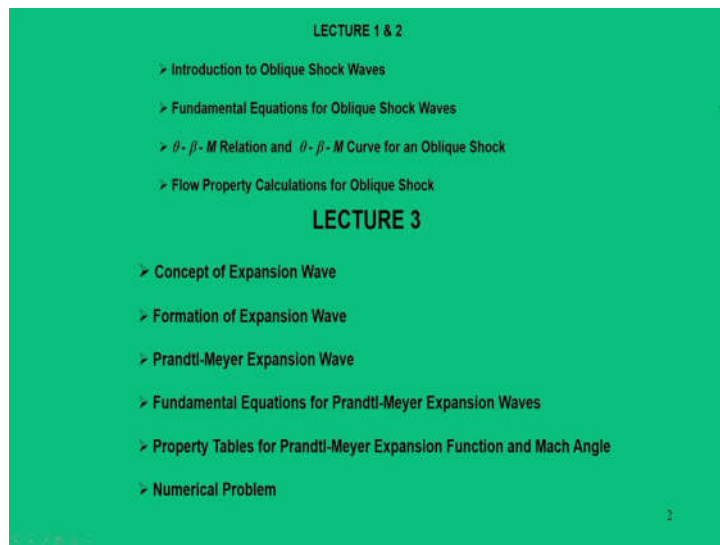


Fundamental of Compressible Flow
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Module – 05
Lecture - 16
Expansion Waves and Oblique Shocks- III

Welcome to this course Fundamentals of Compressible Flow, we are in module 5 that is Expansion Waves and Oblique Shock Waves.

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LECTURE 1 & 2
➤ Introduction to Oblique Shock Waves
➤ Fundamental Equations for Oblique Shock Waves
➤ θ - β - M Relation and θ - β - M Curve for an Oblique Shock
➤ Flow Property Calculations for Oblique Shock
LECTURE 3
➤ Concept of Expansion Wave
➤ Formation of Expansion Wave
➤ Prandtl-Meyer Expansion Wave
➤ Fundamental Equations for Prandtl-Meyer Expansion Waves
➤ Property Tables for Prandtl-Meyer Expansion Function and Mach Angle
➤ Numerical Problem

So, in the previous lectures in this module that is 1 and 2 we have exhaustively covered the oblique shock, its fundamental equations. Then we derived a relations known as θ - β - M relations and based on this θ - β - M relations, a curve was generated which is the array of all such curves for different Mach number and flow deflection angle.

This curve helps us to find graphically the information about the oblique shocks and shock wave angle and moreover, this curve is also very vital in estimating the flow properties across the oblique shocks. And now today we are going to discuss about something just opposite to that of oblique shocks that is known as expansion waves.

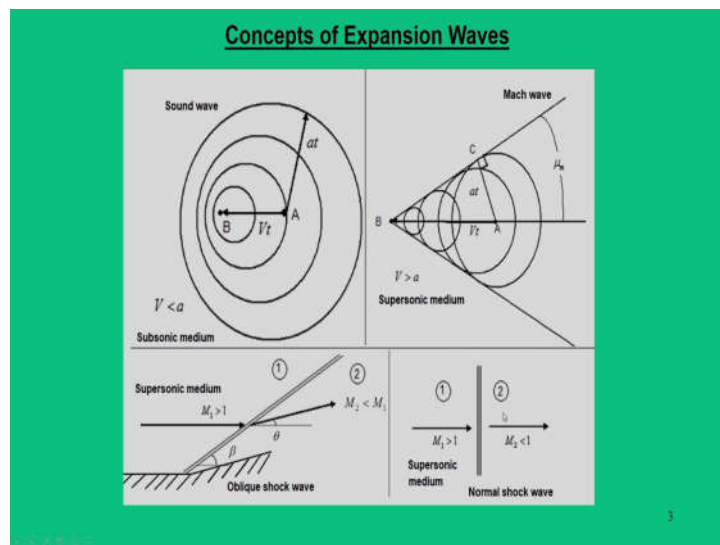
So, while analyzing the oblique shocks we say that when a supersonic flow encounters a compression corner, flow gets deflected towards itself. And in this way the flow becomes compressed and its all other properties like pressure, temperature etc increases.

Now can we have another situations where instead of getting the flow compressed, can the flow expand? So, in such a situation we cannot call this as oblique shocks rather we will tell it as expansion waves.

So, in this particular lecture we will talk in details about the concept of expansion waves, how it forms? Then in particular we will discuss about a very specific expansion wave that is Prandtl - Meyer expansion wave or many a time it is called as centered expansion.

Then for this Prandtl - Meyer expansion waves, we try to derive the fundamental equations and moving further we will discuss about the property tables for the Prandtl - Meyer expansion function and Mach angle. And subsequently we will try to solve some problems based on this Prandtl - Meyer expansion wave.

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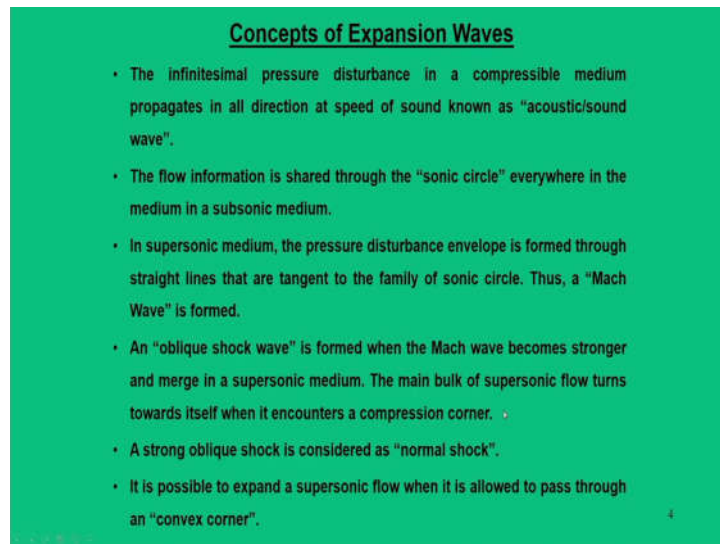
Now, to just to give glimpses of what we have covered till this point of time, we know it is a sound wave, it is a pressure wave and it compresses the medium. The main intention of the sound wave is that it propagates the flow information; it shares the flow information. Now when these pressure waves become stronger and stronger that is possible only when your gas velocity or flight body moves at higher speed than the

sound waves. So, these pressure waves become stronger and they try to form a Mach wave.

Now with further intensity of increase in the pressure waves, all these formation of Mach waves that are getting generated through this body moving at supersonic speed, they try to merge as a shock wave and that shock wave we call as a oblique shock wave.

Now the strength of the oblique shock wave is based on the shock wave angle, now when the shock wave angle is 90° means that the streamlines and the shock waves they are perpendicular to each other. So, in that situation oblique shock becomes a normal shock. Now in all these example we say that all these waves are nothing but the pressure waves and that compresses the medium.

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Concepts of Expansion Waves

- The infinitesimal pressure disturbance in a compressible medium propagates in all direction at speed of sound known as "acoustic/sound wave".
- The flow information is shared through the "sonic circle" everywhere in the medium in a subsonic medium.
- In supersonic medium, the pressure disturbance envelope is formed through straight lines that are tangent to the family of sonic circle. Thus, a "Mach Wave" is formed.
- An "oblique shock wave" is formed when the Mach wave becomes stronger and merge in a supersonic medium. The main bulk of supersonic flow turns towards itself when it encounters a compression corner.
- A strong oblique shock is considered as "normal shock".
- It is possible to expand a supersonic flow when it is allowed to pass through an "convex corner".

