Fundamentals of Compressible Flow
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Module – 01
Review Concepts of Fluid Mechanics and Thermodynamics
Lecture – 02
Review Concepts of Fluid Mechanics and Thermodynamics – II

Welcome to this course, Fundamentals of Compressible Flow. We are in the first module that is Review Concepts of Fluid Mechanics and Thermodynamics.

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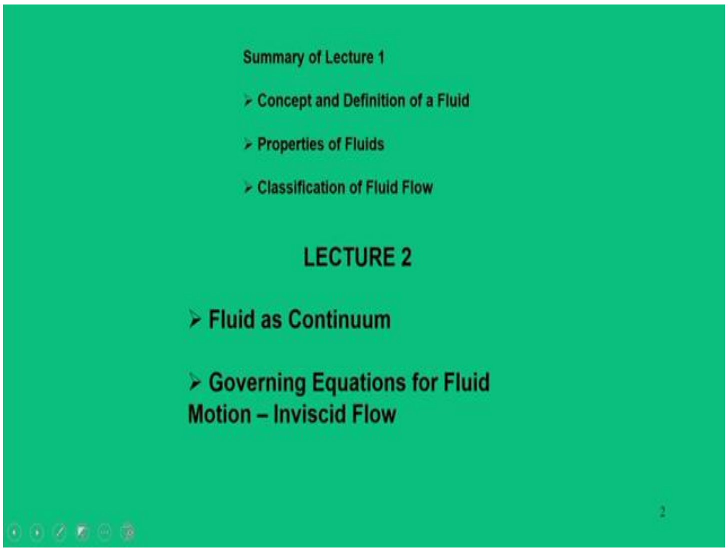

So, this particular module has 2 lectures. In this first lecture, we discussed about the concepts and definition of the fluid and in general for gas. Then, we discussed about the properties of fluids and mostly very specific to gases which are compressible in nature. The last part of this lecture was classifications of fluid flow and there we analyzed various types of fluids that is compressible, incompressible, inviscid, viscous fluid flow, internal flow, external flow etc.

And the and in this particular lecture, we will move ahead with describing two important concepts that is fluid as a continuum and governing equations of fluid motions and in particular, we will debate here about only for inviscid flow.

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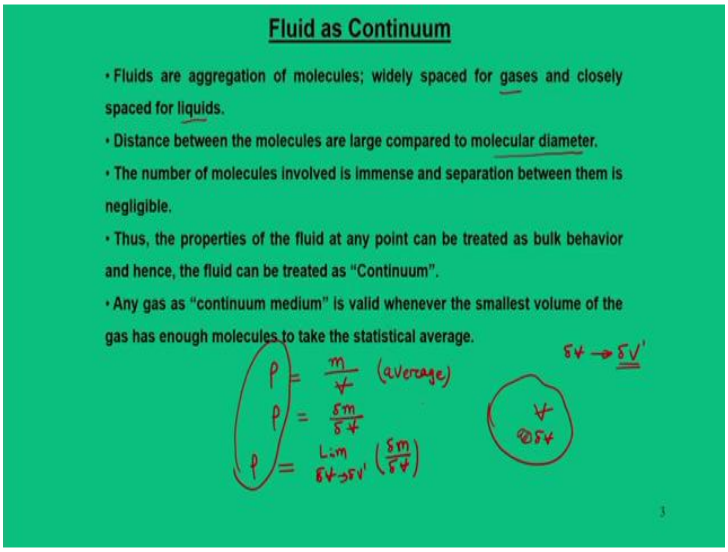

Now, going for the concept of fluid as continuum. So, when you say fluid as a continuum which means that the fluid has certain number of molecules which is occupied in certain volume and these number of molecules are sufficient enough to take the statistical average of any fluid properties.

So, this is also applicable for liquids as well as gases and in general, the molecular motion inside the gases are loose which means that the gases move around inside the closed container and in their moment, they collide each other in such a way that we define a parameter that is called as mean free path.

So, based on these particular concepts, we will describe under what circumstances, we can say fluid is a continuum. So, the first point that is fluids are aggregation of molecules and they are widely spread for gases and closely spaced for liquids. It is quite obvious that distance between the molecules are large, as compared to their molecular diameter. Here, we define a parameter called as molecular diameters.

The number of molecules involved are immense and the separation between them is almost negligible. Now, the properties of fluids at any point can be treated as a bulk behaviour. Hence, the fluid can be treated as a continuum. So, to treat this medium to be continuum, we require sufficient number of molecules to be packed in a closed volume and this closed volume, if you say V and for this volume, if you want to define a particular parameter, let us say density. The first we say the density as a global behaviour which means we can say it is mass times its volume.

So, this means this entire volume occupies certain mass and divided by volume, we call this as a density and this is what we say as a average density. For instance, for gases or in particular, air, we say density of air is 1.2 kg/m^3, which means whether you take 1 kg of air or 1 gram of air, its density remains same. So, now how long we can keep this continuum hypothesis to be valid?

So, for example, you divide this volume in some very small elements so that each volume, each element will have certain volume δV. So, for that case, for that volume if you calculate the mass, we say its mass is $\frac{\delta m}{\delta V}$, but still this value of density remains same.

Now, if we reduce this δV to a very small value $\delta V'$, which means that is the minimum volume that is $\delta V'$ is the minimum volume for which the average behaviour can be defined, which means that you do not have enough number of molecules, if the volume is reduced to a value below this value $\delta V'$. So, for that case, we define this fluid as this density.

For same situation, we define this density $\rho = \lim_{\delta V \to \delta V'} \left(\frac{\delta m}{\delta V} \right)$

So, the same fluid property which is density, but it is defined in three different situations. Hence, it is said to be the fact that density can be considered as an average behaviour or global behaviour of the fluid particles as long as the fluid is treated to be continuum.

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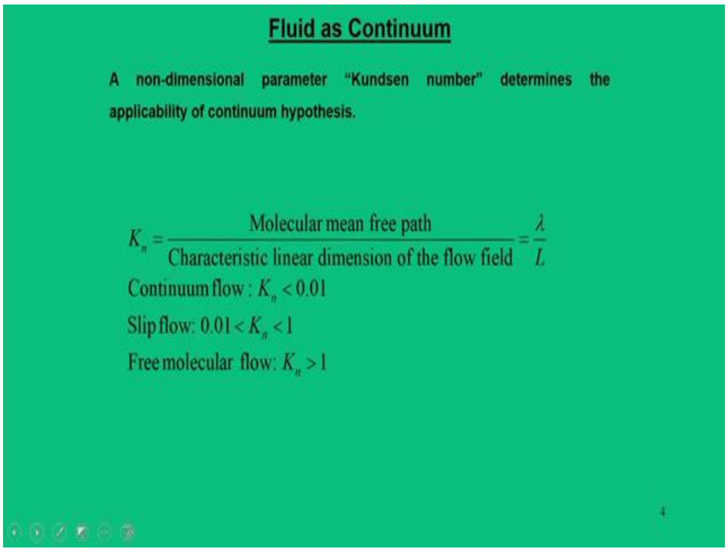

Now, to define this continuum parameter, we quantify it in terms of a number which is known as Knudsen number. This Knudsen number is a very vital parameter that essentially says that whether a certain fluid mass can be treated as a continuum or not. So, what does this mean? So, Knudsen number is defined quantitatively as the ratio of two parameters that is the molecular mean free path and characteristics linear dimension of the flow field. For instance, if we have a flow domain of certain length, width and height; so, its overall or maximum characteristics length can be defined by a number L and the molecular mean free path which is nothing but the distance travelled by the molecule between two successive collisions.

