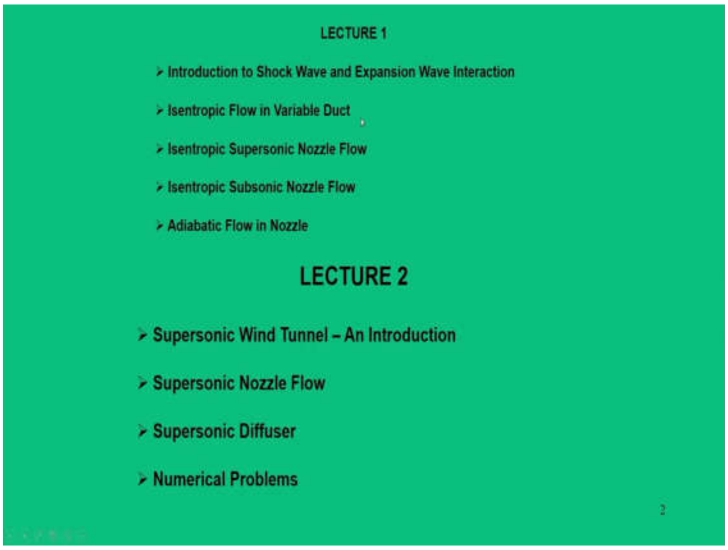
Fundamentals of Compressible Flow
Prof. Niranjan Sahoo
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 06
Lecture - 18
Interaction of Shocks and Expansion Waves

Welcome to this course Fundamentals of Compressible Flow. We are in module 6. That is Interaction of Shocks and Expansion Waves.

(Refer Slide Time: 00:47)



So, in the previous lecture of this module, we discussed about the importance of shock wave and expansion waves and their interactions. So, one such classical example that we can have is flow through a variable area duct.

So, in that concept we discussed about isentropic flow in a variable area duct. We also discussed about if the duct happens to deliver a supersonic flow then what would be its passage. So we call this as a supersonic nozzle flow. Now, if we do not deliver supersonic Mach number at the nozzle exit, it could be due to the pressure difference across the nozzle.

So, in that aspects if adequate pressure difference are not maintained, flow still remains isentropic, but we land of in getting subsonic flow at the exit. So, we call this as a isentropic subsonic nozzle flow.

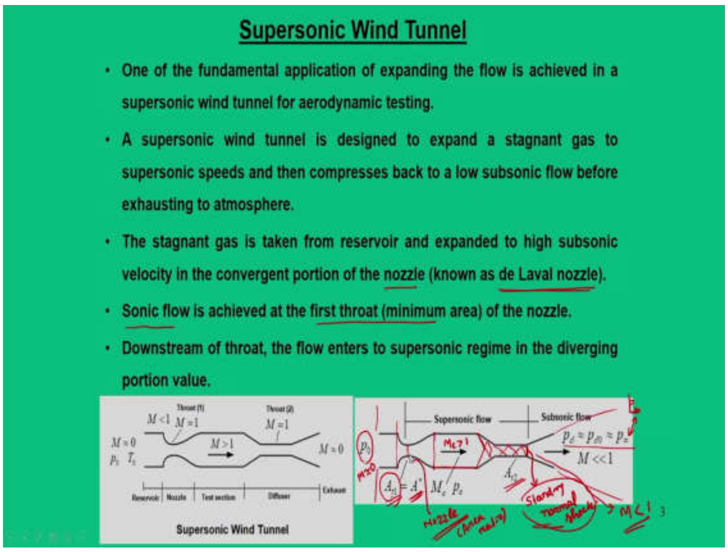
Now, if you see these two aspects; the entire flow field is always isentropic. Now, if the flow is no longer isentropic or there happens to be normal shocks, oblique shocks at any cross section of the nozzle, then those kind of flow is treated as a adiabatic flow in the nozzles.

So, in fact, we discussed exhaustively under what circumstances or what pressure ratio conditions flow regime of different Mach numbers can be obtained for a nozzle flow. Now, in this particular lecture we will mostly focus on the similar aspect, but we will call this as a diffuser flow. So, we call this as a supersonic diffuser.

In fact, whenever we want to have a nozzle flow, diffuser is also integral part of it. One such example we are going to give, it has a supersonic wind tunnel. Where, both supersonic nozzle and supersonic diffuser are integral part of it.

So, in this particular lecture we will mostly discussed about the summary of supersonic nozzle flow and then I will give some introduction about a supersonic wind tunnel, where nozzle and diffusers are integral part of it. And then we will mostly discuss on the fundamental concepts of a supersonic diffuser, how it operates.

(Refer Slide Time: 03:52)



So, just to give a brief introduction about a supersonic wind tunnel. So, most of the aerodynamic facilities operating in high speed regime has Mach number in the supersonic regime and all the testing needs to be done in the test section of the tunnel. So such a tunnel we call this as a supersonic wind tunnel.

So as shown in this figure, the schematic diagram of this particular supersonic wind tunnel has five major sections. So, first section is the reservoir, where we can consider the storage of the gas at very high pressure and temperatures, and such pressures and temperatures are treated to be stagnation pressure and stagnation temperature. And of course, there is no gas velocity, so Mach number is nearly 0.

This flow, when it is allowed to pass through a nozzle and since the flow is at very high pressure and temperature, when it is allowed to pass through a passage which in this case we call this as a nozzle, the flow enters to the nozzle and attains a supersonic Mach number in the test sections.

So, depending on the area ratio of the throat and the exit area of this nozzle, the flow attains supersonic Mach number in the test sections. Now, after desired testing in this test section, the flow is again allowed to pass through a converging potion. This converging portion is similar to that a nozzle, but we call this as a diffuser, and this diffuser has almost similar shapes like a converging shape, some constant area shape, and then a diverging shape.

However, the internal features of a nozzle and diffusers are different. In fact lengths, the throats, all the diameters are different. So what the basic philosophy is that when you operate this supersonic wind tunnel both the nozzle and diffuser are part of it.

