Fundamentals of Compressible Flow
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Module - 07
Lecture – 19
Compressible Flow with Friction and Heat Transfer

Welcome to this course Fundamentals of Compressible Flow. We are in a new module 7. Name of this module is Compressible Flow with Friction and Heat Transfer.

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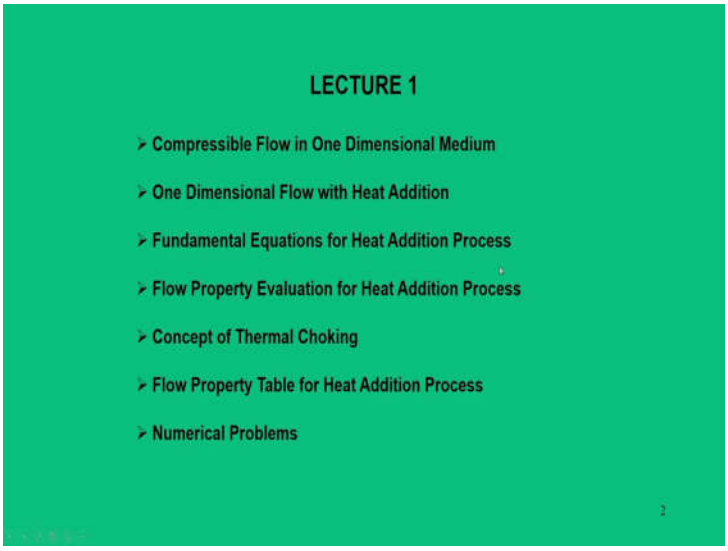

So, in the first lecture of this module, we will deal with the compressible flow with heat additions. And to be very specifics, entire analysis will be done in one-dimensional medium. Like in other situations, we will also derive the fundamental equations for heat addition process, the flow property evaluations when heat is added to the compressible flow. Then we will now introduce a concept of thermal choking.

Normally choking is the word which is used when the flow is sonic. Now, in this case the thermal choking will mean that the sonic flow is achieved through heat transfer process. Now, by introducing this thermal choking, we will see how the flow property parameter gets changed and how it becomes easy for us to define a property table known as flow property table for heat addition process. Now, based on our understanding, we will try to solve couple of problems based on this topic.

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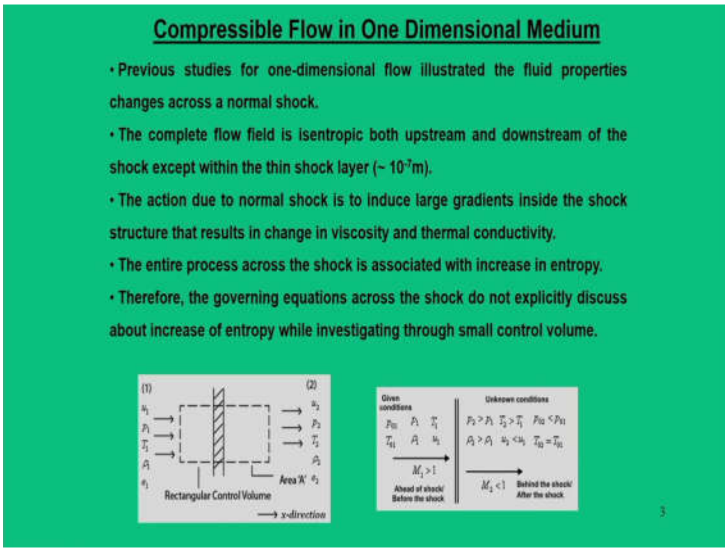

Now, just to revisit that whatever compressible flow in one-dimensional we have studied, it was mainly done with an analysis that we have a rectangular control volume, and it is imaginary in nature. So, we can see it is a kind of a constant area duct, there is some inflow with certain flow conditions and there is some exit outflow at certain flow conditions. And based on these inflow and outflow conditions, we derived the relations for a one-dimensional medium.

And in all earlier cases we never dealt with a heat transfer process in this one-dimensional medium or there was no work transfer also into this medium. So, one such instance we have seen that how the properties change could happen between 1 and 2 by considering a normal shock. So, the entire flow field is very discontinuous across this particular thin region or we call this as a thin shock layer of typically dimension of 10^{-7} m.

And across the shock layer we say there are large gradients in the flow properties inside the shock structure that results in the change in the viscosity and thermal conductivity in the medium. So, in this thin region, the flow happens to be a process that increases the entropy. So, even though the entire flow field is isentropic, but within this region the flow is non-isentropic. And in fact, we have proved that entropy always increases across a normal shock.

And wherever and when you derive these relations for entropy change, the pressure and temperature assumptions are such that it does not matter explicitly. We do not explicitly talk about the increase in entropy, because the already enhanced properties of pressure and temperature has ensured that entropy will increase. But in this process also, we say that entire imaginary duct which consists this normal shock is adiabatic in nature, in fact there is no heat addition into this process.

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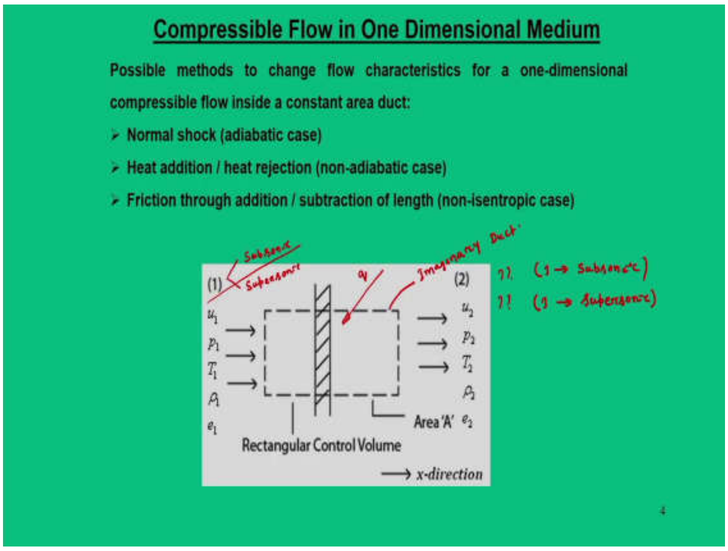

Now, in the similar logic, we will now move to another situations where we will say that we have same one-dimensional medium which is a rectangular control volume. And we can say it is an imaginary duct consisting of large streamlines. But what is the basic difference between earlier system and this that we are going to add some heat into this imaginary duct.

