Fundamental of Compressible Flow
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Module – 05
Lecture - 16
Expansion Waves and Oblique Shocks- III

Welcome to this course Fundamentals of Compressible Flow, we are in module 5 that is Expansion Waves and Oblique Shock Waves.

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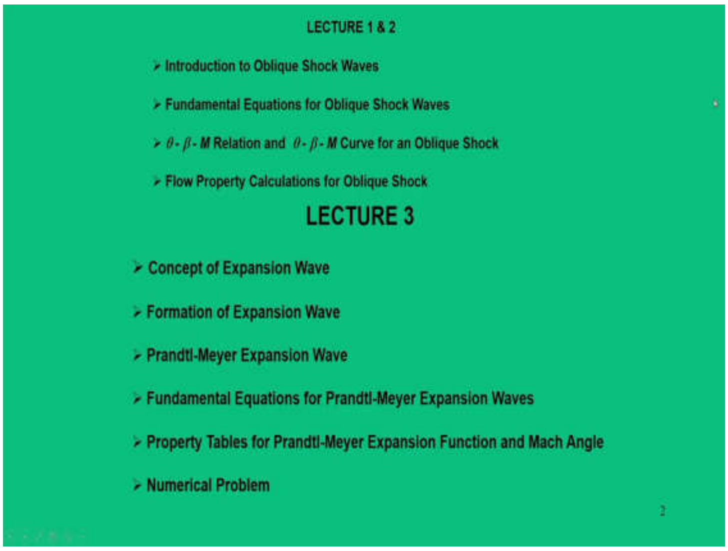

So, in the previous lectures in this module that is 1 and 2 we have exhaustively covered the oblique shock, its fundamental equations. Then we derived a relations known as θ-β-M relations and based on this θ-β-M relations, a curve was generated which is the array of all such curves for different Mach number and flow deflection angle.

This curve helps us to find graphically the information about the oblique shocks and shock wave angle and moreover, this curve is also very vital in estimating the flow properties across the oblique shocks. And now today we are going to discuss about something just opposite to that of oblique shocks that is known as expansion waves.

So, while analyzing the oblique shocks we say that when a supersonic flow encounters a compression coroner, flow gets deflected towards itself. And in this way the flow becomes compressed and its all other properties like pressure, temperature etc increases.

Now can we have another situations where instead of getting the flow compressed, can the flow expand? So, in such a situation we cannot call this as oblique shocks rather we will tell it as expansion waves.

So, in this particular lecture we will talk in details about the concept of expansion waves, how it forms? Then in particular we will discuss about a very specific expansion wave that is Prandtl - Meyer expansion wave or many a time it is called as centered expansion.

Then for this Prandtl - Meyer expansion waves, we try to derive the fundamental equations and moving further we will discuss about the property tables for the Prandtl - Meyer expansion function and Mach angle. And subsequently we will try to solve some problems based on this Prandtl - Meyer expansion wave.

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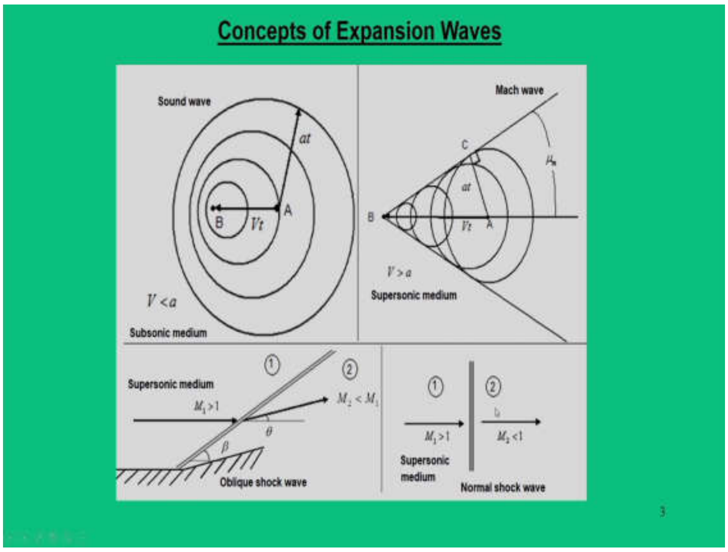

Now, to just to give glimpses of what we have covered till this point of time, we know it is a sound wave, it is a pressure wave and it compresses the medium. The main intention of the sound wave is that it propagates the flow information; it shares the flow information. Now when these pressure waves become stronger and stronger that is possible only when your gas velocity or flight body moves at higher speed than the sound waves. So, these pressure waves become stronger and they try to form a Mach wave.

Now with further intensity of increase in the pressure waves, all these formation of Mach waves that are getting generated through this body moving at supersonic speed, they try to merge as a shock wave and that shock wave we call as a oblique shock wave.

Now the strength of the oblique shock wave is based on the shock wave angle, now when the shock wave angle is 90° means that the streamlines and the shock waves they are perpendicular to each other. So, in that situation oblique shock becomes a normal shock. Now in all these example we say that all these waves are nothing but the pressure waves and that compresses the medium.

