terms of this voltage.

So, in other words for a temperature change of ΔT, a voltage gets generated. This is how the fundamental principle of a thermocouple and it works on the Seebeck principle. So, now based on this category, there are many types of thermocouples.

There are many, these are the scientific names. We call them as E-type, K-type, J-type, N-type; these are the commercial names and scientific names there Chromel-Constantan, Chromel-Alumel, Copper-Constantan and all these things.

And each of these thermocouple depending on the type of material, the temperature range is also specified, but what very basic philosophy is that the word sensitivity here. So, if you look at this magnitude of the sensitivity talks about that a for a one degree of change in the temperature, we can get a about 39 microvolt voltage for a thermocouple of Nicrosil and Nisil. So, likewise every thermocouple needs to be calibrated to know its sensitivity value.

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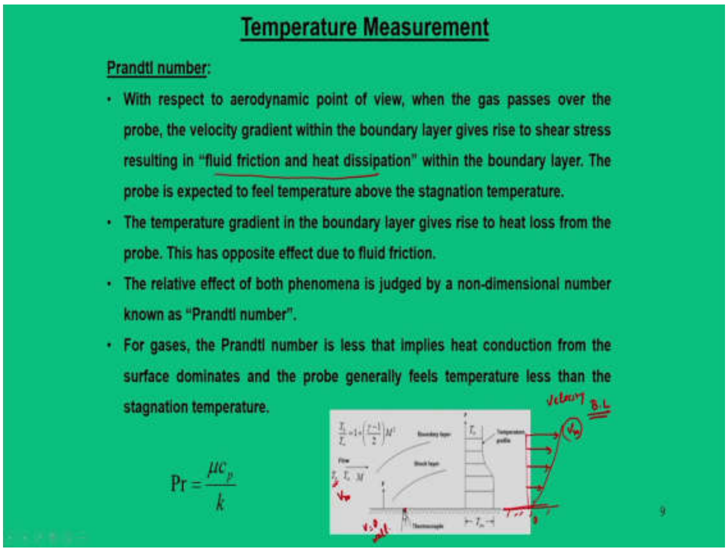

So, this is how the thermocouple becomes a one of the important device to measure the temperatures. Now, let us see that we will go to some aerodynamic view point of this temperature measurement and we will see here, what is the role of Prandtl number in this temperature measurement.

So, if you look at the expression of the Prandtl number, $\mathrm{Pr} = \frac{\mu c_p}{k}$. So, μ is the viscosity of the flow and C_p and k these are thermal parameters. So, it talks about the relative weightage of the viscous effect and the heat conduction effect that is k. Now, where is this term comes in?

So, if you see here in this figure, as I said that we are measuring the temperature in a boundary layer or shock layer domain. So, there will be velocity profile and the velocity is 0 on the wall and in max it approaches the free stream and while making this measurement in this probe, we are seeing the effect of this velocity boundary layer.

Now, what is the effect of velocity boundary layer? This effect of velocity boundary layer it causes fluid friction and heat dissipation. This fluid friction and heat dissipation caused by this velocity boundary layer, it increases the temperature within the boundary layer. So, we are supposed to see first thing is that, your velocity is 0 at the wall. So, if it is 0, we are supposed to measure total temperature T_0.

But, due to this velocity boundary layer and the fluid friction and heat dissipation, the temperature within the boundary layer increases. So, this thermocouple is expected to feel a temperature which is above stagnation temperature, this is one part. And this is happening is due to fluid viscosity μ in this expression. But the other side of the story is that, if you look at the temperature boundary layer, what happens if you can see that the wall has certain temperature and the free stream had has another temperature.

Now, due to this viscous effect the wall temperature suits up. So, why this wall temperature suppose to increase? Because when this increases; obviously, the heat will be taken from the wall. So, this will give a opposing effect to the fluid friction; that means, the temperature gradient in the boundary layer gives rise to heat loss from the probe. So, there is a relative effect between these two phenomena and their relative effect is judged through a non dimensional number known as Prandtl number.

Now, what happens if Prandtl number is high or low? It has been seen that for gases, the Prandtl number is less which implies that heat conduction from the surface dominates; that means, here the k mostly dominates. So, when the k mostly dominates. So, the probe feels a temperature less than the stagnation temperatures.

So, in other words, the final bottom line of this thing although we have a total temperature of T_0 and we are trying to get the information about this T_0through this thermocouple, but it is expected that due to these two important effects our will be expecting a temperature which is less than this T_0. That means I cannot approach to T_0. I will be measuring little bit of lesser than what is supposed to be for the flow.