So, this is what we have discussed so far. So, infinitesimally small pressure disturbance is known as "acoustic or sound waves" and through these sound waves a sonic circle is generated and through this "sonic circle" flow information is shared.

Now a "Mach wave" is formed when the pressure disturbance envelope is formed through straight lines that are tangent to the family of sonic circles; when these Mach waves become stronger the "oblique shock wave" is formed.

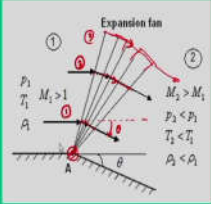
And this is a situation when a supersonic flow encounters a compression corner and the entire flow or main bulk of the flow turns towards itself. And the strength of the oblique shock when increases it becomes a “normal shock.”

So, in all possible cases, it is the medium that gets compressed. Now, for a supersonic flow that has to expand then there is a provision that we can allow the flow to pass through a “convex corner.”

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Concepts of Expansion Waves

- The flow streamlines are deflected downwards from the main bulk of flow with maximum spread of deflection at infinity and minimum deflection in the vicinity of point of origin.
- The flow streamlines are smoothly curved till downstream of the flow becomes parallel to the wall surface. In this way, a supersonic flow can expand to a higher Mach number through turning away from itself across a corner.
- All the flow properties do change smoothly through the expansion fan except at the point of origin.



The diagram illustrates an expansion fan originating from point A on a convex corner. The flow is divided into three regions: Region 1 (upstream, horizontal), Region 2 (between the corner and the fan), and Region 3 (downstream, parallel to the new wall surface). The flow turns through an angle θ . The properties in Region 2 are labeled as $M_2 > M_1$, $P_2 < P_1$, $T_2 < T_1$, and $\rho_2 < \rho_1$. The properties in Region 3 are labeled as $M_3 > M_2$, $P_3 < P_2$, $T_3 < T_2$, and $\rho_3 < \rho_2$. The flow is deflected downwards from the main bulk of flow with maximum spread of deflection at infinity and minimum deflection in the vicinity of point of origin.

So, how is it done we can refer this particular figure; as we say that, in this particular example what has been seen shown here that a expansion corner is created. What does this mean? Means there are essentially there are two surfaces and these two surfaces are meeting at a point A. So, the supersonic flow passes through this surfaces which is horizontal that is the upstream of flow, region 1 and this flow has to turn through an angle θ downwards.

So, in other words the streamlines that has to be in the region 2 must be parallel to this second surface. That means, in other words the flow has to turn an angle θ and which is away from the main bulk of the flow; that means, had this flow what had been undertaken, it would have gone through in this path. So, in this process of turning, the flow was turn to an angle θ and across this θ angle the streamlines are now parallel to the second surface.

Now if we look at this particular events. So, in between this turning process we can view this fact that flow has expanded, because it is seeing a larger area and it is the flow is turning away from itself. And when you view this we can say there are series of expansion waves that gets generated from the point A. Now looking at the streamlines let us say a streamline the first streamline and this is second streamline.

So, for the first streamline, it has to cover a distance in a curved manner and for the second streamline, this flow has to turn also in a curved fashion. And, in this process the distance traveled by second stream line, it covers a larger distance. The streamline 1 has a has to cover a smaller distance likewise if you look at another stream lines maybe somewhere 3 down the far away from this it has to cover a much larger distance.

So, in other words what we can say that at the end of this turning process all the streamlines in the downstream side are always parallel to this second surface. But one interesting thing that happens, the streamline which is just close to the point A has a discontinuity in the medium. So, the flow is as if that streamline does not undergo the turning; which is near to the wall whereas, all other stream lines turns in a smooth manner.

The smaller the distance of the streamline from the wall, smaller is the turning distance; larger is the streamline away from the wall larger is the turning distance. This is how the philosophy of expansion waves happens. And in this process all the flow properties change smoothly through an expansion fan except at the point of origin. So, in this case the point of origin is A.

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Formation of Expansion Waves

- Consider a gas confined in a long tube with a piston inside it. While at rest, one can imagine infinite layers of gas on either side of the piston.
- When the piston in the tube is given a steady velocity towards left, a very weak expansion is generated that propagates to the right at speed of sound relative to the stationary gas ahead of it.
- The propagation of this wave into the medium drops its pressure and temperature.

The diagram illustrates the formation of expansion waves in a gas tube. It shows a piston moving back and forth with increasing velocity (dV , $2dV$, $3dV$). Each movement generates a weak expansion wave that propagates to the right at the speed of sound (a_1 , a_2 , a_3). The waves are shown as regions of lower pressure and temperature. The diagram also shows the wave moving away from the piston at a speed greater than the speed of sound ($a_2 < a_1$, $a_3 < a_2$). The wave is labeled 'Head' and 'Tail'.

Now, to evaluate this particular concept of expansion process we will now discuss about the formation of expansion waves and likewise in previous cases what we do is that we take a piston cylinder device in which a gas is kept. So, in this case also we will say there is a gas which is kept in a piston cylinder medium.

Now while talking about these compression mechanisms we say that piston was given a push towards right, but in this case we will say that push piston is given a push so that a small velocity dV is generated and towards right. So, that means when the gas is at initial position its initial pressure was p_0 and when a piston is given a push towards left the gas is now seeing a larger volume.

So, thereby it tries to expand, when it expands it comes down to the pressure let us say p_1 . So, from this initial pressure p_0 gas gets expanded and in this process what happens is that when the push was given a sound wave is generated that moves a speed a_1 .

Now, in the next instance, we give another incremental in velocity so, then this dV becomes $2dV$. So, increase the velocity by $2dV$. So, thereby we will have the first sound waves as it is and second sound wave will also get generated a_2 as it is. So, now we are looking at this particular event at t is equal to t_1 , in the earlier event t is equal to t_0 and in this process also what happens, the gas also sees a larger volume.

So, thereby its pressure changes from initial pressure p_0 to when the first sound wave is generated through it gets the differential pressure dp and for the another differential pressure dp is also generated. And finally, the same pressure p_2 is formed. In fact, in this case also the pressure difference is dp . So, likewise the gas is now expanded to a larger volume.

Now, in this process also there are lot of expansion waves that would have been generated in addition to the sound waves and if you look at this particular event at time is equal to t_3 . Then, this location of sound wave would have been different and we will have this pressure p_2 and through this process there are many such expansion wave that would have get generated.

But most important point in this situation is that when you look at the first sound wave that speed is a_1 and when you look at the second sound wave a_2 , here a_2 is less than a_1 ; that means, strength of the second sound wave is less than the strength of first sound wave. Why? Because since the medium gets expanded; temperature is less that is because a_2 is nothing but $\sqrt{\gamma RT_2}$ and a_1 is $\sqrt{\gamma RT_1}$. So, T_2 is less than T_1 .

So, since T_2 is less than T_1 because the gas gets cooled through this expansion process; so; obviously, the speed of sound waves of the second sound wave is less than the first sound wave. So, since this is the case though the expansion waves that gets generated through this subsequent sound waves, they move at slower rate.