So; obviously, in this figure if you see that the flow is at stagnation pressure p_0 and it is allowed to pass through this nozzle, the flow become sonic at the throat; that means, it attains the Mach number of one at the throat and then the flow tries to expand further and attains desired supersonic Mach number M_e and corresponding pressure p_e.

So, after required testing in the test section, there are two options that one can directly exhaust the flow to atmosphere or we can pass through a diffuser. So here, the role of diffuser is very important because when you pass this exhaust flow directly to atmosphere, we will show that the losses incurred in this mechanism will be very high.

So that losses or in those cases your stagnation pressure requirement will be also very high. So, in such cases, so it is a possible solution that we need to employ a diffuser, and that diffuser will slow down the supersonic velocity to a very subsonic value. When the flow comes out from this diffuser, it has a very low subsonic or almost 0 velocity.

And at the same time, the pressure that comes out at the exhaust, we call this as a diffuser pressure or static pressure or stagnation pressure, they should be close to the free stream pressure which is prevailing in the ambient conditions.

So, the very basic bottom line of this fact that when the flow enters, it is in the stagnation conditions, but have very high pressure and also 0 velocity. When the flow leaves, it has also very minimal or very less velocity and the exhaust pressure of the diffuser and the ambient pressure should match.

So in other words, if you have ambient pressures p_∞ and this diffuser pressure should match. Whereas, in the in the inlet condition, your p_0 and your M is also tends to 0, there is no Mach number. So these are the basic philosophy.

Now, the challenging part of this design of the supersonic wind tunnel is the diffuser sections, because if you do not incorporate a diffuser, then there will be loss of total pressure. So we will show that how we are going to retain a diffuser of appropriate design.

So the two important parts that needs to be highlighted here, since we say its a nozzle, so when you see this nozzle part, there is one minimum area we call this as a throat area. So we call this is as a first throat and where the flow attains sonic velocity. So, the first throat, we can say A_{t1} is equal to A* that means area at the first throat.

Even we will have a second throat, because in the diffuser the flow is already a supersonic in the test section and this has to encounter a series of oblique shocks that emanates from the inner wall of this test sections. And they try to interact and this process will keep on happening and finally, we land of a standing normal shock that has to be there at the throat of the diffuser. So we say it is a standing normal shock.

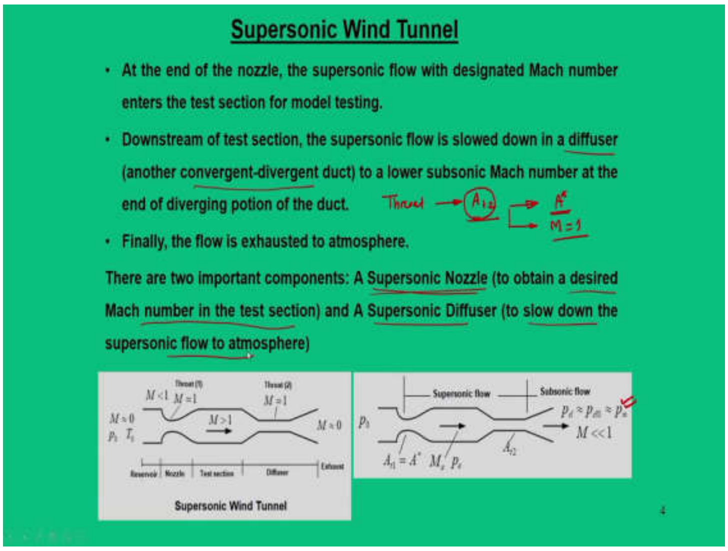
So, why we say standing normal shock? Only there is machines, the flow is already at very high supersonic value in the test section it has to be slow down. So, to slow down the possible mechanism is the normal shock so that downstream of the flow will have subsonic value. So, in this region the flow M is less than 1.

So, when the flow reaches the subsonic value, you give a diverging pressure section or passage, so that the further slowdown is possible with mapping of the total pressure of at the diffuser exist with respect to free stream pressure. So this is how the philosophy of a supersonic wind tunnel works.

And, in our last previous lecture, we exhaustively discussed that how a supersonic flow can be achieved in a nozzle. So, In fact, when you say this nozzle exit, typically it is connected to a test sections; till our last class we are concentrating on this nozzle part, and when you attached this to this test section its a nothing but a constant area duct. And in this test sections we get the desired Mach number. That desired Mach number is decided by the area ratio of this nozzle.

So, this is how the philosophy of supersonic wind tunnel. And, some important point I needs to mention that many a times this nozzle, we call this as a de Laval nozzle. It is a scientific name for a convergent diverging nozzle as a de Laval nozzle. And, there are two throats, first throat which occurs at the minimum area where sonic flow is achieved, so that is in the portion of a nozzle.

(Refer Slide Time: 13:18)

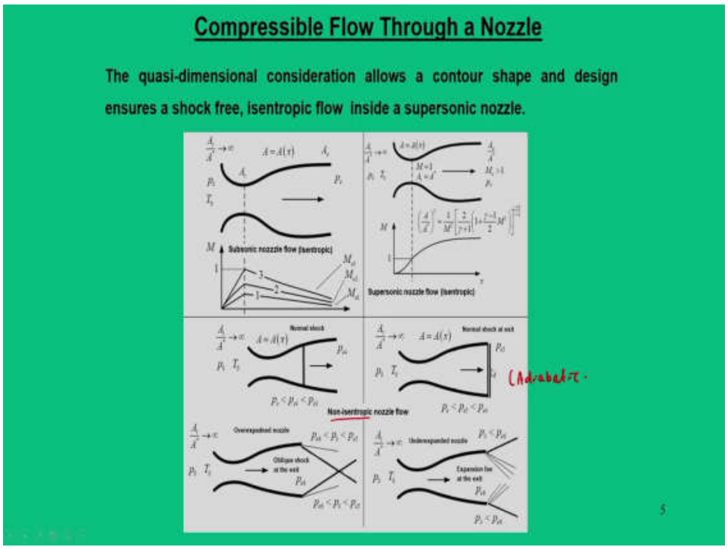


And, when the flow is exhausted to atmosphere it is close to the free stream pressure at the atmospheric conditions. And also, in the diffuser portion, there is another divergent section of the duct which is known as diffuser. And this diffuser has also a throat for which the area is A_{t2}, and here also it is nothing but the choked area or A* and M is also attain 1.