So, in a closed space of fluid elements in a container, the molecules in general move randomly and during their motion, they collide each other and during their collision, we characteristically define a parameter called as mean free path which is λ that is distance travelled between two successive collisions by the molecules; so, that is λ.

So, it is quite obvious that if you have enough number of molecules or sufficient molecules or the medium is very closely packed, then the mean free path is much much smaller than the global length that is L. So, such a situation, the Knudsen number happens to be less than 0.01 and such a flow, we call this as a continuum flow. And now, the other extreme that comes in that if the mean free path length is very large. So, this happens when we see this in particular, this happens in vacuum. So, when the medium is vacuum will hardly find any molecules in certain domains.

So, for instance, when you view these a deep space atmosphere, there are hardly any molecules. So obviously, the characteristics linear dimensions is much much less than the mean free path. So, such a case, we call this as pre molecular flow for which Knudsen number is greater than 1.

Now, sandwiched between these two that is continuum flow and free molecular flow, we also defined another types of flow, Slip flow; means that under certain circumstances, we can treat this behaviour of the fluid as a continuum flow and in some other circumstances, we can with the fluids behave as a free molecular flow.

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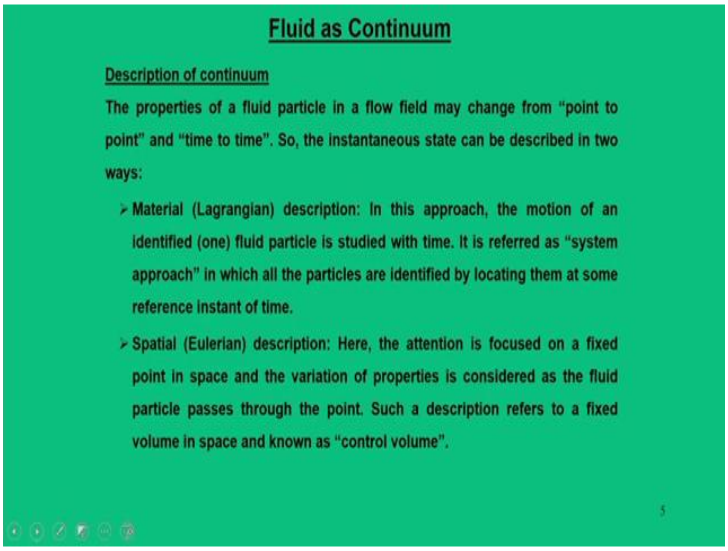

But as far as this particular course is concerned, we will be mostly talking about the fluid to be a continuum. To describe this continuum behaviour, there are two approaches that has been followed so far; one is Lagrangian approach, other is Eulerian approach.

Now, what are they? So, in general, the properties of a fluid particles in a flow field, the in a flow field, the properties of the fluid particles change from point to point or from time to time. Now, when you say it is a point to point, so that means, you are talking about space. When you are talking about time to time that means, you are talking about the different time instances.

So, at any situations, so we can describe this fluid as a continuum medium by these two descriptions; Lagrangian description and Eulerian descriptions. So, in the first category are like that is Lagrangian descriptions, we say that the motion of the fluid particle needs to be studied from time to time.

So, with respect to time we have to see that how this fluid particle changes its positions. Now, the other approach is Eulerian approach. So, here the attention is focused to the fixed point in the space that is fixed domain in the space and the variation of that domain that we will look with respect to the fact that when certain fluid element passes through it, how that medium changes.

Now, this is what the way we represent the continuum medium in Lagrangian Eulerian approach. Now, corollary to this or analogous to this, we have closed system and open systems. So, this is something similar to thermodynamically the way we view it as a closed systems that is which is a Lagrangian approach and open system when it is viewed as a Eulerian description. So, open system in general as a control volume medium.

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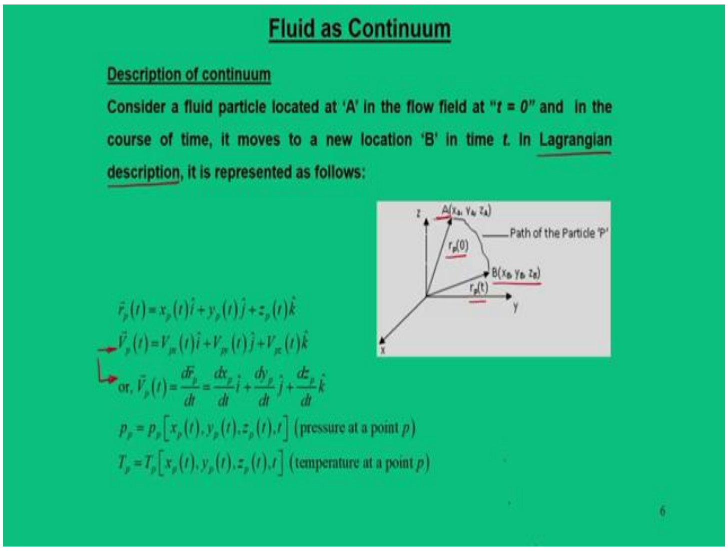

Now, once you say this continuum, now let us go by one by one that how we can say quantitatively in a Lagrangian approach. So, now, we are dealing with Lagrangian descriptions. So, in this Lagrangian descriptions what we want to study is the fact that we define a fluid element at certain locations. Let us say this if in our situation referring to this figure, we have a fluid element located at point A.

Now, with respect to cartesian coordinates x, y, z; we represent this position vector $\vec{r}_p$ for the point A. So, $\vec{r}_p(0)$ means we are defining the position of the fluid particle at point A as its coordinate x_A, y_A, and z_A and which is defined at the time instance t = 0. Now, we allow some time to pass. So, by that time, the position of the particle has changed from A to B. So, when it goes from A to B, the new position vector is defined as $\vec{r}_p(t)$. So, the coordinates of the point B has change to x_B, y_B, and z_B.

So, in this process of changing the position of the fluid particle, it would have undergone at certain velocity which is $\vec{V}_p$ and it would have this $\vec{V}_p$ and $\vec{r}_p$ is related by this differential equations; that is velocity of the particle p can be defined as the differentiation of the position vector with respect to time.

$$\vec{V}_p(t) = \frac{d\vec{r}_p}{dt} = \frac{dx_p}{dt}\hat{i} + \frac{dy_p}{dt}\hat{j} + \frac{dz_p}{dt}\hat{k}$$

So, in this way, we describe this position and when we talk about the position A and B, we also say that at that particular point, the pressure might be p_p as a function of x_p, y_p, z_p and t and also, we can have temperature at point p can be defined in a similar manner.

