Fundamentals of Compressible Flow
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Module – 07
Lecture – 21
Compressible Flow with Friction and Heat Transfer

Welcome to this course Fundamental of Compressible Flow. We are in module 7 that is Compressible Flow with Friction and Heat Transfer. In the last two lectures of this module, we discussed about a one dimensional flow in a compressible medium with heat addition process.

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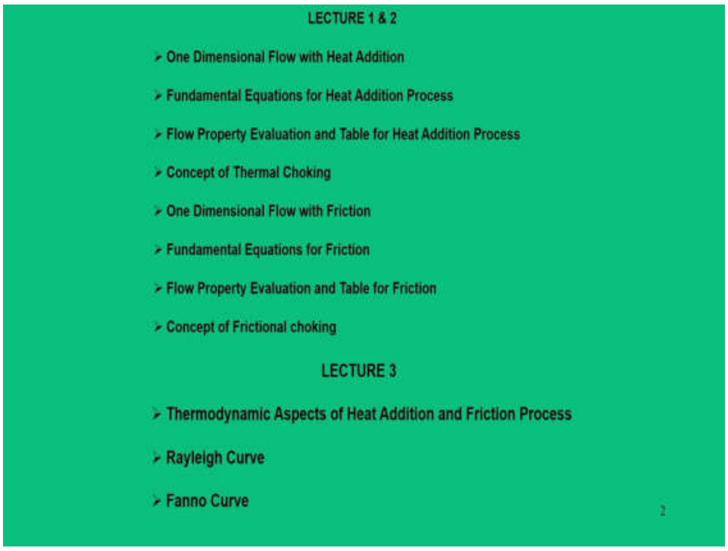

Subsequently, we derived the fundamental equations, then we made this equations to be available for flow property evaluations; based on those equations a table is formed which is called as one dimensional table for heat addition process and while dealing with all these heat transfer process, we introduce a concept of thermal choking that is bringing a compressible flow to a sonic state at Mach number of 1 through heat addition or heat rejection.

Then in a similar aspects, we dealt with the one dimensional flow with friction. So, with friction it means that to increase this friction we have to increase the length also. Now, in a compressible medium when it flows in a duct if I add length to this duct then I can increase the friction and vice versa.

So, in those cases we also found out that what are the fundamental equations that governs that flow path. Based on this equations, we also evaluated the flow property and thereby, we introduce the gas table for friction. Having said this friction, there is another concept which is called as frictional choking. In this process also the compressible flow is made to a sonic state by adding length to the duct or subtracting length from the duct.

So, in all this process we derived all the fundamental equations and in this particular lecture we will talk about some of the important interferences in terms of thermodynamic aspects. In fact, this is one of the important area where in which the gas dynamics course is different from a conventional fluid mechanics course.

So, while dealing with the compressible flow, although it is a fluid related topic; but the thermodynamic approach is also vital to equally understand the flow phenomena. Now, for this heat addition and friction process which is the main focus of this lecture and we will try to give the thermodynamic definitions for this process and those are known as Rayleigh curve and Fanno curve. This Rayleigh curve is for heat addition process and this Fanno curve is for friction process.

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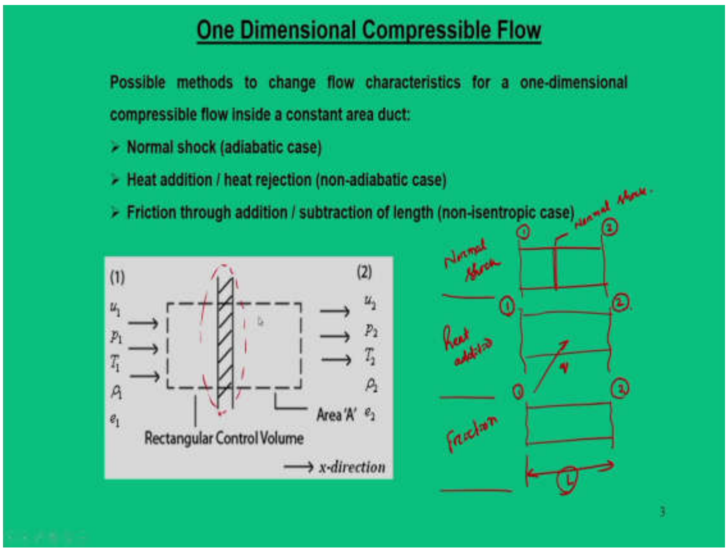

So, in the beginning of this one dimensional flow analysis, we introduced a constant area duct and we said there are three possible ways in which the flow properties from upstream and downstream conditions can be altered.

So, in this first approach it is a normal shock and in fact, it is a adiabatic duct. So, when you say normal shock, in this constant area duct, we simply say that you make the flow conditions in such a way that one normal shock will form and across which these properties going to change.

So, these particular thin region in which your attention is focused; in this first case it is a normal shock and in the second case in this control volume we say that we are going to add heat. So, thereby the conditions upstream and downstream conditions can be altered.

So, in the third case same one dimensional duct region 1 and 2; so, we say that length of the duct can also change the flow phenomena. So, here the driving factor by this length means it is the friction. So, these are the three broad areas in which we talked about compressible flow property change through a normal shock, heat addition, and friction and the very basic fundamental figure is shown here, it is a constant area duct.

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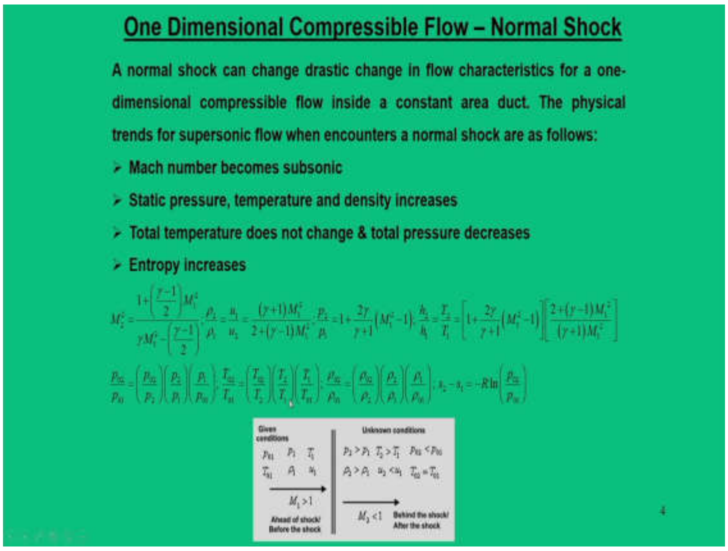

Now, let us talk about something more detail on what happens in a normal shock, as I mentioned and in fact, we have derived this normal shock relations much earlier. So, the very basic summary that we get out of this normal shock relation is that for given conditions or known conditions if you want to correlate the information between unknown condition and known condition then one can frame the fundamental equations as given in the slides.

So, through this equations one can say that the across a normal shock the flow becomes subsonic, static pressure and temperature density, all these numbers will rise.

The total temperature does not change, but the total pressure decreases and entropy increases. So, here the entropy is increased mainly due to the drop in the total pressure. So, all these summary references are shown here and of course, this is also an non isentropic process and this drop in total pressure causes this entropy to increase.