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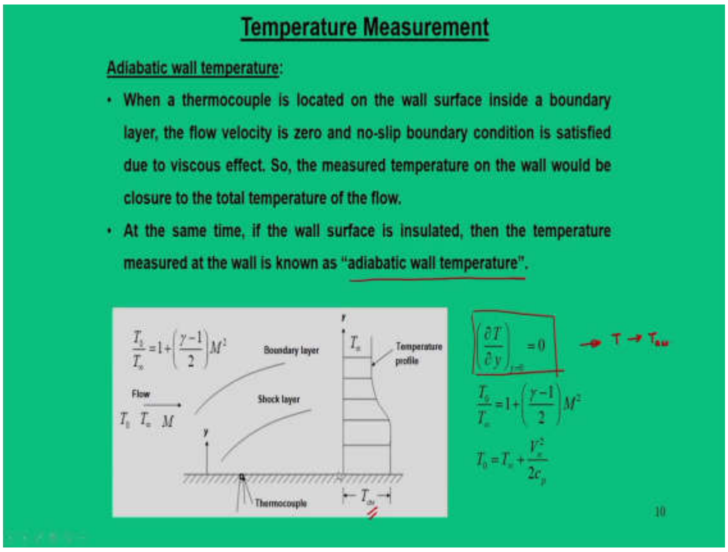

Now, to quantify this, what we have defined that, let us give a definition that what temperature we are trying to measure, that temperature we call this as a adiabatic wall temperature. So, it means that when a thermocouple is located on the wall surface inside the boundary layer, the flow velocity is zero and no-slip condition is satisfied. So, the measured temperature on the wall would be the total temperature of the flow, but at the same time if we keep this wall insulated, the temperature which is measured is known as adiabatic wall temperature.

So, $\left(\frac{\partial T}{\partial y}\right)_{y=0} = 0$ says that, this particular relations gets satisfied on the wall, that is at y is equal to 0, that is $\frac{\partial T}{\partial y} = 0$, that means, we define this temperature on the wall as T_{aw}, T becomes T_{aw}. So, in other words, if you keep everything perfectly insulated wall and keep the thermocouple to this wall; so, at best what measurement we can make is to capture the T_{aw}. Let us see that, what is the deviation within the T_{aw} and T_0.

So, it means that as a measurement point of view, I can design a thermocouple which can give us the information about T_{aw} for a well insulated wall, but whether there is a deviation between this T_{aw} and T_0.

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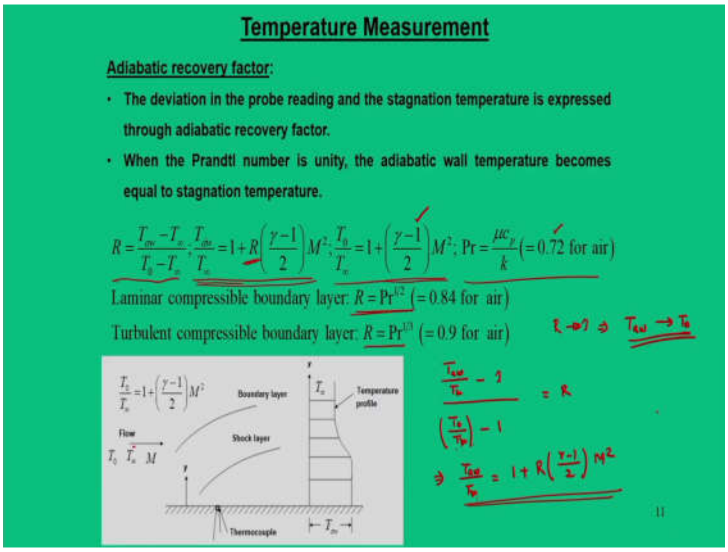

To quantify this parameter, we define a term which we call this as a adiabatic recovery factor. In other words what it says is that, how far we are approaching to the total temperature. So, a relative term that is ratio is defined, that is $R=\frac{T_{aw}-T_\infty}{T_0-T_\infty}$.

So, this says that the, if T_{aw} is equal to T_0, then your R will be 1; so, that means, I am getting closer to T_{aw}; that means, your measurement is quite accurate of the total temperatures. So, relation that can be generated that is $\frac{T_{aw}}{T_\infty}=1+R\left(\frac{\gamma-1}{2}\right)M^2$. So, from this expression what I can say that $\frac{\frac{T_{aw}}{T_\infty}-1}{\frac{T_0}{T_\infty}-1}=R$.

Now, this T_0 - T_∞ is related in this manner. So, when I substitute this here, then I can get $\frac{T_{aw}}{T_\infty}=1+R\left(\frac{\gamma-1}{2}\right)M^2$. So, just substitute this value here and simplify you will arrive at this. So, now, you look closely these two expressions, which is this is this $\frac{T_0}{T_\infty}=1+\left(\frac{\gamma-1}{2}\right)M^2$ we get from this isentropic relations it relates total temperature, static temperature and Mach number.

Now, in a similar lesson, we are going to get here; T_{aw} is your adiabatic wall temperature and T_∞ and Mach number. So, by controlling this parameter R, one can approach when R is equal to 1, this will implies T_{aw} goes to T_0.