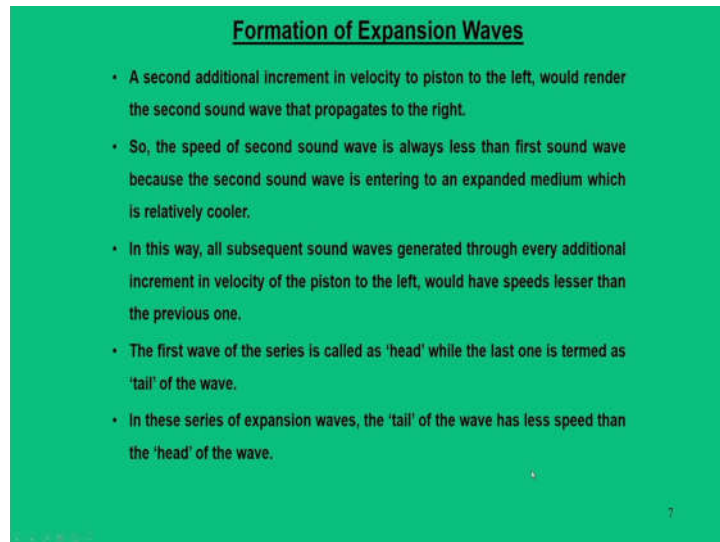
So, in none of the at any cost, the subsequent sound waves cannot overtake the first wave. Whereas, this was the opposite case when the second in a compression medium when the second pressure waves try to overtake the first pressure wave.

So, in this expansion process if I can say the in this process if there are infinite number of sound waves or in other words I will say expansion waves gets generated. So, this is the first expansion wave, second expansion wave, third expansion wave, fourth expansion wave and these expansion waves moves towards left as the piston is given a velocity dV and this expansion waves they try to spread. So, they cannot overtake each other.

So, this will be 1 2 3 4. So, in none of the situations, the expansion wave 4 will overtake the expansion wave 1. So, they try to spread and this first wave is nothing, but we call this as a head of the wave and last one will be tail of the wave. So, this is how the

concept of expansion waves is introduced in a piston cylinder device and that is one dimensional in nature.

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Formation of Expansion Waves

- A second additional increment in velocity to piston to the left, would render the second sound wave that propagates to the right.
- So, the speed of second sound wave is always less than first sound wave because the second sound wave is entering to an expanded medium which is relatively cooler.
- In this way, all subsequent sound waves generated through every additional increment in velocity of the piston to the left, would have speeds lesser than the previous one.
- The first wave of the series is called as 'head' while the last one is termed as 'tail' of the wave.
- In these series of expansion waves, the 'tail' of the wave has less speed than the 'head' of the wave.

So, whatever I told if I summarize that the second additional increment in the velocity to the piston to the left would render the second sound wave to the right. The speed of second sound wave is always less than the speed of first sound waves, because the second sound wave is entering to an expansion medium which is relatively cooler. In this way all subsequent sound waves generated through every additional increment in the velocity to the left, would have speeds less than that of previous one.

This brings a consequence that all the expansion waves, due to the sound waves they move in the direction of the piston movement and they are in the series of waves. And the first wave in this series is called as “head” and the last one is termed as the “tail” of the wave. And; obviously, the “tail” of the wave has less speed than the “head” of the wave. So, that means, tail of the tail of the wave cannot overtake the head of the wave.

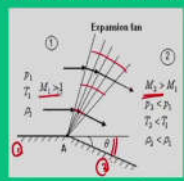
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Expansion Wave in Isentropic Flow

- The expansion waves are antithesis of shock waves.
- Unlike shock waves, it is not possible for merger of expansion waves under any circumstances rather they spread. Thus, creation of finite expansion wave is impossible.
- Hence, the thermodynamic process can be viewed as gradual changes in the flow properties in the medium in isentropic manner through the spreading of expansion wave, with an exception of the wall streamline which changes discontinuously at the center point.

$$\frac{p_1}{p_2} = \left(\frac{T_1}{T_2} \right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_1}{\rho_2} \right)^{\gamma}$$

$$\frac{T_0}{T} = 1 + \left(\frac{\gamma-1}{2} \right) M^2; \quad \frac{p_0}{p} = \left[1 + \left(\frac{\gamma-1}{2} \right) M^2 \right]^{\frac{\gamma}{\gamma-1}}; \quad \frac{\rho_0}{\rho} = \left[1 + \left(\frac{\gamma-1}{2} \right) M^2 \right]^{\frac{1}{\gamma-1}}$$



So, this is how the concept of expansion wave is introduced in the one dimensional medium, but; however, in reality normally the formation of this expansion wave is two dimensional in nature. So, as I showed it in the first example that there are two surfaces 1 and 2 they are inclined at angle θ and the supersonic flow tries to encounter into these two surfaces. So, when it encounters, the streamlines gets deflected and when they get deflected they always try to be in parallel to both the surfaces.

So, in the first instance the stream lines are parallel to the surface 1, in the second instance the streamlines are parallel to the surface 2. And in this process what happens; the Mach number increases, but the static pressure temperature and density drops. But however, the change that occurs is very gradual and why it is gradual because they try to spread and medium is not disturbed.

So, such a process we can view it to be the property change in an isentropic manner, but one important thing is that whether we say that the smooth pattern of this turning is not true at the center A, because at that point of time the changes are discontinuous.

So, in all every instances the flow properties are isentropic, hence the isentropic relations or equations holds good; that means, for the region for the region 1 and region 2. So, we

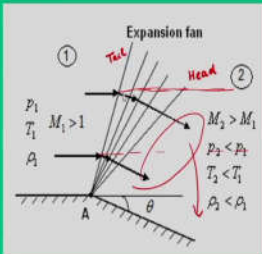
say
$$\frac{p_1}{p_2} = \left(\frac{T_1}{T_2} \right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_1}{\rho_2} \right)^{\gamma}.$$

Now, when I say; the two situations M_1 and M_2 ; so the properties in the region 1 are function of M_1 through this isentropic relation properties, in the region 2 are function of M_2 . Now our main task is to find out after this turning what will be the Mach number M_2 .

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Expansion Wave in Isentropic Flow

- A class of two-dimensional waves, occurring in a supersonic flow can show the opposite effects of oblique shock.
- A supersonic flow can expand to a higher Mach number through turning away from itself across a corner.
- Mathematically, it is represented as infinite continuous waves emanating from a center with a 'head' and 'tail', thus resembles the nature spreading the flow properties.



Since we say that the flow turns smoothly across an expansion wave. So, we can say that medium can be treated to be an isotropic medium.

So, we say that a class of two dimensional waves can be defined in a supersonic flow that has opposite effects to the oblique shocks. And, across this expansion wave the Mach number increases through the turning and this turning process is such that the flow turns away from the main bulk flow. So, if you say the main bulk of the flows is going in this direction, the entire bulk of the flow gets turned away towards this second surface.

And such a process is defined through infinite continuous waves and known as 'head' and 'tail.' So, the first expansion process starts as head of the wave and last one we say tail of the wave. So, when I say head, the flow has initiated turn and the last expansion wave that is at the tail, the flow has completed the entire turning process.

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Prandtl-Meyer Expansion Waves

- A special class of two-dimensional expansion waves originating from a sharp convex corner is a centered expansion commonly known as Prandtl-Meyer (PM) expansion wave.
- The streamlines through the expansion wave are smooth curved lines.
- The PM expansion fans are composed of infinite number of Mach waves originating from a point and bounded by forward Mach line and rearward Mach line. Accordingly, forward and rearward Mach angles are defined.

The diagram illustrates the Prandtl-Meyer expansion wave. It shows a flow with initial conditions $P_1, T_1, M_1 > 1, \rho_1$ approaching a sharp convex corner at point A. The flow turns through an angle θ . The expansion fan is bounded by a forward Mach line and a rearward Mach line. The flow conditions after the expansion are P_2, T_2, M_2, ρ_2 . The forward Mach angle is defined as $\mu_{m1} = \sin^{-1}(1/M_1)$ and the rearward Mach angle is defined as $\mu_{m2} = \sin^{-1}(1/M_2)$. The diagram also shows the Mach number increasing from M_1 to M_2 and the pressure and temperature decreasing from P_1 to P_2 and T_1 to T_2 respectively.