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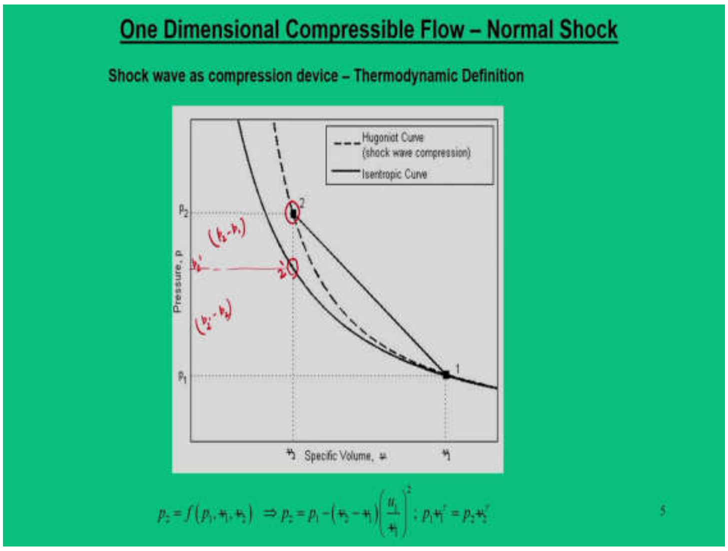

And based on this particular fact, we tried to map the difference between a isentropic curve and shock based compressions. As I mentioned the shock based compression is a known non isentropic phenomena and it is governed by a curve, which is known as Hugoniot curve and this Hugoniot curve gives an indication that it is more efficient than isentropic curve.

So, that tells the information that the shock based compressions is more efficient than isentropic curve. As you see that when you are moving from a point 1 to 2, as and when the volume specific volume decreases, the Hugoniot curve climbs above the isentropic curve.

So, as and when we increase further, the difference becomes higher. So, in other words for given a specific volume if I take two points 2` and 2 then that means, if I come from isentropic curve, then I should have got the pressure difference as $p_2' - p_2$. So, this much compression would have been achieved.

If, I have to come through a shock based compression, then I would have got $p_2 - p_1$; obviously, this number is higher than that of isentropic curve. So, this makes a conclusion that a shock based compression is more efficient than Hugoniot curve; of course, it drops the total pressure.

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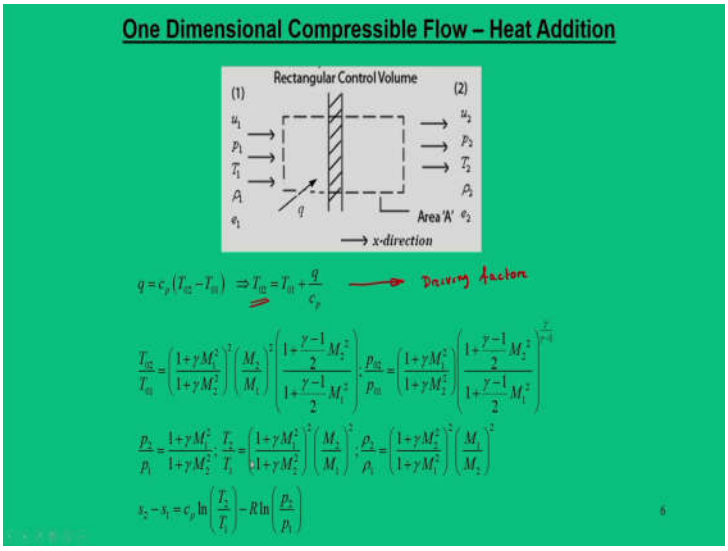

So, in the similar logic when you dealt with one dimensional flow with heat additions; that means, in a one dimensional duct when certain amount of heat is added, it changes the flow properties. So, this heat addition what it does in this case is; increases the total temperatures.

So, this is the driving factor to change the flow properties. So in fact, we earlier we derived the property relations between region 2 and region 1 through these equations and in fact, we have derived them earlier. There is no point to discuss it again and what we were trying to say here what are its consequence.

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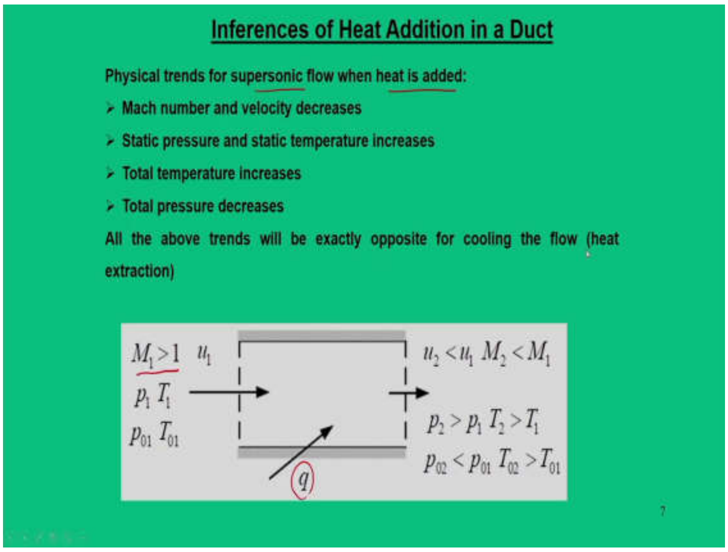

So, from this equations one can talk about two important informations; one is what happens if your upstream condition is a supersonic flow, when heat is added. In fact, there are four possibilities; the first two possibilities could be inflow can be supersonic or subsonic and for a supersonic flow heat can be added or heat can be taken out; so, that means, heating the flow or cooling the flow and in fact, same thing for subsonic flow, one can heat the flow as well as cool the flow.

So, now in the first case we are going to analyze when the inlet condition is supersonic that is M_1 is greater than 1. So, the physical trend for a supersonic flow when heat is added; so, heat is added into a supersonic flow. So, what does this mean for us? As you see in the downstream case, when heat is added in a constant area duct the changes that occurs in the flow properties are as follows.x

So, what we see here from the equation that your Mach number decreases, of course since Mach number decreases velocity also decreases, the static pressure and static temperature increases. The total pressure that drops and total temperature increases, because since heat is being added.

Now in the same supersonic flow if you take a reverse situations like instead of adding the heat, if I take out heat, then these conditions will be exactly opposite that is what it is written here that all the above trends will be exactly opposite for the cooling flow or in heat extraction process.

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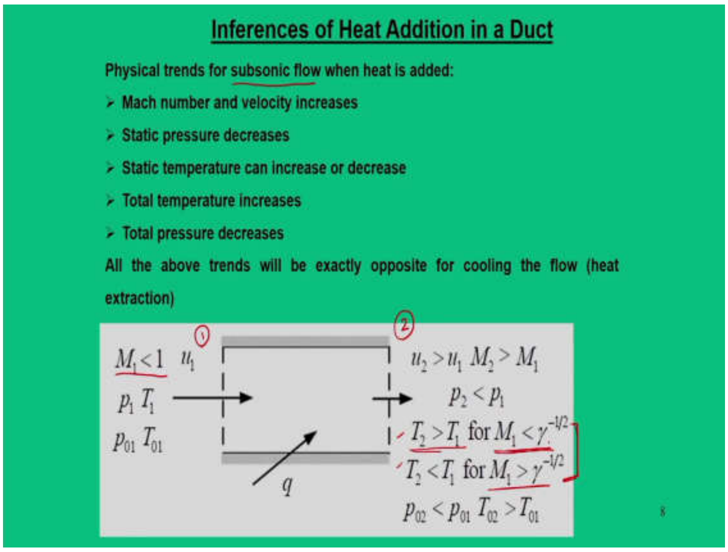

Now, something we talk about the subsonic flow. So, in the same logic when a subsonic flow which is at the inlet condition 1, condition 2 is mentioned here. So, for an inlet condition 1 when the flow is subsonic and it is entering to a constant area duct in which heat is added; so what will it make? In the downstream conditions, it would be such that the Mach number will increase, static pressure will decrease and this is most significant part for this subsonic flow about the temperature.

So, what it has been seen that static pressure can increase or decrease. So, we will try to explain why it is so. So, this is most important inferences that we are going to discuss in this lecture that why it is happening, because it is not following the exactly the opposite trend as it was for a supersonic flow.