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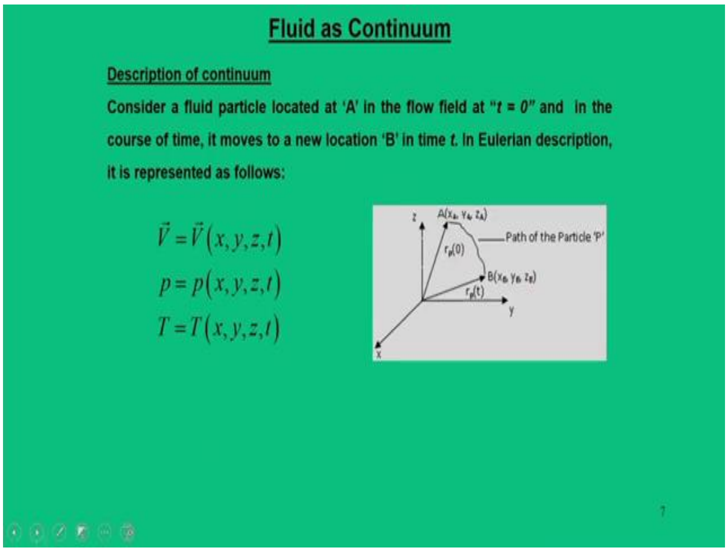

Now, the same situation, when you view as a Eulerian approach; although event is same, but the way of representation of the description changes. So, here the velocity vector is defined with respect to space as well as time; globally, with respect to space and time.

Now, in the event of this fluid particle the change in the fluid particles, there would have a some global change in the medium that we are looking at. So, for that situations the representation has changed as a velocity vector pressure or temperatures and these we see as the global picture with respect to velocity vector with respect to space and time.

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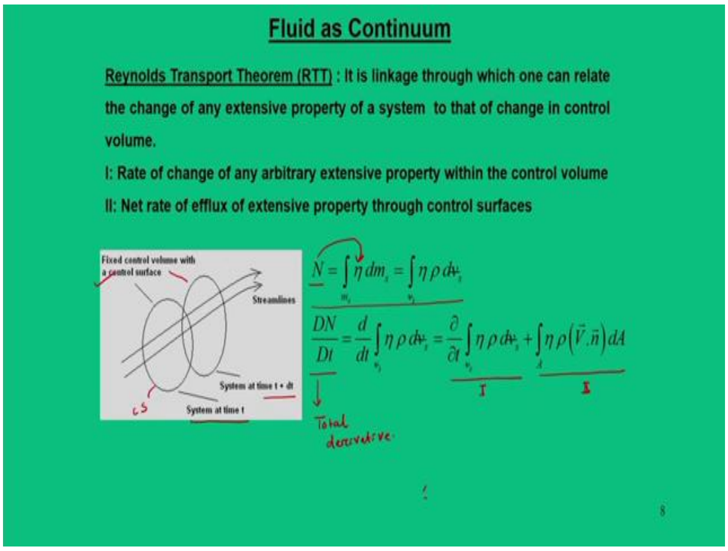

So, once having said this, now we are going to discuss about a very important theorem which is called as Reynolds transport theorem. So, till this point of time, we are essentially focusing that how you want to describes the medium in terms of Lagrangian description or in terms of Eulerian descriptions. But the global effect remains same because we are representing the same event, but in two different forms.

So, to link between these two events or these two forms, a famous theorem that comes into picture that is Reynolds Transport Theorem. So, it is an essentially linkage between the change in a property of the fluid with respect to a control volume. So, control volume is essentially a Eulerian descriptions, but if you very specific to the fluid property of an element that element has certain properties.

So, the particle approach and the control volume approach can be linked together in terms of defining the properties in two different domains. So, this particular figure says that first thing is let us take the first elliptical space which typically we call this as a fixed volume, fixed control volume with this control surface. So, this is nothing but your control surface and this control volume is defined as a system at time t.

Now, what has happened is that there are certain stream of flow which is entering to this control volume. Through this entering process, it would have done some changes in the mass, changes in the energy and in this process, the system is now changed to a another location; but with same fixed volume and that is defined as system at time t+dt. So, eventually, what has happened? System has changed from one time instant to other and through this change, if there is a property N which is defined at particular time t, that property, how it changes when the system has gone to another time instance?

So, in this particular say if N is your any extensive properties for which the corresponding intensive properties is η. So, this can be related in an integral form $\int_{m_s} \eta dm_s$. So, s stands for system and in terms of volume, we can write it as $\int_{V_s} \eta \rho dV_s$. So, this is how the extensive properties is defined.

Now, we see that when this extensive property is going to change based on this Reynolds transport theorem, we defined this as $\frac{DN}{Dt}$. This particular term, we call this as a total derivative and through Reynolds transport theorem, we say that this is equal to the properties that same properties when you when it changes with respect to time and same property, how it changes with respect to space.

$$\frac{DN}{Dt} = \frac{d}{dt}\int_{V_s} \eta\rho dV_s = \frac{\partial}{\partial t}\int_{V_s} \eta\rho dV_s + \int_{s} \eta\rho\left(\vec{V}.\vec{n}\right)dA$$

The first term of this right hand side equation is the rate of change of the arbitrary extensive properties with respect to control volume and the second term represent the net rate of efflux of extensive properties through this control surface. We will not derive this Reynolds transport theorem, but we will only talk about how it is relates a given property of a fluid system.

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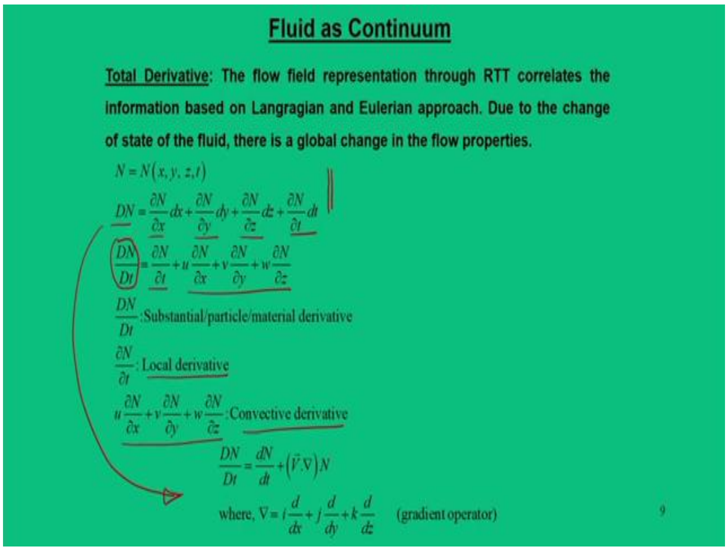

Now, the Reynolds theorem introduce a parameter which is called as $\frac{DN}{Dt}$ and it happens to be a mathematical term which we called is as a Total Derivatives. So, to solve this mathematical term, we have a tool which is mathematically which is represent this DN in terms of dx, dy, dz and dt. So, this is a mathematical expression that comes for a given properties.

$$DN = \frac{\partial N}{\partial x}dx + \frac{\partial N}{\partial y}dy + \frac{\partial N}{\partial z}dz + \frac{\partial N}{\partial t}dt$$

$$\frac{DN}{Dt} = \frac{\partial N}{\partial t} + u\frac{\partial N}{\partial x} + \upsilon\frac{\partial N}{\partial y} + w\frac{\partial N}{\partial z}$$

Now, here this total derivative $\frac{DN}{Dt}$ we call this as a substantial derivative or particle derivative or material derivatives and first term that is $\frac{DN}{Dt}$ is nothing but the local derivative that is with respect to time; the other three terms $u\frac{\partial N}{\partial x}$, $\upsilon\frac{\partial N}{\partial y}$ and $w\frac{\partial N}{\partial z}$ is nothing but your convective derivative and these two are very important parameters that essentially talks about the changes in a fluid system and when we deal with the space as well as time, such a time such a concept, we call this as a field concepts. So, which will come in the subsequent slides.