This is what has been derived here, but as I said that this R is a is due to two effect and one is the viscous dissipation and with other is due to velocity boundary layer and other is the temperature gradient and that is related through a Prandtl number.

And when you are doing the measurements for air, we take this Prandtl number for air is 0.72 and it has been found that for a laminar compressible boundary layer, the value of $R = \Pr^{1/2}$ and for a turbulent compressible boundary layer $R = \Pr^{1/3}$. So, these are the standard numbers that can be routinely used for temperature measurement.

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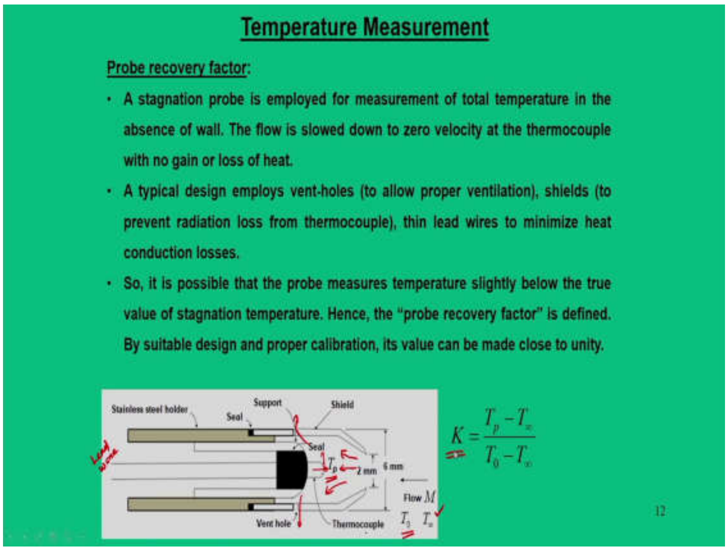

Now, this is one aspect when we do the measurement by putting our probe on the wall. So, as I also mention there are another devices or another type of design in which we do not use this probe on the wall or maybe some certain situation it is not possible to fix the probe on the wall. So, rather we capture the flow in a passage where our temperature sensor is placed.

So, typical design of this, I mentioned there is a thermocouple, it has two lead wires that measures the voltage due to this temperature change between T_∞ and the measurement point and what we see is that the flow is allowed to enter in this passage and it leaves in this passage. So, in this manner the probe experiences the supersonic flow. Now, let us see that what the probe sees then.

Now, this probe sees a temperature what we call as T_p, but the actual total temperature is T_0. Now, whether we are getting this T_p or T_0 same or not, that can be ensured that since the location of measurement and this flow, they are perpendicular to each other; that means, probe is facing the flow directly.

So, the probe is facing the flow directly. So, means that we are supposed to get a stagnation point at the meeting point on the sensing surface of the probe. So, this temperature is measured as T_p not necessarily that this temperature would be equal to T_0.

So, this all depends that how we designed this probe holder; that means how you have designed the seals, how you have designed this support structure, and how the flow that encounters the probe. So, all such things makes a slight deviation in the temperatures.

So, here the typical design have vent holes that allows proper ventilations, the design may have some seals to prevent radiation from the thermocouple, the design may have thin lead wires to minimize heat conduction losses.

So, this possible things could land of having a temperature measurement which is different from the total temperature. So, based on that we define a parameter what is called as probe recovery factor K. So, this is also defined in a similar manner, that we say it is $\frac{T_p - T_\infty}{T_0 - T_\infty}$. This is also defined in line with the adiabatic wall temperature.

But difference is that here we will not have any boundary layer or here we will not have any velocity boundary layer or temperature boundary layer and no effects is there and only thing is that the flow is allowed to come to rest isentropically at this point and whether we are really ensuring the isentropic situation or not, that is defined by the factor probe recovery factor.

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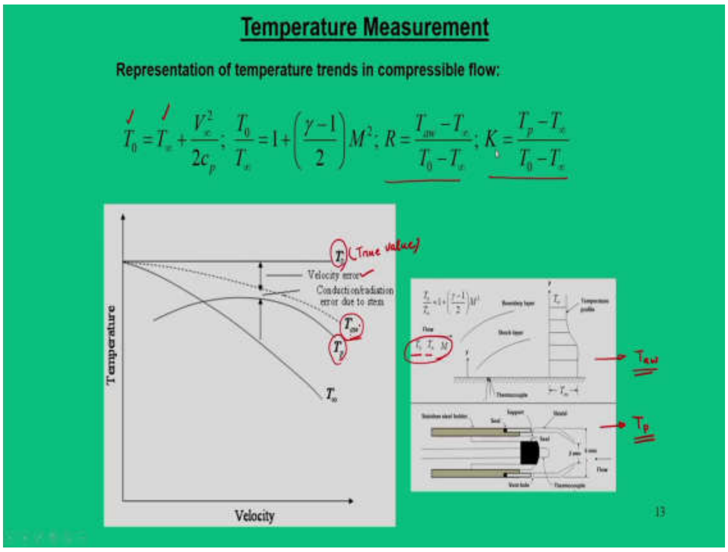

So, we have said that, for a given flow problem, our main intention is to go for total temperature and static temperature for a given Mach number and for this situation, we are doing two types of design, in one design we are getting a information about T_{aw}, that is mounting the thermocouple on the wall, in other design we are bringing the flow to rest, we are getting the information of T_p.