So, for certain situations the static temperature increases and for other situations static temperature drops and mathematically, it can be shown that these are the limits in which the static temperature can increase or static temperature can decrease, but what happens to total temperature? Total temperature should always increase, because heat is added, but the total pressure drops. This is what the conclusion that we get when in a subsonic flow, when heat is added.

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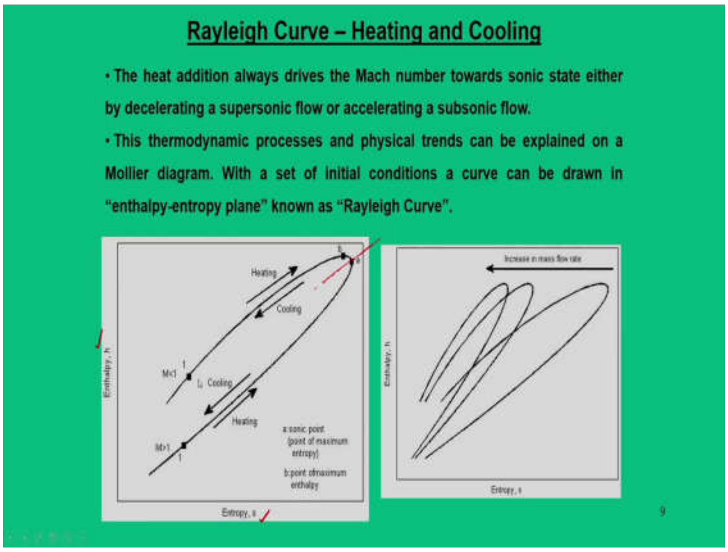

Now, here the most important topic that we are going to cover is the Rayleigh curve and this gives the thermodynamic definitions for a heating and cooling process in a compressible flow through a constant area duct.

So, what it says is that from this mathematical interpretations described in the last two slides, what we found out that heat addition process that means when a heat is added either to a supersonic flow or subsonic flow, it drives the Mach number towards a sonic state.

That means, in a supersonic flow when heat is added the Mach number decreases, but in a subsonic flow when heat is added Mach number increases. So, at some point of time they will reach at a sonic state and such a process thermodynamically we are trying to interpret in a diagram which is called as Mollier diagram enthalpy-entropy diagram.

In fact, this is a very basics basic diagram in the thermodynamics and one can draw these diagrams for different mass flow rates. So, that means for different mass flow rates one can draw this h-s diagram and if you increase the mass flow rate then each of the curve can be traced or plotted.

So, as you can see here in the right hand side of the figure with increase in the mass flow rate, the curve traverse towards the left in the enthalpy entropy plot. Now, let us take one particular curve from this series, because there may be infinite number of such plots and let us take one particular curve.

In fact, to give the name of the curve it is known as Rayleigh curve; so, that means, for different mass flow rate there may be infinite number of Rayleigh curves. We are going to take one particular Rayleigh curve and which is plotted in the left hand side of the figure. When I say it is one particular Rayleigh curve so I mean that this curve is drawn for one particular mass flow rate.

So, as you see here that this is the curve which talks about for a given mass flow rate. So, here I will explain about some of the important points is that the arrow indicates that we are moving in a direction either heating or cooling, similarly in this case also.

As I mentioned that there is a point ‘a’ which is known as sonic point that is point of maximum entropy. So, point of maximum entropy divides this curve as two halves; one is the subsonic flow, other is the supersonic flow.

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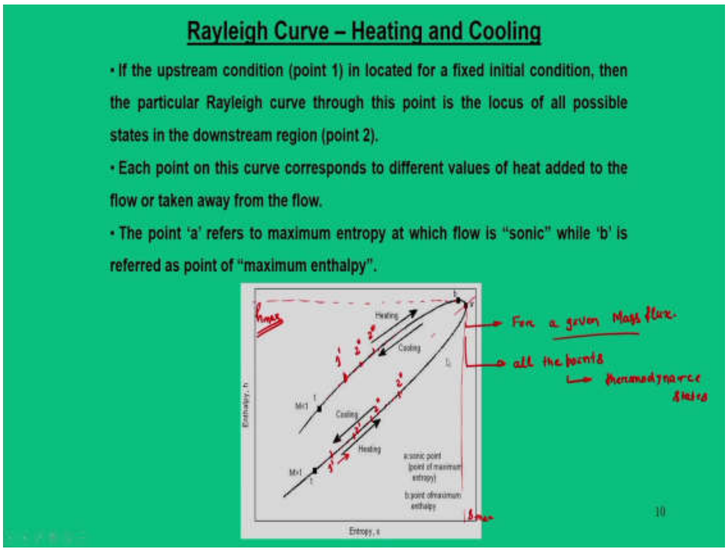

Now, moving further, here we are now going to talk about what are the different points on this particular curve is meant for. So, this particular curve is meant for a given mass flux.

So, first thing is that this curve Rayleigh curve is drawn for a given mass flux. So, in this curve I can choose any points and all these points denote thermodynamic states. All the points implies some thermodynamic states, which means let us we have 1`, if I add heat then I can move to 2` or 2`` or 2```. So, this heat addition process will drive me towards point a.

Now, in this region when I am in the lower half of the curve the heat addition will take me to point a. Now, reverse is also true. Suppose, I am at point 2``, if I instead of adding heat, if I cool the flow I can bring the state back to 1 also. Similarly, when we are in the upper half of the curve that is when we are in subsonic region, when heat addition will also drive the flow towards point 'a'.

So, heat addition will drive the flow towards point 'a' and again if I take out heat or I cool the flow, I can trace back the path in the exactly opposite manner. But, one important point here is that there are two cardinal points of importance; one is point of maximum entropy that is a and that is another point is point of maximum enthalpy that is point b. So, this point refers to s_{max} and point b refers to h_{max}.

So, this is a very important conclusion here, that when I am in the upper half of the curve that means when I am in the subsonic region either heating or cooling, the heating will drive me towards the point a; but before I reach the point a, I should pass through the point b; this makes the critical point that why static temperature should increase and then drop.

So, this is what we are going to explain. And similarly, but when we are in the lower half of the curve the heating will take me directly into point a. So, the question of maximum enthalpy does not arise when we are in the bottom part of this curve. So, these are the some tricky points that needs to be analyzed carefully.

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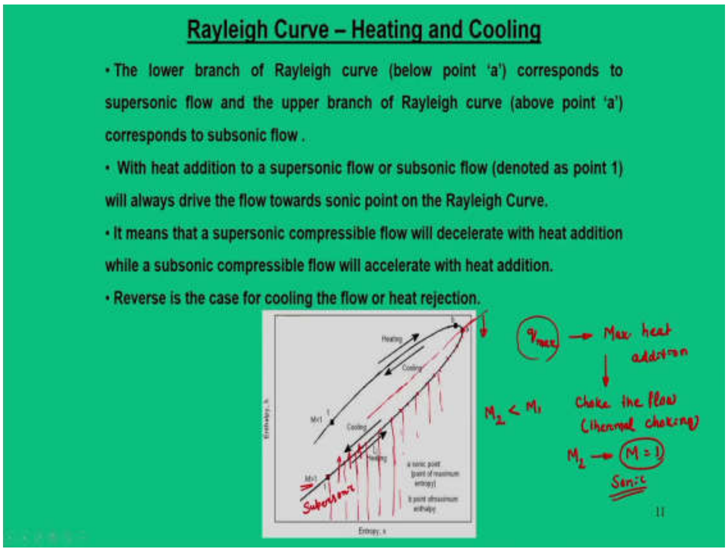

Now, let us talk about one by one; that means, we are in the lower branch of the curve. So, I am dividing this curve as two branch and I am in the lower branch of this curve. So, when I am lower branch of the curve that means obviously, we are in the supersonic region.