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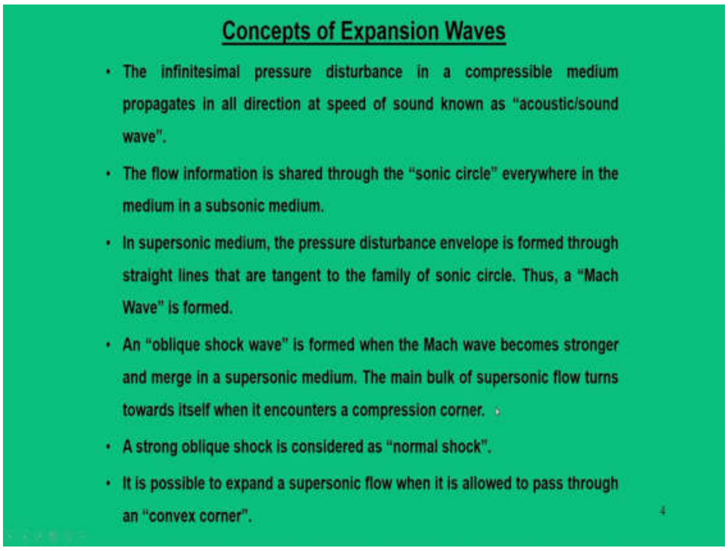

So, this is what we have discussed so far. So, infinitesimally small pressure disturbance is known as "acoustic or sound waves" and through these sound waves a sonic circle is generated and through this "sonic circle" flow information is shared.

Now a "Mach wave" is formed when the pressure disturbance envelope is formed through straight lines that are tangent to the family of sonic circles; when these Mach waves become stronger the "oblique shock wave" is formed.

And this is a situation when a supersonic flow encounters a compression corner and the entire flow or main bulk of the flow turns towards itself. And the strength of the oblique shock when increases it becomes a “normal shock.”

So, in all possible cases, it is the medium that gets compressed. Now, for a supersonic flow that has to expand then there is a provision that we can allow the flow to pass through a “convex corner.”

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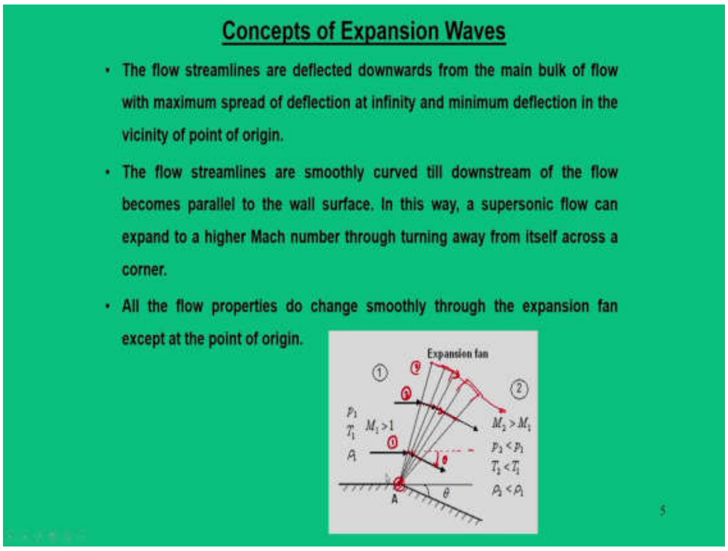

So, how is it done we can refer this particular figure; as we say that, in this particular example what has been seen shown here that a expansion corner is created. What does this mean? Means there are essentially there are two surfaces and these two surfaces are meeting at a point A. So, the supersonic flow passes through this surfaces which is horizontal that is the upstream of flow, region 1 and this flow has to turn through an angle θ downwards.

So, in other words the streamlines that has to be in the region 2 must be parallel to this second surface. That means, in other words the flow has to turn an angle θ and which is away from the main bulk of the flow; that means, had this flow what had been undertaken, it would have gone through in this path. So, in this process of turning, the flow was turn to an angle θ and across this θ angle the streamlines are now parallel to the second surface.

Now if we look at this particular events. So, in between this turning process we can view this fact that flow has expanded, because it is seeing a larger area and it is the flow is turning away from itself. And when you view this we can say there are series of expansion waves that gets generated from the point A. Now looking at the streamlines let us say a streamline the first streamline and this is second streamline.

So, for the first streamline, it has to cover a distance in a curved manner and for the second streamline, this flow has to turn also in a curved fashion. And, in this process the distance traveled by second stream line, it covers a larger distance. The streamline 1 has a has to cover a smaller distance likewise if you look at another stream lines maybe somewhere 3 down the far away from this it has to cover a much larger distance.

So, in other words what we can say that at the end of this turning process all the streamlines in the downstream side are always parallel to this second surface. But one interesting thing that happens, the streamline which is just close to the point A has a discontinuity in the medium. So, the flow is as if that streamline does not undergo the turning; which is near to the wall whereas, all other stream lines turns in a smooth manner.

The smaller the distance of the streamline from the wall, smaller is the turning distance; larger is the streamline away from the wall larger is the turning distance. This is how the philosophy of expansion waves happens. And in this process all the flow properties change smoothly through an expansion fan except at the point of origin. So, in this case the point of origin is A.

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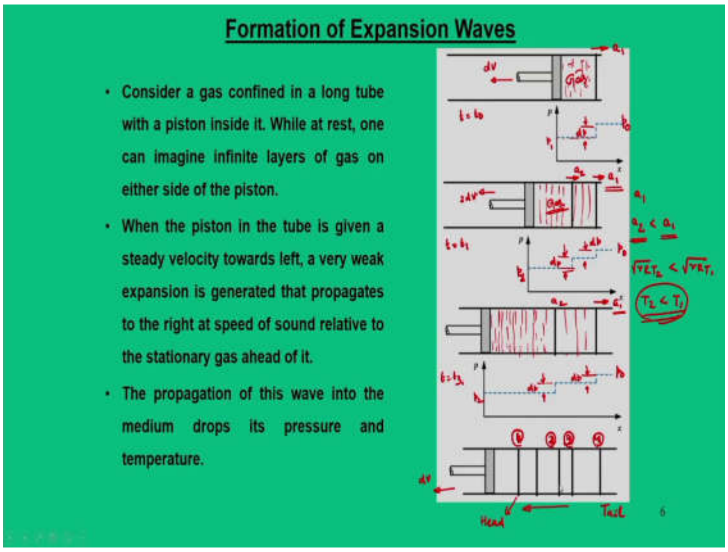

Now, to evaluate this particular concept of expansion process we will now discuss about the formation of expansion waves and likewise in previous cases what we do is that we take a piston cylinder device in which a gas is kept. So, in this case also we will say there is a gas which is kept in a piston cylinder medium.