So very basic difference between nozzle throat and diffuser throat is that at the nozzle throat the flow attains sonic velocity and thereby it accelerates and get a supersonic flow. But in a diffuser part, the flow is already supersonic.

It is decelerated to a sonic value in the second throat and then to a very low subsonic Mach number at the exit. So, our main intention is that two components that is supersonic nozzle flow that gives desired Mach number in the test section. Other is the supersonic diffuser that slows down the supersonic flow to the atmosphere.

(Refer Slide Time: 15:00)



Many a times what will happen that when we discussed about the supersonic flow, we found that when the flow is completely isentropic that is the nozzle is shock free. But what happens? If the flow is non isentropic, if the flow is non isentropic or in our term we call this as adiabatic then, we land of having getting normal shocks, and this normal shock is controlled through pressure ratio between the reservoir and the exit pressure.

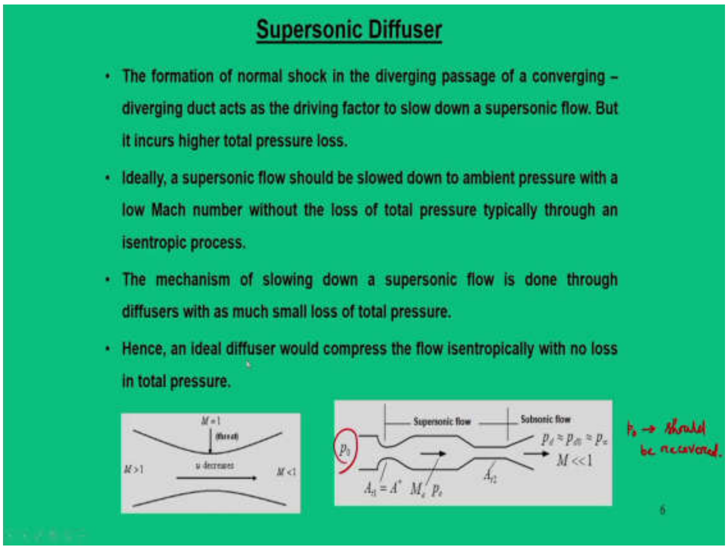
So, if sufficient pressure ratio is maintained so that the normal shock is driven by the flow and that comes out. So, at one particular instant when the pressure at the exit is p_4 and it is dropped to a further lower value; you can see that this normal shock which was at some portion in the diverging part, in fact, it starts with throat gets driven by the flow and some point down the line in the diverging section, it stands at some pressure and if pressure is reduced further, it goes to the exit. So that normal shock now stands at exit.

Now if you again further reduce this pressure from p_5 to p_6, then we land off in getting an oblique shock; that means, strength of this normal shock gets reduced. And when this process keeps on that we further reduce the pressure to a value p_b which is p_{e6}; that means, the flow is fully expanded and pressure is such that everywhere the flow will see isentropic flow and it lands of having expansion fan at the exit.

So in this particular situation the flow will always be shock free for any pressure between the value for which the normal shock stands at the exit and the expansion fan that is initiated at the exit. So any pressure in this range will make that nozzle shock free.

But, we do not want this nozzle to be shock free when it is integrated with a diffuser. The shock has to pass through the diffuser so that we can further slow down its velocity.

(Refer Slide Time: 18:06)

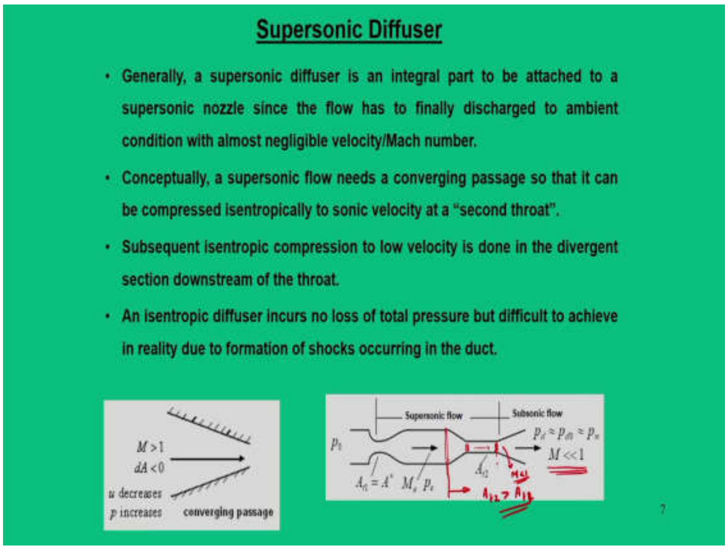


So, the formation of normal shock in a diverging passage of a converging diverging duct act as a driving factor to slow down the supersonic flow, but it incurs a higher total pressure loss.

So, the ideal way of looking at to slow down the supersonic flow to ambient pressure at low Mach number is without the loss of total pressure. The loss of total pressure I mean, whatever flow has total pressure at the inlet should map with total pressure at the exhaust.

So this p_0 should be recovered. So, if you are able to recover this; that means, we are doing our role in designing an ideal supersonic wind tunnel. Hence, the mechanism of slowing down the supersonic flow is done through diffuser with as much small loss in the total pressure. Thus, an ideal diffuser would compress the flow isentropically with no loss of total pressure, but this is not the reality.

(Refer Slide Time: 19:28)



So in the reality what we are going to achieve is that we have to incorporate a diverging passage at the end of the test sections part. So if you see here, the test section parts ends at this point. So, when it is at this exit of this test section, the flow is already supersonic, so since the flow is already supersonic it has to enter a converging passage to slow down its velocity.