So, the initial points can be anything. So, let me start with the point 1. So, heating will take me from point 1 to point a. So, through this process I can trace many points in this line on a given Rayleigh curve. So, each point we will talk about amount of heat that gets added, that increases. So, I can say q_1, q_2, q_3.

So, likewise from point 1, for different value of heat addition I can move along this curve. So, one particular time when I am at point a, that is the maximum possible of heat addition that can be given to a supersonic flow. So, this is the maximum amount that can be given to a supersonic flow so that it becomes sonic in the downstream.

So, that heat addition is nothing but q_{max} that is the maximum heat addition and this maximum heat addition will the choke the flow and this choking will known as thermal choking. By choking I mean heat addition; so, obviously when as you see in this figure when I add heat to a supersonic flow, Mach number decreases, M_2 is less than M_1.

So, one particular instant, M_2 will reach as 1 that is sonic. When this condition is reached for a given q_{max} then the flow is called as thermally choked. Now, similarly I can also reverse the situation that if I am at any intermediate point if I reject heat; that means, I cool the flow I can trace back to that point 1. So, this Rayleigh curve is also reversible in nature in which either heating or cooling can be plotted.

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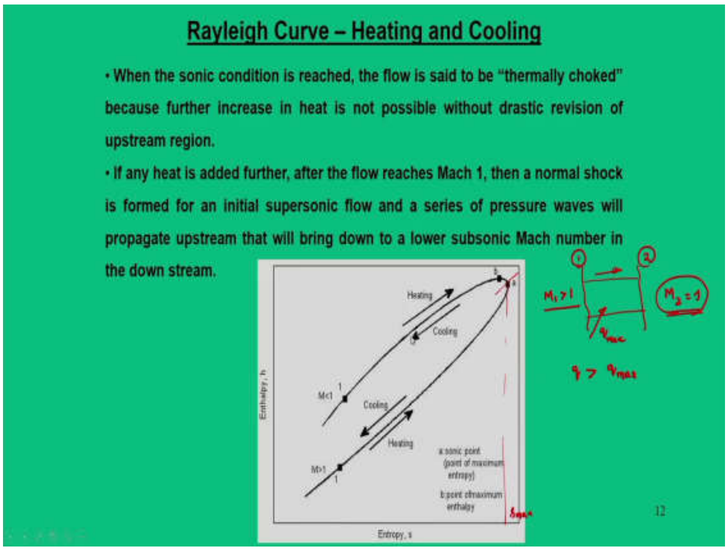

Now, what will happen? once we have reached point a; that means, in a duct with region 1 and region 2 when I am adding q_{max} heat, so this condition M_2 becomes 1 for a supersonic flow.

So, further if q heat addition is greater than q_{max}; that means, which is more than choking the flow then what will happen that if you look at this particular curve, the beyond this point; that means, we are already at point of maximum entropy, then entropy will try to drop.

Since the flow is proceeding from 1 to 2 through this increasing order of the entropy, at one particular instance when q greater than q_{max} when these flow is sonic then further heat addition will not take it further increase in the Mach number or decrease in the Mach number; that means, the flow for which the entropy and beyond this point a will drop. So, this will violet the second law of thermodynamics.

So; that means, I cannot continuously add heat to a supersonic flow that will cross the point a that point of maximum entropy. So, this will violate the second law of thermodynamics, which means then what will happen to the flow; then a normal shock will be formed for the initial supersonic flow. So, in that case the flow will adjust itself in such a way that a normal shock will be formed for which that will bring down the supersonic flow to a subsonic Mach number.

So, this is a case when continuously adding heat to a supersonic flow is not possible, however, from point a if I cool, I can trace back the path in a subsonic domain.

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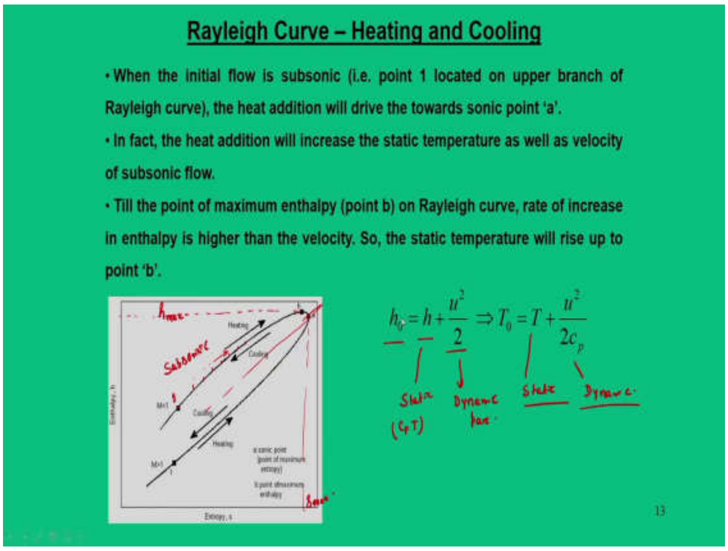

Now, let us see that what happens when heat addition process takes place for a subsonic flow. So, that means, when we are talking about the subsonic flow, we are now in the upper half of the curve. So, here we have subsonic. So, for an arbitrary states point 1 if I locate a point 1 here; so, heat addition will drive me to go in this path and cooling this flow will take me in the opposite path.

So, while going from 1 to any other points on this curve through heating then the flow will be driven towards the sonic point, but interestingly when it goes to the sonic point it has to cross the point b.

So, when it has to cross the point b; that means, we are denoting this point as maximum enthalpy, h_{max} and point a is maximum entropy. So, obviously when you add heat, the enthalpy will rise, because the static temperature will rise.

$$h_0 = h + \frac{u^2}{2} \Rightarrow T_0 = T + \frac{u^2}{2c_p}$$

Now, if you look at this equation what you can see here the total enthalpy of the flow has two components that is one is static part which is h and this is nothing, but c_p times T, other is the dynamic part.

So, what we see here is that the total enthalpy has two components static part and dynamic part. So, heat addition will increase both temperature as well as velocity. So, when we are at point 1 and we are moving towards point of maximum entropy a. Now, between the point 1 and b, the static temperature increases as much faster rate.

So, heat addition will drive the static temperature to rise in the first part; so, mostly in the first part, but what will happen at point b; that means, once it has reached to maximum enthalpy; that means, flow has reached to its say maximum point; that means, you cannot take out any heat energy further. So, since it cannot take any heat energy, the static temperature is not going to rise.

So, but whatever amount that heat is given to this flow, it will increase its velocity. So, when it increases the velocity. So, between the point b and a, the velocity which tends to rise, but its static temperature will drop, because it has to maintain the total temperature to remain constant.

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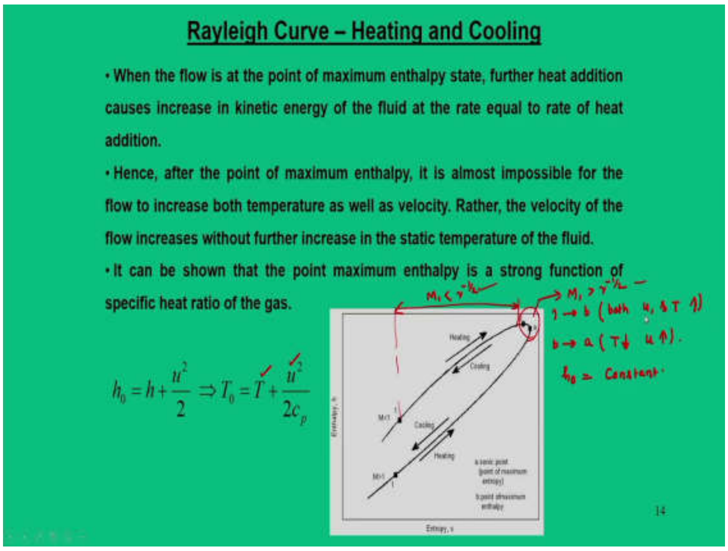

So, this is how the curve is drawn here. So, I can write from point 1 to b we can say that both u and T increases. So, as you can see that temperature as well u increase; between b and a, temperature cannot increase.