Now while talking about these compression mechanisms we say that piston was given a push towards right, but in this case we will say that push piston is given a push so that a small velocity dV is generated and towards right. So, that means when the gas is at initial position its initial pressure was p_0 and when a piston is given a push towards left the gas is now seeing a larger volume.

So, thereby its tries to expand, when it expands it comes down to the pressure let us say p_1. So, from this initial pressure p_0 gas gets expanded and in this process what happens is that when the push was given a sound wave is generated that moves a speed a_1.

Now, in the next instance, we give another incremental in velocity so, then this dV becomes 2dV. So, increase the velocity by 2dV. So, thereby we will have the first sound waves as it is and second sound wave will also get generated a_2 as it is. So, now we are looking at this particular event at t is equal to t_1, in the earlier event t is equal to t_0 and in this process also what happens, the gas also sees a larger volume.

So, thereby its pressure changes from initial pressure p_0 to when the first sound wave is generated through it gets the differential pressure dp and for the another differential pressure dp is also generated. And finally, the same pressure p_2 is formed. In fact, in this case also the pressure difference is dp. So, likewise the gas is now expanded to a larger volume.

Now, in this process also there are lot of expansion waves that would have been generated in addition to the sound waves and if you look at this particular event at time is equal to t_3. Then, this location of sound wave would have been different and we will have this pressure p_2 and through this process there are many such expansion wave that would have get generated.

But most important point in this situation is that when you look at the first sound wave that speed is a_1 and when you look at the second sound wave a_2, here a_2 is less than a_1; that means, strength of the second sound wave is less than the strength of first sound wave. Why? Because since the medium gets expanded; temperature is less that is because a_2 is nothing but $\sqrt{\gamma R T_2}$ and a_1 is $\sqrt{\gamma R T_1}$. So, T_2 is less than T_1.

So, since T_2 is less than T_1 because the gas gets cooled through this expansion process; so; obviously, the speed of sound waves of the second sound wave is less than the first sound wave. So, since this is the case though the expansion waves that gets generated through this subsequent sound waves, they move at slower rate.

So, in none of the at any cost, the subsequent sound waves cannot overtake the first wave. Whereas, this was the opposite case when the second in a compression medium when the second pressure waves try to overtake the first pressure wave.

So, in this expansion process if I can say the in this process if there are infinite number of sound waves or in other words I will say expansion waves gets generated. So, this is the first expansion wave, second expansion wave, third expansion wave, fourth expansion wave and these expansion waves moves towards left as the piston is given a velocity dV and this expansion waves they try to spread. So, they cannot overtake each other.

So, this will be 1 2 3 4. So, in none of the situations, the expansion wave 4 will overtake the expansion wave 1. So, they try to spread and this first wave is nothing, but we call this as a head of the wave and last one will be tail of the wave. So, this is how the concept of expansion waves is introduced in a piston cylinder device and that is one dimensional in nature.

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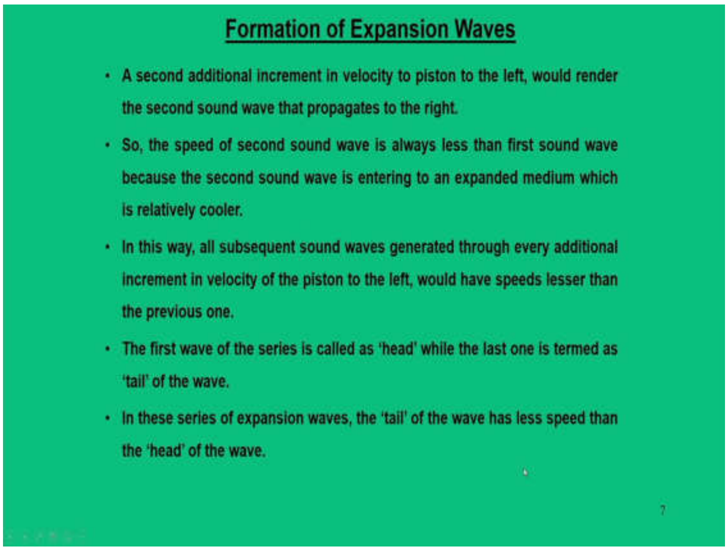

So, whatever I told if I summarize that the second additional increment in the velocity to the piston to the left would render the second sound wave to the right. The speed of second sound wave is always less than the speed of first sound waves, because the second sound wave is entering to an expansion medium which is relatively cooler. In this way all subsequent sound waves generated through every additional increment in the velocity to the left, would have speeds less than that of previous one.

This brings a consequence that all the expansion waves, due to the sound waves they move in the direction of the piston movement and they are in the series of waves. And the first wave in this series is called as "head" and the last one is termed as the "tail" of the wave. And; obviously, the "tail" of the wave has less speed than the "head" of the wave. So, that means, tail of the tail of the wave cannot overtake the head of the wave.