So, this is one way that flow property can change. Other mechanism in which the flow properties can be altered is through friction. So, in general there are three possible methods in a one-dimensional medium for a compressible flow in a constant area of duct. So, those methods are a normal shock which you have already covered, and that is in adiabatic situation.

And next method is heat addition or rejections. Obviously, it is a non adiabatic case since heat is being added or rejected. So, this is the main theme of this lecture. And last one is the friction through addition and subtraction of length which is also a non-isentropic case because it involves frictions. So, we will see these particular aspects in the subsequent class.

So, our main focus is that what will happen to a compressible flow when heat is added. So, the possible situation could be your condition 1 could be subsonic or supersonic. Now, if we say that it is a subsonic flow what will happen to downstream situations?

And we again if the condition 1 is supersonic what will happen to the downstream situation? So, these are the few questions we are going to answer from our analysis. So, to do all this analysis, we have to revisit the fundamental equations again.

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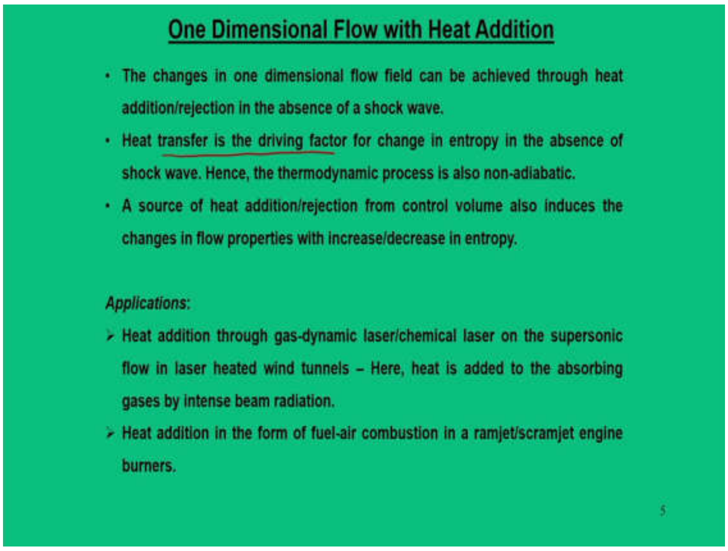

Now, just to say that what is the mechanism of heat addition process in a one-dimensional flow, we can say that in this particular duct, we are looking at the flow phenomena through an heat addition process. Of course, there is no normal shock which is sitting in the flow which we could have said that there are drastic change in properties.

But what change the flow properties is the heat transfer. So, as I mentioned that if our inflow can be subsonic, we will try to find out answer that if I add heat or subtract heat what will happen to the downstream condition 2. Or, in vice versa, if the inflow condition is supersonic, what will happen if I add or subtract heat to the downstream conditions. So, these are the few questions which you are going to answer in this analysis.

So, the very basic philosophy of bottom line is the fact that heat transfer is the driving factor. But since there is a addition of heat, so we can say the entropy is going to increase or decrease. If it is added entropy is going to increase; if it is taken out from the flow entropy is going to decrease. So, addition of heat will increase the entropy; or rejection of heat will decrease the entropy.

So, this heat transfer addition into a one-dimensional flow has many applications. One such application is that the next generation type a ground facilities like laser heated wind tunnels. So, researcher had thinking of to develop a laser heated wind tunnel in which the property conditions of the flow can be altered through the heat addition process using a laser beam.

So, when heat is added through this laser beam radiations, the gases that gets heated up, and they change their flow properties. And this is one type of application where the next generation aerodynamic facilities are built with.

The next type of heat addition mechanism happens during a fuel air combustion process. Normally high speed aerospace vehicles use a different kind of engines known as ramjet or scramjet engines, in which we consider the fuel layer combustion. So, in such process when you combust a fuel and air, the enormous amount of heat is generated into the medium.

So, thereby it alters the flow properties of the mixtures. So, these are the some of the applications in which the heat addition has shown its significance for a compressible flow.

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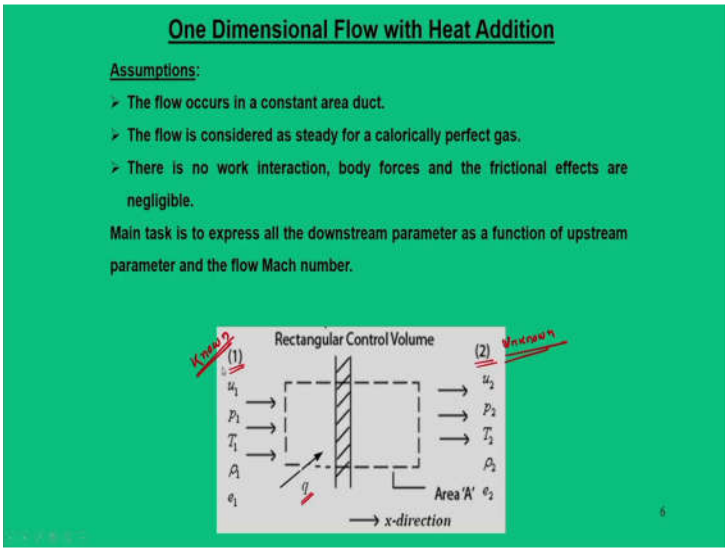

Now, let us discuss about the heat addition process in detail. So, the first thing that we are going to discuss is the heat addition process which is in a one-dimensional medium. And we consider a rectangular control volume with certain inflow conditions which is denoted by region 1. The outgoing flow conditions are denoted by the region 2. But what changes the properties is the heat transfer which is certain amount of heat is added into this medium.