Now, having said this we will now define a special class of expansion wave known as Prandtl - Meyer expansion waves. So, these Prandtl - Meyer expansion waves are the a class of waves where we call this as a expansion fan and here the head and tail of the wave are defined through a Mach line. Or in other words we can say these are the waves that generated from a center point and there are infinite waves and all these infinite waves are known as Mach waves.

Now when I say Mach waves; the head and tail are now defined by a line which is known as rearward Mach line and forward Mach line. When I say this forward Mach line, this forward Mach line is defined through a Mach angle $\mu_{m1} = \sin^{-1}\left(\frac{1}{M_1}\right)$. So, the location of this forward Mach line is initiated from the horizontal at angle μ_{m1} and that is defined by $\sin^{-1}\left(\frac{1}{M_1}\right)$. This is what they call as forward Mach line.

And the rearward Mach line; that means, at that Mach line the flow has completed the turning. So, when we say completed the turning that Mach line will be decided by the Mach number corresponding to condition 2 that is M_2 . So, that Mach line makes an angle μ_{m2} from the second surface. So, if this is the surface first surface and this is the second surface; for the forward Mach line this angle we say μ_{m1} and for the rearward Mach line this angle is μ_{m2} and in this process the flow has turned through an angle θ .

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Prandtl-Meyer Expansion Waves

- The expansion takes place through a continuous succession of Mach waves so that there is no change in entropy of the flow across each Mach wave so that entire expansion process can be considered as isentropic.
- Only Mach number increases while static flow parameters (pressure, temperature and density) decreases. The corresponding stagnation parameters do not change.

Forward Mach angle: $\mu_{n1} = \sin^{-1}(1/M_1)$
Rearward Mach angle: $\mu_{n2} = \sin^{-1}(1/M_2)$

And since this is how I have explained all these things, the expansion waves takes place through continuous succession of Mach waves. So, there is no change in the entropy of the flow across each Mach waves and we say all these are Mach waves.

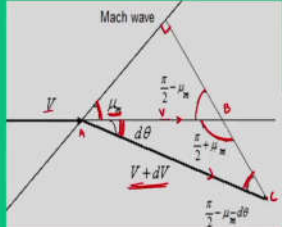
So, through this turning process there is no change in the entropy. And what is the net effect? The net effect is the only Mach number increase while the static flow properties such as pressure temperature density, they drop; whereas, stagnation properties also do not change means total conditions p_{01} and p_{02} are same T_{01} and T_{02} are same.

In other words, even within this region also no change in stagnation properties p_0 and T_0 ; whereas, this was the not the case when the supersonic flow encounters a oblique shock, the total pressure drops although temperature does not change, but the total pressure drops. But in a Prandtl - Meyer expansion the stagnation properties do not change and in other words the entire flow field is isentropic in nature.

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Fundamental Equations for Prandtl-Meyer Expansion Wave

- The quantitative analysis for expansion waves involves flow properties determination downstream of expansion fan for a given upstream conditions.
- Consider an infinitesimal changes of flow properties across a very weak wave (Mach wave).

$$\frac{V + dV}{V} = \frac{\sin\left(\frac{\pi}{2} - \mu_m\right)}{\sin\left(\frac{\pi}{2} - \mu_m - d\theta\right)} \Rightarrow 1 + \frac{dV}{V} = \frac{\cos \mu_m}{\cos \mu_m \cos d\theta - \sin \mu_m \sin d\theta} = \frac{\cos \mu_m}{\cos \mu_m - d\theta \sin \mu_m} = \frac{1}{1 - d\theta \tan \mu_m}$$


Law of Sine: ΔABC
 $\frac{V + dV}{V} = \frac{\sin(\theta + \mu_m)}{\sin(\mu_m - d\theta)}$
 $\approx \frac{\sin(\theta + \mu_m)}{\sin(\mu_m - d\theta)}$
 $\approx \frac{\sin(\theta + \mu_m)}{\sin(\mu_m - d\theta)}$
 $\theta \rightarrow 0, \sin d\theta = d\theta$

Now we will move to the fundamental equations for Prandtl - Meyer expansion waves. So, to start with the fundamental equation of Prandtl - Meyer expansion waves what we are going to see is that as I mentioned that the event of supersonic flow in an expansion process occurs through series of infinite Mach waves, we will take one such Mach waves.

So, here in the this figure we can say it is one of the Mach waves and this θ , the entire flow deflections we have defined that streamline as if it has undergone a very small deflection $d\theta$. And through this deflection, the flow velocity instead of V it is get increased by $V + dV$ in one such Mach waves. Likewise one can say that if I integrate this θ over this angle, then I can eventually obtain the final velocity in the region 2. So, to do that analysis we started one particular expansion waves or Mach wave. Now let us understand this figure.

So, as I mentioned that across this Mach wave the velocity vector become $V + dV$ and this Mach wave makes an angle μ_m that is the Mach angle this angle is the flow deflection angle and we can drop a perpendicular from this streamline vector $V + dV$ on to this Mach wave.

So, once you drop a perpendicular to it, we will have all these geometrical information in place. So, now, I can say that we have a triangle that consist of ABC that consists of the

So, law of sine tells that I can write $\frac{V+dV}{V}$, then the right hand side of this equation will

have the angle corresponding angles that is $\frac{\sin\left(\frac{\pi}{2} + \mu_m\right)}{\sin\left(\frac{\pi}{2} - \mu_m - d\theta\right)}$ or in other words for this

So, this is how it is written here $\frac{V+dV}{V} = \frac{\sin\left(\frac{\pi}{2} + \mu_m\right)}{\sin\left(\frac{\pi}{2} - \mu_m - d\theta\right)}$. So, this equation gets now

simplified, the numerator becomes $\cos \mu_m$; denominator one can expand $\cos \mu_m \cos d\theta - \sin \mu_m \sin d\theta$. So, this can be expanded in this form.

Then one thing we needs to note here that in this term $\sin d\theta$ when θ tends to 0, $\sin d\theta$ we can write as $d\theta$. So, that we can write this. Now from this equation if you divide $\sin \theta$ in this numerator and denominator we get $\frac{1}{1-d\theta \tan \mu_m}$. So, the equation becomes

$$1 + \frac{dV}{V} = \frac{1}{1 - d\theta \tan \mu_m}.$$

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So, here this particular expression, we have to use some mathematical trick or expansion series, which means any equation $\frac{1}{1-x}$, where x less than 1 in an expansion series, we can write $\frac{1}{1-x} = 1 + x + x^2 + \dots$. So, here replace x as $d\theta \tan \mu_m$. So, when I replace this so I can write $1 + d\theta \tan \mu_m + \dots$. So, since $d\theta$ is small. So, this means higher order series are neglected.

So, in other words we can write that $1 + \frac{dV}{V} = 1 + d\theta \tan \mu_m$. So, we get $d\theta = \frac{(dV/V)}{\tan \mu_m}$. Now, we also know that $\sin(\mu_m) = \frac{1}{M}$. So, we can find out $\tan \mu_m$; will be $\sqrt{M^2 - 1}$. So, these are the trigonometric information we can put.

So, we get now $d\theta = \sqrt{M^2 - 1} \frac{dV}{V}$. So, here the question still it is not known is what is $\frac{dV}{V}$? Although, we know the Mach number, but you do not know the $\frac{dV}{V}$, to do this we have to recall that we have to represent this velocity in the form of Mach number.

