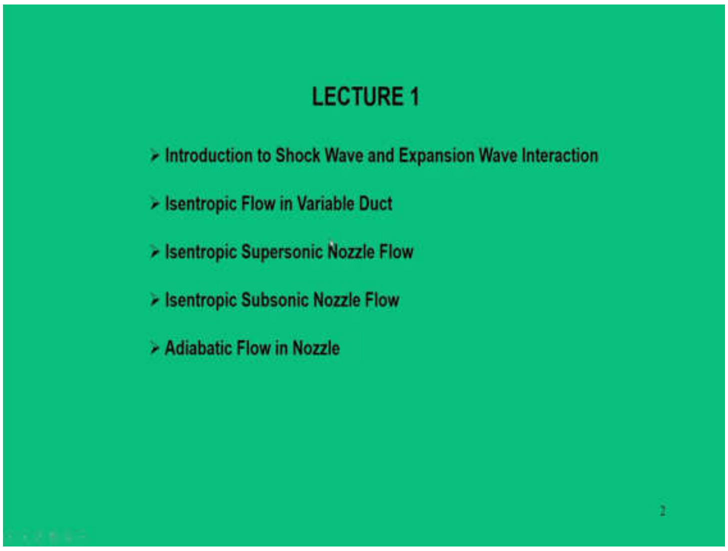
Fundamentals of Compressible Flow
Prof. Niranjan Sahoo
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 06
Lecture - 17
Interaction of Shocks and Expansion Waves

Welcome to this course Fundamentals of Compressible Flow. We are in a new module 6, the title of this module is Interaction of Shocks and Expansion Waves.

(Refer Slide Time: 00:44)



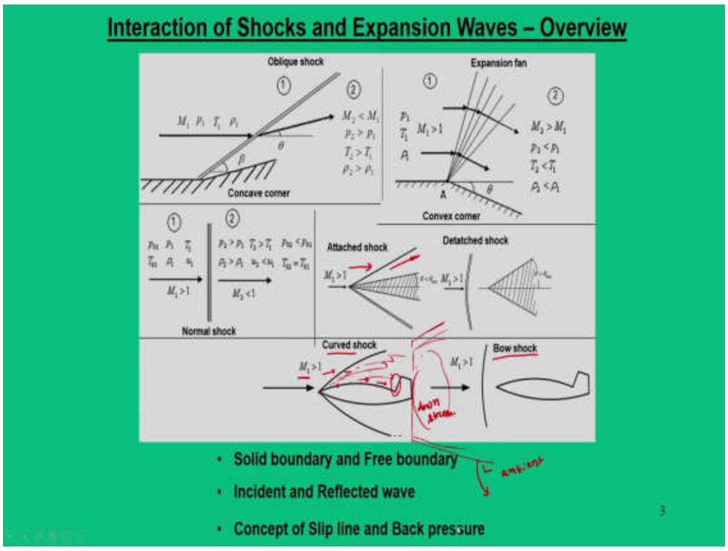
So, in this lecture of this module, we will talk about some physical phenomena in which we can see the encounter between shock waves and expansion waves. So, we call it as a shock wave and expansion wave Interactions.

Then, we will to analyze this shock wave and expansion wave interactions. We will try to revisit the flow in a variable duct and in particular, we will be explaining the fact with respect to isentropic flow. After this, we will discuss about the very important topics in the compressible flow which is known as nozzle.

So, in this category, we will discuss about isentropic supersonic nozzle flow and also, we will discuss about isentropic subsonic nozzle flow. In fact, nozzle is one of the integral components that accelerates the flow and this is a very important device in which flow can be accelerated to any Mach number values.

Now, apart from that, all these were treated in a isentropic ways; but we will relax this isentropic analysis and we will talk about an adiabatic flow in the nozzle, means what happens if the flow is no longer isentropic. So, this is the brief introduction about this particular lectures.

(Refer Slide Time: 02:43)



So, the first topic that we are going to discuss is the interaction of shocks and expansion waves. What does this mean? Just to give a brief overview, till this point of time, we know that an oblique shock is formed whenever a supersonic flow encounters a concave corner or it is a compressive corner.

Now, this supersonic flow is diverted or the flow is deflected after encountering this oblique shocks. Now, when it diverges path, it makes a flow deflection at angle θ and for that θ angle and that supersonic flow at the inlet, there will be an oblique shock sitting at this compressive corner.

So, this shock wave angel, we call this as a β. So, this is how a oblique shock is formed in a two-dimensional plane. Now, when the shock wave becomes stronger and stronger, we call it as a normal shock and this two-dimensional flow field can be relooked into one-dimensional flow.

And when it is a normal shock, after this normal shock, the flow becomes highly subsonic. So, this is how the oblique shock and normal shock is defined. Now, with this philosophy we define the concept of attached shock and detached shocks.

So, for a given flow deflection angle flow across a wedge, we will have an attached shock and when the supersonic flow gets deflected across this shockwave. Now, if the flow does not see sufficient angle for which the oblique shocks solution is not possible, the shock wave is no longer attached.

So, it becomes a detached shocks. So, this is what we call this has a flow over an wedge and moving further, when you are going for a realistic applications like the flow field over an aircraft, so this is a schematic sketch that the flow field over an aircraft, we can have a curved shock.

So, why it is a curved shock? Because the surface, the aircraft surface is no longer as a straight line as you view in the case of wedge. So, we will have a shape that changes continuously on the surface. So, based on that, we may have a attached shock, we may have a detached shock.

So, many a times, since the flow fields are always continuously changes across this curved shock, so we say instead of attached shock, we say it is a curved shock because the surface of the geometry changes continuously; whereas, in the case of wedge the surface is maintained at a fixed angle θ, which is not the case when it is for a surface which changes continuously in its geometry.

So, also the other name that is given for a detached shock is bow shock. Before I go for this, now if the same supersonic flow, if it encounters an expansion corner or a convex corner, the flow gets deflected in a exactly reverse manner. So, where in which the flow try to expands that is Mach number increases.

When the flow tries to expands, we will have a series of expansion fan generated from the center point. So, the very basic philosophy between the oblique shock and expansion fan is that in both the cases flow gets deflected. But in oblique shock the flow gets deflected towards the main flow; but in a expansion fan, the flow deflects away from the main bulk of the flow. So, this is the basic difference.

Now, what may happen in a realistic situation is that this kind of shock waves and expansion waves, normal shock, attached shock, detached shock, they can be formed externally on the surface of the body or this kind of things can also happen in an internal flow. And while doing so, the shape and geometric of the bodies is such that like in this particular case, if you look at this particular point, for the main flow, this point is a compression corner.

But however, with respect to this point, if you chose another point in on the surface, it is a expansion because the surface is such that with respect to this point, the flow experiences an expansion at this point. Likewise, it will keep on happening and with relative to this point, all the properties that are going to change decrease with respect to this particular point at the nose.

Similarly, here also the main flow here tries to gets deflect in the same fashion as with respect to surface of the body. So, at any point the flow tries to be parallel to that point and many times, when it is at this particular corner, the flow depending on the upstream and downstream conditions.

So, we say if this is upstream and this is downstream condition. We may arrive at the point as an expansion point or a compression point. So, likewise one may have physical solid boundary or you may have a free boundary. Free boundary is something like that when the flow comes out from this vicinity, it sees a complete boundary which is divided from the main flow by ambient conditions.