Now, since these expressions when we represent in this form that is derivative form, the other way of representing the same expressions in vectorial form or that is

$$\frac{DN}{Dt} = \frac{dN}{dt} + \left(\vec{V}.\nabla\right)N$$

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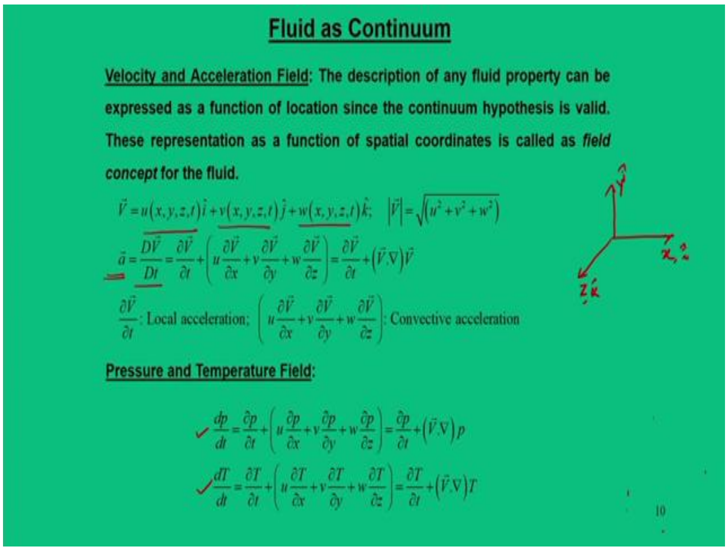

Now, when you say it is a field concept, so we will now introduce that these properties N whether it is a scalar property or vector properties. Now, if it is a scalar property, we can represent either as a pressure or temperature by using the same expressions and if it is a vector properties, then we have to again orient these properties with respect to cartesian coordinates x, y and z and unit vector for each orthogonal directions, we say $\hat{i}$, $\hat{j}$ and $\hat{k}$.

So, the velocity vector has three components u which is again a function of x, y, z and t, v and w. Both u, all the terms u, v and w they are all functions of x, y, z and again, this velocity vector and in our case, when this velocity vector is differentiated in terms of a total volume derivatives, we represent we call this as a acceleration vectors. So, this is how the importance of this total derivative is all about that it captures different or it expresses different flow field nature.

$$\vec{V} = u(x,y,z,t)\hat{i} + \upsilon(x,y,z,t)\hat{j} + w(x,y,z,t)\hat{k}$$

$$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \left(u\frac{\partial \vec{V}}{\partial x} + \upsilon\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} \right) = \frac{\partial \vec{V}}{\partial t} + \left(\vec{V}.\nabla\right)\ \vec{V}$$

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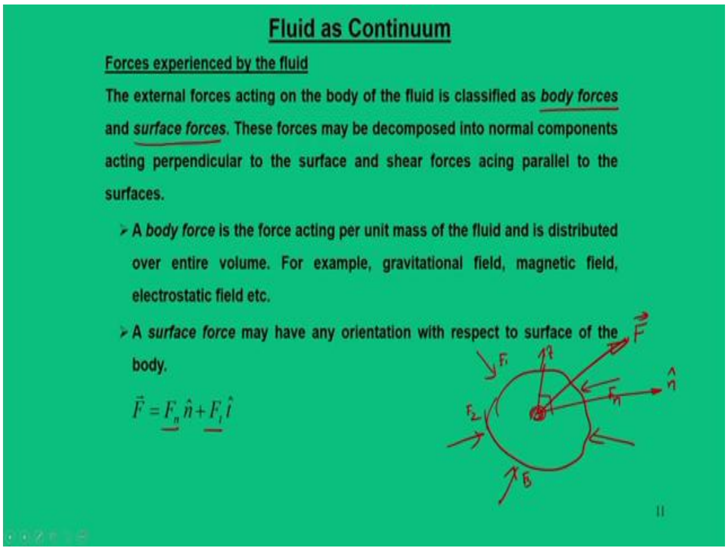

See once you know these velocities, then another important parameter fluid as a continuum is the force. Typically, in a fluid domain, there are two types of forces; one is external forces, other is internal forces. So, and in general and we are looking at the external forces; because we are talking about the global behaviour. These external forces has two components; one is the body forces, other is the surface forces. Body forces are generally by virtue of its own weight or volume; whereas, surface forces are the forces that acts on the boundaries of the fluid domain.

So, in general, if you say there are in a domain, they are many arbitrary forces acting on it. So, we say F_1, F_2, F_3 likewise and we can say there are n number of forces acting on it and this n number of forces has a resultant force vector $\vec{F}$ and these resultant force vectors acts at some points which you typically call as centroid. So, with this point, if you just take an certain small area and draw a normal to it, we say area vector normal to the surface.

Now, perpendicular to it, if you draw another vector $\vec{t}$, we say the tangential to it. So, irrespective of the fact what directions force comes, whether x, y, z, we now represent in terms of the normal forces and tangential forces. So, this is how we say F_n and F_t.

$$\vec{F} = F_n \hat{n} + F_t \hat{t}$$

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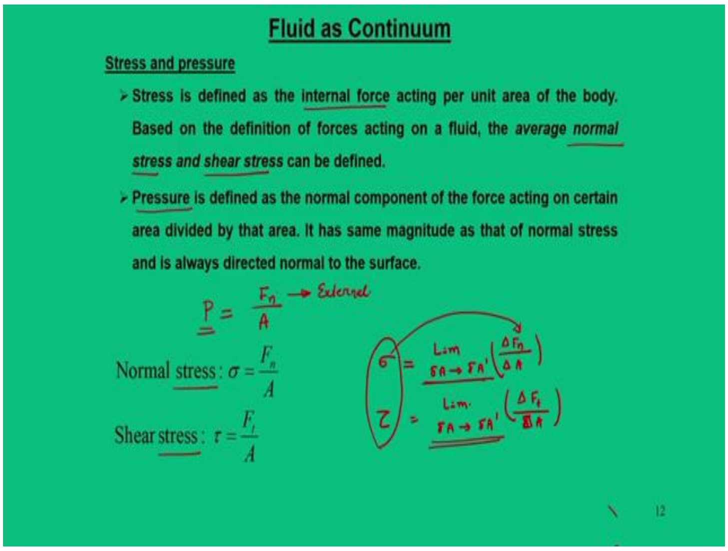

Now, if you represent same behaviour F_n and F_t, we can say that we can define these normal forces and this tangential forces in this following manner that we call these normal forces. The net effect of normal forces is nothing but it induces an internal force which is expressed as a stress and even for tangential force, we can also represent another term which is called as shear stress. So, the stress is nothing but the force per unit area. So, the stress is defined as the internal force acting per unit area of the body. So, based on the definition of forces acting on the fluid, we call this as a normal stress or shear stress.