So, temperature will drop but velocity will increase, because h_0 has to be constant. So, it can be shown that from point 1 to b in this zone, your M_1 will be less than $\gamma^{-\frac{1}{2}}$, mathematically it can be shown and in this zone your M_1 is greater than $\gamma^{-\frac{1}{2}}$.

So, under this condition when temperature increases your M 1 is less than $\gamma^{-\frac{1}{2}}$. So, these two expressions holds good.

So, it can be shown that for this reason if your M_1 greater than $\gamma^{-\frac{1}{2}}$ the static temperature will drop, velocity will increase so as to enthalpy remains constant, but between the region 1 to b both static temperature and velocity is going to increase.

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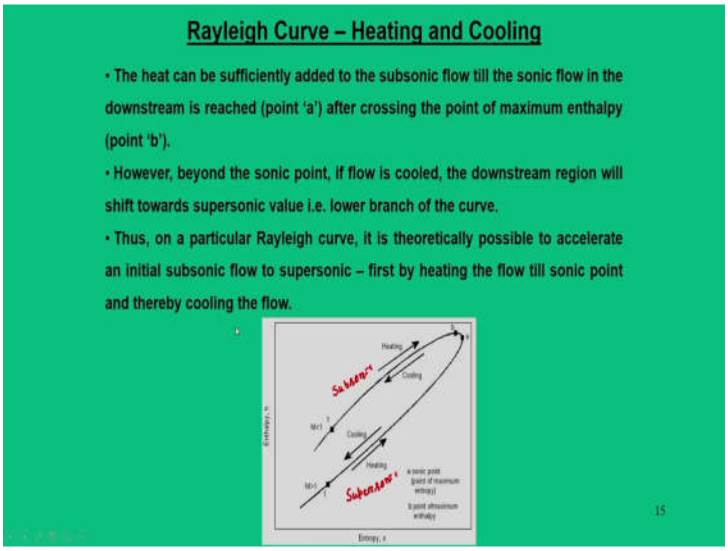

So, this is the philosophy that happens in the case of Rayleigh curve. So, another inference that can be done irrespective, whether you are in the subsonic region that is in the upper half curve or supersonic region that is lower half of the curve.

So, when I move from the supersonic region that is when I am in the lower half of the curve, the heat addition; that means, heating the flow will bring the point 1 towards point a that is sonic point, but thereby further heat addition is not possible because it is already choked. What I can do? By cooling the flow I can trace back the subsonic, I can go into the subsonic domain.

Similarly, other cases also a subsonic flow by heating can be reached to a sonic state and again by cooling it can be brought back to the supersonic state. These are the some important inferences that the Rayleigh curve gives us that it is theoretically possible to accelerate the initial subsonic flow to supersonic first by heating the flow till sonic point and thereby cooling the flow.

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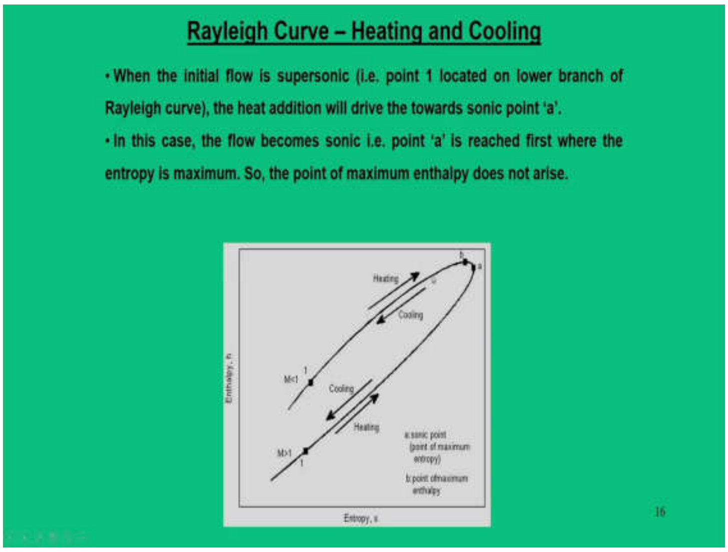

So, this point also I am already mentioned. When you are initial flow is supersonic the heat addition will drive the flow towards the sonic point and for this case there is no question of maximum enthalpy on its path, but it is not true when it is the initial condition is subsonic.

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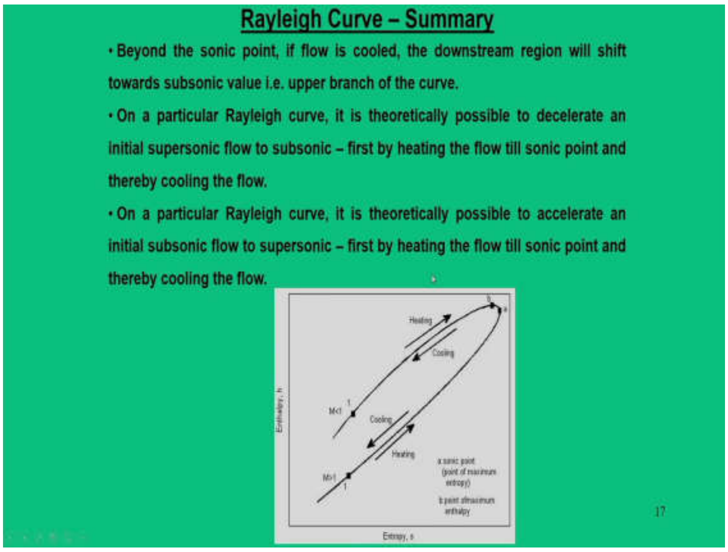

So, finally, we can summarize from Rayleigh curve that it is theoretically possible to decelerate a initial supersonic flow to subsonic flow; first by heating the flow till sonic point thereby, cooling the flow. In particular Rayleigh curve it is also theoretically possible to accelerate an initial subsonic flow to a supersonic flow first by heating till sonic point thereby, cooling the flow.

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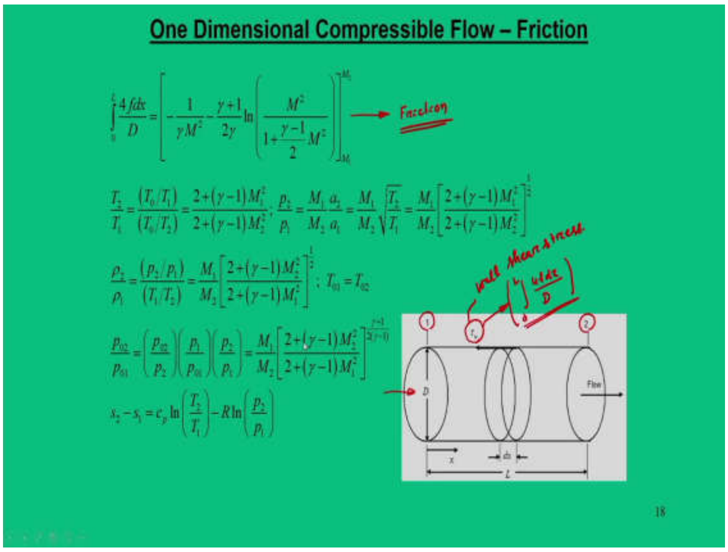

This is what we get from the Rayleigh curve. Now, with respect to friction, the next important topic is that other flow conditions can be altered through friction as well. So, here we talked about a constant area circular pipe of certain length.