So, $V = M a$ you take logarithm of that. So, we get $\ln V = \ln M + \ln a$, from this equation we can find out $\frac{dV}{V} = \frac{dM}{M} + \frac{da}{a}$. So, that means the dθ equation we introduce one more term $\frac{dM}{M}$, but still it is not clear what is $\frac{da}{a}$. So, we have to bring this $\frac{da}{a}$ as a function of M, because all our information are related to the upstream flow Mach number.

But surprisingly, since entire process is isentropic we have one relation $\frac{T_0}{T} = 1 + \left(\frac{\gamma-1}{2}\right)M^2$ and this $\frac{T_0}{T} = \frac{a_0^2}{a^2}$ because $a = \sqrt{\gamma RT}$. So, these equations now is replaced with this. So, we can write $a^2 = \frac{a_0^2}{1 + \left(\frac{\gamma-1}{2}\right)M^2}$.

We have to now differentiate this equation so that a_0 will get canceled. So, it will be now replaced as a function of Mach number.

$$\frac{da}{a} = -\left(\frac{\gamma-1}{2}\right) M \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} \frac{dM}{M}$$

So, what you do now, in this main equation which is $d\theta = \sqrt{M^2 - 1} \left(\frac{dM}{M} + \frac{da}{a} \right)$. So, we are now in a position that we know $\frac{da}{a}$, we also know $\frac{dM}{M}$; when you replace this θ we arrive at this particular equations.

(Refer Slide Time: 40:45)

Fundamental Equations for Prandtl-Meyer Expansion Wave

$$\int_{\theta_1}^{\theta_2} d\theta = \int_{M_1}^{M_2} \frac{\sqrt{M^2 - 1}}{M} \frac{dM}{M}; \quad \frac{dV}{V} = \frac{1}{1 + \left(\frac{\gamma-1}{2}\right) M^2} \frac{dM}{M}$$

$$\int_{\theta_1}^{\theta_2} d\theta = \theta_2 - \theta_1 = \int_{M_1}^{M_2} \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma-1}{2} M^2} \frac{dM}{M}$$

$$v(M) = \int \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma-1}{2} M^2} \frac{dM}{M} \Rightarrow v(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} (M^2 - 1)} - \tan^{-1} \sqrt{M^2 - 1}$$

P.M. Function

$$\Rightarrow \int_{\theta_1}^{\theta_2} d\theta = \theta_2 - \theta_1 = v(M_2) - v(M_1) \Rightarrow \theta = v(M_2) - v(M_1)$$

Forward Mach angle: $\mu_{M_1} = \sin^{-1}(1/M_1)$
 Rearward Mach angle: $\mu_{M_2} = \sin^{-1}(1/M_2)$

So, putting $\frac{dV}{V}$ as a function of $\frac{dM}{M}$, then we can replace that equation and now we can integrate that equations from θ_1 to θ_2 . So, θ_1 to θ_2 in our case is nothing but our initial angle is 0. So, many a times θ_1 is equal to you can say it is 0. So, you can go up to θ_2 . So, we get a integral of this equations.

$$\int_{\theta_1}^{\theta_2} d\theta = \theta_2 - 0 = \int_{M_1}^{M_2} \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma-1}{2} M^2} \frac{dM}{M}$$

So, this particular integral has a meaning to it and we give a meaning to this equation as $v(M)$, $v(M)$ is known as Prandtl - Meyer function. And in fact, this function is a angle and this angle we can find out as a function of Mach number and in fact, this function is an integral and this integral has a value as given by this expression.

$$\nu(M) = \int \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma - 1}{2} M^2} \frac{dM}{M} \Rightarrow \nu(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1} \sqrt{\frac{\gamma - 1}{\gamma + 1} (M^2 - 1)} - \tan^{-1} \sqrt{M^2 - 1}$$

So, knowing this particular Prandtl - Meyer function at a given Mach number we can

evaluate the $\int_{\theta_1}^{\theta_2} d\theta = \theta_2 - \theta_1 = \nu(M_2) - \nu(M_1)$ because we are taking integral from M_1 to

M_2 . So, finally, we arrived that the supersonic flow when gets deflected through an angle θ , that θ is nothing but the difference in Prandtl - Meyer function corresponding to their respective Mach numbers.

So, this is the very important summary that we can find out that the flow deflection angle is a function of Prandtl - Meyer functions. Thus, the entire information or entire deflection is now left with evaluation of these particular integral and let us see how we are going to find out. So, although this expression is a big expression and it is a function of Mach number, but it is difficult to evaluate through manually.

(Refer Slide Time: 43:31)