So, by virtue of the definition of normal forces and shear stress, we say that this normal stress which is the average behaviour. And if we talk in terms of small elemental force, then we represent this normal stress as $\sigma = \lim_{\delta A \to \delta A'} \left(\frac{\delta F_n}{\delta A} \right)$ and the shear stress $\tau = \lim_{\delta A \to \delta A'} \left(\frac{\delta F_t}{\delta A} \right)$. So, this is how the continuum hypothesis needs to be satisfied to define this normal stress and shear stress and this contribution comes from the force acting per unit area and in this case, this force is the internal force. Analogous to this, we also come across the term pressure. This pressure also P also represented by normal force per unit area; but the basic difference between pressure and stress is that for pressure calculations, we take this F_n which is external. So, but while stress calculations, we say the force acting is the internal force acting per unit area. So, this is how the definition is slightly different.

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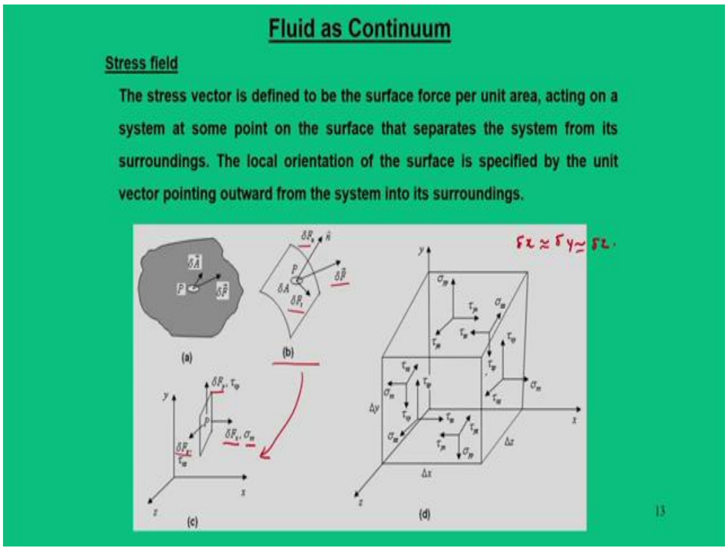

Now, moving further, these pressure concept, force concept gives us and stress concept and with respect to a continuum medium, we behave this stress or we represent the stress in a more global manner that is called as stress field. So, for to define a stress field, what we look at is that for a given surface, we see that there is a force perpendicular to it and there is a force tangential to it. So, essentially, this normal force which is perpendicular to the surface gives rise to normal stress and the tangential force which is perpendicular to this normal force gives rise to shear stress.

So, for a given surface or a given domain like this as shown in the figure, if you take an elemental area dA and for that elemental area the force vectors $\delta\vec{F}$ and along this normal and along the tangential directions, we say $\delta\vec{F}_n$ and $\delta\vec{F}_t$ and once you say this, so essentially this is a unit vector which is normal to this is $\vec{n}$ and tangential to this is $\vec{t}$ and now, let us see that same representation of the force as shown in (b), we now bring it to orthogonal coordinates x, y, z.

So, for that given space, we can see that for this for the same point P will have three different forces; δF_x, δF_y and δF_z. But these net three forces actually gives rise to three stresses on this particular plane.

So, out of these three stress; one is normal to this that is σ_{xx} and other is the tangential to this. So, one may be one is due to δF_y that is τ_{xy} and other is τ_{xz}. So, likewise, we can imagine this size of this element δx, δy and δz for a particular q for which we have six surfaces. For these six surfaces, one can define these normal stresses and shear stresses for each particular face.

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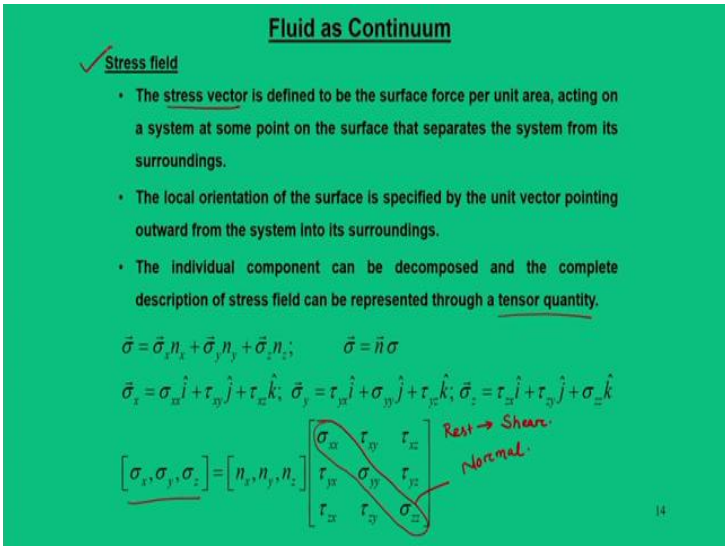

And finally, it turns out to be in a matrix form which is

$$[\sigma_x, \sigma_y, \sigma_z] = [n_x, n_y, n_z] \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

n_x, n_y and n_z that is unit vector in respective coordinate directions and this matrix is very important that is it contains all the nine stresses. So, out of which there is only the diagonal vector or diagonal part of this matrix is the normal component and rest are shear component.

So, this is how we define these stress. Although we initially call it as a vector, it has a particular directions, but since it has lot of there are nine components that drops in all the six surfaces. So, it is referred to as a tensor quantity and this complete descriptions, we call this as a stress field.

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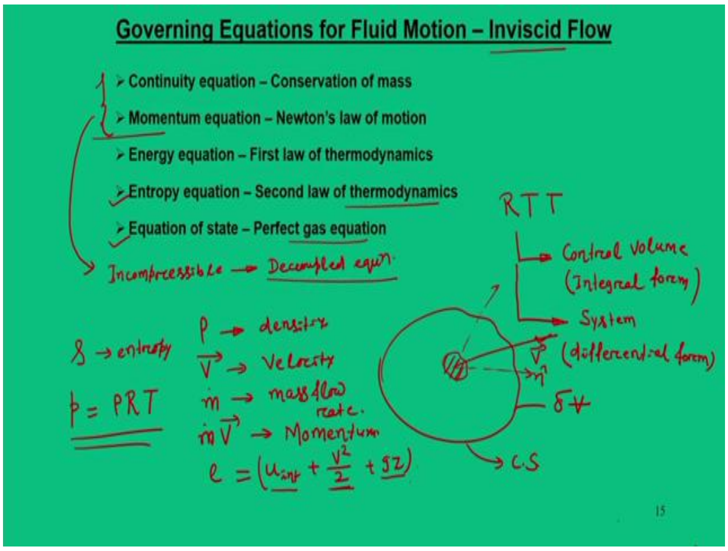

Now, moving further, we are now going to discuss about the governing equations of fluid motions and in particular, it is inviscid flow. Till this point of time, we know that the governing equations are based on Continuity that is Conservation of mass, Momentum equation that is Newtons law of motion, Energy equation which is the First law of thermodynamics and in particular for incompressible, the first three equations are mostly important and this are called as decoupled equation which means this energy equations can be treated separately from the continuity and momentum equations.