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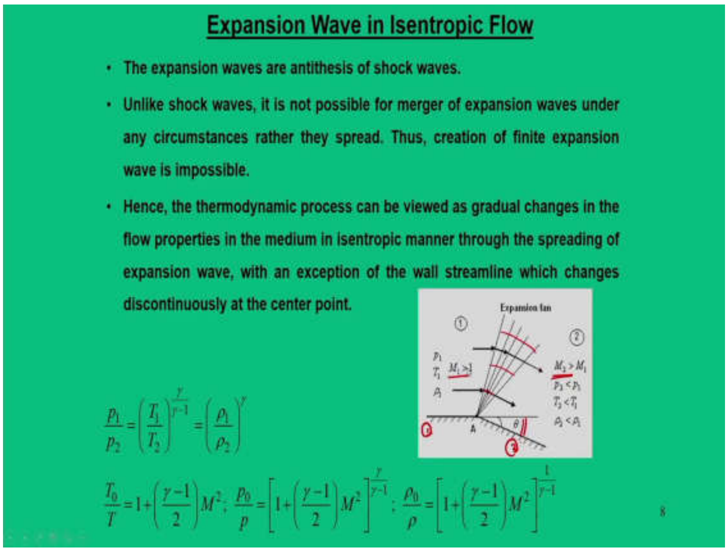

So, this is how the concept of expansion wave is introduced in the one dimensional medium, but; however, in reality normally the formation of this expansion wave is two dimensional in nature. So, as I showed it in the first example that there are two surfaces 1 and 2 they are inclined at angle θ and the supersonic flow tries to encounter into these two surfaces. So, when it encounters, the streamlines gets deflected and when they get deflected they always try to be in parallel to both the surfaces.

So, in the first instance the stream lines are parallel to the surface 1, in the second instance the streamlines are parallel to the surface 2. And in this process what happens; the Mach number increases, but the static pressure temperature and density drops. But however, the change that occurs is very gradual and why it is gradual because they try to spread and medium is not disturbed.

So, such a process we can view it to be the property change in an isentropic manner, but one important thing is that whether we say that the smooth pattern of this turning is not true at the center A, because at that point of time the changes are discontinuous.

So, in all every instances the flow properties are isentropic, hence the isentropic relations or equations holds good; that means, for the region for the region 1 and region 2. So, we say $\frac{p_1}{p_2} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_1}{\rho_2}\right)^{\gamma}$.

Now, when I say; the two situations M_1 and M_2; so the properties in the region 1 are function of M_1 through this isentropic relation properties, in the region 2 are function of M_2. Now our main task is to find out after this turning what will be the Mach number M_2.

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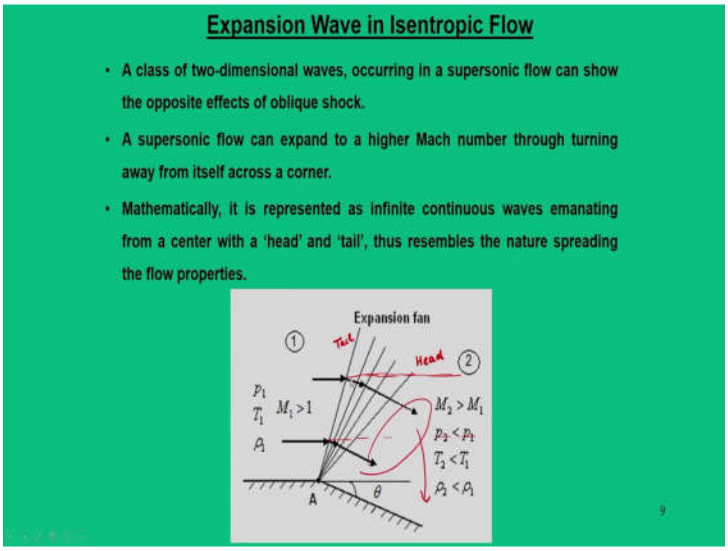

Since we say that the flow turns smoothly across an expansion wave. So, we can say that medium can be treated to be an isotropic medium.

So, we say that a class of two dimensional waves can be defined in a supersonic flow that has opposite effects to the oblique shocks. And, across this expansion wave the Mach number increases through the turning and this turning process is such that the flow turns away from the main bulk flow. So, if you say the main bulk of the flows is going in this direction, the entire bulk of the flow gets turned away towards this second surface.

And such a process is defined through infinite continuous waves and known as 'head' and 'tail.' So, the first expansion process starts as head of the wave and last one we say tail of the wave. So, when I say head, the flow has initiated turn and the last expansion wave that is at the tail, the flow has completed the entire turning process.

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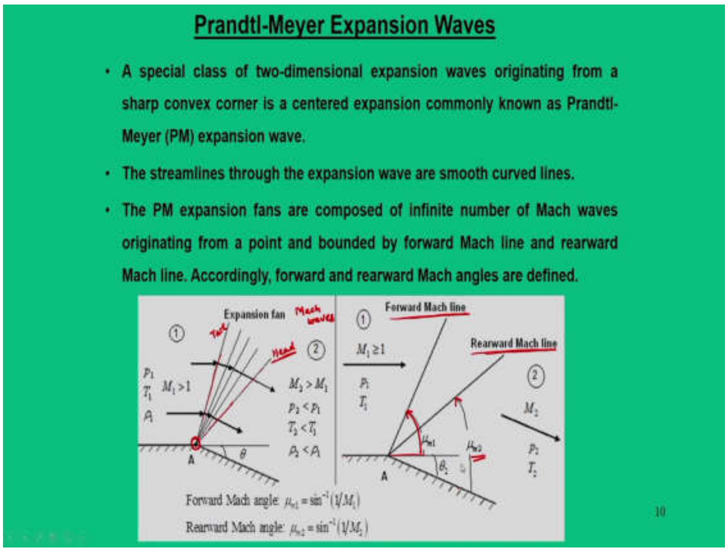

Now, having said this we will now define a special class of expansion wave known as Prandtl - Meyer expansion waves. So, these Prandtl - Meyer expansion waves are the a class of waves where we call this as a expansion fan and here the head and tail of the wave are defined through a Mach line. Or in other words we can say these are the waves that generated from a center point and there are infinite waves and all these infinite waves are known as Mach waves.