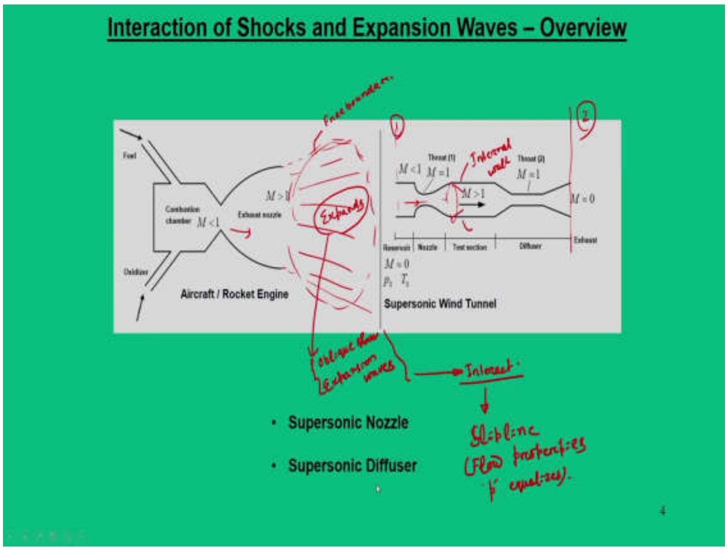
Like for example, if you can distinguish between this phase and this phase, we may have a ambient conditions which is for this boundary and in this particular vicinity, the flow will be entirely governed by the downstream condition of the properties.

Similarly, here also this is one such case, what I can say that one can have a free boundary or one can have a solid boundary and in some instances, we will say that we can have an incident shock or a reflected shock. So, incident shock means that this oblique shock is forming in one compressive corner like this.

Suppose, we will have a analogous compressive corner on the upper surface, then we will have another shock which is can come from the upper surface. So, such a case we can have a incident shock and we can have a reflected shock.

Likewise, down the line we will define some concept like slip line and back pressure in which we are trying to see that whenever the oblique shocks and expansion waves interact, the flow conditions are generally governed by the back pressure and in order to maintain the pressure equalization across a imaginary line, we call it as a slip line. So, those particular concept will be discussed.

(Refer Slide Time: 11:38)



Now, to give some practical utility or applications for this case, I will try to explain some kind of applications in which we will use two types of internal flow; one is flow in a supersonic nozzle and flow in a supersonic diffuser and where we find them? So, the nozzle and diffuser, they are integral part of the compressible flow devices during experimental conditions.

So, one can find a nozzle, many of us are aware that we have a rocket nozzle, the flow from the exhaust which is nothing but the fuel and air when it mixed in a combustion chamber, they try to expands through a nozzle and we call this as a exhaust nozzle and this exhaust nozzle, the flow inside this nozzle we can say it as a internal flow.

Now, when the flow comes out, we may have a imaginary boundary which is called as free boundary and this gas that goes as a exhaust. So, this flow expands. Now, during this expansion process, we may have oblique shock, expansion waves and they try to interact. Now, when they try to interact in this process, an imaginary boundary line is formed which is known as slip line across which the flow properties match.

These flow properties I mean mainly pressure equalizes. So, this particular things is a very complicated phenomena, which we are not going to deal in this particular course. But what we will be going to deal is that what happens when there is a flow inside the nozzle. This is one application. Another typical application, we come across is a Supersonic wind tunnel.

So, in a supersonic wind tunnel, the schematic diagram says that we have a main reservoir. There will be a nozzle; there is a test section; there is a diffuser and exhaust. So, these are the components involved in a supersonic wind tunnel. So, what happens from the reservoir; that means, where the flow is stagnant, but we have a very high pressure and temperature source.

So, when we allow that flow to pass through a nozzle, they try to expand. So, while expansion, we get a desired Mach number in the test sections. Now, and in this process, since we have a very high pressure stream in the reservoir and you are allowing it to a small passage. So, we may come across different types of features like we may have a normal shock, we may have oblique shock, if you do not maintain appropriate downstream condition.

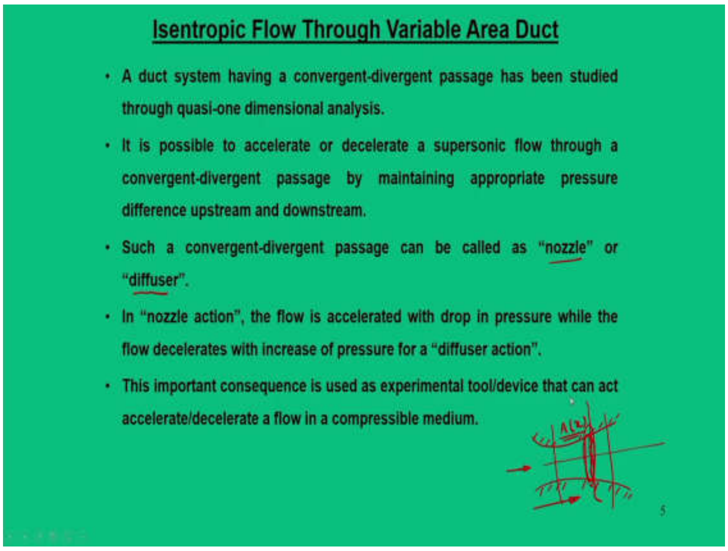
So, in our philosophy, we can say this is the region 1 and this is the region 2. So, in this region 1 and 2, there may be the formation of oblique shocks; there may be formation of expansion wave; there may be formation of normal shock and many times the flow that tries to see there are two types of wall; one is this internal wall, so that means, what we are showing is that the flow encounters the entire vicinity of the wall.

And whenever there is a oblique shock or normal shock formation, they try to reflect and all such interactions could be possible, if the appropriate conditions between 1 and 2 are not maintained. So, for this type of philosophy and in fact, a very good supersonic wind tunnel design requires that it should be completely shock free. Shock free means there should not be any shock wave in its path because its shock wave, loses in the total pressure; there will be loss in total pressure.

So, means that when we take the gas from the reservoir, our responsibility is that we should also make the gas to go out of atmosphere with almost zero velocity or like a stagnant gas. So, this is how a very good supersonic wind tunnel philosophy requires that we device adequate components so that we will make this device completely shock free.

So, in this case it is a kind of an internal flow. In this case, we can have a internal flow and as well as the external flow. So, the importance of shocks and expansion waves can be realized only in this kind of applications; but however, we will speak to our basic definition that we will talk about two specific important devices which we call as a nozzle and diffuser in this particular lecture.

(Refer Slide Time: 17:51)



So, before we start the nozzle and diffuser, we will just revisit some fundamental concepts of isentropic flow through variable area duct. In fact, we have discussed this particular topics, mostly in one of our module that is quasi one-dimensional flow.

So, I will just take out some important points from those module. So, there what we have discussed about a flow path that involves a converging and diverging area where area is a function of x. So, it is no longer a constant area. So, area is a function of x. So, x is the direction in which the flow is taking place.

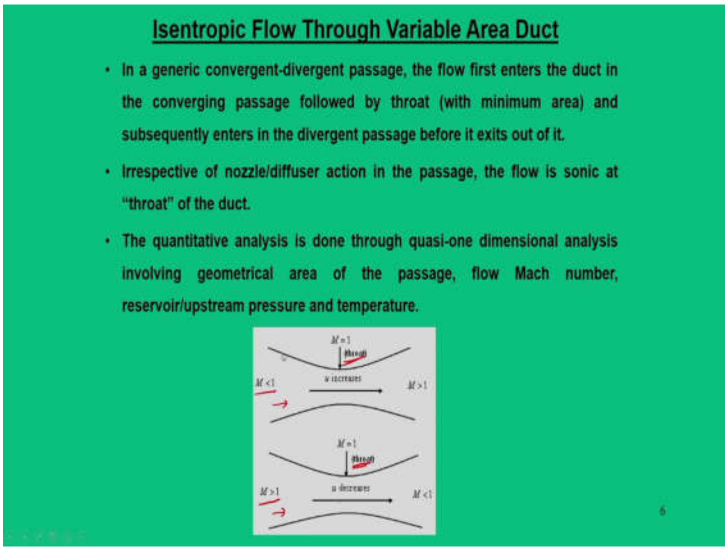
So, this particular concept, we call this as a quasi one-dimensional flow and the main intention for this that when you deal with quasi one-dimensional flow, if you take any cross section, whatever properties you define, they are mostly same across that cross section.