So, based on these, we say we have defined an adiabatic recovery factor and probe recovery factor. But your true value of temperature is T_0, that is total temperature and static temperature is T_∞. So, if you just plot a some kind of trend it may look like that, this is the true value of T_0, irrespective whether it is a shock wave or not, the total temperature remains same, but the static temperature may change.

So, total temperature line remains constant, but static temperature line may change. If it is a shock wave, it also have a same number or the way the density changes in the flow field. Now, through this we will have another number, if you measure by this method, we will land of our measurement in the range which is we call this as a T_{aw}, but if you do in this method, we may land of another type value T_p.

So, the relative judgment one can make that through both the approaches one can find an error and when you do this the error between T_0 and T_{aw} is termed as velocity error. And, the error between T_0 and T_p that is due to conduction and radiation error due to stem; that means, we are unable to look into these effect, the losses due to conduction and radiation in this methods.

So, likewise a suitable design of a probe will help that how far we are approaching this T_p value with respect to its true value T_0. Typically a number of more than 0.9 or so is ideal for temperature measurements; that means, recovery factor should be as close as to its true value.

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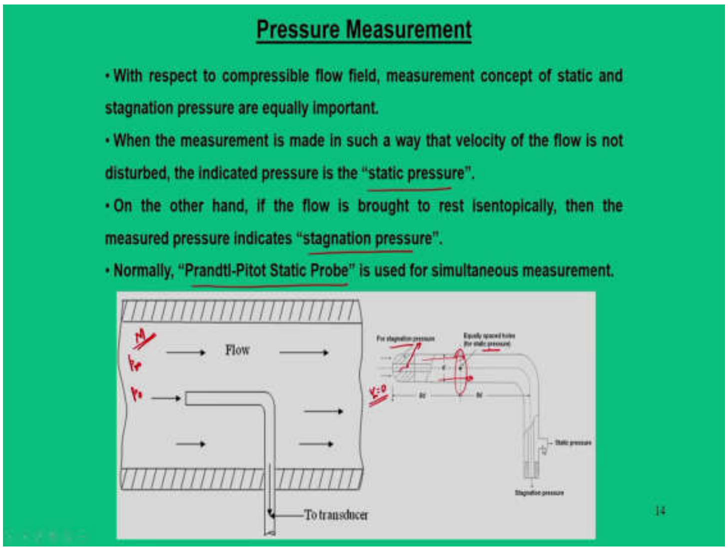

Now, moving further in the similar to temperature measurements, we also can think of pressure measurements. But, here in the pressure measurement, there are two types of pressure, one we can say static pressure, other may be will be the stagnation pressure.

So, the flow may have p_∞ and p_0 for Mach number M. But while doing we can follow the similar strategy for pressure measurement as well, but a classical approach is that you design a Prandtl-Pitot Static Probe. What it does is that, this is similar to a stagnation temperature probe, but here the design is little bit of different. So, here what you do have is that a Prandtl Pitot static tube have provision for static pressure and stagnation measurement simultaneously.

So, at the nose part of this probe the flow is allowed to come to rest. So, for which this your V_∞ is equal to 0. So, the sensor which is mounted at this location, this measures the stagnation pressure, but on the circumference, there are multiple static holes and on the circumference some flow is allowed to pass through continuously on this sides wall and through this we get the information of static pressure measurements.

So, finally, what you see here is that, the lead wires for stagnation pressure probe and lead wires for the static pressure probe. So, in this way both static and stagnation pressures are measured simultaneously.