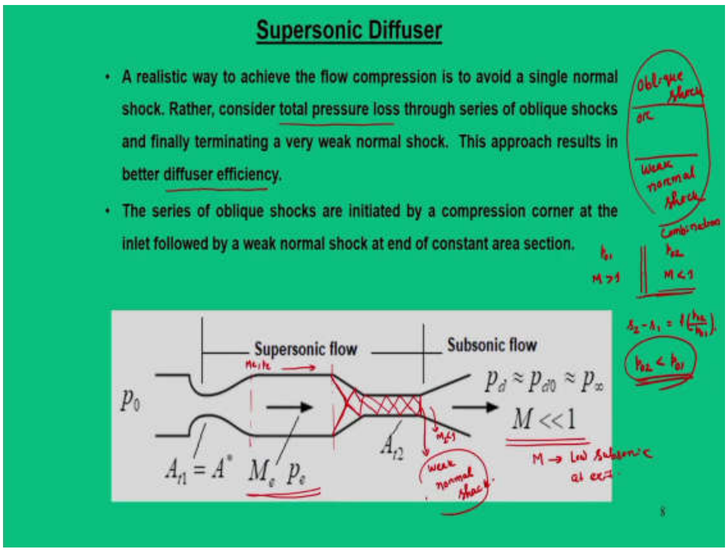
So, that is the theory of quasi one dimensional flow for which you need to decrease the velocity. Hence, a converging passage is integrated after this test section. Now, after this converging passage we also need to incorporate a minimum area and it has to be again attached to a diverging passage.

The role of this area is such that, this particular area should be higher than the first throat. And, this minimum area, maybe later on, we will find that this area at the diffuser portion should be higher than the first throat. This one we are going to prove down the line.

So once you put this; that means, a supersonic flow when it sees a diverging portion its velocity gets reduced, so it the flow Mach number become sonic in this throat and finally, after the flow is sonic Mach number becomes 1. We land of having a normal shock. Normal shock may start here, but it may gets pushed up due to this pressure difference.

So, whenever we say it is a weak normal shock at the exit of the throat then you see a diverging passage. So obviously, since it is a normal shock, Mach number in this region is less than 1 and when the Mach number is less than 1 and the flow sees a diverging section, the speed further reduces to Mach number much much less than 1. So, almost it gets slow down to a very low value of velocity.

(Refer Slide Time: 22:00)



So, this is how I have explained, but to quantify the role of a diffuser, we need to define a term called as diffuser efficiency. So, diffuser efficiency talks about a quantification of a parameter, the total pressure loss. To the total pressure loss should be as minimum as possible.

And in fact, you have a strong normal shock, because loss and total pressure is typically achieves through a normal shock. So we have already discussed that when you have a normal shock, p_{01} and p_{02} so your Mach number is always supersonic, Mach number is always of subsonic, and this normal shock leads to; that means, $s_2 - s_1$ is a function of $\frac{p_{02}}{p_{01}}$.

And you will see that, p_{02} is always less than p_{01}; so that means, there is always a drop in total pressure, and this drop in total pressure is maximum for a normal shock. And when you strength of this normal shock increases, this loss also increases.

So the ideal way of looking at a right diffuser is to consider a weak normal shock or oblique shock. So, these are the two ways that we can think of or combination of both. So, this is how the philosophy of regime happens.

So, what happens? That at the end of test section, we will have oblique shocks that comes, this is similar to the oblique shock that forms at the exit of a nozzle.

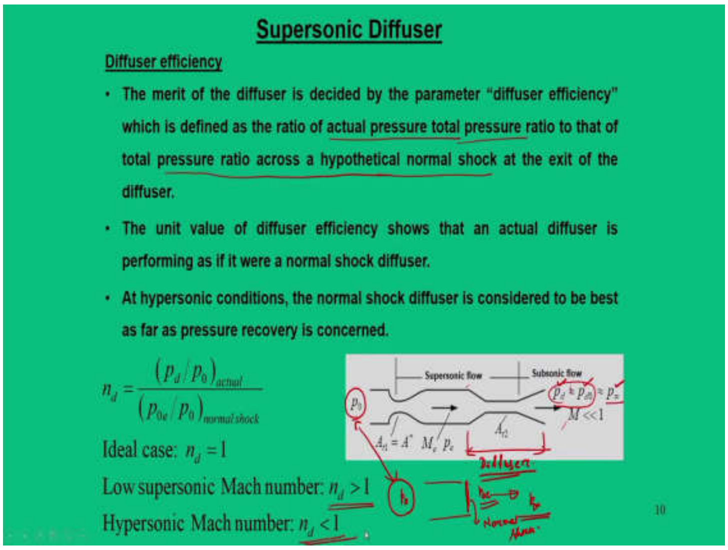
In fact, we can say all though they exit of the nozzle is here, but we are maintaining a same constant area and the flow always remains at that value of M_e. So you can say whatever M_e and p_e at this location that same conditions prevails till the end of the test sections and there the oblique shock starts forming.

And in the process they keep on interacting each other, and then on the wall and finally, in the diffuser section, and in the end it lands of having a normal shock at the exit. So this normal shock is a very weak normal shock.

So what we started is, we started with series of oblique shock terminated with a weak normal shock. And when we say weak normal shock your M_2 is also less than 1 and it sees a diverging portion, so Mach number becomes low subsonic at the exit.

But, to quantify this entire process of the mechanism of oblique shock phenomena and their interaction we have to define a parameter called as diffuser efficiency. So, the diffuser efficiency is decided by how much gain in total pressure we can have.

(Refer Slide Time: 26:21)



So, that part when you define this as the diffuser efficiency, we say that the merit of a diffuser is decided by a parameter diffuser efficiency which is defined as the ratio of actual pressure loss of total pressure to that of total pressure loss in a hypothetical normal shock situations.

So, for instance, if you look at this figure we say that we have added a diffuser part, when you add this diffuser part, we landed of in having a pressure at the exit which is close to p_d or is equal to p_{d0} is equal to p_∞. This is free stream pressure. This is static pressure at the diffuser exit p_d and this is the total pressure at the diffuser exit p_{d0}.

So if you employ a diffuser, so the pressure ratio would have been $\frac{p_d}{p_0}$. In this actual case, p_0 is your reservoir pressure at the inlet. But, this is what the term which appears in the numerator. But in the denominator what happens, that if we do not employ a diffuser and allow the gas at the end of the section pass directly to atmosphere, and it has to have a normal shock.