So, that is the concept of quasi one-dimensional flow and in fact, we dealt with the isentropic flow in a variable area duct system. So, just to summarize some of the salient features, I can read out as a duct system having convergent divergent passage is studied through quasi one-dimensional analysis.

So, in that analysis, it was found that one can accelerate or decelerate a supersonic flow through a converging convergent divergent passage by maintaining appropriate pressure difference in the upstream and downstream and such a device or passage, we call this as a either a nozzle or a diffuser.

Now, when you talk about a nozzle action, the flow is accelerated with a drop in the pressure; while the flow decelerates with increase in the pressure for a diffuser action. So, this important consequence of this nozzle and diffuser actions makes us to built a laboratory tool or experimental tool that can accelerate and decelerate the flow in a compressible medium.

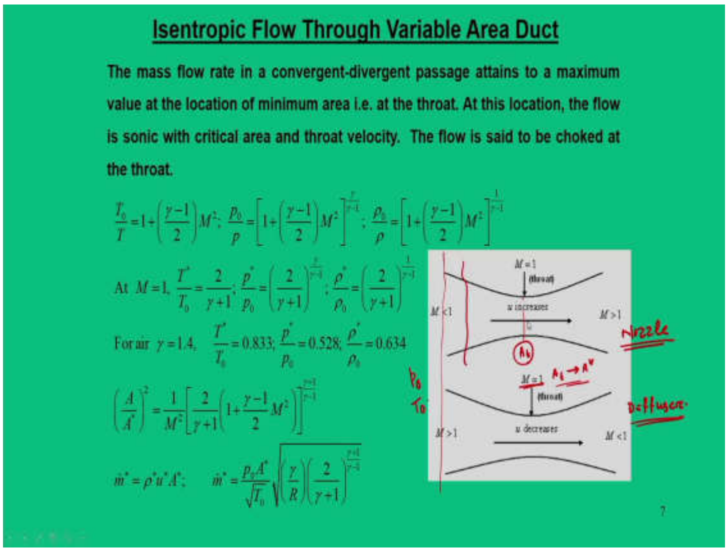
(Refer Slide Time: 20:30)



Now, when you deal with this variable area passage, irrespective of the fact whether your inflow is subsonic or inflow is supersonic and when it enters in a converging diverging passage, it sees a location where the area is minimum, which is known as throat.

So, at the throat, the area is minimum. So, in fact, we did through quasi dimensional analysis about the calculation of upstream pressure and temperatures, flow properties, flow Mach number for this convergent divergent passage.

(Refer Slide Time: 21:19)



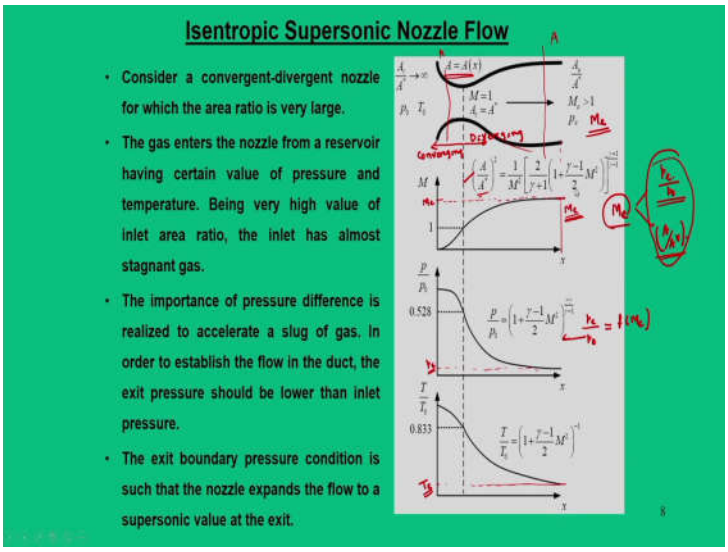
Now, these are the summary of table, what we get that irrespective of the fact whether the flow is subsonic or supersonic, the flow encounters a minimum area that is throat and one can bring a subsonic flow to a supersonic flow and also, one can bring a supersonic flow to a subsonic flow.

So, this is a nozzle and this is a diffuser. Now, irrespective of the fact whether is a nozzle or a diffuser and what happens when we look at this particular throat area. And this is known as area at the throat which is the minimum at this location and for this if the flow condition is such that we get sonic flow, then the minimum area we represent as A*.

So, if you get a sonic flow, we get a minimum area as A* and for a given nozzle, one can find out area Mach number relations. This area means, any arbitrary area to this throat area that gives you the area ratio and Mach number relation. Now, the flow field is entirely isentropic, it is governed by this isentropic relations that is ratio of stagnation temperature to static temperature, stagnation pressure to static pressure, stagnation density to static density.

And once the flow is choked or at the throat, we call this as a choked mass flow rate $\dot{m}^*$. So, this can be calculated from the reservoir conditions. Here, the reservoir conditions are typically we say p_0 and T_0 and one can evaluate this mass flow rate that is happening at this particular area known as throat.

(Refer Slide Time: 23:46)



Now with this background, let us see that how we can achieve a supersonic flow in a nozzle having the process to be isentropic means we are talking about isentropic supersonic nozzle flow in a variable passage. So, if you look at this particular figure, so what has been done is that we have created a passage which looks like a nozzle. It has a converging section which is located in the left hand side of this line and other side, we say it is a diverging. So, we have a converging section as well as diverging section.

Now, while you while designing this converging and diverging session, they are attached with a area which is minimum and known as the throat area. In fact, in most of the situation, its not an attachment; its a continuous passage which is maintained for the flow internally.

Now, the converging part is designed such a way that entry of the converging part that is inlet area to this minimum area goes to infinity is very high. That means, flow is coming from a reservoir which is maintained at condition of stagnation pressure p_0 and T_0.

Now, what we are going to do is that we are going to say that this particular nozzle, we want to achieve a supersonic flow. So, for a supersonic flow, that means, at the exit we are going to achieve a supersonic Mach number of M_e. Now, to get the supersonic Mach number of M_e and we know that at any location to the throat area ratio is related with respect to Mach number.

For example, this particular equation talks about area ratio and Mach number relations. Here, A stands for any arbitrary area; that means, it may be at the converging section or diverging section. So, it is a function of x and A* is the area which is at the throat. So, this particular relation will tell you that if you want to get a supersonic flow, then we can based on the Mach number, we may trace a path how the Mach number can vary along x.

So, with this area relation, one may fix the Mach number M_e which is this value at this location x. So that means, at this location, I am expecting a Mach number of M_e and with this Mach number of M_e, I can get the shape of this converging and diverging passage.

The main issue is that for this slug of mass to accelerate, we need a pressure difference. So, for this things, we have to look into the fact that we must maintain a pressure ratio between the exit and the reservoir such that flow should happen.

So, in this case, we should maintain a pressure ratio p_e, we should maintain a value p_e such that we get this Mach number M_e. So, this follows the relation that one can get the exit pressure, like here we can have $\frac{p_e}{p_0}$, one can write from this equation as function of Mach number from using this equations.

So, to achieve this Mach number, we must have a pressure value p_e at the exit as shown in this figure and similarly, for temperature we may have the temperature T_e that needs to be maintained at the exit so that we can get the desired Mach number.

So, this makes a conclusion that to achieve a supersonic flow at the exit, we must follow area Mach number relations and we must maintain a desired pressure ratio and this value is very unique, since we are using this for isentropic relations. And this value is fixed value and any pressure which is below p_e will always give a desired Mach number M_e.