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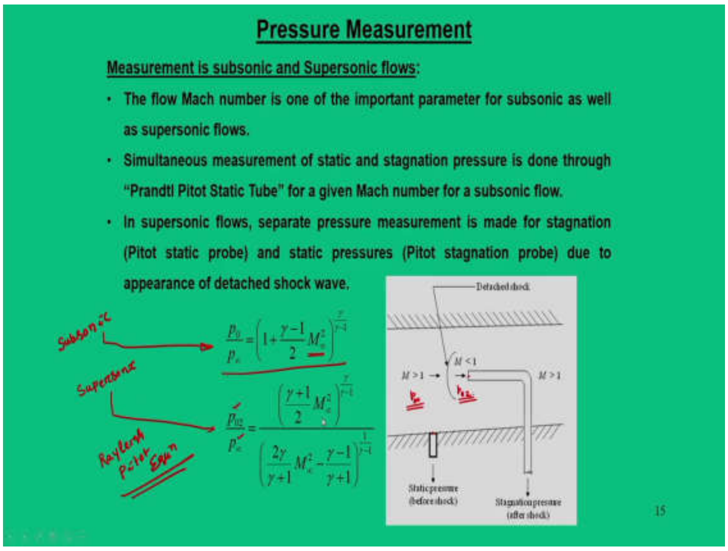

And there are two types of situation that may have, one may be subsonic flow. So, when we have a subsonic flow, the static pressure and stagnation pressures are related with respect to Mach number. Now, when we have a supersonic flow; that means, we measure the static pressure measurement or we can directly use the Prandtl Pitot tube to measure both the information and find out the Mach number, other ways that you do a measurement in a supersonic flow.

So, this we say it is a supersonic and the relation between the total pressure and the static pressure is defined across the shock wave which is supposed to have. So, in one situation, when you are trying to capture the information of the pressure, the measurement is typically done after the shock. So, we say that whatever value this pressure probe gives, it will give the total pressure after the shock.

So, it is p_{02}, but before that we have free steam pressure p_∞ and this particular $\frac{p_{02}}{p_\infty}$ is related to Mach number and in fact we already derived in the one of their calculations we called as Rayleigh Pitot equation.

In fact, if you refer the normal shock table, this particular ratio is mentioned there, directly getting ratio from here will lead to the calculation of Mach number.

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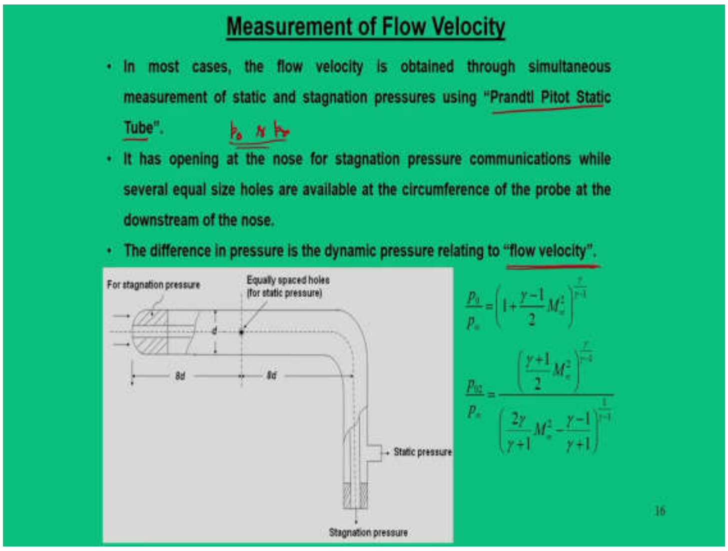

This is how we talked about pressure measurement and this static pressure measurement can give you the information about flow velocity. So, the Prandtl-Pitot static tube gives p_0 and p_∞. Difference between these p_0 and p_∞ can be related to flow velocity.

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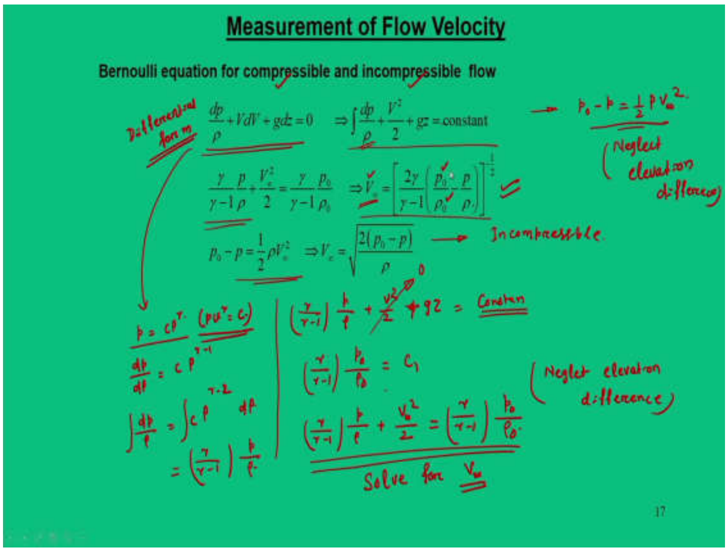

So, how we are going to see here, let us talk about the fundamental equations that, in a compressible flow how this velocity is related to total pressure and density.