So, when we say it has certain length and the flow is entering into the pipe from region 1 and is leaving at region 2. Since, in such cases the friction that the flow is going to encounter for this length is expressed in terms of τ_w that is wall shear stress.

Now, this wall shear stress can be mathematically related to a factor which is $\int_0^L \frac{4fdx}{D}$.

So, quantification of shear stress is given by this integral term and that integral term is nothing but it is a function of Mach number. So, we say that friction is the driving factor for the condition to be altered upstream and downstream.

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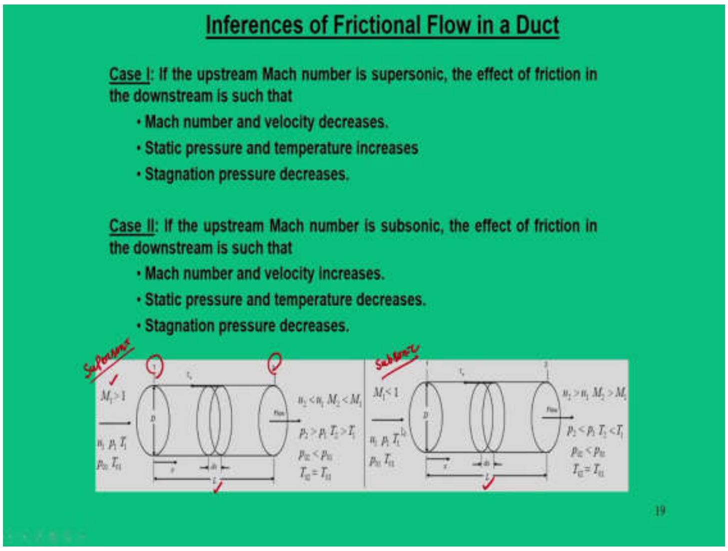

So, based on this fact we derived the fundamental equations for one dimensional compressible flow with frictions in earlier slides. So, the summary for those fundamental equations tells us here. So, the possibilities that one can have is that for a given length of duct L, one can have a supersonic flow inlet or we can have subsonic inlet flow.

Now, let us take the case 1, if the upstream Mach number is supersonic the effect of friction or the length L in the downstream that is for region 2 is such that the Mach number and velocity decreases; the static pressure and temperature increases, the stagnation pressure drops and the total temperature remains same; these inferences we can get from the fundamental equations of friction.

The other situation is that if the upstream Mach number is subsonic, the downstream condition will have the effect of the friction such that the Mach number and velocity increases, static pressure and temperature decreases, total pressure drops, stagnation temperature does not change, because there is no heat added or taken out into the system. So, these are the derivation or summary that we can get from the fundamental equations.

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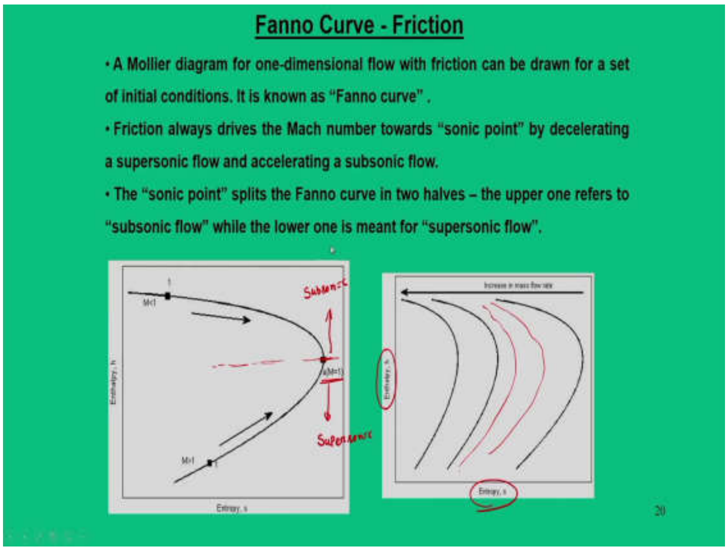

Let us see how we can give a thermodynamic definition to this. So, such definition is possible through a curve which is known as Fanno curve. So, the Fanno curve is nothing but Mollier diagram, which is drawn for a one dimensional flow with friction. And this particular curve shows here that in the similar way it is plotted in enthalpy entropy plane and for a given mass flux, there can be a Fanno curve. So, likewise different Fanno curves can be drawn for different mass flux rate.

So, if you increase in this directions, then we can have infinite number of such Fanno curves. Each Fanno curve will be for a particular mass flux. So, let us take one particular Fanno curve for a given mass flow flux and it is shown here.

So, this Fanno curve has two sections; one is upper half and other is lower half. So, upper half refers to subsonic region, lower half refers to supersonic region and both upper half and lower half of the curve is differentiated by a point `a`, which is known as sonic point. So, this sonic point splits the Fanno curve into two halves; upper one is subsonic flow, lower one is supersonic flow.

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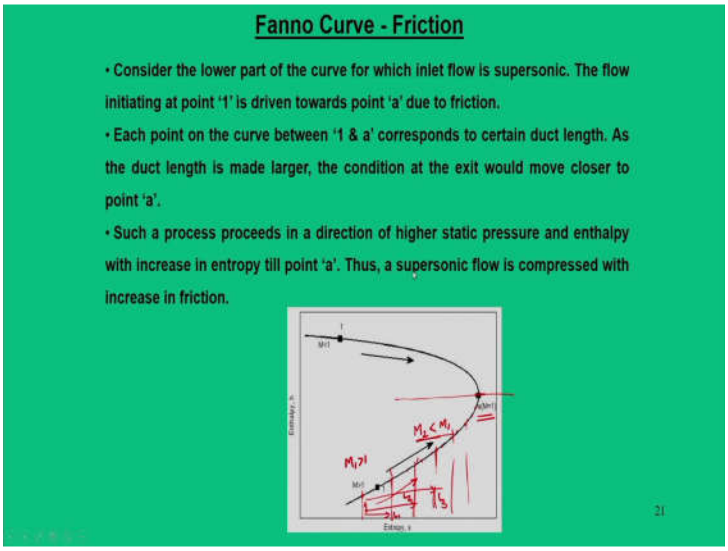

Now, let us see that what we can get out the information from this Fanno curve. Now, let us say that we are in the lower half of the curve. So, each point in this lower half of the curve denotes different thermodynamic states.

So, when I am start with point 1`, I can move towards the sonic point through on this particular curve and each point on the curve means different values of L; L_1, L_2, L_3, and so on. So, that means if I add length to a duct, I can bring this different thermodynamic states in the downstream.

So, addition of length or friction brings the subsonic with brings the supersonic flow towards sonic state. When I am moving from supersonic flow towards sonic state means Mach number decreases. So, if your initial Mach number M_1 is greater than 1. So, the Mach number at any points on this curve is always less than M_1 and finally, I arrived at a point `a`, which is sonic.

So, here it says that the each point on the curve 1 and a corresponds to certain duct length and these duct length is made larger, the condition at the exits would be closer to the point a.

And in this process when I am going from this point to point a, it leads to increase in the static pressure and enthalpy with increase in the entropy as well. So, a supersonic flow is compressed with increase in the friction.