So, we say that the flow occurs in a constant area duct, the flow is considered to be steady for a calorically perfect gas, there is no work interaction, body forces and frictional effects are negligible. So, these are the assumption based on which the one-dimensional flow equations are governed. So, the entire idea for this analysis is that the conditions 1, they are commonly known as known conditions; and the region 2, these are the unknown condition.

So, our main task like in earlier cases we did is that we express all the downstream parameters like all unknown parameters as a function of the upstream parameter and the flow Mach number. So, all output conditions for the region 2, we have to express as a function of region 1. So, the property parameters are defined with respect to known parameters for the region 1.

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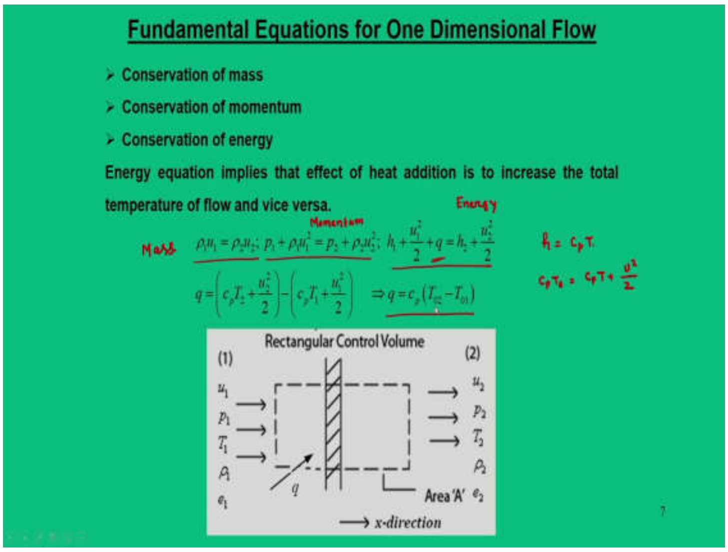

So, the next aspect we are going to study about this is that with this control volume of one-dimensional medium. We are going to write the fundamental equations. So, those equations are conservation of mass. And so this first equation which is written here is the $\rho_1 u_1 = \rho_2 u_2$. So, it is a mass conservation.

This is second equation which is used is $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$; this is the momentum equation and last equation that is $h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$. So, this is the equation that is going to change in our earlier situations, because q is introduced as a heat which is being added into the flow.

Now, from this energy equations, so we can write the enthalpy for calorically perfect gas as $h = c_p T$. So, for condition 1 and 2, from the energy equation if I take out q, then I can write $q = \left(c_p T_2 + \frac{u_2^2}{2} \right) - \left(c_p T_1 + \frac{u_1^2}{2} \right)$. So, we know that $c_p T_0 = c_p T + \frac{u^2}{2}$.

So, from this energy equation we can derive this particular value of $q = c_p \left(T_{02} - T_{01} \right)$. So, in other words, the heat addition is expressed as a function of total temperature. So, this is the first step.

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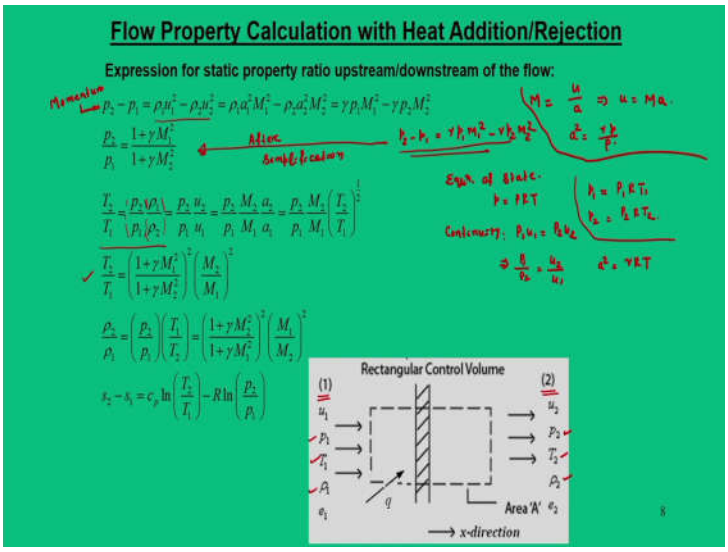

And next step what we are going to look at the flow property calculations. And here we will deal with the static property ratio. By static property ratio, I mean $\frac{p_2}{p_1}$, $\frac{T_2}{T_1}$, and $\frac{\rho_2}{\rho_1}$. So, to start with the static property ratio, we have to consider the momentum equations.

So, this particular equation is considered from momentum equation. So, from this momentum equation, we write this $p_2 - p_1 = \rho_1 u_1^2 - \rho_2 u_2^2$. Then what I use this relation Mach number as u/a. So, this will implies u is equal to Mach number times a.

So, for condition 1 and 2, if when I put u as M times a, so I can write $\rho_1 a_1^2 M_1^2 - \rho_2 a_2^2 M_2^2$. So, again we know that $a^2 = \frac{\gamma p}{\rho}$; when you put this relation here, we are now able to express a^2 as a function of pressures.

Now, the main equations which was we are here, we now write $p_2 - p_1 = \gamma p_1 M_1^2 - \gamma p_2 M_2^2$. So, now, we are able to find out pressure ratios as a function of Mach number M_1 and M_2. The second property which we are going to evaluate is temperature ratio.

To know the temperature ratio, we will have to recall that the equation of state. So, equation of state states that $p = \rho RT$. So, for condition 1 and 2, I can write $p_1 = \rho_1 RT_1$, $p_2 = \rho_2 RT_2$. So, this will give a relation that $\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2}$.

So, now in this equation, we already know the pressure ratio $\frac{p_2}{p_1}$ from the above. We now require $\frac{\rho_1}{\rho_2}$. So, this rho 1 by rho 2 we will get from continuity equation like $\rho_1 u_1 = \rho_2 u_2$. So, this will tell you that $\frac{\rho_1}{\rho_2} = \frac{u_2}{u_1}$. Now, again u_2 is equal to Mach number times a, we can write this. And also here we will now write $\frac{a_2}{a_1}$ as also a function of $\frac{T_2}{T_1}$.