So, to do that let us recall this Bernoulli's equation in differential form. So, here while talking about this Bernoulli's equation, we can say this can be applicable for compressible as well as incompressible flow, but how this main equation is going to change for compressible flow or incompressible flow. So, one is a differential form then we can do the integration. So, we say this is the one of the form of Bernoulli's equations.

$$\frac{dp}{\rho} + VdV + gdz = 0 \quad \Rightarrow \int \frac{dp}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

Now, from this equations, when you put $p = C\rho^{\gamma}$, that is for isentropic flows $p\upsilon^{\gamma} = C$, and try to get $\frac{dp}{d\rho} = C\rho^{\gamma-1}$, then you require $\int \frac{dp}{\rho}$.

So, for that you rewrite this expression as $\frac{dp}{\rho} = C\rho^{\gamma-2} d\rho$. Now, if you take the integral, then we will land of this particular expression. So, if you take the integral, this will give you a value $\frac{\gamma}{\gamma-1}\frac{p}{\rho}$.

Now, when you substitute this equation here, the expression now becomes $\frac{\gamma}{\gamma-1}\frac{p}{\rho}+\frac{V^2}{2}+gz=\text{constant}$. Now, to find out this constant, we say that you approach this V tends to 0; that means, when V tends to 0, the pressure will be p_0, then we can say $\frac{\gamma}{\gamma-1}\frac{p_0}{\rho_0}$ is this constant.

Now, this constant when you rewrite this equation again. So, what we can write that $\frac{\gamma}{\gamma-1}\frac{p}{\rho}+\frac{V_\infty^2}{2}=\frac{\gamma}{\gamma-1}\frac{p_0}{\rho_0}$. So, here we have neglected elevation difference. Now, once you do this we get this expression. Now, what you do? Solve for V_∞. To get when you solve for V infinity you will arrive at this expression.

$$V_\infty=\left[\frac{2\gamma}{\gamma-1}\left(\frac{p_0}{\rho_0}-\frac{p}{\rho}\right)\right]^{-\frac{1}{2}}$$

So, in this expression what you see that V_∞ has the information about the stagnation pressure, stagnation density, static pressure and static density. So, in other words the information of stagnation pressure and static pressure with the knowledge of these density value can lead to the velocity information.

But if the flow would have been incompressible, then this density would have been kept constant in this Bernoulli's equation. So, this equation would have been $p_0-p=\frac{1}{2}\rho V_\infty^2$ which is here and here of course, we neglect elevation. So, this philosophy of doing this indirect measure of velocity is done through pressure measurements.

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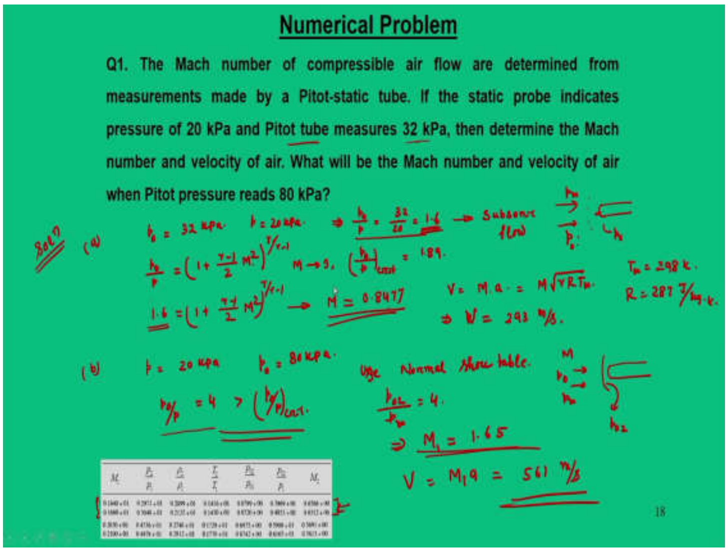

So, this is all about this for this lecture today, but before you close let us try to solve some numerical problems which we pertained in this particular lecture. And these problems are based on the pressure measurements and temperature measurements of course, flow velocity measurements. The first question gives that Mach number of a compressible flow air is determined from pitot-static tube.

The static probe indicates that 20kpa, pitot tube measures 32kpa; determine the Mach number and the velocity of the air. So, here we know we have two information, one is total pressure p_0, it is done through pitot tube that is 32 kPa and static pressure p it is 20 kPa. Now, we can find out this p_0/ p ratio that is 32/ 20 that is 1.6.

Now, before you proceed further we have to see whether the flow is sonic or not, because that will tell you whether these measurements involves after the shock wave or not; that means, when I put this probe whether there is a shock wave or not, to ensure that we find the critical value of pressure.

So, in the first relation of isentropic relation we can write $\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$, when M goes to 1, this $\frac{p_0}{p}$ becomes critical value, that becomes 1.89, but our case is 1.6. So, that means, flow is subsonic; so that means, this pressure ratio is for subsonic flow, means there is no shock waves.

So, rather we say it is a p_∞ and this pressure will be p_0. When I say it is a subsonic flow and flow entire flow field isentropic, then one can rewrite this value as $1.6 = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$, this will say that our real Mach number of the flow is 0.8477.