So, essentially in most of the situations, we solve continuity and momentum equations simultaneously and energy equation is treated as a separately, that is what we call this as a decoupled equations.

Now, when you when you move to compressible flow, we will bring two more equations into system that is entropy equations, where we bring the Second law of thermodynamics and that is equation of state that is Perfect gas equations. It is quite obvious that when you deal with the compressible flow, we mostly deal with gases and all five equations needs to be solved in such a way that it gives the complete description of the systems. This is one aspect.

The second important aspects that while dealing with these equations, we treat a particular parameter to be importance. Like in the continuity equation, we say this mass flow rate remains constants. So, in the Newton's law of motion that is a momentum equation, we say it is a momentum is the main parameter; in the energy equation, we say it is a the total energy contained in the system is the main parameter and in the entropy equation, it is the entropy and from the second law, the important property is entropy gives the direction of the certain process to happen.

Now, while talking about these equations and we will not derive those equations, we will I will just give you the end results of those equations with some important remarks. So, for deriving those equations which is based on these fundamental theorem that is Reynolds transport theorem which is analysed in two approaches; one is as a control volume approach. When you say control volume approach, we say it is a integral from integral form of equation. Now, when you say the system approach, it is a differential form of equations. However, both the form of equations are equally importance; but it all depends how you deal with a particular flow problem.

Now, for all the cases, we say there is a control volume $\delta \forall$. It is bounded by a control surface. Now, we say some fluid element or we take some elemental area for which these velocity vector is $\vec{V}$ and this velocity vector is normal to this area vector. We say that velocity vector is the resultant vector in which fluid is moving, we also define ρ to be density and we say this $\vec{V}$ to be velocity.

And this continuity equation will bring about this mass flow rate; momentum equation, will talk about $m\vec{V}$; energy equation will talk about the energy total energy that is consist of internal energy plus per unit mass. Its energy per unit mass consists of internal energy plus kinetic energy plus potential energy that is $u_{\text{int}} + \frac{V^2}{2} + gz$ and for entropy equation, we say s and for equation of state, we say $p = \rho RT$. So, essentially, while using these equations the Reynolds transport theorem that talks about the extensive property 'n' is now replaced with these parameters that is first case, it is mass; second case, it is momentum; third case, it is energy.

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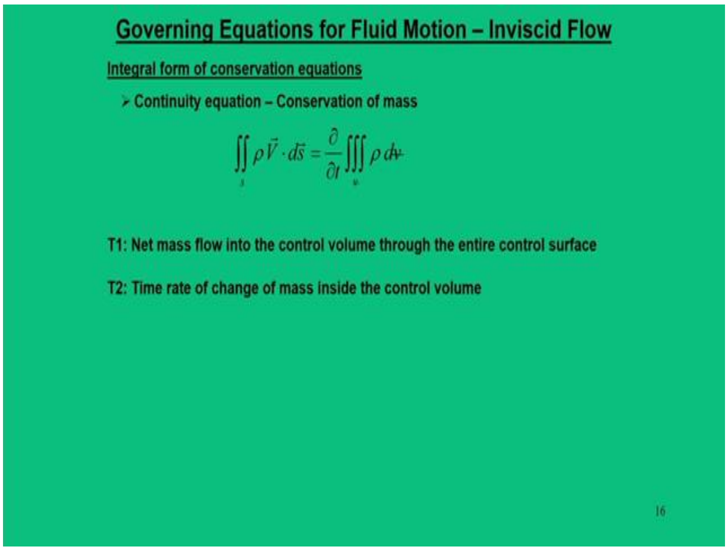

Now, when you deal with this, when you do this and apply this equation; then, the integral form of conservation equation that is conservation of mass as two terms; left hand side and right hand side. The left hand side term represents a surface integral which is the net mass flow rate into the control volume through the entire control surface. The second term or right hand side of this term is a volume integral which is the time rate of change of mass inside the control volume.

$$\iint_s \rho\vec{V}.d\vec{s} = \frac{\partial}{\partial t}\iiint_{V} \rho dV$$

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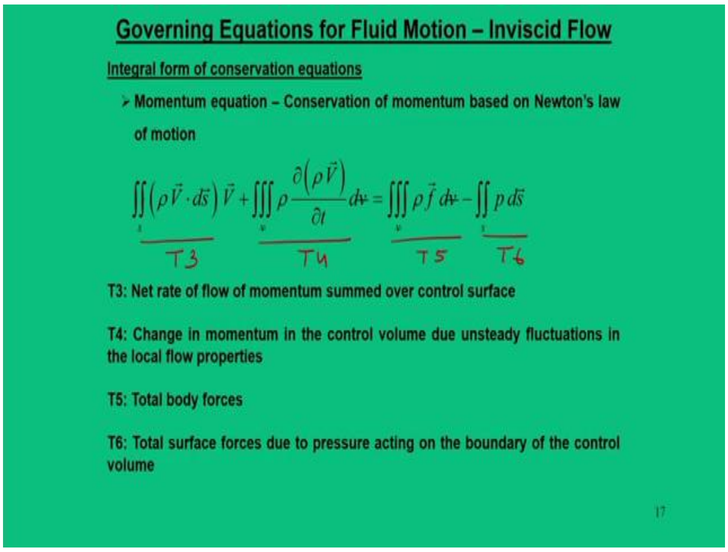

Now, moving further, when you same equation, we talk about integral form of equations for momentum. So, here the momentum is the main parameter.

$$\iint_{s} \left(\rho\vec{V}.d\vec{s}\right)\vec{V} + \iiint_{V} \rho \frac{\partial\left(\rho\vec{V}\right)}{\partial t} dV = \iiint_{V} \rho\vec{f} dV - \iint_{s} p d\vec{s}$$

It has this equation has four terms; T_3, T_4, T_5 and T_6. So, T_3 is the rate of flow of momentum summed over the control surface. So, it is a surface integral. T_4 is the change of momentum in the control volume due to unsteady fluctuations in the local flow properties and it is a volume integral.

And T_5 term is the total body forces because the left hand side term talks about the rate of change of momentum; whereas, the right hand side term talks about the that is equals to the force. So, that is what the T_5 term is talked about the total body forces and T_6 term the total force that arises due to the pressures acting on the boundaries of the control surfaces.