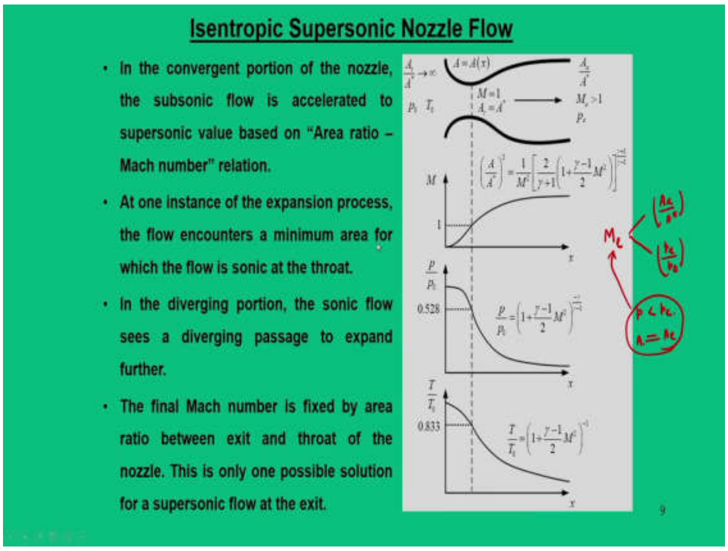
So, it means that if the pressure is maintained below this p_e value, your Mach number will also get M_e. Also, suppose for example, this Mach number at the exit is a function of two parameters; one is $\frac{p_e}{p_0}$, other is $\frac{A}{A^*}$ So, both way, we can achieve this.

Now, to achieve this M_e, we need to have this both the conditions to be satisfied and any if any of this condition is not satisfied, we were not able to get a Mach number M_e; that means, we will not get an isentropic solutions. So, this is how the basic philosophy of a isentropic supersonic nozzle flow. So, whatever I have explained, if I can summarize them, then I can readout them as follows.

We are considering a converging diverging nozzle, which has area ratio very large; that means $\frac{A_i}{A^*}$. The gas enters from a reservoir with certain value of pressure and temperature and since it has a very high value, the inlet has almost stagnant gas.

The pressure difference you have to maintain in order to accelerate this slug of mass through this passage. The exit boundary condition that is p_e is such that the nozzle expands the flow to a supersonic value that means we are allowing this area Ratio and Mach number relation to get satisfied.

(Refer Slide Time: 31:32)



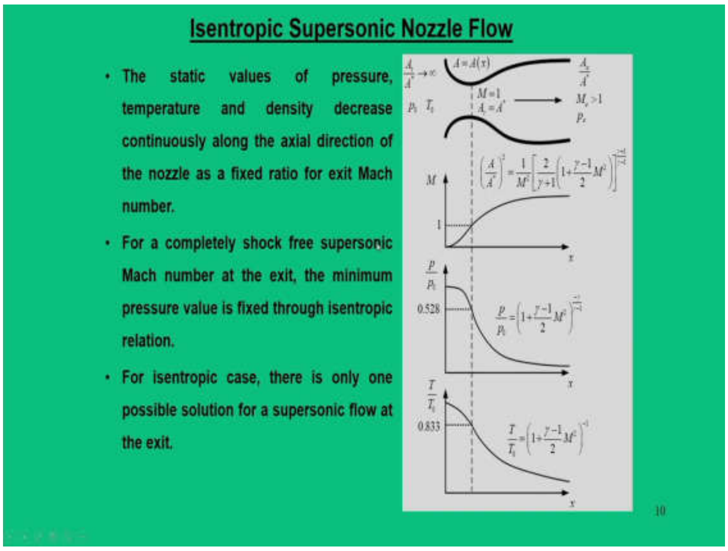
Now, in the process of the flow from converging passage to the throat, the flow encounters a minimum area that is throat and at this point, the flow is sonic. Now, when the flow is sonic and it sees a adequate pressure difference at the exit, the flow tries to expand further because it sees both the conditions are satisfied; that means it is a diverging passage as well as this is a adequate value of exit pressure so that it can expand.

So, this is the critical point that whether we want to accelerate the flow or not, this depends on the exit pressure. So, in the diverging portion the sonic flow sees a diverging passage to expand further. Hence, the final Mach number is fixed by the area ratio between the exit and throat of the nozzle.

So, this is the only one possible solution for the supersonic flow at the exit. So, the very basic bottom line is that to get M_e supersonic at the exit, we must satisfy two conditions $\frac{A_e}{A^*}$ and second thing is that $\frac{p_e}{p_0}$. If you satisfy both, it is possible to have isentropic solutions in this manner.

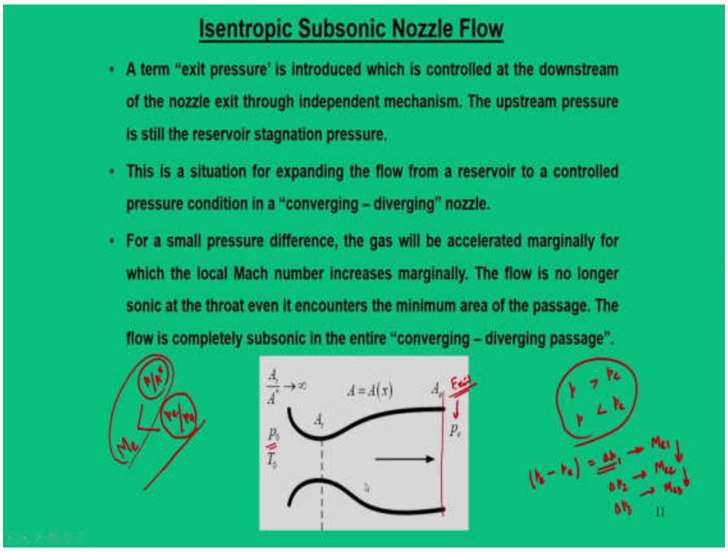
Now, any value p_e which is less than p_e or any value of A which is equal to A_e, will give you the Mach number M_e. So, this is the inferences from the isentropic supersonic flow.

(Refer Slide Time: 33:30)



Now, why we say it is a isentropic supersonic flow? Because the entire flow, we do not see any shock waves in its path. So, it is a shock free. So, it is a completely shock free with supersonic Mach number at the exit and similarly, the pressure should drop temperature is also drops. All these numbers relations are fixed for this isentropic supersonic flow.

(Refer Slide Time: 34:06)



Now, let us see that if we do not have a supersonic flow what are the other possibilities. So, we said that the Mach number M_e is a function of two parameters; one is area ratio and exit pressure, this two conditions are satisfied. But still if this two conditions are satisfied, we say we can achieve a supersonic solution.

Now, let us see that we gave this area ratio to get this Mach number; but we do not maintain this pressure ratio, but still the flow will be isentropic, but it would not be supersonic at the exit. So, this is what the real challenge that for a slug of mass to accelerate a flow in a passage, we need a pressure difference. So, if the pressure difference is not maintained, the flow cannot be accelerated.

So, for this situation, what we are analyzing that if you do not maintain this pressure ratio, what is going to happen? So, we land of a solution for which the Mach number is subsonic. So, we call this as a isentropic subsonic nozzle flow. So, here also, you have a similar passage, same nozzle with very high value of area ratio at the inlet $\frac{A_i}{A^*}$ goes to infinity with same reservoir having pressure p_0 and T_0 with minimum area A_t.

Here, we have exit area A_e and exit pressure p_e. If we maintain the adequate value of p_e, then we will get a Mach number M_e. Now, if you do not maintain p_e; then, what will happen? Then, one can have a actual value of p which is may be higher than p_e or one can have value of p which is lower than p_e.