So, when I say Mach number is this, I can say velocity is Mach number into speed of sum; that is $V = M\sqrt{\gamma RT}$. So, take this T_∞ as 298 K, R is equal to 287 J/kgK. So, this will give you velocity as 293m/s. This is one part.

Now, second part of the problem says that, what will be the Mach number and velocity of air, if the Pitot pressure reads 80 kpa; that means, we have p is 20 kPa and p_0 now here is 80 kpa. When you take this ratio $\frac{p_0}{p}$ is equal to 4 that is greater than $\frac{p_0}{p}$ critical.

So, obviously, this probe will have a bow shock sitting on this, where we have M, p_0, p_∞ and here we will have p_{02}. Then, since for this pressure ratio, we will have a bow shock. So, you use the normal shock table. When you use a normal shock table for $\frac{p_0}{p}$ is equal to 4 p here will be something $\frac{p_{02}}{p_\infty}$ to be 4 and closely it is in this range. So, this will gives roughly Mach number as 1.65.

So, when I know Mach number of 1.65. So, I can say V is equal to M times a; for same value of a, the velocity becomes 561m/s. So, this is the way we measure the static pressure, we measure the Mach number and velocity from the pitot readings.

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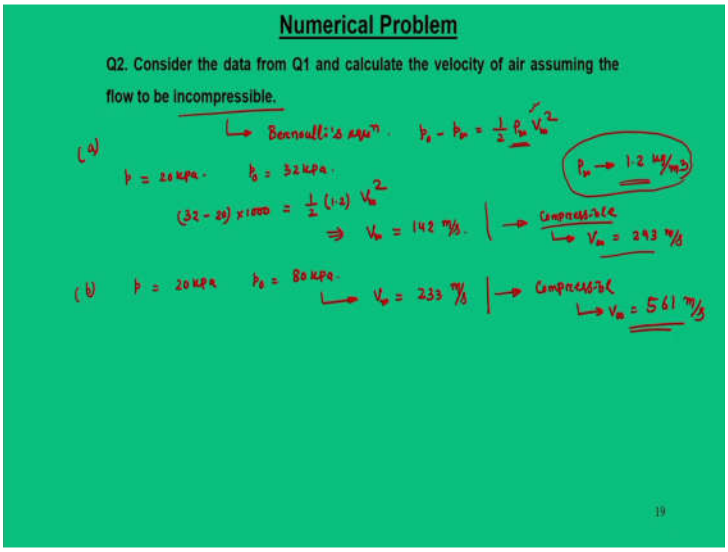

Now, second problem says that, now take the data for question 1, calculate the velocity of air assuming the flow to be incompressible. So, here we are treating that, if you use this incompressible flow assumption what would have been your Mach number and velocity? So, let us take the first case. So, for incompressible flow, we can write Bernoulli's equation, which says that $p_0 - p_\infty = \frac{1}{2}\rho_\infty V_\infty^2$.

So, our role is to find V_∞. So, what we require? ρ_∞; ρ_∞ for standard air is 1.2 kg/m^3. So, assuming the flow to be incompressible density of air is treated as 1.2 kg/m^3. Now, in the first case our static probe reads 20 kpa and total pressure reads 32 kpa. So, I can say $(32-20)\times 1000 = 0.5(1.2)V_\infty^2$

So, this will give you V_∞ value as 142m/s. If the flow would have been compressible flow, in the last problem we found these V_∞ value as 293m/s.

So, that means, your Bernoulli's equation for incompressible calculation under predicts, when the gas becomes compressible. So, it is about 142m/s when it is incompressible calculation and it becomes 293 when it becomes compressible calculations. In fact, this will also have similar situation for the second case, where static pressure is 20kpa and total pressure is 80kpa.

So, this will also give V_∞ value incompressible calculation as 233m/s and same value for compressible, this value becomes to 561m/s. We can see that how deviation can happen if you neglect the compressible nature of the gas or air.

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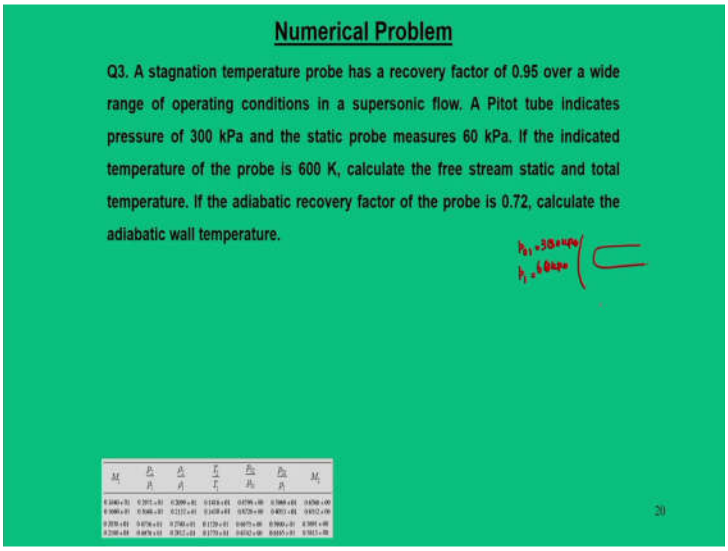