Now when I say Mach waves; the head and tail are now defined by a line which is known as rearward Mach line and forward Mach line. When I say this forward Mach line, this forward Mach line is defined through a Mach angle $\mu_{m1} = \sin^{-1}\left(1/M_1 \right)$. So, the location of this forward Mach line is initiated from the horizontal at angle μ_{m1} and that is defined by $\sin^{-1}\left(1/M_1 \right)$. This is what they call as forward Mach line.

And the rearward Mach line; that means, at that Mach line the flow has completed the turning. So, when we say completed the turning that Mach line will be decided by the Mach number corresponding to condition 2 that is M_2. So, that Mach line makes an angle μ_{m2} from the second surface. So, if this is the surface first surface and this is the second surface; for the forward Mach line this angle we say μ_{m1} and for the rearward Mach line this angle is μ_{m2} and in this process the flow has turned through an angle θ.

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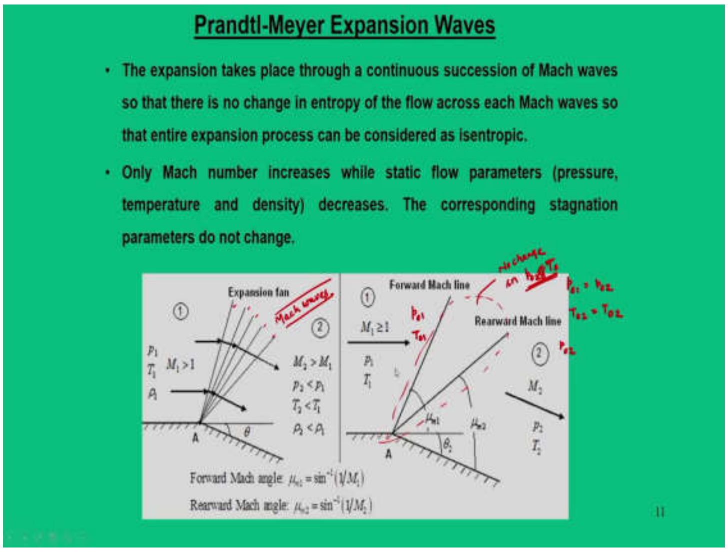

And since this is how I have explained all these things, the expansion waves takes place through continuous succession of Mach waves. So, there is no change in the entropy of the flow across each Mach waves and we say all these are Mach waves.

So, through this turning process there is no change in the entropy. And what is the net effect? The net effect is the only Mach number increase while the static flow properties such as pressure temperature density, they drop; whereas, stagnation properties also do not change means total conditions p_{01} and p_{02} are same T_{01} and T_{02} are same.

In other words, even within this region also no change in stagnation properties p_0 and T_0; whereas, this was the not the case when the supersonic flow encounters a oblique shock, the total pressure drops although temperature does not change, but the total pressure drops. But in a Prandtl - Meyer expansion the stagnation properties do not change and in other words the entire flow field is isentropic in nature.

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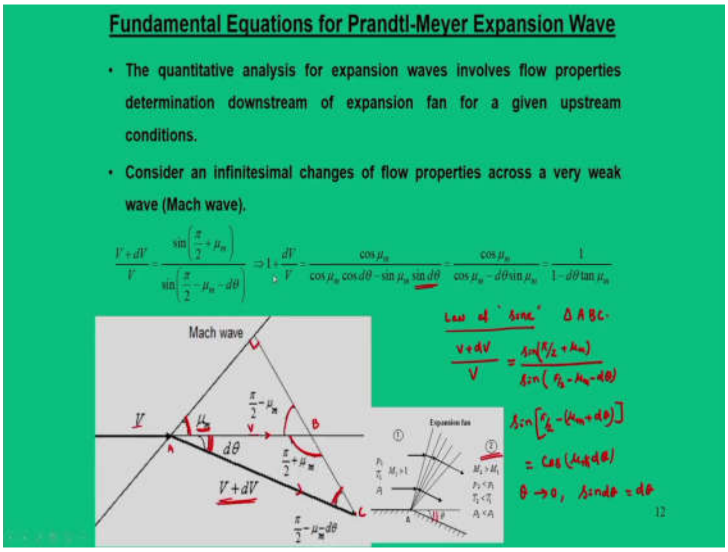

Now we will move to the fundamental equations for Prandtl - Meyer expansion waves. So, to start with the fundamental equation of Prandtl - Meyer expansion waves what we are going to see is that as I mentioned that the event of supersonic flow in an expansion process occurs through series of infinite Mach waves, we will take one such Mach waves.

So, here in the this figure we can say it is one of the Mach waves and this θ, the entire flow deflections we have defined that streamline as if it has undergone a very small deflection dθ. And through this deflection, the flow velocity instead of V it is get increased by V + dV in one such Mach waves. Likewise one can say that if I integrate this θ over this angle, then I can eventually obtain the final velocity in the region 2. So, to do that analysis we started one particular expansion waves or Mach wave. Now let us understand this figure.

So, as I mentioned that across this Mach wave the velocity vector become V + dV and this Mach wave makes an angle μ_m that is the Mach angle this angle is the flow deflection angle and we can drop a perpendicular from this streamline vector V + dV on to this Mach wave.

So, once you drop a perpendicular to it, we will have all these geometrical information in place. So, now, I can say that we have a triangle that consist of ABC that consists of the velocity vector V and V + dV and we use the law of sine. So, we can write for the triangle ABC.

So, law of sine tells that I can write $\frac{V+dV}{V}$, then the right hand side of this equation will have the angle corresponding angles that is $\frac{\sin\left(\frac{\pi}{2}+\mu_m\right)}{\sin\left(\frac{\pi}{2}-\mu_m-d\theta\right)}$ or in other words for this particular triangle once we know all geometrical angles, we can use the law of sine in this manner.