So, under those circumstances, what are the possible solution that we can land on? Now, just to start with if at all you want to use it as a convergent divergent passage or nozzle flow, so the flow to happen, we must maintain a very small difference; that means, we should have p_e should be slightly less than p_0 or we must maintain a pressure difference $p_0 - p_e$ which is Δp_e which is very small.

So, initially you start with a very small pressure difference, so a slug of mass will try to enter. Now, if you increase this pressure difference; so, obviously you are going to accelerate the flow. Now, if you keep on increasing the pressure difference, we will have a different Mach number at the exit.

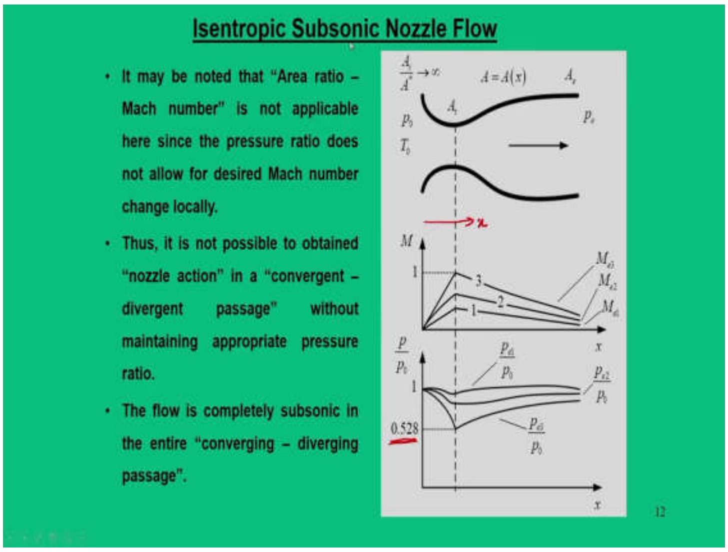
So, entire flow is isentropic, but we will not get a supersonic Mach number M_e, we will get some other Mach number. So, this is the philosophy or concept of an isentropic sub subsonic nozzle flow. So, we define a term which is exit pressure and that is introduced to control the downstream condition of the nozzle exit.

So, if this is your nozzle exit, we say it is a exit. So, exit means its it has a corresponding pressure p_e The upstream pressure is still stagnation pressure p_0. So, this is a situation that when you are expanding the flow from a reservoir to a control pressure condition in a converging diverging passage.

Now for a small pressure difference, the gas will be accelerated marginally for which the local Mach number increases marginally; that means, for a small pressure difference Δp, we may get some Mach number, we may say M_{e1}. If you increase further for one value of Δp_1, we get M_{e1}; another value of Δp_2, we will get slightly higher Mach number M_{e2} which increases likewise another value of Δp_3, we will get another Mach number M_{e3}.

So, as and when you increase this pressure difference, the Mach number is going to increase; but the flow is no longer sonic. But one important point is although you increase this Mach number, but the flow is no longer sonic.

(Refer Slide Time: 39:18)



So, this is shown here that we are using the same nozzle, we are trying to plot the flow passage; flow is along the x direction that is along the direction of the flow, how the pressure ratio and Mach number varies along x. So, for a small pressure ratio $\frac{p_{e1}}{p_0}$, we get a Mach number M_{e1}.

This Mach number further increases when we maintain a higher pressure ratio; and when the pressure difference decreases, the Mach number further increases.

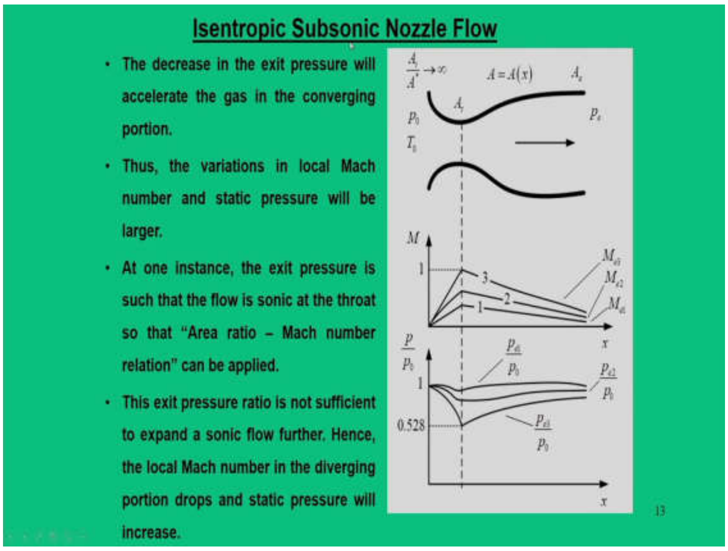
What you see here that even though we have a minimum area, but for the situation 1 and 2, we do not have a sonic flow at the throat. So, the concept of area ratio Mach number relation does not sounds good here, rather you have to use the simple continuity equations, the flow across any cross section to find out the flow rate or mass flow rate.

But what will happen is that for one particular case that is case 3, when you have a substantial higher value of pressure ratio and this pressure ratio happens to be 0.528 that is the isentropic relation number, if this ratio is maintained; then, we may land of a Mach number of 1 at the throat. But this same pressure ratio does not carry this flow all though we get a sonic flow, this sonic flow does not see sufficient pressure difference in the downstream to expand further.

So, the Mach number further drops because it does not see the required pressure difference. The Mach number in the diverging section also drops. Although Mach number in the converging section increases, Mach number in the diverging passage falls down because it does not see the pressure difference.

So, this is the philosophy that if you do not maintain the pressure ratio, we will have this solution. So, in fact in the entire situation, the flow is isentropic; but there an infinite number of solutions or there infinite number of possibilities of Mach number at the exit. On all these Mach numbers are subsonic. Hence, we call this as a subsonic nozzle flow.

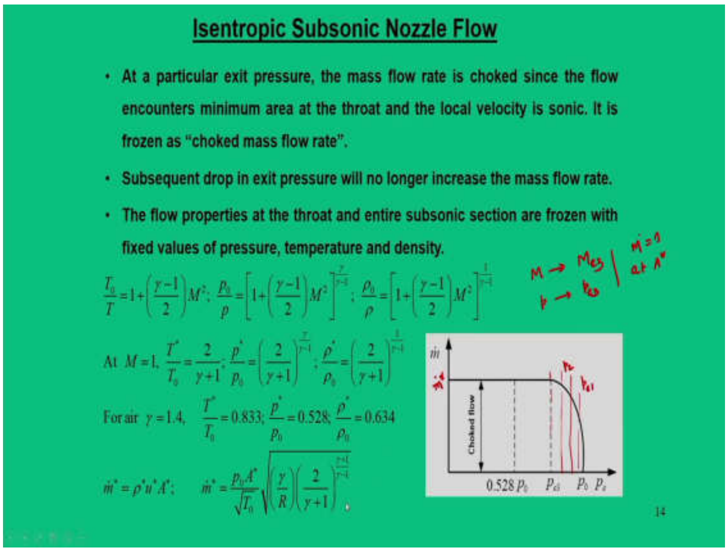
(Refer Slide Time: 41:53)



So, this is what I have summarized here, the decrease in the exit pressure will accelerate the gas in the converging portion. Thus, the variation of local Mach number and static pressure will be larger.