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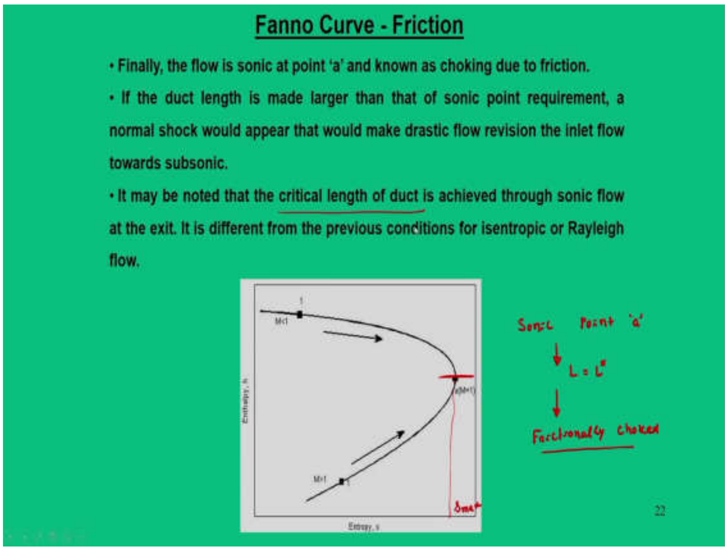

Now finally, when I reach the sonic point, the flow is choked. Sonic point a we say L becomes L^*, the flow is frictionally choked, which means physical inference that adding length or increasing the friction in a supersonic flow will lead you a sonic state.

I cannot travel back beyond this point, because it could violate second law of thermodynamics, because this point a, the entropy will drop. So, this point is known as the critical length of the duct.

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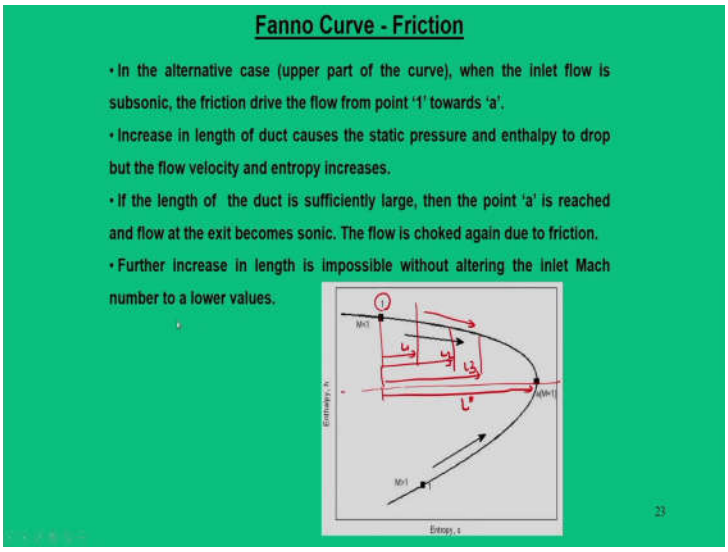

The alternative case when the flow is in the upper part of the curve.. So, likewise in this upper part of the curve when I am at point a, the length addition will take me towards the sonic point. So, each point will give different L_1, L_2, L_3 and finally, I will come back to this point as L^*.

So, flow is again choked, but it is frictionally choked. So, when this flow is frictionally choked, I reach a point where the friction causes the static pressure and enthalpy to drop, but with increase in the flow velocity and entropy. Further increase in this length beyond L^* is not possible without altering the inlet Mach number to a lower values.

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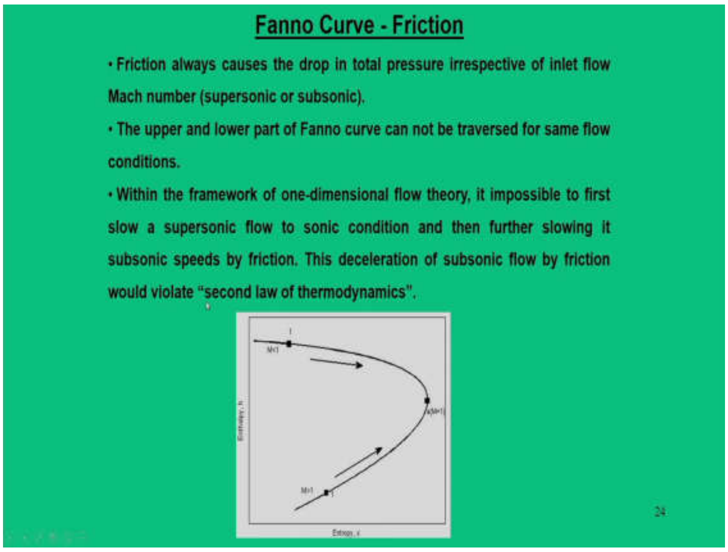

So, one can finally summarize that the friction always drives the flow towards sonic state with a drop in total pressure irrespective whether the flow is supersonic or subsonic at the inlet state. The upper part and the lower part of the Fanno curve cannot be traversed back for same flow conditions.

It is important to note here, that friction process is always a irreversible process. This is not like what we did for heating and cooling that with in case of heat transfer process. What happens here, that the main intention of this sentence is that by when I am moving from subsonic or supersonic region to sonic state; I can go from point 1 to point a and beyond that I cannot go back, because friction will not allow me to cross this point.

Similarly, when we are in the subsonic region one can come from point 1 to maximum point a. I cannot traverse back, whether you add or extract length from this duct. So, in other words what it says is that in Rayleigh curve the way you put heating or cooling here, we cannot put it, because it will violate the second law of thermodynamics and since the friction is always an irreversible process.

So, what can be concluded here is that within the framework of one dimensional theory, it is impossible to slow a supersonic flow to a sonic conditions then further slowing to subsonic speed by friction because the deceleration of subsonic flow by friction would violate the second law of thermodynamics.

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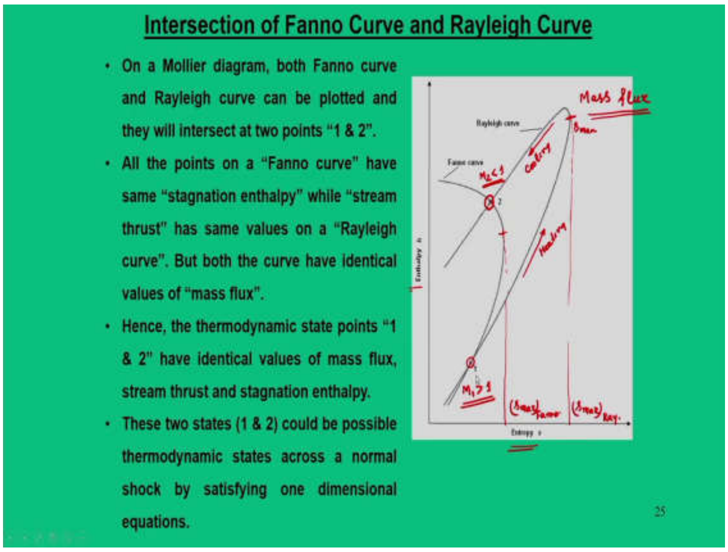

So, having said both Rayleigh curve and Fanno curve. Let us try to make certain correlation among between them. So, what correlation? What you can get from here that we are now putting both Rayleigh curve and Fanno curve on a single domain or single plane.

So, in enthalpy and entropy plane we are representing both Fanno curve and Rayleigh curve and of course, when you are representing both Fanno curve and Rayleigh curve, they must have same mass flux.

So, for same mass flux one can get one Rayleigh curve and also we can get one Fanno curve. What can be shown is that both the curves intersects at point 1 and 2 and if you look at the curves closely, the point 1 and 2 happens to be at two other streams. It means if you take the Rayleigh curve that is s max that is point of maximum entropy, I will put as Rayleigh and point of maximum entropy for Fanno curves I can put as $(s_{max})_{Fanno}$.

So, the point 1 in both Rayleigh and Fanno curve happens to be in the supersonic region and similarly, point 2 is always subsonic region. Now, if I want to go from 1 to 2 then along a Rayleigh curve then I must go first by heating till the point of maximum entropy s_{max}, then I can trace reverse the path.