So, this is how it is written here $\frac{V+dV}{V}=\frac{\sin\left(\frac{\pi}{2}+\mu_m\right)}{\sin\left(\frac{\pi}{2}-\mu_m-d\theta\right)}$. So, this equation gets now simplified, the numerator becomes $\cos\mu_m$; denominator one can expand $\cos\mu_m\cos d\theta-\sin\mu_m\sin d\theta$. So, this can be expanded in this form.

Then one thing we needs to note here that in this term $\sin d\theta$ when θ tends to 0, $\sin d\theta$ we can write as $d\theta$. So, that we can write this. Now from this equation if you divide $\sin\theta$ in this numerator and denominator we get $\frac{1}{1-d\theta\tan\mu_m}$. So, the equation becomes $1+\frac{dV}{V}=\frac{1}{1-d\theta\tan\mu_m}$.

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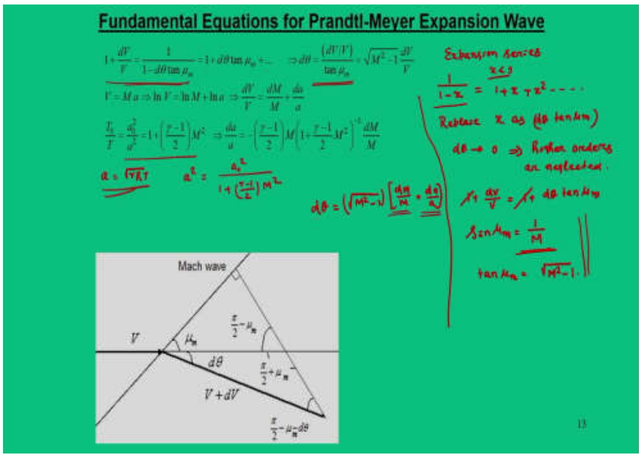

So, here this particular expression, we have to use some mathematical trick or expansion series, which means any equation $\frac{1}{1-x}$, where x less than 1 in an expansion series, we can write $\frac{1}{1-x}=1+x+x^2+........$ So, here replace x as $d\theta\tan\mu_m$. So, when I replace this so I can write $1+d\theta\tan\mu_m+...$. So, since $d\theta$ is small. So, this means higher order series are neglected.

So, in other words we can write that $1+\frac{dV}{V}=1+d\theta\tan\mu_m$. So, we get $d\theta=\frac{(dV/V)}{\tan\mu_m}$. Now, we also know that $\sin(\mu_m)=\frac{1}{M}$. So, we can find out $\tan\mu_m$; will be $\sqrt{M^2-1}$. So, these are the trigonometric information we can put.

So, we get now $d\theta=\sqrt{M^2-1}\frac{dV}{V}$. So, here the question still it is not known is what is $\frac{dV}{V}$? Although, we know the Mach number, but you do not know the $\frac{dV}{V}$, to do this we have to recall that we have to represent this velocity in the form of Mach number.

So, $V=M\,a$ you take logarithm of that. So, we get $\ln V=\ln M+\ln a$, from this equation we can find out $\frac{dV}{V}=\frac{dM}{M}+\frac{da}{a}$. So, that means the dθ equation we introduce one more term $\frac{dM}{M}$, but still it is not clear what is $\frac{da}{a}$. So, we have to bring this $\frac{da}{a}$ as a function of M, because all our information are related to the upstream flow Mach number.

But surprisingly, since entire process is isentropic we have one relation $\frac{T_0}{T}=1+\left(\frac{\gamma-1}{2}\right)M^2$ and this $\frac{T_0}{T}=\frac{a_0^2}{a^2}$ because $a=\sqrt{\gamma RT}$. So, these equations now is replaced with this. So, we can write $a^2=\frac{a_0^2}{1+\left(\frac{\gamma-1}{2}\right)M^2}$.

We have to now differentiate this equation so that a_0 will get canceled. So, it is it will be now replaced as a function of Mach number.

$$\frac{da}{a} = -\left(\frac{\gamma-1}{2}\right) M \left(1+\frac{\gamma-1}{2}M^2\right)^{-1} \frac{dM}{M}$$

So, what you do now, in this main equation which is $d\theta = \sqrt{M^2-1}\left(\frac{dM}{M}+\frac{da}{a}\right)$. So, we are now in a position that we know $\frac{da}{a}$, we also know $\frac{dM}{M}$; when you replace this θ we arrive at this particular equations.

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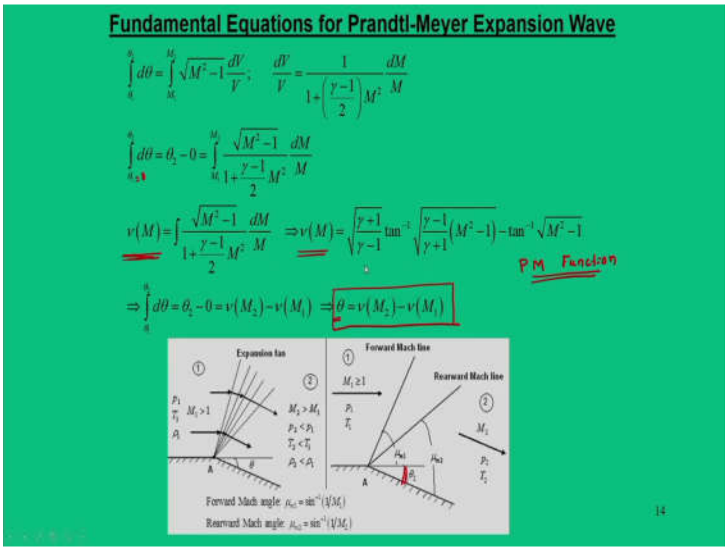