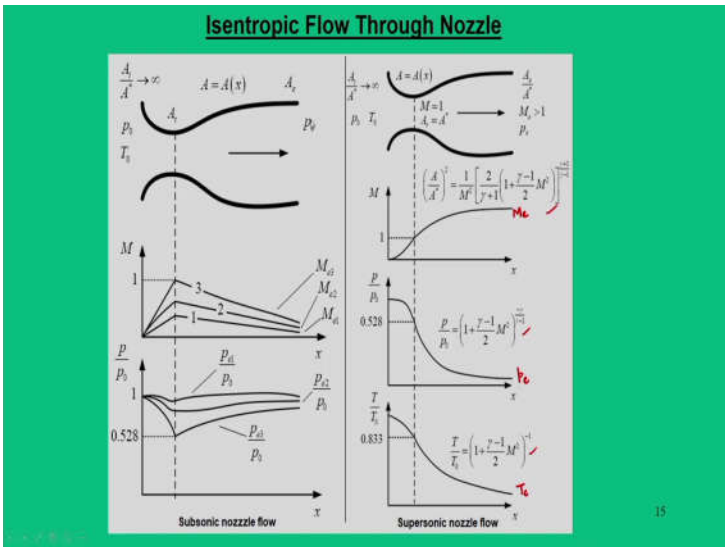
At one instant, the exit pressure is such that flow is sonic at the throat and area Mach number relation can be applied. But this exit pressure ratio is not sufficient to expand the sonic flow further. Hence, the local Mach number in the diverging portion drops and the static pressure increases.

(Refer Slide Time: 42:26)



So, if this is the case that one particular Mach number M_{e3}, if you can just plot. Now, when we say Mach number is sonic at the throat, when we have Mach number M goes to M_{e3} and pressure is at p_{e3}, then we can get M is equal to 1 at A* and that situation the flow we will define as $\dot{m}^*$ that is choked mass flow rate. And for this p_{e3} and all other pressure value, say that we have p_{e1}, we have p_{e2} that any intermediate point flow is not choked at all. So, when we have a choked flow rate, then we can directly use this particular relations.

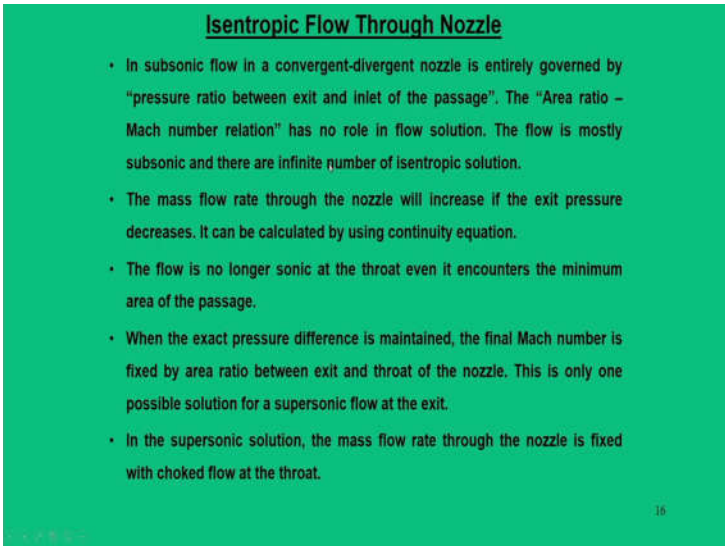
(Refer Slide Time: 43:28)



So, if you just summarize both isentropic flow in the nozzle; in one case, we have a subsonic nozzle flow; other case, we have a supersonic nozzle flow. When we have a supersonic nozzle flow, we will have only one possible solutions for a given Mach number, following the area ratio Mach number relations provided adequate pressure ratio p_e is maintained and we have M_e and we have T_e and they use this expressions to fix the value at the exit.

But if you do not have this isentropic supersonic solutions or you do not maintain both area ratio or pressure, then we may land of infinite number of possible subsonic solutions having Mach number M_{e1}, M_{e2} and M_{e3} at the exit. So, likewise there may be many possibilities we may have infinite subsonic solutions. But one very basic bottom line is in the both the situations the flow is isentropic is in nature.

(Refer Slide Time: 44:40)

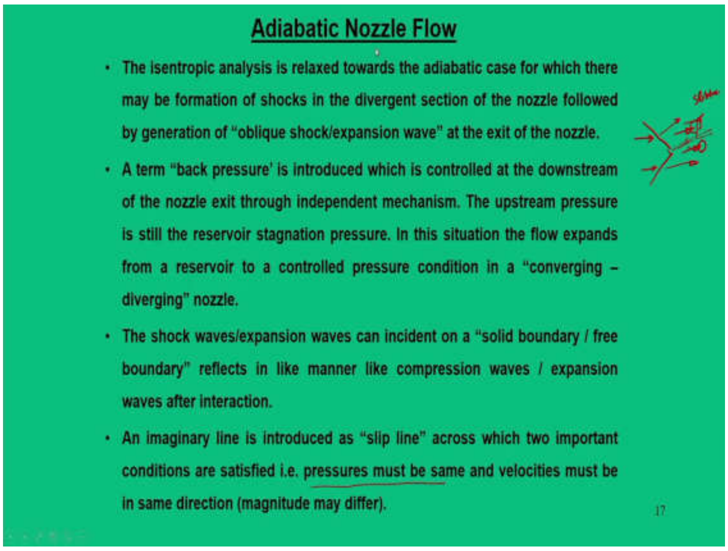


So, if you summarize these things, so a subsonic flow in a convergent divergent nozzle is governed by the pressure ratio at the exit and inlet of the passage; area ratio Mach number relation has no role in the flow solutions, rather we have to use simple continuity equations. The flow is mostly subsonic and there are infinite number of isentropic solutions and the mass flow rate through the nozzle will increase if the exit pressure decreases and this can be calculated simply by continuity equations.

The flow is no longer sonic at the throat, even it encounters a minimum area. Not necessarily it needs to be sonic, but when exact pressure ratio is maintained, the final

Mach number is fixed by the area ratio between the exit and the throat of the nozzle and this is the only one supersonic solutions at the exit and for the supersonic solution, the mass flow rate is fixed by choked condition; that means, flow is choked at the throat.

(Refer Slide Time: 45:52)



Now, we will move to the next class of nozzle flow which is a adiabatic nozzle flow. So, in this case, we are going to relax that if we do not have an isentropic process that is happening inside the nozzle, we will have a non-isentropic process and in particular, such kind of application when we deal with nozzle flow, a realistic way of defining the thermodynamic process is adiabatic in nature.

In fact, we do not allow any heat to be get added when the flow is taking place in a variable passage. So, we call this as a adiabatic type nozzle flow. So, when you deal with this kind of things, there may be a possibilities of generation of oblique shocks and expansion waves, we will see how they are going to happen and we will introduce a term which is called as a back pressure and this back pressure is introduced to control the pressure at the exit.

So, it is a independent mechanism in which we physically control this flow so that the way we want the flow to happen, we can have it. For example, if you want to have a oblique shock, then you can have it; if you want to have a normal shock in the nozzle, we can have it; but in all this process, the flow is no longer isentropic.

So, that is what with the concept of back pressure is introduced. Now, in the process when you have a normal shock and expansion wave that forms in the inside the nozzle, they may interact.

So, there are the possibilities that they can incident on a solid boundary; that means, inside surface of the nozzle or it may be a free boundary, when the flow comes out of the nozzle, it does not see any physical existence of the boundary rather it sees a ambient conditions. So, we call this ambient conditions which can be treated as a imaginary boundary called as free boundary.

So, there may have reflections of compression waves, expansion wave and many possible complex flow phenomena and in this process also, there will be an imaginary line which is known as slip lines across which two conditions are satisfied; that means, the pressures must be same and the velocities are in the same direction.