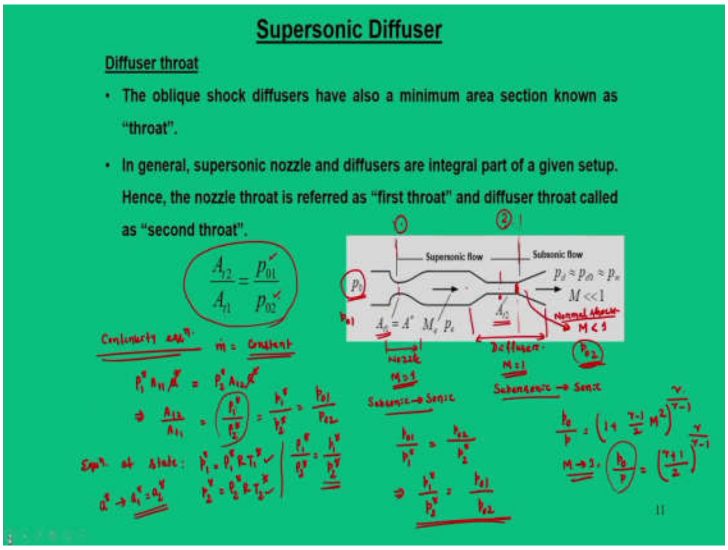
So; obviously, we will have the total pressure p_{0e} and we have total pressure p_0 which is same as that of here. And if there is a normal shock the ratio of the total pressure loss would have been $\frac{p_{0e}}{p_0}$, if it is a normal shock.

Effectively, what remain same is total pressure at the inlet p_0, what does not change is the actual pressure when we employ a diffuser and normal shock when we do not employ a diffuser. And, it has been seen that in ideal case, the diffuser efficiency is close to 1; that means, actual diffuser performs as if it is a normal shock diffuser.

And, in a low supersonic situations for a wind tunnel cases we typically considered the diffuser efficiency to be greater than 1. So, in other words when you say supersonic wind tunnel, we consider this diffuser efficiency should be higher than 1. But when you say hypersonic wind tunnel, we will see that the diffuser efficiency is less than 1, because at those cases employing diffuser may not be a right choice.

So, hardly people consider a hypersonic wind tunnel in this particular situation. When Mach number increases much much higher than value of hypersonic situation then the role of a diffuser is such that its efficiency should be less than 1.

(Refer Slide Time: 30:50)



Now, we will talk about the some of the important aspect of the diffuser design and nozzle design. So, what will we look at here is that this is schematic of this same wind tunnel what we have, and in this portion we have nozzle and in this portion we have diffuser.

As I mentioned, there are two throats or two locations of minimum area; one is A_{t1} for nozzle and A_{t2} for diffuser. And In fact, at both the locations your Mach number is sonic. So these two conditions should hold good.

But only difference here, the Mach number reaches sonic from subsonic to sonic value. Here, the Mach number reaches from supersonic to sonic value. That means, flow is accelerated and flow is decelerated. So, here from supersonic to sonic.

So this is the basic two difference that happens at this two locations. But what remains constant, if I can say that continuity equation; we say mass flow rate remains constant.

So, when you say mass flow rate remains constant, I can apply the continuity equations at two locations; 1 and 2. So, what I can say is that $\rho_1^* A_{t1} a^* = \rho_2^* A_{t2} a^*$. So, a^* will get cancel. So what we land of is $\frac{\rho_1^*}{\rho_2^*} = \frac{A_{t2}}{A_{t1}}$.

Now here I can say that what is this $\frac{\rho_1^*}{\rho_2^*}$. So I can recall that equation of state. We says that at this two location I can write $p_1^* = \rho_1^* R T_1^*$ and $p_2^* = \rho_2^* R T_2^*$. R happens to be remain same. And In fact, since a^* is same. So, this means T^* also remain same.

So, we this will imply that $\frac{\rho_1^*}{\rho_2^*} = \frac{p_1^*}{p_2^*}$. But what is this p_1^* and p_2^*? One can recall that, $\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}$. So when M goes to 1, we write $\frac{p_0}{p} = \left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma}{\gamma-1}}$, because M is equal to 1.

So, one can rewrite this equations for state 1 and state 2 for p_1 and p_2. So, from this equation, one can find out that $\frac{p_{01}}{p_1^*} = \frac{p_{02}}{p_2^*}$.

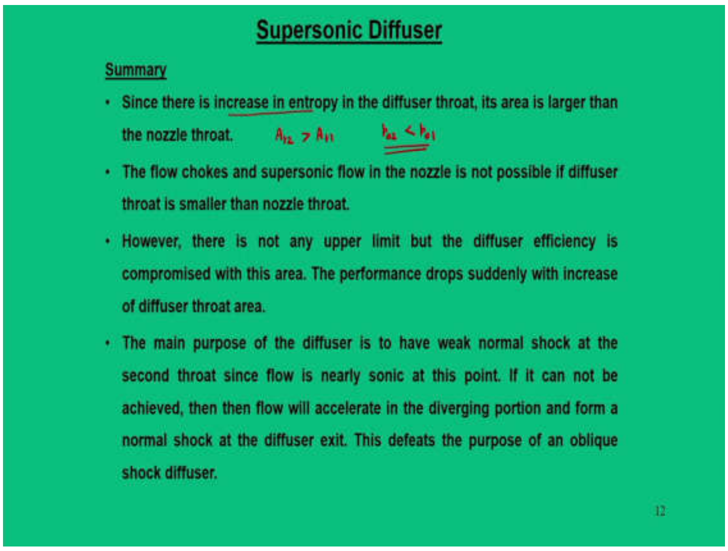
So this will say that $\frac{p_{01}}{p_{02}} = \frac{p_1^*}{p_2^*}$. So, we land of having this equations. So, we have left with $\frac{A_{t2}}{A_{t1}} = \frac{\rho_1^*}{\rho_2^*} = \frac{p_1^*}{p_2^*}$, then we prove that $\frac{p_1^*}{p_2^*} = \frac{p_{01}}{p_{02}}$.