So, putting $\frac{dV}{V}$ as a function of $\frac{dM}{M}$, then we can replace that equation and now we can integrate that equations from θ_1 to θ_2. So, θ_1 to θ_2 in our case is nothing but our initial angle is 0. So, many a times θ_1 is equal to you can say it is 0. So, you can go up to θ_2. So, we get a integral of this equations.

$$\int_{\theta_1}^{\theta_2} d\theta = \theta_2 - 0 = \int_{M_1}^{M_2} \frac{\sqrt{M^2-1}}{1+\frac{\gamma-1}{2}M^2}\frac{dM}{M}$$

So, this particular integral has a meaning to it and we give a meaning to this equation as $\nu(M)$, $\nu(M)$ is known as Prandtl - Meyer function. And in fact, this function is a angle and this angle we can find out as a function of Mach number and in fact, this function is an integral and this integral has a value as given by this expression.

$$\nu(M) = \int \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma - 1}{2} M^2} \frac{dM}{M} \quad \Rightarrow \nu(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \sqrt{\frac{\gamma - 1}{\gamma + 1}(M^2 - 1)} - \tan^{-1} \sqrt{M^2 - 1}$$

So, knowing this particular Prandtl - Meyer function at a given Mach number we can evaluate the $\int_{\theta_1}^{\theta_2} d\theta = \theta_2 - 0 = \nu(M_2) - \nu(M_1)$ because we are taking integral from M_1 to M_2. So, finally, we arrived that the supersonic flow when gets deflected through an angle θ, that θ is nothing but the difference in Prandtl - Meyer function corresponding to their respective Mach numbers.

So, this is the very important summary that we can find out that the flow deflection angle is a function of Prandtl - Meyer functions. Thus, the entire information or entire deflection is now left with evaluation of these particular integral and let us see how we are going to find out. So, although this expression is a big expression and it is a function of Mach number, but it is difficult to evaluate through manually.

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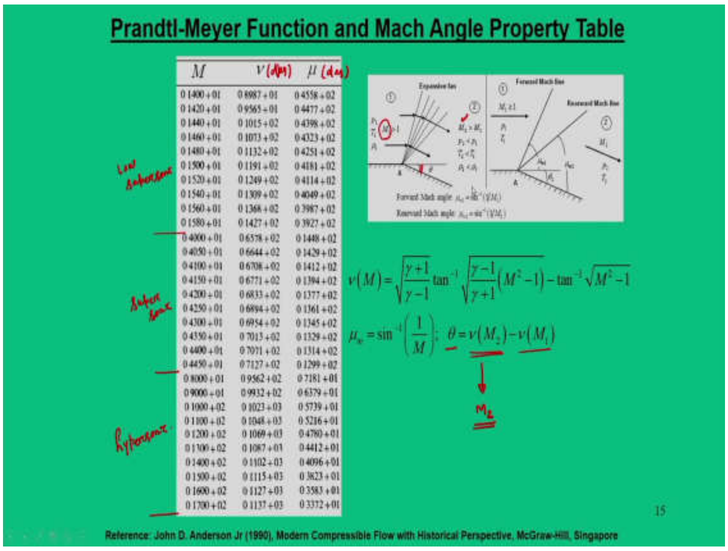

So, we take the help of a table which is known as property table that defines the Prandtl - Meyer function and Mach angle property table. So, what we see this table like, this is similar to all other tables we have talked about for normal shock, for isentropic table and here this is meant for the Prandtl - Meyer functions. Here there are three columns, the first column denotes the Mach number, second function denotes the Prandtl - Meyer function, third column denotes the Mach angle. So, this value of μ or ν are in degree.

What we have seen here you can say there are three regimes I have put one is low supersonic, second category is maybe supersonic in the range of 4 to 4.5 and this is hypersonic. All these extracts are taken from this book John D. Anderson, Modern Compressible Flow with Historical Prospective McGraw Hill. So, this is some extract, I have just taken some snapshot of the some extract of this data.

And what it says is that now to solve this particular problem what your approach should be is that for a given flow information we know the upstream conditions M_1, when you know $M_{1,}$ we know θ then we can find out. correspondingly its Prandtl - Meyer functions we know the Prandtl - Meyer function, μ_m; we know the θ.

So, once we know μ_{m1} and θ, μ_{m2} can be found out once you have μ_{m2}; this μ_{m2} will tell you the corresponding value of Mach number, new Mach number which is in the down streams. So, the M_2 is now known. So, once we know Mach number at M_1 and M_2 all other isentropic equations can be used.

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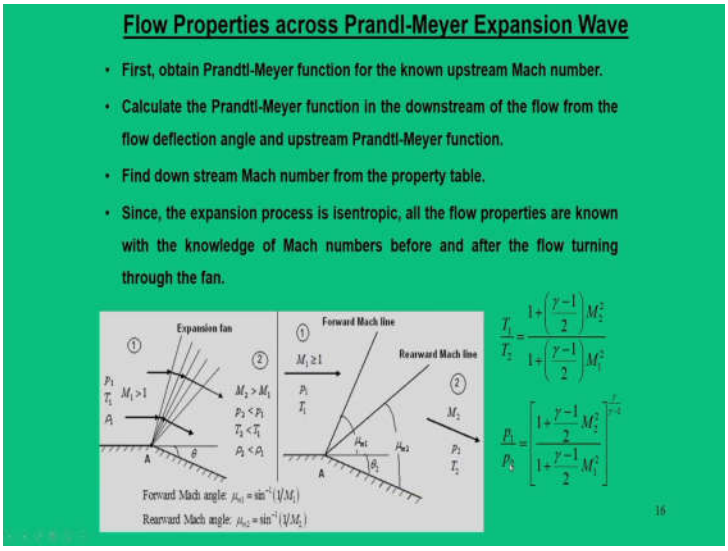

So, this is how the summary of the flow property evaluation across a Prandtl - Meyer expansion waves. So, first, you obtain the Prandtl - Meyer function for a given upstream

Mach number. Calculate the Prandtl - Meyer function in downstream with the help of flow deflection angle and upstream Prandtl - Meyer function.