So, we can write that $a^2 = \gamma RT$. So, for condition 1 and 2 this is expressed in the form of $\left(\frac{T_2}{T_1}\right)^{\frac{1}{2}}$. So, final expression now stands as $\frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{M_2}{M_1}\left(\frac{T_2}{T_1}\right)^{\frac{1}{2}}$.

So, we can bring this particular term here. We also know the pressure ratios from these equations. So, ultimately, we get a net results, and the temperature ratio by this form.

$$\frac{T_2}{T_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \left(\frac{M_2}{M_1}\right)^2$$

So, once we know the temperature ratio, we can also find the density ratio. So, density ratio is $\frac{\rho_2}{\rho_1}$. Again using equation of state we can write $\left(\frac{p_2}{p_1}\right)\left(\frac{T_1}{T_2}\right)$. So, prior to this we all know the expressions of these values. So, after simplification we can arrive at this.

$$\frac{\rho_2}{\rho_1} = \left(\frac{1+\gamma M_2^2}{1+\gamma M_1^2}\right)^2 \left(\frac{M_1}{M_2}\right)^2$$

So, in other words what is I have done here for pressure, temperature, and density, at the upstream and downstream conditions. They are expressed as M_1 and M_2. So, obviously, once when you know the pressure, temperature, and densities, we can also evaluate what is happening to the change in the entropy.

Now, effectively this particular expression does not explicitly focus about the heat addition process, but these heat addition q has already changed the properties ratio $\frac{p_2}{p_1}$ and $\frac{T_2}{T_1}$. And these ratios are mentioned here. And since it is a heat addition process, so entropy will increase. This is how we get to know that how we are going to evaluate the static flow properties during this process.

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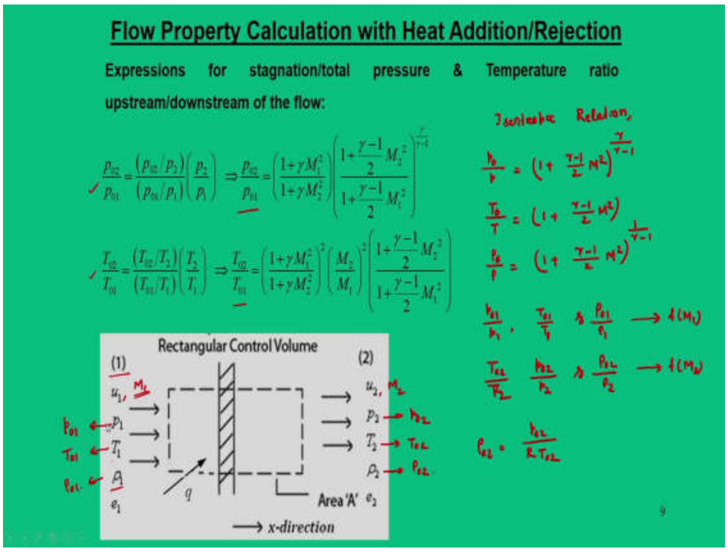

Now, we will move onto the flow property calculations for stagnation conditions. For stagnation conditions, we are now left with two parameters; one is stagnation pressure, other is the stagnation temperature.

So, the stagnation properties, what I can say is for the condition 1 which is we can say u_1 or Mach number M_1 for the pressure p_1, we can define p_{01}. And for temperature T_1 and Mach number M_1, we can also define T_{01}. For M_1 and ρ_1, we can also define ρ_{01}.

So, and similarly for u_2 and M_2, this will talk about p_{02}, T_{02} and ρ_{02}. Here mostly an parameter is p_{02} and T_{02}.

So, we can write the $\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$; $\frac{T_0}{T} = \left(1 + \frac{\gamma - 1}{2} M^2\right)$ and $\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$. So, these are arbitrary properties.

We can now define p_{01}/p_1, T_{01}/T_1, and ρ_{01}/ρ_1. Similarly, we can write p_{02}/p_2, T_{02}/T_2 and ρ_{02}/ρ_2. So, here it is a function of M_2, this is a function of M_1. But we are now require the ratio between $\frac{p_{02}}{p_{01}}$. So, we write this $p_{02} = \left(p_{02}/p_2\right) p_2$; and in the denominator it is $\left(p_{01}/p_1\right) p_1$.

So, these p_{02}/p_2, we get from these expressions where M is replaced with M_2, and p_{01}/p_1 is replaced by same expression where M is replaced by M_1. Also in the static property relations, we already know what is the value ratio of $\frac{p_2}{p_1}$. So, after putting this, we are in now in a position to simplify a relation which is $\frac{p_{02}}{p_{01}}$.

So, in the similar manner, one can evaluate the stagnation property ratio $\frac{T_{02}}{T_{01}}$, and they are expressed as both function of M_1 and M_2. So, this is how we are going to evaluate the stagnation properties. Of course, once you know p_{02} and T_{02}, one can also evaluate also ρ_{02}, where ρ_{02} can also be written as $\frac{p_{02}}{RT_{02}}$. So, likewise all properties of downstream conditions are expressed as function of upstream parameters.

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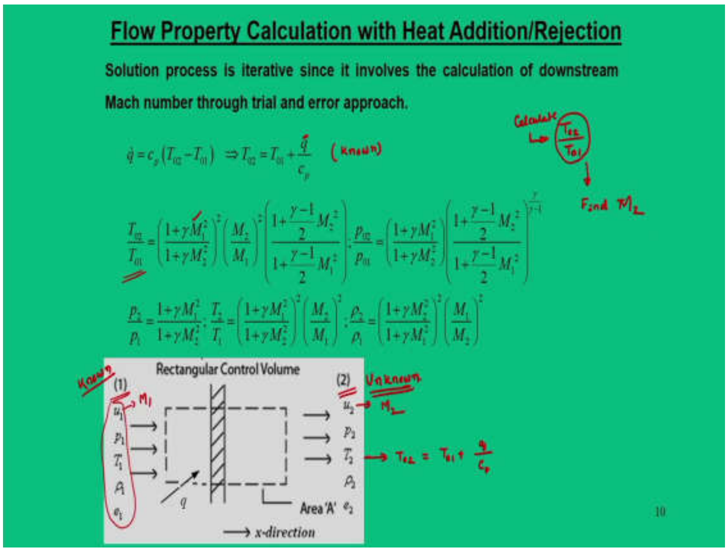

So, with these equations we are now summarize that how we are going to evaluate the downstream flow properties for a heat addition process in a one-dimensional compressible medium.