Prandtl-Meyer Function and Mach Angle Property Table

M	ν (deg)	μ (deg)
0.1400 + 01	0.8887 + 01	0.4556 + 02
0.1420 + 01	0.9565 + 01	0.4477 + 02
0.1440 + 01	0.1015 + 02	0.4398 + 02
0.1460 + 01	0.1073 + 02	0.4323 + 02
0.1480 + 01	0.1132 + 02	0.4251 + 02
0.1500 + 01	0.1191 + 02	0.4181 + 02
0.1520 + 01	0.1249 + 02	0.4114 + 02
0.1540 + 01	0.1309 + 02	0.4049 + 02
0.1560 + 01	0.1368 + 02	0.3987 + 02
0.1580 + 01	0.1427 + 02	0.3927 + 02
0.1600 + 01	0.1487 + 02	0.3869 + 02
0.1620 + 01	0.1546 + 02	0.3813 + 02
0.1640 + 01	0.1606 + 02	0.3759 + 02
0.1660 + 01	0.1666 + 02	0.3707 + 02
0.1680 + 01	0.1726 + 02	0.3656 + 02
0.1700 + 01	0.1786 + 02	0.3607 + 02
0.1720 + 01	0.1846 + 02	0.3559 + 02
0.1740 + 01	0.1906 + 02	0.3513 + 02
0.1760 + 01	0.1966 + 02	0.3468 + 02
0.1780 + 01	0.2026 + 02	0.3424 + 02
0.1800 + 01	0.2086 + 02	0.3381 + 02
0.1820 + 01	0.2146 + 02	0.3339 + 02
0.1840 + 01	0.2206 + 02	0.3298 + 02
0.1860 + 01	0.2266 + 02	0.3258 + 02
0.1880 + 01	0.2326 + 02	0.3219 + 02
0.1900 + 01	0.2386 + 02	0.3181 + 02
0.1920 + 01	0.2446 + 02	0.3144 + 02
0.1940 + 01	0.2506 + 02	0.3107 + 02
0.1960 + 01	0.2566 + 02	0.3071 + 02
0.1980 + 01	0.2626 + 02	0.3036 + 02
0.2000 + 01	0.2686 + 02	0.3002 + 02
0.2020 + 01	0.2746 + 02	0.2968 + 02
0.2040 + 01	0.2806 + 02	0.2935 + 02
0.2060 + 01	0.2866 + 02	0.2903 + 02
0.2080 + 01	0.2926 + 02	0.2871 + 02
0.2100 + 01	0.2986 + 02	0.2840 + 02
0.2120 + 01	0.3046 + 02	0.2810 + 02
0.2140 + 01	0.3106 + 02	0.2780 + 02
0.2160 + 01	0.3166 + 02	0.2751 + 02
0.2180 + 01	0.3226 + 02	0.2722 + 02
0.2200 + 01	0.3286 + 02	0.2694 + 02
0.2220 + 01	0.3346 + 02	0.2666 + 02
0.2240 + 01	0.3406 + 02	0.2639 + 02
0.2260 + 01	0.3466 + 02	0.2612 + 02
0.2280 + 01	0.3526 + 02	0.2586 + 02
0.2300 + 01	0.3586 + 02	0.2561 + 02
0.2320 + 01	0.3646 + 02	0.2536 + 02
0.2340 + 01	0.3706 + 02	0.2512 + 02
0.2360 + 01	0.3766 + 02	0.2488 + 02
0.2380 + 01	0.3826 + 02	0.2465 + 02
0.2400 + 01	0.3886 + 02	0.2442 + 02
0.2420 + 01	0.3946 + 02	0.2420 + 02
0.2440 + 01	0.4006 + 02	0.2398 + 02
0.2460 + 01	0.4066 + 02	0.2377 + 02
0.2480 + 01	0.4126 + 02	0.2356 + 02
0.2500 + 01	0.4186 + 02	0.2336 + 02
0.2520 + 01	0.4246 + 02	0.2316 + 02
0.2540 + 01	0.4306 + 02	0.2297 + 02
0.2560 + 01	0.4366 + 02	0.2278 + 02
0.2580 + 01	0.4426 + 02	0.2259 + 02
0.2600 + 01	0.4486 + 02	0.2241 + 02
0.2620 + 01	0.4546 + 02	0.2223 + 02
0.2640 + 01	0.4606 + 02	0.2205 + 02
0.2660 + 01	0.4666 + 02	0.2188 + 02
0.2680 + 01	0.4726 + 02	0.2171 + 02
0.2700 + 01	0.4786 + 02	0.2154 + 02
0.2720 + 01	0.4846 + 02	0.2137 + 02
0.2740 + 01	0.4906 + 02	0.2121 + 02
0.2760 + 01	0.4966 + 02	0.2105 + 02
0.2780 + 01	0.5026 + 02	0.2089 + 02
0.2800 + 01	0.5086 + 02	0.2074 + 02
0.2820 + 01	0.5146 + 02	0.2059 + 02
0.2840 + 01	0.5206 + 02	0.2044 + 02
0.2860 + 01	0.5266 + 02	0.2030 + 02
0.2880 + 01	0.5326 + 02	0.2016 + 02
0.2900 + 01	0.5386 + 02	0.2002 + 02
0.2920 + 01	0.5446 + 02	0.1988 + 02
0.2940 + 01	0.5506 + 02	0.1975 + 02
0.2960 + 01	0.5566 + 02	0.1961 + 02
0.2980 + 01	0.5626 + 02	0.1948 + 02
0.3000 + 01	0.5686 + 02	0.1935 + 02
0.3020 + 01	0.5746 + 02	0.1922 + 02
0.3040 + 01	0.5806 + 02	0.1910 + 02
0.3060 + 01	0.5866 + 02	0.1897 + 02
0.3080 + 01	0.5926 + 02	0.1885 + 02
0.3100 + 01	0.5986 + 02	0.1873 + 02
0.3120 + 01	0.6046 + 02	0.1861 + 02
0.3140 + 01	0.6106 + 02	0.1850 + 02
0.3160 + 01	0.6166 + 02	0.1838 + 02
0.3180 + 01	0.6226 + 02	0.1827 + 02
0.3200 + 01	0.6286 + 02	0.1816 + 02
0.3220 + 01	0.6346 + 02	0.1805 + 02
0.3240 + 01	0.6406 + 02	0.1794 + 02
0.3260 + 01	0.6466 + 02	0.1784 + 02
0.3280 + 01	0.6526 + 02	0.1773 + 02
0.3300 + 01	0.6586 + 02	0.1763 + 02
0.3320 + 01	0.6646 + 02	0.1753 + 02
0.3340 + 01	0.6706 + 02	0.1743 + 02
0.3360 + 01	0.6766 + 02	0.1733 + 02
0.3380 + 01	0.6826 + 02	0.1724 + 02
0.3400 + 01	0.6886 + 02	0.1714 + 02
0.3420 + 01	0.6946 + 02	0.1705 + 02
0.3440 + 01	0.7006 + 02	0.1695 + 02
0.3460 + 01	0.7066 + 02	0.1686 + 02
0.3480 + 01	0.7126 + 02	0.1677 + 02
0.3500 + 01	0.7186 + 02	0.1668 + 02
0.3520 + 01	0.7246 + 02	0.1659 + 02
0.3540 + 01	0.7306 + 02	0.1650 + 02
0.3560 + 01	0.7366 + 02	0.1641 + 02
0.3580 + 01	0.7426 + 02	0.1632 + 02
0.3600 + 01	0.7486 + 02	0.1623 + 02
0.3620 + 01	0.7546 + 02	0.1614 + 02
0.3640 + 01	0.7606 + 02	0.1605 + 02
0.3660 + 01	0.7666 + 02	0.1596 + 02
0.3680 + 01	0.7726 + 02	0.1587 + 02
0.3700 + 01	0.7786 + 02	0.1578 + 02
0.3720 + 01	0.7846 + 02	0.1569 + 02
0.3740 + 01	0.7906 + 02	0.1560 + 02
0.3760 + 01	0.7966 + 02	0.1551 + 02
0.3780 + 01	0.8026 + 02	0.1542 + 02
0.3800 + 01	0.8086 + 02	0.1533 + 02
0.3820 + 01	0.8146 + 02	0.1524 + 02
0.3840 + 01	0.8206 + 02	0.1515 + 02
0.3860 + 01	0.8266 + 02	0.1506 + 02
0.3880 + 01	0.8326 + 02	0.1497 + 02
0.3900 + 01	0.8386 + 02	0.1488 + 02
0.3920 + 01	0.8446 + 02	0.1479 + 02
0.3940 + 01	0.8506 + 02	0.1470 + 02
0.3960 + 01	0.8566 + 02	0.1461 + 02
0.3980 + 01	0.8626 + 02	0.1452 + 02
0.4000 + 01	0.8686 + 02	0.1443 + 02
0.4020 + 01	0.8746 + 02	0.1434 + 02
0.4040 + 01	0.8806 + 02	0.1425 + 02
0.4060 + 01	0.8866 + 02	0.1416 + 02
0.4080 + 01	0.8926 + 02	0.1407 + 02
0.4100 + 01	0.8986 + 02	0.1398 + 02
0.4120 + 01	0.9046 + 02	0.1389 + 02
0.4140 + 01	0.9106 + 02	0.1380 + 02
0.4160 + 01	0.9166 + 02	0.1371 + 02
0.4180 + 01	0.9226 + 02	0.1362 + 02
0.4200 + 01	0.9286 + 02	0.1353 + 02
0.4220 + 01	0.9346 + 02	0.1344 + 02
0.4240 + 01	0.9406 + 02	0.1335 + 02
0.4260 + 01	0.9466 + 02	0.1326 + 02
0.4280 + 01	0.9526 + 02	0.1317 + 02
0.4300 + 01	0.9586 + 02	0.1308 + 02
0.4320 + 01	0.9646 + 02	0.1299 + 02
0.4340 + 01	0.9706 + 02	0.1290 + 02
0.4360 + 01	0.9766 + 02	0.1281 + 02
0.4380 + 01	0.9826 + 02	0.1272 + 02
0.4400 + 01	0.9886 + 02	0.1263 + 02
0.4420 + 01	0.9946 + 02	0.1254 + 02
0.4440 + 01	1.0006 + 02	0.1245 + 02
0.4460 + 01	1.0066 + 02	0.1236 + 02
0.4480 + 01	1.0126 + 02	0.1227 + 02
0.4500 + 01	1.0186 + 02	0.1218 + 02
0.4520 + 01	1.0246 + 02	0.1209 + 02
0.4540 + 01	1.0306 + 02	0.1200 + 02
0.4560 + 01	1.0366 + 02	0.1191 + 02
0.4580 + 01	1.0426 + 02	0.1182 + 02
0.4600 + 01	1.0486 + 02	0.1173 + 02
0.4620 + 01	1.0546 + 02	0.1164 + 02
0.4640 + 01	1.0606 + 02	0.1155 + 02
0.4660 + 01	1.0666 + 02	0.1146 + 02
0.4680 + 01	1.0726 + 02	0.1137 + 02
0.4700 + 01	1.0786 + 02	0.1128 + 02
0.4720 + 01	1.0846 + 02	0.1119 + 02
0.4740 + 01	1.0906 + 02	0.1110 + 02
0.4760 + 01	1.0966 + 02	0.1101 + 02
0.4780 + 01	1.1026 + 02	0.1092 + 02
0.4800 + 01	1.1086 + 02	0.1083 + 02
0.4820 + 01	1.1146 + 02	0.1074 + 02
0.4840 + 01	1.1206 + 02	0.1065 + 02
0.4860 + 01	1.1266 + 02	0.1056 + 02
0.4880 + 01	1.1326 + 02	0.1047 + 02
0.4900 + 01	1.1386 + 02	0.1038 + 02
0.4920 + 01	1.1446 + 02	0.1029 + 02
0.4940 + 01	1.1506 + 02	0.1020 + 02
0.4960 + 01	1.1566 + 02	0.1011 + 02
0.4980 + 01	1.1626 + 02	0.1002 + 02
0.5000 + 01	1.1686 + 02	0.0993 + 02
0.5020 + 01	1.1746 + 02	0.0984 + 02
0.5040 + 01	1.1806 + 02	0.0975 + 02
0.5060 + 01	1.1866 + 02	0.0966 + 02
0.5080 + 01	1.1926 + 02	0.0957 + 02
0.5100 + 01	1.1986 + 02	0.0948 + 02
0.5120 + 01	1.2046 + 02	0.0939 + 02
0.5140 + 01	1.2106 + 02	0.0930 + 02
0.5160 + 01	1.2166 + 02	0.0921 + 02
0.5180 + 01	1.2226 + 02	0.0912 + 02
0.5200 + 01	1.2286 + 02	0.0903 + 02
0.5220 + 01	1.2346 + 02	0.0894 + 02
0.5240 + 01	1.2406 + 02	0.0885 + 02
0.5260 + 01	1.2466 + 02	0.0876 + 02
0.5280 + 01	1.2526 + 02	0.0867 + 02
0.5300 + 01	1.2586 + 02	0.0858 + 02
0.5320 + 01	1.2646 + 02	0.0849 + 02
0.5340 + 01	1.2706 + 02	0.0840 + 02
0.5360 + 01	1.2766 + 02	0.0831 + 02
0.5380 + 01	1.2826 + 02	0.0822 + 02
0.5400 + 01	1.2886 + 02	0.0813 + 02
0.5420 + 01	1.2946 + 02	0.0804 + 02
0.5440 + 01	1.3006 + 02	0.0795 + 02
0.5460 + 01	1.3066 + 02	0.0786 + 02
0.5480 + 01	1.3126 + 02	0.0777 + 02
0.5500 + 01	1.3186 + 02	0.0768 + 02
0.5520 + 01	1.3246 + 02	0.0759 + 02
0.5540 + 01	1.3306 + 02	0.0750 + 02
0.5560 + 01	1.3366 + 02	0.0741 + 02
0.5580 + 01	1.3426 + 02	0.0732 + 02
0.5600 + 01	1.3486 + 02	0.0723 + 02
0.5620 + 01	1.3546 + 02	0.0714 + 02
0.5640 + 01	1.3606 + 02	0.0705 + 02
0.5660 + 01	1.3666 + 02	0.0696 + 02
0.5680 + 01	1.3726 + 02	0.0687 + 02
0.5700 + 01	1.3786 + 02	0.0678 + 02
0.5720 + 01	1.3846 + 02	0.0669 + 02
0.5740 + 01	1.3906 + 02	0.0660 + 02
0.5760 + 01	1.3966 + 02	0.0651 + 02
0.5780 + 01	1.4026 + 02	0.0642 + 02
0.5800 + 01	1.4086 + 02	0.0633 + 02
0.5820 + 01	1.4146 + 02	0.0624 + 02
0.5840 + 01	1.4206 + 02	0.0615 + 02
0.5860 + 01	1.4266 + 02	0.0606 + 02
0.5880 + 01	1.4326 + 02	0.0597 + 02
0.5900 + 01	1.4386 + 02	0.0588 + 02
0.5920 + 01	1.4446 + 02	0.0579 + 02
0.5940 + 01	1.4506 + 02	0.0570 + 02
0.5960 + 01	1.4566 + 02	0.0561 + 02
0.5980 + 01	1.4626 + 02	0.0552 + 02
0.6000 + 01	1.4686 + 02	0.0543 + 02
0.6020 + 01	1.4746 + 02	0.0534 + 02
0.6040 + 01	1.4806 + 02	0.0525 + 02
0.6060 + 01	1.4866 + 02	0.0516 + 02