Then find out the free stream Mach number from the property table with respect to downstream Mach number. And once you know all the upstream Mach number and downstream Mach number, we can say the properties are related respectively because you know M_1 and M_2, we also know T_1. So, we can find T_2 similarly we can find out p_2.

So, this is how the properties are evaluated.

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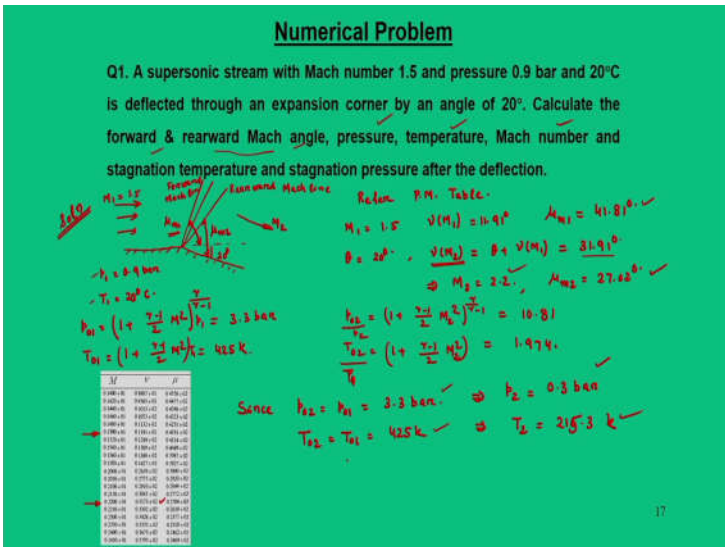

Now, whatever basic things if I just have to summarize that how we are going to use this property table, I have framed a problem which is for a supersonic stream when it encounters a compression corner.

So, the problem is defined that, a supersonic stream with Mach number of 1.5 initial pressure 0.9 bar and temperature 20°C, it is deflected through an expansion corner by an angle 20°. Calculate the forward and rearward Mach angle, pressure, temperature, Mach number and stagnation temperature and stagnation pressure after the deflections.

So, the solution process for this can have the following aspects. So, first we have to draw that what is the actual flow phenomena that is happening. So, here we can say it is a expansion corner. So, we can draw a schematic diagram of a expansion corner with an angle 20° which is known. And since it is a expansion corner this supersonic flow has initial value M_1 which is 1.5 when it sees this expansion corner it has to undergo a centered expansion or through Prandtl - Meyer expansion.

So, we can have a rearward Mach line and we will have a forward Mach line. So, these angles can be defined as μ_{m1} and this angle can be defined as μ_{m2}, this μ_{m2} will be function of M_2. So, once it undergoes this deflection, the downstream stream line becomes parallel to this second surface. So, we have M_2. So, this is the problem; in addition to this, other parameters given that is p_1 as 0.9 bar and T_1 as 20°C. So, since we know this Mach number and the pressure information; so, one can find out the stagnation pressure also is equal to $p_{01} = p_1\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}$.

Similarly we can find out $T_{01} = T_1\left[1+\left(\frac{\gamma-1}{2}\right)M^2\right]$. So, putting gamma 1.4 and M_1 is equal to 1.5; p_{01} can be evaluated as 3.3 bar, T_{01} will be evaluated as 425 K. So, what we require now is condition at 2, to do that let us start with the solution.

So, to start the solution what we now require to refer Prandtl - Meyer table. So, we know M_1 as 1.5. So, when I say refer this particular table with M_1 as 1.5, I can say $\nu\left(M_1\right)$ that is Prandtl - Meyer function. For this Mach number M_1 is 11.91 and for the Mach angle that is μ_{m1} that is the forward Mach angle now becomes 41.81.

Now we know θ as 20°. So, we can write $\nu\left(M_2\right)$ that is Prandtl - Meyer function corresponding to Mach number M_2 would be $\theta+\nu\left(M_1\right)$. So, when I do this. So, we will have another value of Prandtl - Meyer function that is 31.91°. So, I have to refer this particular table again for this Prandtl - Meyer functions.

So, I have to refer to a value close to this 31.91° in this particular table. So, a close number that is seen here is 31.73, I can take this value. So, this will imply Mach number as 2.2, when I say Mach number as 2.2, this will also μ_{m2} that is rearward Mach angle becomes 27.02°. So, we know forward Mach angle and rearward Mach angle. So, what is not known to us.

Now, once you know M_2. So, we can evaluate $\frac{p_{02}}{p_2} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M_2^2\right]^{\frac{\gamma}{\gamma-1}}$, $\frac{T_{02}}{T_2} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M_2^2\right]$. So, from this we can find out p_{02} since we know M_2. So, this number would be 10.81. So, $\frac{T_{02}}{T_2}$ will be 1.974.

So, since $p_{02} = p_{01}$ that is 3.3 bar. So, this will give you form this p_2 as 0.3 bar; $T_{02} = T_{01} = 425$ K. So, this will give you T_2 as about 215.3 K. So, we get all the information. So, we know p_2 pressure, temperature, Mach number, we have stagnation pressure, p_2, T_2, p_{02}, T_{02} forward and Mach angle μ_{m1} and μ_{m2}.

So, this is how we get all the information when the supersonic flow gets deflected by an angle θ_2. I think with this I am able to explain how we are going to look the property table of Prandtl - Meyer expansion. With this I will conclude my lecture for this module 5.

Thank you for your attention.