But if you look at actually the these equations what we have seen is that let us take a particular equation let us say $\frac{T_{02}}{T_{01}}$, so if we want to find out T_{02} then we have to know T_{01} M$_1$ and M$_2$, then only it will be able to find T_{02}.

But in general the region 1 is generally known condition, and region 2 is unknown condition. So, effectively we do not know, effectively we have two unknown parameters in the region 2 that is T_{02} and M$_2$.

So, even this also is true for all other relations where we need to require at least one parameter in the downstream either Mach number or any of the static properties. So, here, but to do that, we do not have any other choice rather we only know that some heat is getting added into the medium.

So, to do that what we see is that we can express this $q = c_p\left(T_{02} - T_{01}\right)$. So, from this equation, we can evaluate $T_{02} = T_{01} + \frac{q}{c_p}$. So, in this equation what we know is T_{01}, q and

c_p. So, this is a known quantity. So, with this known upstream parameter, we only know one downstream parameter $T_{02} = T_{01} + \frac{q}{c_p}$

So, now next step is that you calculate $\frac{T_{02}}{T_{01}}$. When you calculate $\frac{T_{02}}{T_{01}}$, for this ratio, we are now in a position that to use these equations of $\frac{T_{02}}{T_{01}}$, where these is a known parameter, this ratio is a known parameter we have one known parameter M_1.

So, iteratively we have we have to do a trial and error approach to find M_2. So, this is how the iterated approach is involved to calculate Mach number M_2. Now, we are in a position that we know M_1, we also know M_2. So, then all other parameters can be calculated from this process. So, entire calculations involves the trial and error process only for the calculation of Mach number.

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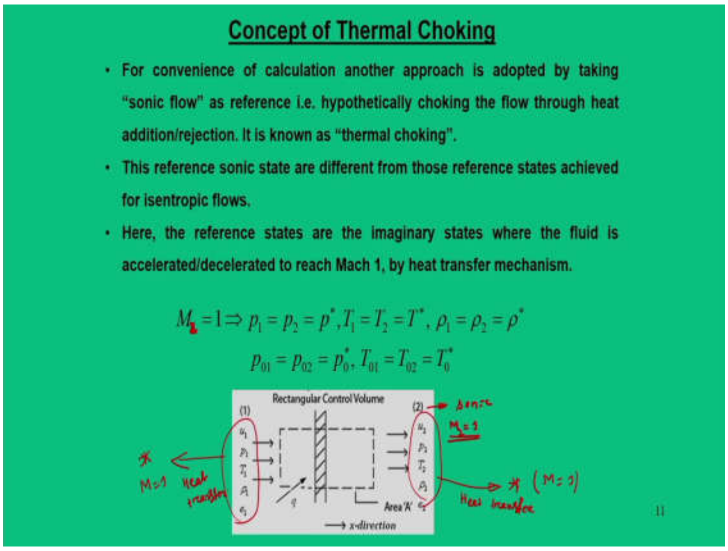

To avoid such calculations, what we are now going to propose a concept with respect to a reference conditions. So, what does this mean that had the flow been sonic at the exit, what would have been its condition? Like for a given arbitrary condition of 1, if the flow is going to be sonic.

If we say condition 2 happens to be sonic means I can say at this case we are saying Mach number as 1, if we add continuously heat into this duct such that the condition 2 for which Mach number is sonic, then we can define a reference condition as p^* T^* ρ^* p_0^* and T_0^*.

This is also possible that we can bring this condition to star conditions. When I say star condition that point I will say its Mach number will be 1, here I say Mach number will be 1, but how this star condition is reached, through heat transfer.

So, it means that the sonic flow reference condition can be achieved hypothetically by choking the flow through heat addition or rejection; such a method is known as thermal choking.

So, here we introduce a word thermal choking because the choking conditions or sonic condition is achieved through heat transfer. In all other previous instances, we also define these choking conditions, but that was with reference to the mass flow rate. So, that is the basic difference that how thermal choking is different from the mass flow rate choking.

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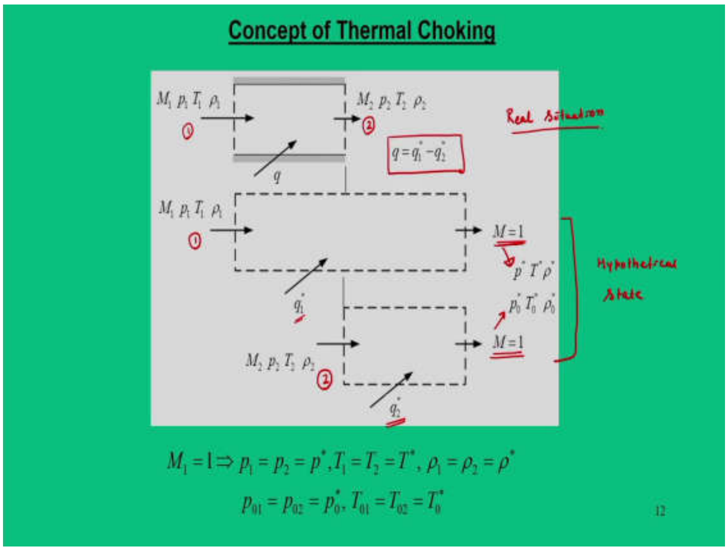

Now, to simplify this analysis, let me cite this example by using a pictorial representation. For instance, we have a duct one-dimensional duct where the condition 1 is defined. For a given condition we have some downstream condition 2 like M_2 p_2 and ρ_2. So, the change in the flow properties is achieved through a heat transfer is q.

Now, what we imagine, so this is a real condition. And in a hypothetical situation what we assume that if I want to go from the condition 1 to a sonic state through heat transfer, so I have to add a magnitude of heat that is q_1* such that your downstream Mach number will be unity. So, under those circumstances, I can define these pressure conditions in the downstream as p* T^* ρ* p_0* $T_0^*$and ρ_0*.