So, instead of heating I should go as cooling, then I can reach this point. But, if I am in the Fanno curve, I cannot go from 1 to 2 because it will violate the second law of thermodynamics, but what most important point common here is that both the point have same mass flux.

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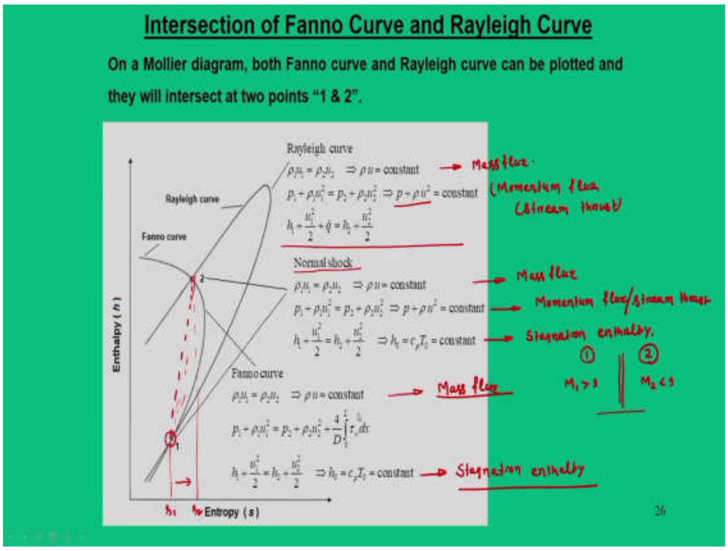

So, let us look closely these equations. So, here I have plotted a more explicitly manner about the enthalpy entropy diagram for a Rayleigh curve and Fanno curve and mathematically what can be proven is that on a Rayleigh curve, if you look at what are the equations.

So, first equation is the continuity equations, we say $\rho u = \text{constant}$; so we say it is a mass flux that remains constant. Even $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$; so, $p + \rho u^2$ we say it is a momentum flux.

So, its known as stream thrust. So, these two parameter remains constant. Since, it is a heat addition process, stagnation temperature does not remain constant, but when you look at a Fanno curve what remains constant is mass flux and stagnation enthalpy.

These two parameter remains constant, but interestingly when these two curves intersect at point 1 and 2 means all the equations of Rayleigh curve and Fanno curve of equations are satisfied; that means, which was not satisfied in Rayleigh curve is stagnation temperature which was not satisfied in the Fanno curve is nothing but moment of flux.

So, these two are satisfied at the intersection points, but interestingly when we found out if such a things happens under what circumstances all the three conditions are satisfied, it happens to be a normal shock. Now, if you look at the normal shock you see that across a normal shock we satisfy mass flux, we satisfy momentum thrust or momentum flux or stream thrust, we also satisfy stagnation enthalpy.

So, the intersection points happens to be possible thermodynamic states across a normal shock. Now, which can be thought of at two different thermodynamic states of a normal shock and which one can be supersonics and which one will be subsonic. So, M_1 has to be supersonic and M_2 has to be subsonic, because spontaneous process from 1 to 2 is possible and since it is in the direction of increase in the entropy.

So, spontaneous process from 2 to 1; that means, if I want to go as a across a normal shock then I can draw a dotted line that touches this point 1 and 2. So, this thermodynamic states of normal shock can be represented as s_1 and s_2.

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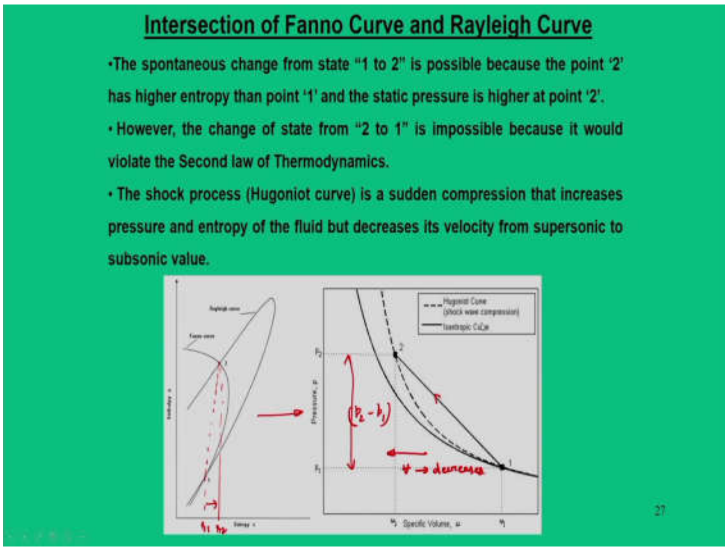

This is how we are going to represent here that the spontaneous change from 1 to 2 is possible, because one can draw a dotted line from point 1 and 2 in the direction of increasing the entropy. From 2 to 1, it is almost impossible to come, because it will violate the second law of thermodynamic process, but what we can infer from these when I am going from 1 to 2, then if you correlate same information from Hugoniot curve, we can say which was in the beginning we emphasized that when I am going from

1 to 2 in a spontaneous process then it increases the pressure as p_2 - p_1 which is nothing, but the static pressure rise across a normal shock, velocity will drop and Mach number become subsonic. Finally, it will decrease in the specific volume.

So, this is the link between the Fanno curve, Rayleigh curve and subsequently we brought the Hugoniot curve into picture. Intersection of Fanno curve and Rayleigh curve is a possible thermodynamic states that follows the Hugoniot equation or Hugoniot curve.

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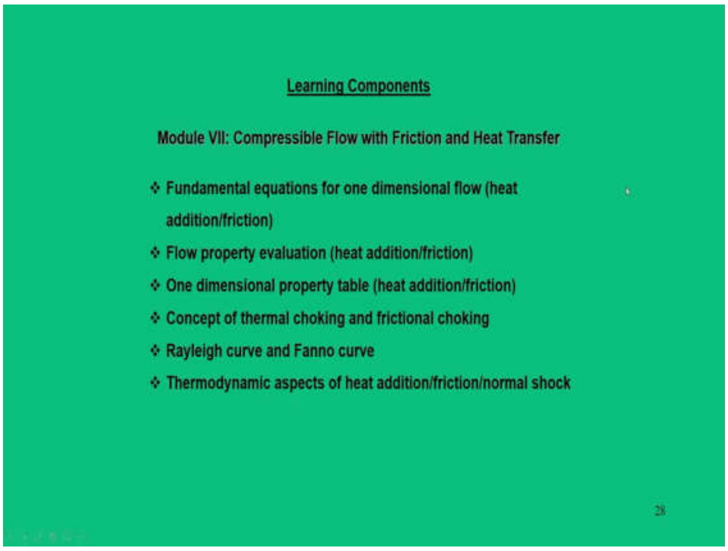

So, with this I conclude this module that is compressible flow with friction and heat transfer and some of the important inferences of this module or learning components can be summarized as follows. So, in this module we talked about fundamental equations for one dimensional flow separately; one for heat addition, other for fiction, then we try to evaluate the properties upstream and downstream for a heat addition process and for a friction process.

Then you introduced a property table; one for a heat addition process, other for friction process. Then you introduce the concept of thermal choking as well as frictional choking to give some thermodynamic definitions for heat addition and friction process. We defined a Rayleigh curve and Fanno curve and try to see that what are the different thermodynamics states that are possible; how a process can proceed in certain direction.

So, all these things were answered on a Rayleigh curve for heat addition process and on a Fanno curve for a friction process and finally, we tried to correlate that the most important aspects of compressible flow heat and friction with respect to normal shock analysis. So, try to gave a thermodynamic definitions for heat addition, friction process, and across a normal shock. So, with this I will conclude this module.

Thank you for your attention.