first column denotes the Mach number, second function denotes the Prandtl - Meyer function, third column denotes the Mach angle. So, this value of μ or ν are in degree.

What we have seen here you can say there are three regimes I have put one is low supersonic, second category is maybe supersonic in the range of 4 to 4.5 and this is hypersonic. All these extracts are taken from this book John D. Anderson, Modern Compressible Flow with Historical Prospective McGraw Hill. So, this is some extract, I have just taken some snapshot of the some extract of this data.

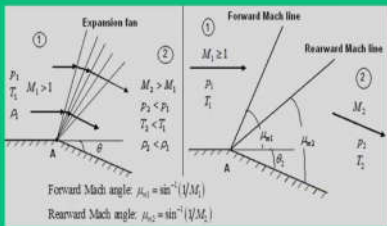
And what it says is that now to solve this particular problem what your approach should be is that for a given flow information we know the upstream conditions M_1 , when you know M_1 , we know θ then we can find out. correspondingly its Prandtl - Meyer functions we know the Prandtl - Meyer function, μ_m ; we know the θ .

So, once we know μ_{m1} and θ , μ_{m2} can be found out once you have μ_{m2} ; this μ_{m2} will tell you the corresponding value of Mach number, new Mach number which is in the down streams. So, the M_2 is now known. So, once we know Mach number at M_1 and M_2 all other isentropic equations can be used.

(Refer Slide Time: 46:13)

Flow Properties across Prandtl-Meyer Expansion Wave

- First, obtain Prandtl-Meyer function for the known upstream Mach number.
- Calculate the Prandtl-Meyer function in the downstream of the flow from the flow deflection angle and upstream Prandtl-Meyer function.
- Find down stream Mach number from the property table.
- Since, the expansion process is isentropic, all the flow properties are known with the knowledge of Mach numbers before and after the flow turning through the fan.



$$\frac{T_2}{T_1} = \frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2}$$

$$\frac{P_2}{P_1} = \left[\frac{1 + \frac{\gamma-1}{2} M_1^2}{1 + \frac{\gamma-1}{2} M_2^2} \right]^{\frac{\gamma}{\gamma-1}}$$

Forward Mach angle: $\mu_{c1} = \sin^{-1}(1/M_1)$
Rearward Mach angle: $\mu_{c2} = \sin^{-1}(1/M_2)$

So, this is how the summary of the flow property evaluation across a Prandtl - Meyer expansion waves. So, first, you obtain the Prandtl - Meyer function for a given upstream

Mach number. Calculate the Prandtl - Meyer function in downstream with the help of flow deflection angle and upstream Prandtl - Meyer function.