So, with the same logic, if I will go from condition 2 to get this star condition, so that the flow is sonic at the downstream, then I have to add q_2* amount of heat. Thus since both the states are a hypothetical state. But the very basic fact that by assuming these hypothetical states, I am able to calculate the amount that I am going to add since the reference conditions are same for the both the states.

So, this will give me a equation that from this pictorial representation, I can say the amount of heat that I am going to add in a real situation is nothing but the hypothetical heat transfer q_1* for the state 1 and state 2. And in fact, we will now show that how this particular concept will help us in simplifying our flow calculation properties.

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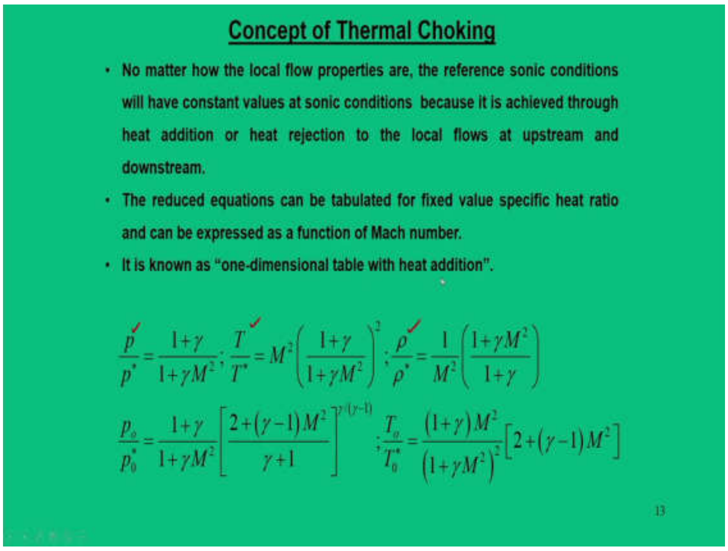

So, if I do that all governing equations for the static properties ratios or stagnation properties ratio, like if you say that for the star condition if you put Mach number as 1, tthose equations can be simplified in this fashion. And in fact what we see in this equation that p, T, ρ, are any arbitrary properties for any real situation; and star parameters, they are the corresponding imaginary states when the flow is brought to sonic state through heat addition.

So, in such cases, the equations get simplified. And what we see is they are only function of Mach number. So, this gives an indication that we can form a table just by changing the Mach number and find these ratios so that it will help us in simplifying our calculations.

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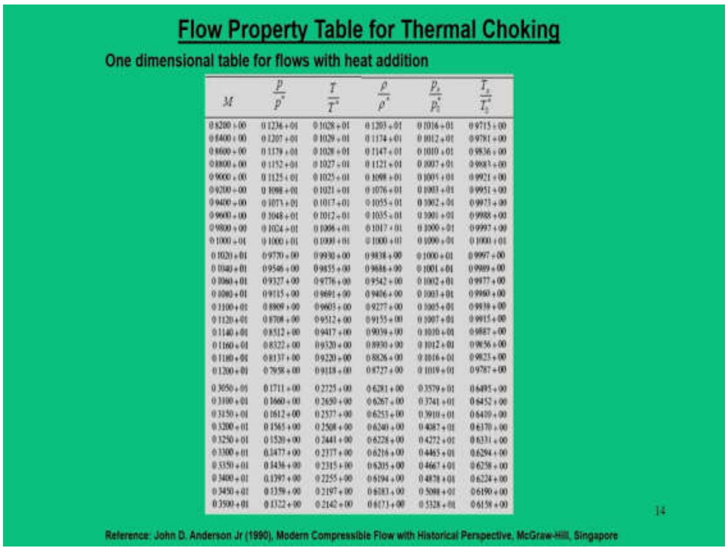

Flow Property Table for Thermal Choking

One dimensional table for flows with heat addition

M	$\frac{p}{p^*}$	$\frac{T}{T^*}$	$\frac{\rho}{\rho^*}$	$\frac{p_o}{p_o^*}$	$\frac{T_o}{T_o^*}$
0.8200+00	0.1236+01	0.1028+01	0.1203+01	0.1016+01	0.9715+00
0.8400+00	0.1207+01	0.1029+01	0.1174+01	0.1012+01	0.9781+00
0.8600+00	0.1179+01	0.1028+01	0.1147+01	0.1010+01	0.9836+00
0.8800+00	0.1152+01	0.1027+01	0.1121+01	0.1007+01	0.9883+00
0.9000+00	0.1125+01	0.1025+01	0.1098+01	0.1005+01	0.9921+00
0.9200+00	0.1098+01	0.1021+01	0.1076+01	0.1003+01	0.9951+00
0.9400+00	0.1073+01	0.1017+01	0.1055+01	0.1002+01	0.9973+00
0.9600+00	0.1048+01	0.1012+01	0.1035+01	0.1001+01	0.9988+00
0.9800+00	0.1024+01	0.1006+01	0.1017+01	0.1000+01	0.9997+00
0.1000+01	0.1000+01	0.1000+01	0.1000+01	0.1000+01	0.1000+01
0.1020+01	0.9770+00	0.9930+00	0.9838+00	0.1000+01	0.9997+00
0.1040+01	0.9546+00	0.9855+00	0.9686+00	0.1001+01	0.9989+00
0.1060+01	0.9327+00	0.9776+00	0.9542+00	0.1002+01	0.9977+00
0.1080+01	0.9115+00	0.9691+00	0.9406+00	0.1003+01	0.9960+00
0.1100+01	0.8909+00	0.9603+00	0.9277+00	0.1005+01	0.9939+00
0.1120+01	0.8708+00	0.9512+00	0.9155+00	0.1007+01	0.9915+00
0.1140+01	0.8512+00	0.9417+00	0.9039+00	0.1010+01	0.9887+00
0.1160+01	0.8322+00	0.9320+00	0.8930+00	0.1012+01	0.9856+00
0.1180+01	0.8137+00	0.9220+00	0.8826+00	0.1016+01	0.9823+00
0.1200+01	0.7958+00	0.9118+00	0.8727+00	0.1019+01	0.9787+00
0.3050+01	0.1711+00	0.2725+00	0.6281+00	0.3579+01	0.6495+00
0.3100+01	0.1660+00	0.2650+00	0.6267+00	0.3741+01	0.6452+00
0.3150+01	0.1612+00	0.2577+00	0.6253+00	0.3910+01	0.6410+00
0.3200+01	0.1565+00	0.2508+00	0.6240+00	0.4087+01	0.6370+00
0.3250+01	0.1520+00	0.2441+00	0.6228+00	0.4272+01	0.6331+00
0.3300+01	0.1477+00	0.2377+00	0.6216+00	0.4465+01	0.6294+00
0.3350+01	0.1436+00	0.2315+00	0.6205+00	0.4667+01	0.6258+00
0.3400+01	0.1397+00	0.2255+00	0.6194+00	0.4878+01	0.6224+00
0.3450+01	0.1359+00	0.2197+00	0.6183+00	0.5098+01	0.6190+00
0.3500+01	0.1322+00	0.2142+00	0.6173+00	0.5328+01	0.6158+00