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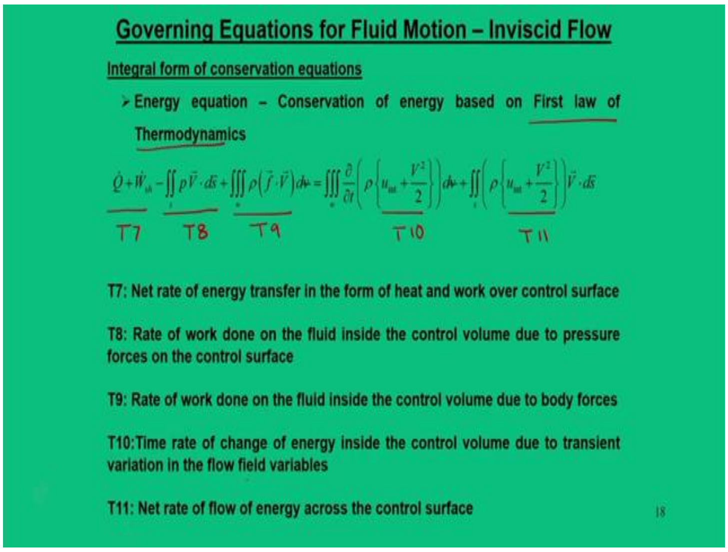

And the next part is the energy equations, which is based on the first law of thermodynamics. So, if you look at these expressions, here we are the term that is energy consisting of internal energy, kinetic energy plus potential energy. Here, we have not mentioned about the potential energy because we are not we will not be dealing with this.

$$\dot{Q}+\dot{W}_{sh}-\iint_{s} p\vec{V}.d\vec{s}+\iiint_{\forall}\rho\left(\vec{f}.\vec{V}\right)d\forall=\iiint_{\forall}\frac{\partial}{\partial t}\left(\rho\left\{u_{\text{int}}+\frac{V^2}{2}\right\}\right)d\forall+\iint_{s}\left(\rho\left\{u_{\text{int}}+\frac{V^2}{2}\right\}\right)\vec{V}.d\vec{s}$$

So, this part comes as the total energy that is in the right hand side equations and the left hand side of this equation consists of energy transfer in terms of work or work or heat. Then, in terms of the work done with respect to pdv work; thermodynamically, it is pdv work and the work transfer due to control volume.

So, here T_7 terms is the net rate of energy transfer in the form of heat and work; T_8 term is the rate of work done on the fluid inside the control volume due to pressure forces; T_9 term is the rate of work done on the fluid inside the control volume and T_{10} term is the rate of change of energy in the control volume due to transient variation of the field variables; T_{11} term is the net rate flow of energy across the control surface. So, we want to do derive these equations and, but however, we will make some corollary of these equations which are much more beneficial of for our analysis.

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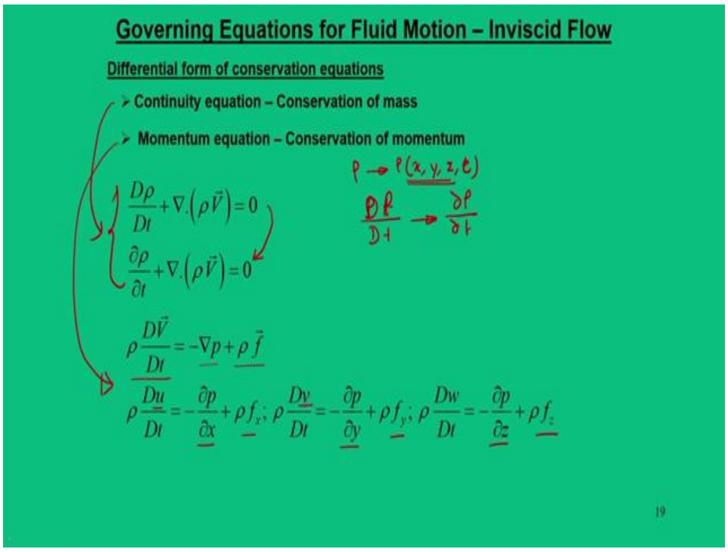

Now, we will talk about this the differential form of conservation of equations and that is continuity and momentum equations. In this slide, there are two equations given one is for continuity equations and the second one is the momentum equations.

So, in certain problems, they are addressed through differential approach. So, in which we represent them in differential form. So, in the first equations that is continuity equations, we say the total derivative contains the density. But however, or when you say density as a function of x, y, z and t and from the very basic assumption of the continuum, the density we say it is as a global properties and it do not change with space. So, we say it density can change only with time. So, this $\frac{D\rho}{Dt}$ happens to be change with respect to $\frac{\partial \rho}{\partial t}$. So, from these equations, we can directly write this

$$\frac{D\rho}{Dt}+\nabla.\left(\rho\vec{V}\right)=0$$

$$\frac{\partial\rho}{\partial t}+\nabla.\left(\rho\vec{V}\right)=0$$

$$\rho\frac{D\vec{V}}{Dt}=-\nabla p+\rho\vec{f}$$

$$\rho\frac{Du}{Dt}=-\frac{\partial p}{\partial x}+\rho f_x;\rho\frac{D\upsilon}{Dt}=-\frac{\partial p}{\partial y}+\rho f_y;\rho\frac{Dw}{Dt}=-\frac{\partial p}{\partial z}+\rho f_z$$

And for the momentum equations, we talks about velocity vector multiplied by density in the form that is left hand side of these equations and right hand side expressions are in terms of pressure force and body force.

Of course, these equations are based on certain unit volume and this velocity vector has three terms; one in x directions, in y direction and z directions for which respective forces are f_x, f_y and f_z. Similarly, pressure forces are $\frac{\partial p}{\partial x}$, $\frac{\partial p}{\partial y}$ and $\frac{\partial p}{\partial z}$. And with respect to velocity, the corresponding velocity in x direction is u, corresponding velocity in y direction is v and corresponding velocity in z direction is w.

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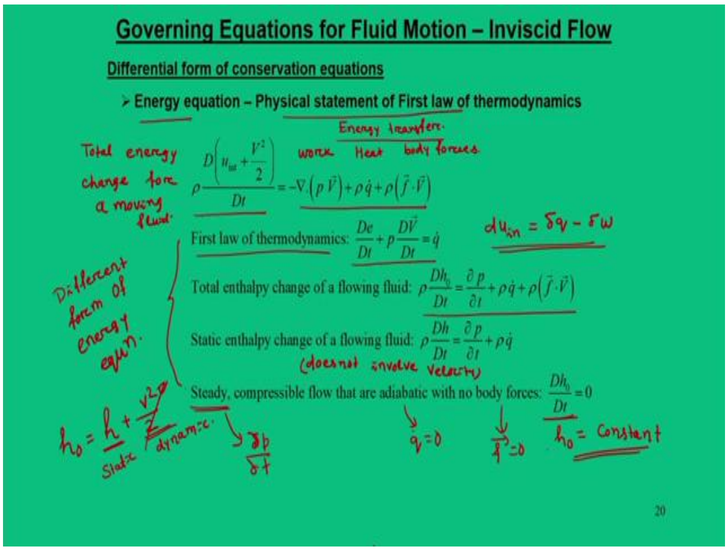

$$\rho \frac{D\left(u_{\text{int}} + \frac{V^2}{2}\right)}{Dt} = -\nabla.\left(p\vec{V}\right) + \rho\dot{q} + \rho\left(\vec{f}.\vec{V}\right)$$

And the most important part of the differential form of this equation is the energy equations. This energy equation, as I mentioned earlier is the physical statement of first law of thermodynamics. Now, when you deal with see this equations in differential form, this is essentially the term that comes from the change of the energy within the differential fluid element. So, this is total energy change for a moving fluid.

And the right hand side, there are three terms; all these terms refers to the energy transfer. Now, in the first term, the first term is the due to work; second term is due to heat and third term is due to body forces.