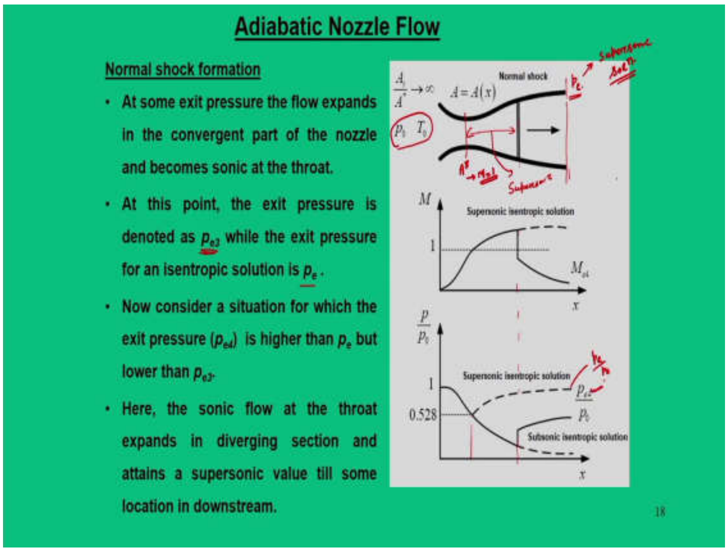
That means, whenever there is a flow for which there will be possible interaction of two shock waves, but after the shock wave, they should go in one particular directions.

For example, if you have two oblique shocks; then, the flow is encountering. But this oblique shock will deflect the flow in one direction, this flow may deflect in another direction and what may happen in such a way that we may have a slip line and the flow tries to be parallel to this streamline.

So, this slip line for which the pressures must match; that means, across this slip line the pressure of the upper part of this flow and pressure in the lower part of the flow should match and the velocities must be in the same direction.

This is how I am trying to explain. So, this may happen in this kind of situations, but all this things are very complicated; but I will try to explain that mainly how a normal shock and oblique shock or expansion waves are formed in case of an adiabatic nozzle flow.

(Refer Slide Time: 49:48)



So, the first thing that we are going to see that in a adiabatic nozzle flow, how a normal shock can be formed. Now, here we can say that for this particular nozzle, same nozzle with reservoir conditions p_0 and T_0 and this exit pressure is p_e and this exit pressure is fixed by supersonic number.

So, this is taken as the reference. Now, if a normal shock is formed at one particular passage, under what circumstances it can form? So, we have said that in our previous analysis that is isentropic subsonic nozzle flow, one particular pressure ratio or exit pressure, will have a sonic Mach number that is at the throat. At the A*, we have M is equal to 1.

Now, at that condition, if your pressure ratio is reduced from that value, what is going to have is that it tries to expand in a diverging passage. Why the isentropic subsonic solution was made? Because the flow did not find a sufficient pressure ratio to expand in the diverging passage. But if you allow the flow to see a sufficient pressure ratio, it will try to expand.

So, one particular instance, when we say that we maintain certain pressure such that a normal shock is formed. So, what condition it may form? So, what we can summarize is that at some exit pressure, the flow expands in the convergent part of the nozzle and become sonic at the throat. At this point the exit pressure is denoted as a p_{e3} in our earlier example.

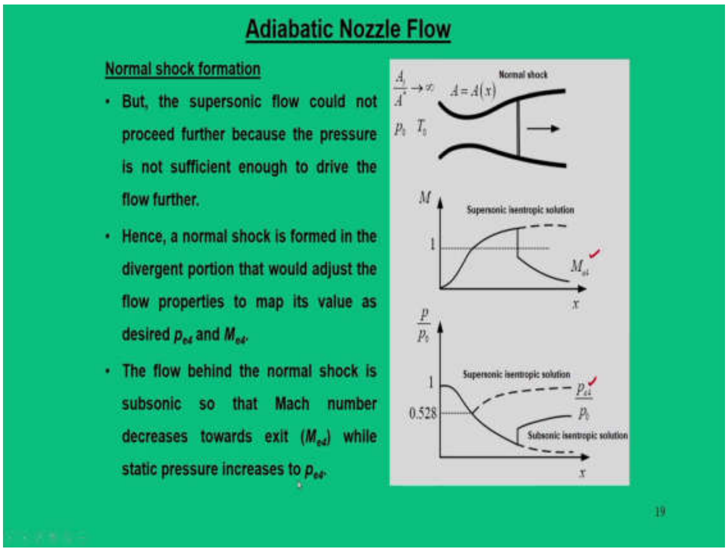
Now, while the exit pressure for an isentropic value is p_e. So, isentropic solution is p_e, but the subsonic solution value is p_{e3} for which flow is sonic. Now, consider a situation for which the exit pressure p_{e4}; that means, we maintain some independent mechanism such a way that we maintain a exists pressure of p_{e4} which is higher than p_e, but lower than p_{e3}.

Then in that case, what will happen? The sonic flow which was at the throat which is already formed when it is at p_{e3}, it tries to expand in the diverging passage because it sees a pressure difference and it tries to attains a supersonic value at some location in the downstream.

So, in this particular case, since it is a normal shock flow inside up to this point is supersonic and once there is a normal shock the Mach number drops. So, when the Mach number drops, the rest of the part of the flow the Mach number become subsonic. So, we will never get a supersonic Mach number at the exit.

So, Mach number become subsonic. So, if you look at this pressure plot here, what it is seen is that had the process been a supersonic, the pressure plot would have taken from the throat point, at this point it would have taken this passage dotted line and this dotted line, we say it is a $\frac{p_e}{p_0}$. But since there is a shock wave forming at one of this location, the static pressure will rise which is $\frac{p_{e4}}{p_0}$. So, had the process been completely subsonic, it would have gone in this manner. Since there is a normal shock, static pressure across the normal shock increases. So, we land of in having a value $\frac{p_{e4}}{p_0}$.

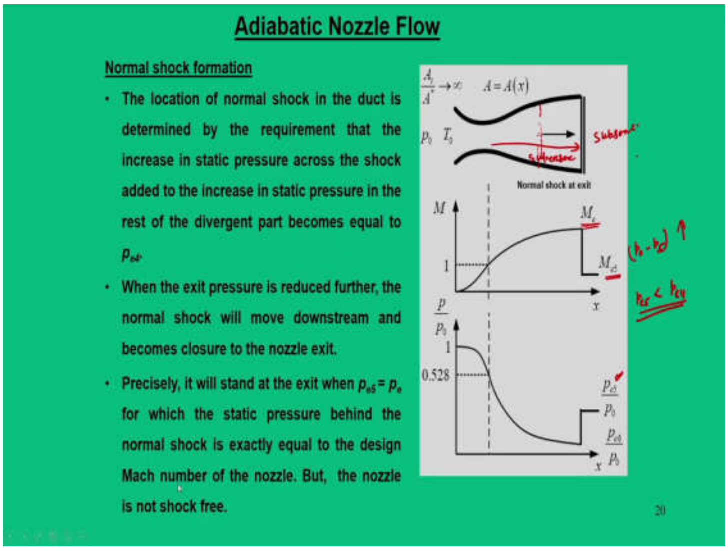
(Refer Slide Time: 54:32)



And correspondingly for this p_{e4} and M_{e4}, the entire solution is depend on what Mach number we are going to get at the exit. So, at the pressure supersonic flow could not proceed further, because pressure different is not sufficient to drive the flow further.

So, normal shock is formed in the divergent portion and the entire flow property is just to map the value as desired at p_{e4} and M_{e4}. The flow behind the normal shock is subsonic so that Mach number decreases at the exit, while the static pressure increases. That is quiet obvious, static pressure across the normal shock increases.

(Refer Slide Time: 55:27)



Now, we now land of the pressure value p_{e4}. We are now moving that if you bring this p to another pressure which is p_{e5}; that means, this p_{e5} is less than p_{e4}. So, obviously, your p_0- p_{e5} that is this value will rise.