Then find out the free stream Mach number from the property table with respect to downstream Mach number. And once you know all the upstream Mach number and downstream Mach number, we can say the properties are related respectively because you know M_1 and M_2 , we also know T_1 . So, we can find T_2 similarly we can find out p_2 .

So, this is how the properties are evaluated.

(Refer Slide Time: 47:15)

Numerical Problem

Q1. A supersonic stream with Mach number 1.5 and pressure 0.9 bar and 20°C is deflected through an expansion corner by an angle of 20°. Calculate the forward & rearward Mach angle, pressure, temperature, Mach number and stagnation temperature and stagnation pressure after the deflection.



Solution:

Given: $M_1 = 1.5$, $p_1 = 0.9 \text{ bar}$, $T_1 = 20^\circ\text{C}$, $\theta = 20^\circ$

Find: Forward & rearward Mach angle, pressure, temperature, Mach number and stagnation temperature and stagnation pressure after the deflection.

Calculation:

From P.M. Table:

$M_1 = 1.5$, $\nu(M_1) = 11.91^\circ$, $M_{n1} = 41.81^\circ$

$\theta = 20^\circ$, $\nu(M_2) = \theta + \nu(M_1) = 31.91^\circ$

$\Rightarrow M_2 = 2.2$, $M_{n2} = 27.42^\circ$

$\frac{p_2}{p_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} = 10.81$

$\frac{T_2}{T_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right) = 1.974$

Since $p_{02} = p_{01} = 425 \text{ k}$, $p_2 = 0.3 \text{ bar}$

$T_{02} = T_{01} = 425 \text{ K}$, $T_2 = 215.3 \text{ K}$

M	ν	M_n
1.0	0.0000	0.0000
1.1	1.0000	1.0000
1.2	2.0000	2.0000
1.3	3.0000	3.0000
1.4	4.0000	4.0000
1.5	5.0000	5.0000
1.6	6.0000	6.0000
1.7	7.0000	7.0000
1.8	8.0000	8.0000
1.9	9.0000	9.0000
2.0	10.0000	10.0000
2.1	11.0000	11.0000
2.2	11.9100	11.9100
2.3	12.8000	12.8000
2.4	13.6800	13.6800
2.5	14.5500	14.5500
2.6	15.4100	15.4100
2.7	16.2600	16.2600
2.8	17.1000	17.1000
2.9	17.9300	17.9300
3.0	18.7600	18.7600
3.1	19.5800	19.5800
3.2	20.4000	20.4000
3.3	21.2100	21.2100
3.4	22.0200	22.0200
3.5	22.8200	22.8200
3.6	23.6200	23.6200
3.7	24.4100	24.4100
3.8	25.2000	25.2000
3.9	25.9800	25.9800
4.0	26.7600	26.7600
4.1	27.5400	27.5400
4.2	28.3100	28.3100
4.3	29.0800	29.0800
4.4	29.8400	29.8400
4.5	30.6000	30.6000
4.6	31.3500	31.3500
4.7	32.1000	32.1000
4.8	32.8400	32.8400
4.9	33.5800	33.5800
5.0	34.3200	34.3200

Now, whatever basic things if I just have to summarize that how we are going to use this property table, I have framed a problem which is for a supersonic stream when it encounters a compression corner.

So, the problem is defined that, a supersonic stream with Mach number of 1.5 initial pressure 0.9 bar and temperature 20°C, it is deflected through an expansion corner by an angle 20°. Calculate the forward and rearward Mach angle, pressure, temperature, Mach number and stagnation temperature and stagnation pressure after the deflections.

So, the solution process for this can have the following aspects. So, first we have to draw that what is the actual flow phenomena that is happening. So, here we can say it is a expansion corner. So, we can draw a schematic diagram of a expansion corner with an angle 20° which is known. And since it is a expansion corner this supersonic flow has

initial value M_1 which is 1.5 when it sees this expansion corner it has to undergo a centered expansion or through Prandtl - Meyer expansion.

So, we can have a rearward Mach line and we will have a forward Mach line. So, these angles can be defined as μ_{m1} and this angle can be defined as μ_{m2} , this μ_{m2} will be function of M_2 . So, once it undergoes this deflection, the downstream stream line becomes parallel to this second surface. So, we have M_2 . So, this is the problem; in addition to this, other parameters given that is p_1 as 0.9 bar and T_1 as 20°C. So, since we know this Mach number and the pressure information; so, one can find out the stagnation

pressure also is equal to $p_{01} = p_1 \left[1 + \left(\frac{\gamma-1}{2} \right) M^2 \right]^{\frac{\gamma}{\gamma-1}}$.

Similarly we can find out $T_{01} = T_1 \left[1 + \left(\frac{\gamma-1}{2} \right) M^2 \right]$. So, putting gamma 1.4 and M_1 is equal to 1.5; p_{01} can be evaluated as 3.3 bar, T_{01} will be evaluated as 425 K. So, what we require now is condition at 2, to do that let us start with the solution.

So, to start the solution what we now require to refer Prandtl - Meyer table. So, we know M_1 as 1.5. So, when I say refer this particular table with M_1 as 1.5, I can say $\nu(M_1)$ that is Prandtl - Meyer function. For this Mach number M_1 is 11.91 and for the Mach angle that is μ_{m1} that is the forward Mach angle now becomes 41.81.

Now we know θ as 20°. So, we can write $\nu(M_2)$ that is Prandtl - Meyer function corresponding to Mach number M_2 would be $\theta + \nu(M_1)$. So, when I do this. So, we will have another value of Prandtl - Meyer function that is 31.91°. So, I have to refer this particular table again for this Prandtl - Meyer functions.

So, I have to refer to a value close to this 31.91° in this particular table. So, a close number that is seen here is 31.73, I can take this value. So, this will imply Mach number as 2.2, when I say Mach number as 2.2, this will also μ_{m2} that is rearward Mach angle becomes 27.02°. So, we know forward Mach angle and rearward Mach angle. So, what is not known to us.

Now, once you know M_2 . So, we can evaluate $\frac{p_{02}}{p_2} = \left[1 + \left(\frac{\gamma-1}{2} \right) M_2^2 \right]^{\frac{\gamma}{\gamma-1}}$,
 $\frac{T_{02}}{T_2} = \left[1 + \left(\frac{\gamma-1}{2} \right) M_2^2 \right]$. So, from this we can find out p_{02} since we know M_2 . So, this
number would be 10.81. So, $\frac{T_{02}}{T_2}$ will be 1.974.

So, since $p_{02} = p_{01}$ that is 3.3 bar. So, this will give you from this p_2 as 0.3 bar;
 $T_{02} = T_{01} = 425$ K. So, this will give you T_2 as about 215.3 K. So, we get all the
information. So, we know p_2 pressure, temperature, Mach number, we have stagnation
pressure, p_2 , T_2 , p_{02} , T_{02} forward and Mach angle μ_{m1} and μ_{m2} .

So, this is how we get all the information when the supersonic flow gets deflected by an
angle θ_2 . I think with this I am able to explain how we are going to look the property
table of Prandtl - Meyer expansion. With this I will conclude my lecture for this module
5.

Thank you for your attention.