So, what we have seen is the total energy change is related to the energy transfer in terms of work heat or body force in this form of equations. Now this is what we say is a corollary of first one of the form of energy equation that is first law what we say is $dq = du + dw$. So, if you recall this change in the internal energy $du_{\text{int}} = \delta q - \delta w$, which is the basic form of first law of equations.

So, what we see these are the different forms of energy equations. In fact, these equations are most usable equations for our analysis. So, that is what we will talk about only end results. So, in one of the form of this complicated equation which is $\frac{De}{Dt} + p\frac{D\vec{V}}{Dt} = \dot{q}$, is the form of first law of thermodynamics.

Another parameter which we will be using in a compressible flow is the total enthalpy change in a flowing fluid. So, total enthalpy change consists of two parameters; one is the static enthalpy plus dynamic enthalpy. So, static enthalpy which we will discuss later. But for the time being if I can write this expression for $h_0 = h + \frac{V^2}{2}$; it has two components which is static part h, other is the dynamic part.

So, the static part of the equation h and when the fluid is at rest this part is not 0 or this dynamic part becomes important for a moving fluid. So, in some instances, if you are interested in this total enthalpy change of the fluid, so, the energy equations turns out to be of this form $\rho\frac{Dh_0}{Dt} = \frac{\partial p}{\partial t} + \rho\dot{q} + \rho\left(\vec{f}.\vec{V}\right)$.

Now, if for this total enthalpy change, if you do not consider the dynamic part, so, it is a static enthalpy, then we do not talk about the velocity vector. So, hence, this part that is for static fluid means we say does not involve velocity. Then, this particular equation again further simplified $\rho\frac{Dh}{Dt} = \frac{\partial p}{\partial t} + \rho\dot{q}$. And the most important part is that further simplification can be done for a steady flow in which we say $\frac{\partial p}{\partial t}$ =0 in this above equations and flow is adiabatic we say $\dot{q}$ =0 and there is no body force, we say $\vec{f}$ = 0.

So, the total derivative of stagnation enthalpy or total enthalpy $\frac{Dh_0}{Dt} = 0$ which means h_0 is equal to constant. This is one of the important consequences of energy equations which is very vital for the compressible flow analysis.

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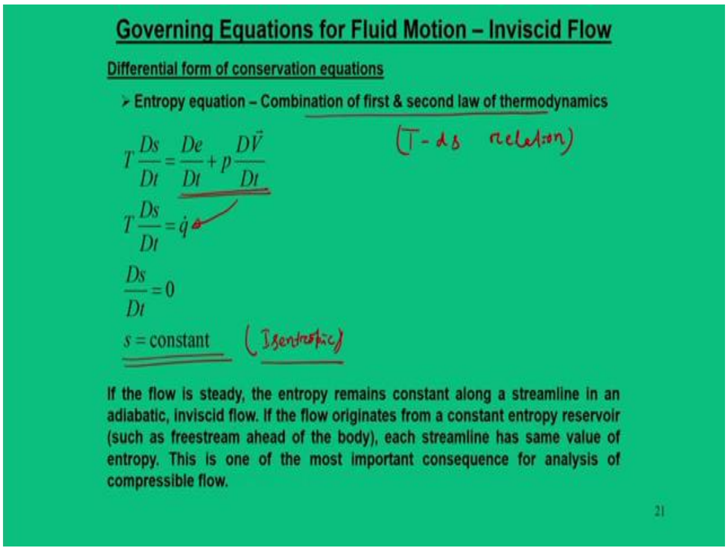

Now, moving further, the last part that we needs to be solved is the entropy equations. and when we say combine first law and second law of thermodynamics, this relations gives us the Tds relations.

This Tds relations are the fundamental parts that we do when we combine first law and second law of thermodynamics and that Tds equations is given by this form $T\frac{Ds}{Dt} = \frac{De}{Dt} + p\frac{D\vec{V}}{Dt}$.

Now, from the first law, this particular term $\frac{De}{Dt} + p\frac{D\vec{V}}{Dt}$ is nothing but your $\dot{q}$. So, from this when you simplify, we say $\frac{Ds}{Dt} = 0$ or s=const. So, this entropy equation, entropy remains constant. If it is entropy remains constant, we say it is a isentropic case. So, why we are particular about isentropic case? Because most of our compressible flow analysis is based on the reference situation as isentropic.

So, the conclusion from this analysis, what we can say from this the analysis of these equations that if a flow is steady, the entropy remains constant along a stream line in an adiabatic, inviscid flow. If the flow originates from a constant reservoir such as free steam ahead of the body, that means, from if it is originates, from a free stream, then each stream line has a same value of entropy. So, it is a definition for isentropic case. So, this is one of the important consequence for the analysis of compressible flow.

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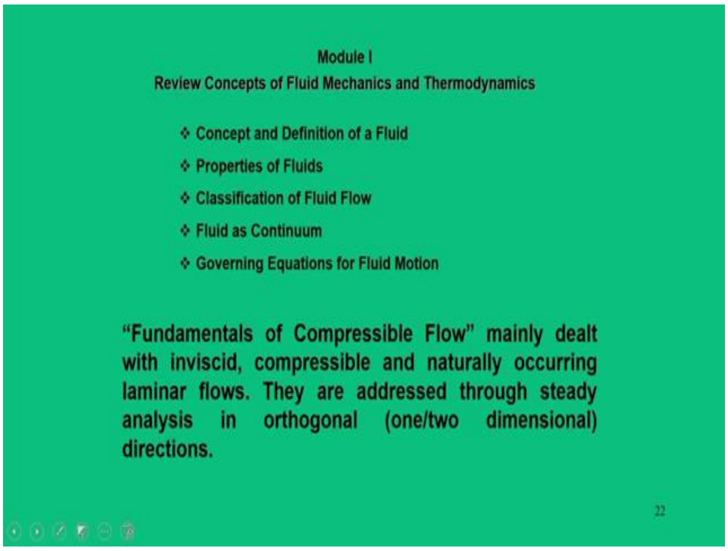

Now, with this, I am going to close this module 1. So, if I summarize what we have discussed in this module that this particular module was focused on some review aspects of the fluid mechanics and thermodynamics. Why we say fluid mechanics and thermodynamics? Because we are mainly deal with gases and the properties of fluid as well as thermodynamic behaviour is equally importance.

Now, while doing so, we discussed the concept and definition of the fluid in particular gas. We debated about the properties of fluids. There are many properties, but some properties are most important with respect to gas.

Then, we also talked about different classification of the fluid. Now, having said that when we do all this analysis, we have to represent the fluid motion mathematically. So, to study this fluid behaviour, we have to assume the fact that fluid has to be treated as a continuum. And finally, we discussed about the governing equation of the fluid motions and in particular, we highlighted that how these equations are important with respect to compressible flow.

So, in the all subsequent modules our main fundamental compressible theory will come into picture and in that fundamental compressible theory, we will take this governing equations as a background to study this compressible flow behaviour.

Now, our attention will be mainly dealt with inviscid compressible flow and it is a naturally flow and that has to be addressed through steady analysis and in terms of particularly, orthogonal direction that is one-dimensional or two-dimensional situations. So, I hope, I have clarified most of the fundamental things for the module 1.

Thank you very much. So, we will see in the next module. Have a good day.