So hence, a relation that is developed that $\frac{A_{t2}}{A_{t1}} = \frac{p_{01}}{p_{02}}$. So, what happens means, that the area ratio in the diffuser and this nozzle has a particular ratio which is followed with respect to total pressure p_{01} and p_{02}.

So what is this p_{01} and what is the p_{02} means? We showed that we will have a normal shock at the diffuser throat, and when we have this diffuser throat at this normal shock, till this point of time the entire flow is completely isentropic. So this $p_0 = p_{01}$. Now when we have a normal shock here, your Mach number becomes subsonic and the total pressure becomes p_{02}, because it is after this normal shock.

So in other words, we say, the pressure that remains at the end of the diffuser is total pressure p_{02}; the pressure which is relevant or prevalent in the reservoir is p_{01}; and the geometry of diffuser throat and nozzle throat has area ratio $\frac{A_{t2}}{A_{t1}}$ and they bear a definite ratio.

(Refer Slide Time: 39:11)



So, this is what the summary of a supersonic diffuser that the increase in the entropy in the diffuser throat that happens due to normal shock. Its area is larger than the nozzle throat; that means, we say $A_{t2} > A_{t1}$, because p_{02} is less than p_{01}.

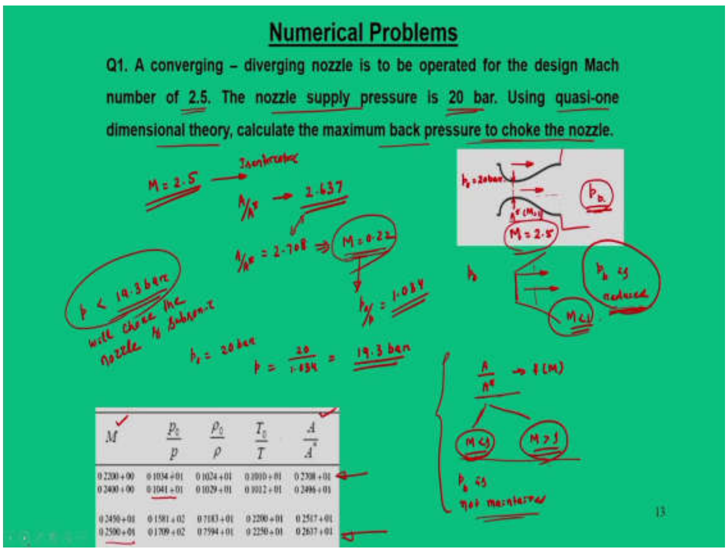
So, we say p_{02} is less than p_{01} So this number is higher, so $A_{t2} > A_{t1}$. So this loss in total pressure is due to increase in the entropy.

The flow chokes, the supersonic flow in the nozzle is not possible if diffuser throat is smaller than the nozzle throat. That means, one cannot deliver a supersonic flow in the nozzle. However, there is no upper limit for the diffuser throat because A_{t2} can be as much as large possible, but the performance drops with very large diffuser area throat.

But the main purpose of the diffuser is to have a weak normal shock; that means, as long as we get weak normal shock at the exit, we do not arbitrarily increase the area, rather we stop the diffuser area throat at that point and at that point the flow is sonic. If it cannot be achieved, then flow will accelerate in the diverging portion and it will form normal shock in the diffuser exit. So, this defeats the purpose of a oblique shock diffuser.

So, as long as we do not have a weak normal shock at the exit, then the role of diffuser is not attained.

(Refer Slide Time: 41:24)



Now with this, we will try to see some numerical problems based on this nozzle and diffuser which we discussed in this module.

So, the first problem that talks about a converging diverging nozzle is to be operated for a design Mach number of 2.5. So, this is how the schematic diagram of a nozzle. So we say, its design Mach number is 2.5. We say, the supply pressure is p_0. The supply pressure is nothing but your reservoir pressure; that is 20 bar.

So what we are going to see is that, using quasi one-dimensional theory, calculate the maximum back pressure to choke the flow. So; obviously, although we have maintain a pressure, now, if this back pressure is close to 20 bar; there would not be flow in the nozzle. Even though we have designed a supersonic nozzle, but if a pressure is close to 20 bar, it will not flow.

So, for this reason, the back pressure needs to be controlled, so that we get the sonic supersonic flow at the exit. And second point needs to be emphasized here, that this nozzle has to be completely shock free. And the flow has to choke at the throat section, then only the flow can entered into the diverging section.

Now, when the flow is choked at the throat; obviously, we will have a normal shock and that will try to gets pushed off in the diverging portion. But the main problem is, that if your back pressure is reduce, and we have your p_0, so it get sufficient range to get pushed off.

But if you do not do this, then the normal shock will either stand at the minimum area or it may so happen if it does not see the required pressure down the line, it will become subsonic. But this purpose will be lost, because we are considering the Mach number of 2.5.

Our main intention is that we have to calculate the maximum back pressure to choke the flow. Now, if you look at this area ratio is which is a function of Mach number. And for a given area ratio there are two solutions; one is Mach number of less than 1, one is Mach number of greater than 1.

Now, if you have a supersonic solution; Mach number is greater than 1, the entire flow nozzle flow will be sonic. But, the flow will be subsonic if required back pressure is not maintained.

Now, with this philosophy from the given data what we are trying to achieve is that, let us calculate that for M is equal to 2.5, what will be the area ratio? So, from this area ratio we will get isentropic table. Now from this area ratio Mach number relation, for 2.5 this is what the extract we get from this book. For a Mach number of 2.5, the area ratio A* has to be 2.637.