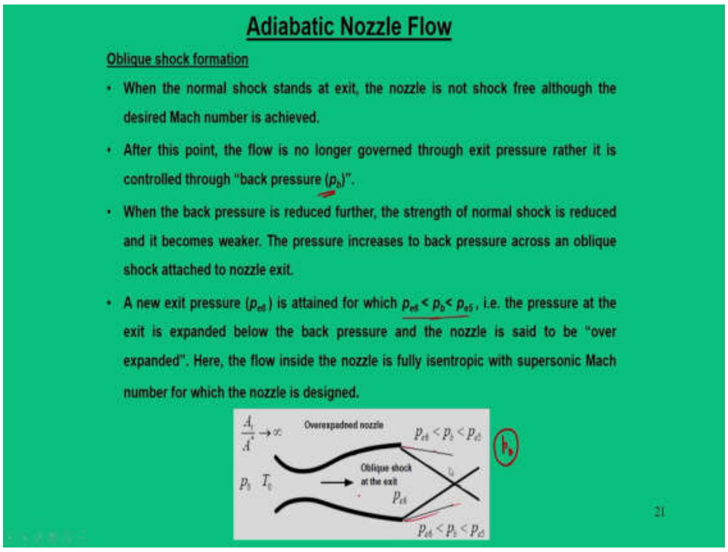
When this is going to rise, we will have enough strength to push this normal shock which was earlier at some location here and it is trying to push this normal shock away. Now, this p_{e5} condition is such that the normal shock stands at the exit. So that means, in the entire process we will have a exit Mach number M_e, flow inside the nozzle is completely supersonic; that means, earlier situation the flow up to this point, it was supersonic and from here it was subsonic.

Now, what happens when the normal shock remains at the exit, the entire flow within this nozzle is supersonic; whereas, after the shock, it becomes subsonic. But after the shock when become subsonic, we will have a normal shock and this Mach number again drops to a subsonic value.

So, from its theoretical value M_e it drops to M_{e5} subsonic value; Mach number drops to this value. When this Mach number drops, the static pressure will raise p_{e5}. So, this is exactly what happens when there is a shock waves that stands at the exit. So, when the exit pressure is reduced further, the normal shock will move downstream and becomes closer to the nozzle exit.

Precisely, it will stand as exit; whereas, p_{e5} is equal to p_e for which the static pressure behind the normal shock is exactly equal to the design Mach number of the nozzle, but nozzle is not shock free.

(Refer Slide Time: 57:49)

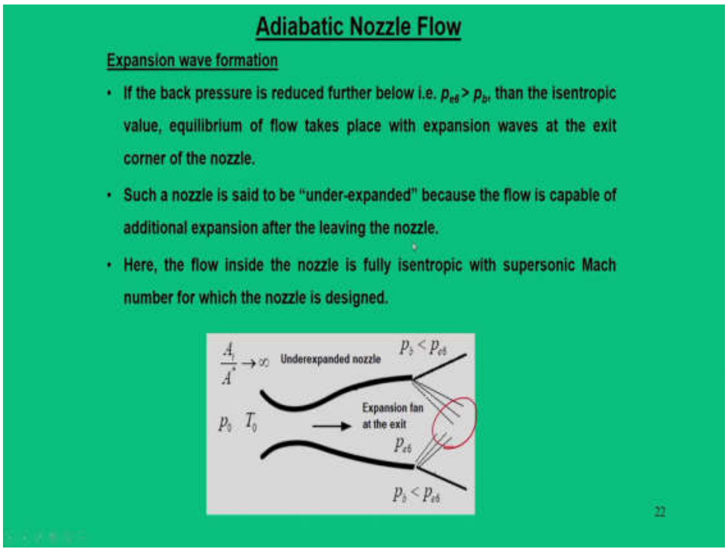


Then, in another instances, we will see that how an oblique shock can be formed in a adiabatic nozzle flow. Now moving further, that here we will introduce a term which is called as "back pressure" p_b.

So, that back pressure p_b is maintained here, when we have a back pressure p_b, which is controlled in a manner such that the normal shock which was at the exit, the strength of the normal shock becomes reduced. When the strength of the normal shock becomes reduced, it sees a imaginary compression corner which is given by this line.

And across this line, we will have the oblique shocks and these oblique shocks try to meet at one point. So, in this process, a new exit pressure which is maintained for which $p_{e6} < p_b < p_{e5}$. Now, at this location the pressure at the exit is expanded below the back pressure and nozzle is set to be over expanded; that means, here the flow is fully isentropic, but with supersonic Mach number, but there is an oblique shock which is formed at the exit.

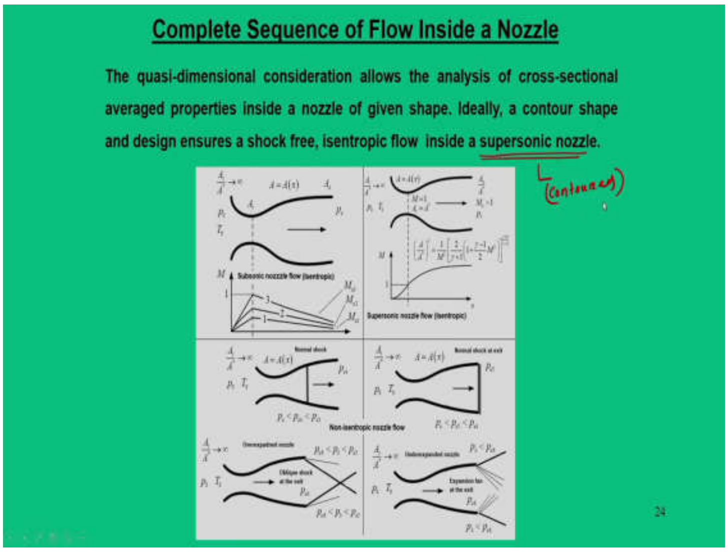
(Refer Slide Time: 59:24)



The last phenomena that may going to happen is that if we reduce this exit pressure such that the back pressure is sufficiently lower than what is supposed to be for the oblique shock, then the we will have a equilibrium flow; that means, we have a pressure value which is much less than the isentropic value and the expansion waves are formed at the exit of the nozzle; that means, flow has sufficient pressure to expand further.

So, since it is a corner, so we may have kind of a expansion wave that gets generated at the exit of the nozzle and down the line these expansion waves can also further interact and complicate the flow phenomena. So, here the flow inside a nozzle is fully isentropic for which the nozzle is designed and such a nozzle is known as “under-expanded” nozzle because the flow is capable for additional expansion after leaving the nozzles.

(Refer Slide Time: 60:40)



So, finally, this is the complete sequence of flow inside a nozzle, where we have categorized in two aspects; one is isentropic flow, other is non isentropic flow. Now, when we have isentropic flow, we have possibilities that flow can be a subsonic or flow can be a supersonic. When the flow is supersonic, there is a unique solution; when the flow is subsonic, there are infinite number of solutions.

Other possible practical cases where non isentropic flow can have, we may land of in having a normal shock formation. This normal shock can move along this nozzle and one point of time, it may stand at the exit, one point of time it may come out. When it come out, it come out as a oblique shock format and finally, the when the pressure difference are such that the flow has sufficient pressure to even expand further even after encountering this diverging passage.

And finally, this particular study will tell us that how the quasi one-dimensional consideration allows the cross sectional average properties inside a nozzle for a given shape. So, it is the main effort for the designer to have a contour type of nozzle which is shock free, entirely isentropic and such a nozzle is known as supersonic nozzle or we say supersonic contour nozzle.

So, that means, its internal geometry changes from point to point. So, it is a contour type. So, the surface will have a smooth surface inside so that the flow sees a very negligible drop in pressure so that the flow sees a very smooth passage in its expansion process and such a process is knows as a contour nozzle.

So, with this, I will conclude this talk for today.

Thank you for your attention.