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Reference: John D. Anderson Jr (1990), Modern Compressible Flow with Historical Perspective, McGraw-Hill, Singapore

So, that table which is known as one dimension table with heat additions. So, this table is presented here. So, it is taken from the extract of the book John D. Anderson that is Modern Compressible Flow with Historical Prospective McGraw-Hill. What has been shown here I have just taken some extract from this.

So, there are about 6 columns. So, the first column talks about the Mach number. That means, if you know the Mach number then we can find out these property ratios – $\frac{p}{p^*}$, $\frac{T}{T^*}$, $\frac{\rho}{\rho^*}$, $\frac{p_o}{p_0^*}$, $\frac{T_o}{T_0^*}$.

So, this table can be prepared. In fact, if we are not really supposed to know the or remember those equations big equations rather just by referring the table, just by knowing one value in this table we can find out the complete information in that row.

For instance if I know $\frac{p_o}{p_0^*}$ as maybe 1, then I can say that all the numbers in that row are known. So, this gives a very easier method to evaluate the flow properties in a heat transfer situation in a compressible flow.

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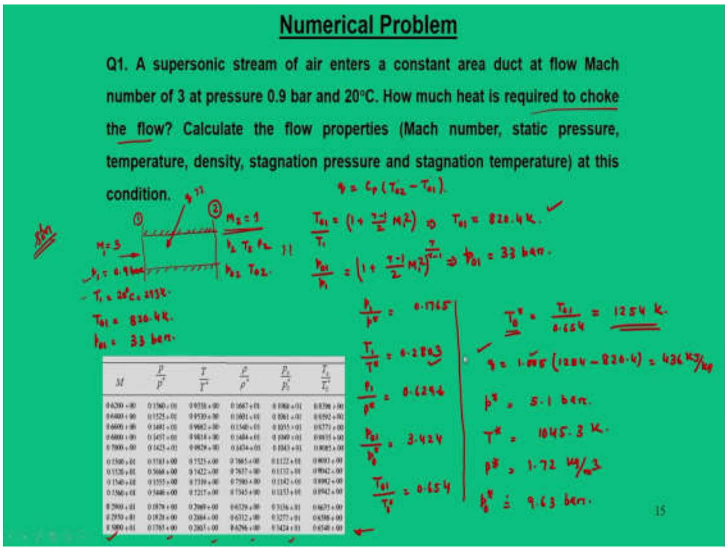

Now, with this logic, we will now try to solve some simple problems which we will talk about how one has to refer a gas table for a given flow situations. So, the question that is given is that a supersonic stream of air enters in a constant area duct at a flow Mach number of 3 at a pressure 0.9 bar, and 20°C.

So, the question that is asked is how much heat is required to choke the flow? Calculate the flow properties the in the downstream. In the downstream means what properties you need to calculate the Mach number, static pressure, static temperature, density, stagnation pressure and stagnation temperature.

So, what the problem that is given to us is that we have constant area duct. So, you draw these situations whatever the flow is entering from region 1 and here the conditions that are given as M is equal to 3 or M_1 is equal to 3, p_1 is equal to 0.9 bar T_1 is equal to 20°C or 293 K.

What is required at the condition 2, we say that the condition 2 is achieved by choking. So, choking the condition means here we say M_2 is nothing but 1. So, for this condition 1, we have to find out to get this condition 2, how much q we are going to add? So, we also have to require p_2, T_2, ρ_2, and p_{02}, T_{02}. So, these are the important parameter which needs to be found out.

But before we start this, we can recall that this $q = c_p\left(T_{02} - T_{01}\right)$. Now, since this is not known, we have to find out, and T_{01} is known to us. So, how do you first find out T_{01}?

So, $T_{01} = \left(1 + \frac{\gamma - 1}{2} M_1^{\,2}\right) T_1$. So, you know M_1 as 3, then this will give you T and T_1 as 293 K. So, this will tell you T_{01} would be 820.4 K; just put M_1 is 3. So, I can write T_{01} as 820.4 K.

Similarly, we can write $\frac{p_{01}}{p_1} = \left(1 + \frac{\gamma - 1}{2} M_1^{\,2}\right)^{\frac{\gamma}{\gamma - 1}}$. So, this will say p_{01} would be 33 bar. So, I say p_{01} is 33 bar.

So, what I am now going to see that if for a upstream Mach number of 3 by adding heat if I want to choke the flow, then I should refer the Mach number M as 3 in this particular column of Mach number. So, here we can see the3 at this last row, we can get at this number as Mach number of 3.

So, for that I can say $\frac{p_1}{p^*}$ as 0.1765. I can note down all the values in this row $\frac{T_1}{T^*}$ because I have put for this condition 1. So, p is replaced as p_1, T is replaced as T_1. This is 0.2803 $\frac{\rho_1}{\rho^*}$ is 0.6296, $\frac{p_{o1}}{p_0^*}$ 3.424, $\frac{T_{o1}}{T_0^*}$ as 0.654.