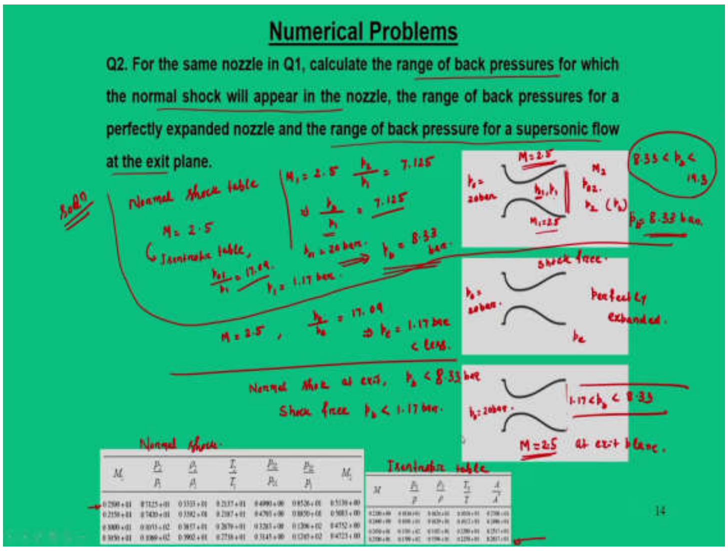
But would may so happen, for close to this value of area ratio we may have a subsonic solutions. For example, for $\frac{A}{A^*}$ is equal to 2.708 which is close to this number. This will implies a Mach number of 0.22, we can see here.

So, here I have trying to explain these point for a given area ratio there are two possibilities; we have a subsonic solution, we have a supersonic solution, then roll of back pressure comes into picture. So, if you do not maintained this desired back pressure then the flow will choke and supersonic flow is not possible.

So flow will land of in subsonic regime. When you have Mach number 0.22, a supersonic flow will not happen. So, if this is the situation, then your $\frac{p_0}{p}$ would be 1.041.

Now, in our case your p_0 is 20 bar. So p would be 20 by 1.034. So this number will be 19.3 bar. So the answer is that if your p is less than 19.3 bar or any value of less p than 19.3 bar, it will choke the nozzle and it will be entirely subsonic. So this is how we have.

(Refer Slide Time: 49:09)



Now, in the same question we are going to little bit of modify for the same nozzle in question 1; calculate the range of back pressure for which normal shock will appear in the nozzle exit. So we say here that your p_0 is 20 bar, M is equal to 2.5. We are going to have a normal shock at the exit.

So when you have a normal shock at the exit, then downstream pressure would be p_{02} here, the upstream pressure will be p_{01} and p_1 and here p_2 and your Mach number will be M_2.

Now, of course, if there is a normal shock, your M_1 would be 2.5. So, we have two solution, we have to refer normal shock table where for M_1 is equal to 2.5. So for M_1 is equal to 2.5, if you see its a normal shock table, then it will give $\frac{p_2}{p_1}$ is equal to 7.125.

Now, this p_2 in our case will be p_b. So we can write $\frac{p_b}{p_1}$ is 7.125. But what is our p_1? p_1 corresponds to Mach 2.5 for which total pressure is p_{01} that is 20 bar. So, p_1 can be calculated; as M is equal to 2.5, so we say isentropic table for which Mach number is 2.5, $\frac{p_{01}}{p_1}$ 17.09.

So, p_{01} is 20 bar. So from these two equation, one can find out p_b is equal to 8.33 bar. Of course, it will give p_1 as 1.17 bar and p_b is this. So, if you have a normal shock then your p_b is 8.33 bar. Any pressure at this number will give you normal shock at the exit.

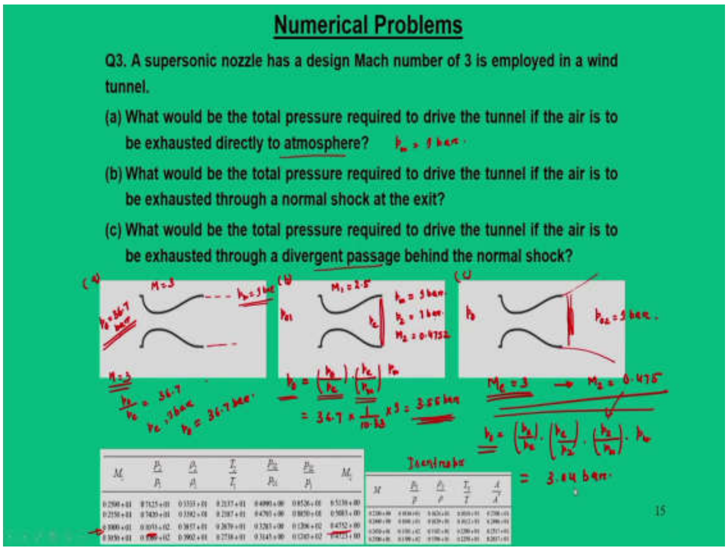
So the first question we are going to answer; the range of normal pressure that will appear in the nozzle. So, 8.33< p_b <19.3 bar, the flow will choke this nozzle, and any pressure in between this will keep this normal shock which were supposed to develop at throat, it will get pushed off and at 8.33 bar it will stand at the exit.

The next question is that range of back pressure for a perfectly expanded nozzle. Now for a perfectly expanded nozzle, we have p_e so that means it is a shock free. So when it is a shock free, same pressure is 20 bar. So, at Mach number of 2.5 one can calculate $\frac{p_0}{p_e}$ as 17.09. So, p_e will be 1.17 bar. So, any value this or less, we will make this nozzle perfectly expanded.

And in third case, it have says that range of back pressure for a supersonic flow in the exit plane. So, now to have a normal shock in the exit. So till exit plane if you want to have a flow to be supersonic, then as I mentioned our earlier calculations, so normal shock at exit will have p_b less than 8.33 bar, p_0 20 bar. Now, it is shock free, your p_b less than 1.17 bar.

So, any pressure between 1.17< p_b< 8.85, we will make this always supersonic flow Mach of 2.5. So, this is how one needs to compute that how you can achieve supersonic flow in the nozzle.

(Refer Slide Time: 56:47)



Now, next problem that is with respect to a wind tunnel in particular, but very specific to a supersonic nozzle as a design Mach number of 3 is to be employed in a wind tunnel. What would be the total pressure required to drive the tunnel if the air is to be exhausted to directly to the atmosphere?.

So, this particular problem is framed with respect to my explanation that I give; what is the role of diffuser; why you require a diffuser in a supersonic wind tunnel.