So, third problem we are going to talk about a temperature probe. We have a stagnation temperature probe of recovery factor of 0.95 over wide range of operating conditions in a supersonic flow and it is used for temperature measurements, but side by side we also measure the pitot tube that measures pressure 300 kPa which is stagnation and a static probe it measure 60 kPa which is static pressure.

If the indicated temperature of the probe is 600 K, calculate the free stream static and total temperature. If the adiabatic recovery factor of the probe is 0.72, calculate the adiabatic wall temperatures.

So, the very basic philosophy of the problem that we have a temperature probe, this temperature probe is used in a supersonic flow; obviously, we will have a some shockwave, since it is a supersonic flow and the total pressure is 300 kPa, static pressure is 60 kPa. Total pressure that is measured by the probe at this point will be p_{02} that is 300 kPa.

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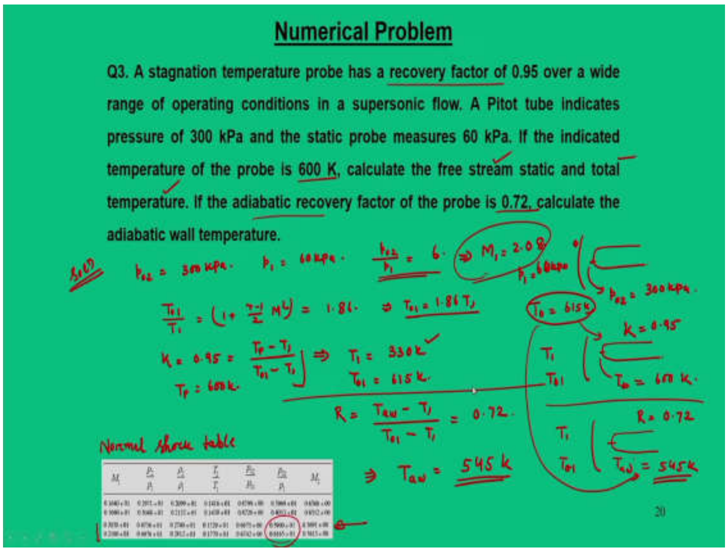

Now, in the same flow you are going to make the temperature measurements. So, the temperature measurement in the first one, we are talking about a probe recovery factor. So, we talk here about T_p and here you talk about T_1 and T_0.

In the second case we talk about adiabatic recovery factor. So, here you talk about T_{aw} and we say T_1 and T_0 and we say R adiabatic recovery factor of 0.72 and here we say K is equal to 0.95. This is the summary of the problem and to address this problem first we have to find out from this pressure data, what is the Mach number of the flow?

So, we start the solution saying that p_{02} as 300 kPa, p_1 is 60 kPa, calculate $\frac{p_{02}}{p_1}$ that happens to be 6. Now, for this pressure ratio, what will be Mach number? So, we refer normal shock table for this $\frac{p_{02}}{p_1}$ ratio close to 6, one can find out this M_1. So, this will be M_1 2.08. Now, we say we can find out $\frac{T_{01}}{T_1} = \left(1 + \frac{\gamma - 1}{2} M^2\right)$, this ratio becomes 1.86.

So, the first problem gives K, this means $T_{01} = 1.86 T_1$. So, we say K is equal to 0.95. So, by definition $K = \frac{T_p - T_1}{T_0 - T_1}$. So, we know $T_{01} - T_1$. So, this will give you and T_p say 600 K. So, from this we can get T_1 as 330 K, T_{01} becomes 1.86 time that, so, 615 K.

So, we get free stream static temperature and total temperature. Now, moving to the second problems for which we define R that is same probe is used and another aspect, where we try to capture adiabatic wall temperature, we say $R = \frac{T_{aw} - T_1}{T_{01} - T_1}$, that number is 0.72.

So, we all know that true value of T_{01}. So, this will give T_{aw} as by putting this value T_1 as 330, T_{01} as 615, we can say T_{aw} is 545 K. So, as we can say your true value is 615 K, but through this measurements we get T_p as 600 K. So, we seems to be more accurate and through this approach we can achieve T_{aw} as 545 K.

So, that means, the a static probe of this sorts, where we try to make the probe in a very well designed manner that can capture the total temperature close to the main flow. So, this is how the typical temperature probe works in a supersonic flow field. Now, with this I conclude this lecture for today.

Thank you for your attention.