So, I can find out T_0^* as T_{o1}/0.654. T_{o1} is known. So, we can find out this value as 1254K. So, we can say q is equal to c_p that is 1.005(1254-820.4). So, this number will be about 436 kJ/kg, because c_p is 1.005 kJ/kg. So, thus we are able to find out how much heat is required.

The next one, we know all property parameters – p_1, T_1, P_{01} and T_{01}. So, from these ratios I can say we can say p^* as 5.1 bar, T^* can be calculated from this expression that is 1045.3 K, then $\rho^*$is equal to 1.72 kg/m^3.

We also can know that as 9.63 bar. So, these numbers are very simple because we know this ratio, we know all the upstream parameters. So, the star conditions can be found out. But the most important segment is the value of q. And here the important information that is required is T_0^* to calculate q.

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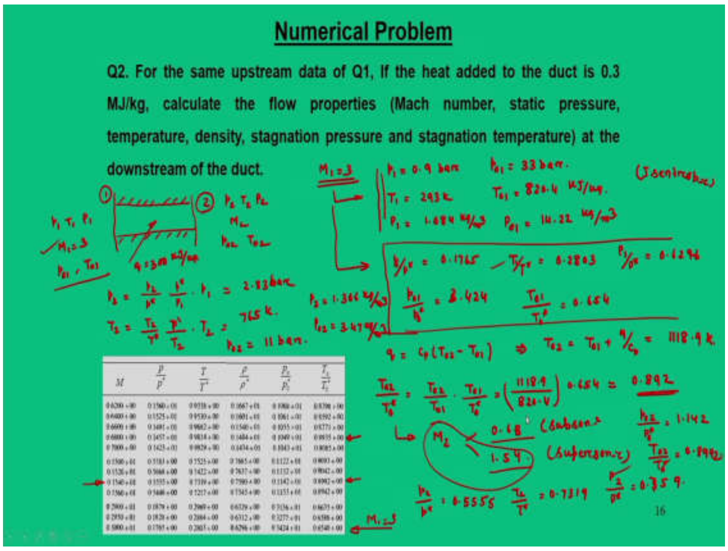

In the second problem, what has been given with the same upstream data if heat added is 0.3 MJ/kg. We have shown that in this case your heat added to choke the flow is required is 436. Now, the question says here now we do not want to add 436 kJ/kg, we are going to add only 300 kJ/kg or 0.3 MJ/kg.

So, in that case what is going to happen in the flow properties? So, here the condition 2 is not a star condition, the conditions 2 is any real situation. So, we define this as p_2, T_2, ρ_2, M_2, p_{02}, T_{02}; condition 1 as p_1, T_1, ρ_1, M_1 that is 3, p_{01} and T_{01}. So, heat is added q is equal to 300 kJ/kg. From the last question data, we can gather the information for M_1 is equal to 3.

For M_1 is equal to 3, we say the data which are known to us p_1 – 0.9 bar, p_{01} – 33 bar, T_1 is 293 K, T_{01} happens to be 820.4 K, and ρ_1 1.084 kg/m^3, ρ_{01} would be 14.22 kg /m^3. So, these information are with respect to Mach number 3 from isentropic relation.

Now, referring to this table for this Mach number 3, when heat is added to choke the flow we can say from this row we can say $\frac{p_1}{p^*}$ as 0.1765. $\frac{T_1}{T^*}$ is 0.2803, $\frac{\rho_1}{\rho^*}$ is 0.6296, then $\frac{p_{o1}}{p_0^*}$ 3.424, $\frac{T_{o1}}{T_0^*}$ as 0.654. So, this is the data which we get.

But what is actually added is 300 kJ/kg. So, now, I can write $q = c_p\left(T_{02} - T_{01}\right)$. So, this will give you $T_{02} = T_{01} + \frac{q}{c_p}$, and that number I can find out 1118.9 K. Then I can find out what is $\frac{T_{o2}}{T_0^*}$.

So, this is nothing but $\left(\frac{T_{o2}}{T_{o1}}\right)\left(\frac{T_{o1}}{T_0^*}\right)$. So, I can write this is 1118.9, T_{01} is 820.4. This ratio $\frac{T_{o1}}{T_0^*}$ is 0.654. So, this will give you a value 0.892.

Now, I am in a position to refer this table for this ratio. If you look at this number, then we will find that there are two numbers which is close to it; one is 0.8935, other is 0.899. So, this will give you your M_2 can be 0.68, or M_2 can be 1.54. So, there are two values; one is subsonic, other is supersonic.

So, what has been seen here that the subsonic solution will not be possible. Whereas, supersonic solution will be possible, why I will come to this in the next class because the supersonic flow without coming back to Mach sonic flow, it cannot again go back to subsonic flow, that means, it is not possible to bring a supersonic flow to subsonic flow without reaching a sonic state, so that is not possible.

So, in this case, we cannot reach the sub sonic state directly. So, one solution that possible is your M_2 should be 1.54. Now, when I say M_2 is 1.54, then I can compute p_2.

So, I have to take all the data as this. So, I can say for M_2 is equal to 1.54, we can say $\frac{p_2}{p^*}$ is 0.5555, $\frac{T_2}{T^*}$ 0.7319, $\frac{\rho_2}{\rho^*}$ 0.759, $\frac{p_{o2}}{p_0^*}$ as 1.142, and $\frac{T_{o2}}{T_0^*}$ as 0.8992. So, once we know this, we can find $p_2 = \left(\frac{p_2}{p^*}\right)\left(\frac{p^*}{p_1}\right)p_1$. So, this number would be 2.83 bar.

Similarly, we can say $T_2 = \left(\frac{T_2}{T^*}\right)\left(\frac{T^*}{T_1}\right)T_1$. So, we all know the property values here, here, and here. So, we can say T_2 is 765 K. Likewise, I can compute p_{02} as 11 bar, ρ_2 as 1.306 kg/m^3, and ρ_{02} as 3.47 kg/m^3. So, as we see that how this table makes the calculation simple. So, you do not have to explicitly use these equations.

So, this is all about for this today's lecture. I hope I have made you understand for the content of this lecture.

Thank you for your attention.