So, main reason is this. This is formulated in this manner. Now in the first case, we are saying that the flow is directly exhausted to the atmosphere in a nozzle; second case the flow is directly exhausted to the atmosphere through a normal shock; third case we are saying that we are going to attach a diverging passage across the normal shock.

Now, in all these cases, what are the changes we are going to see? So, in the first case we say your Mach number is 3, and we say we have a directly atmosphere. So, atmosphere I can assume as 1 bar, p_∞ is 1 bar. So I can assume that the flow is adjusted to the atmosphere in the 1 bar.

So in the first case, we say that the total pressure required to drive the tunnel when it is directly exhausted to the atmosphere. So, what is going to happen if it is 1 bar? And here for M is equal to 3, I can refer this table; isentropic table $\frac{p_0}{p_e}$ is 36.7 bar.

So that means, if p_e is 1 bar, p_0 will be 36.7 bar. So in other words, we require a pressure of 36.7 bar to operate this wind tunnel. But let us pause a moment that what better we can do. So next better you are going to do is that we are saying let us have a normal shock, let us consider a normal shock at the exit.

When I say consider a normal shock I can say p_∞ is 1 bar, now when I say p_∞, this will be nothing but p_2. So, when I have a normal shock we have a static pressure p_2 and your M_2 will be corresponding to M_1 as 2.5. So this normal shock table, we will have M_2 would be 0.4752.

So, when you have say M_1 is this, we require p_{01}. So, we can say $p_0 = \left(\frac{p_0}{p_e}\right)\left(\frac{p_e}{p_\infty}\right)p_\infty$.

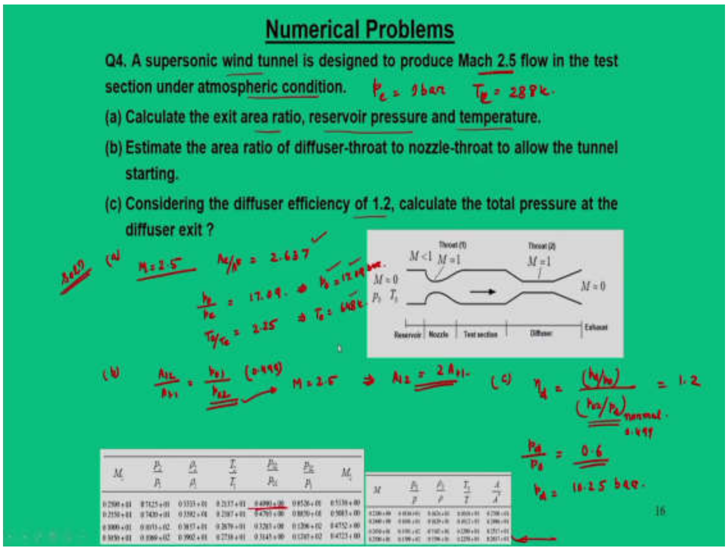
So, $\frac{p_e}{p_\infty}$ 10.33. So I can say, this corresponds to Mach number of 3. So, this is about 36.7*1/10.33*1. So, this will turns out to be 3.55 bar.

So, we can say that the requirement of total pressure now drops from 36 bar to 3.55 bar. Now, further tuning is possible instead of assuming p_2 as 1 bar we say p_{02} as 1 bar. And we will have a same normal shock, but it will be weak. So to make it weak, we have to make it by attaching a diverging passage.

So in same case, and one can rewrite that M_e is equal to 3 it will have M_2 is equal to 0.475. So, in this case one can write $p_0 = \left(\frac{p_0}{p_e}\right)\left(\frac{p_e}{p_2}\right)\left(\frac{p_2}{p_\infty}\right)p_\infty$. This corresponds to Mach number of 4.75, all these numbers you already know, and one can find out this pressure will be 3.04 bar.

So in this entire sequence of process, what we can observe is that if you allow a diverging passage the requirement at the upstream or reservoir pressure significantly drops from 36.7 bar to close to 3 bar. So in other words, if you add a diverging portion we can operate a wind tunnel with a total pressure of 3.04 bar instead of 36.7 bar.

(Refer Slide Time: 1:03:43)



So, last part of this problem is about a diffuser. It is a very simple problem. This is also same thing that we have a Mach 2.5 flow. Here, we need to find the exit area of the diffuser. So we say, the wind tunnel is operated at atmospheric conditions. We can say atmospheric condition as p_e as 1 bar and atmospheric condition as T_e as 288 K.

With this assumptions, and for this Mach number of 2.5. So it will give $\frac{A_e}{A^*}$ as 2.637. We can find out here, and $\frac{p_0}{p_e}$ as 17.09, $\frac{T_0}{T_e}$ as 2.25. So this will tell you that p_0 requirement would be 17.09 bar and T_0 would be 648 K.

So we need exit area ratio 2.637, reservoir pressure 17.9 bar, temperature 648 K. Estimate the area ratio of diffuser throat to nozzle throat to allow tunnel starting. So as I mentioned the diffuser throat to nozzle throat area $\frac{A_{t2}}{A_{t1}}$ is governed by pressure ratio $\frac{p_{02}}{p_{01}}$. This ratio can be found out for M is equal to 2.5 and this value is 0.499.

So, this will close to give you $\frac{A_{t2}}{A_{t1}}$. Considering the diffuser efficiency of 1.2, calculate the total pressure ratio at the diffuser exit. So you can say diffuser efficiency is

$$\frac{\left(p_d/p_0\right)}{\left(p_{02}/p_0\right)_{normal}}$$. In fact, this value is given as 1.2 so you can say p_d/p_0 as this value is 0.499, so we can say this value p_d/p_0 will be close to 0.6 or p_d is equal to 10.25 bar.

Where, p_0 we can say 17.09 bar. So, this is how the diffuser part is considerd.

(Refer Slide Time: 1:07:28)

Now, with this I will conclude this with certain learning components. So, we discussed about a supersonic wind tunnel. We discussed about isentropic flow for supersonic and subsonic case. We also discussed in the module about adiabatic nozzle flow. We talked about supersonic diffuser, nozzle throat and diffuser throat. And finally, diffuser efficiency.

With this I hope I have made you understand about this module of nozzle and diffuser.

Thank you for your attention.