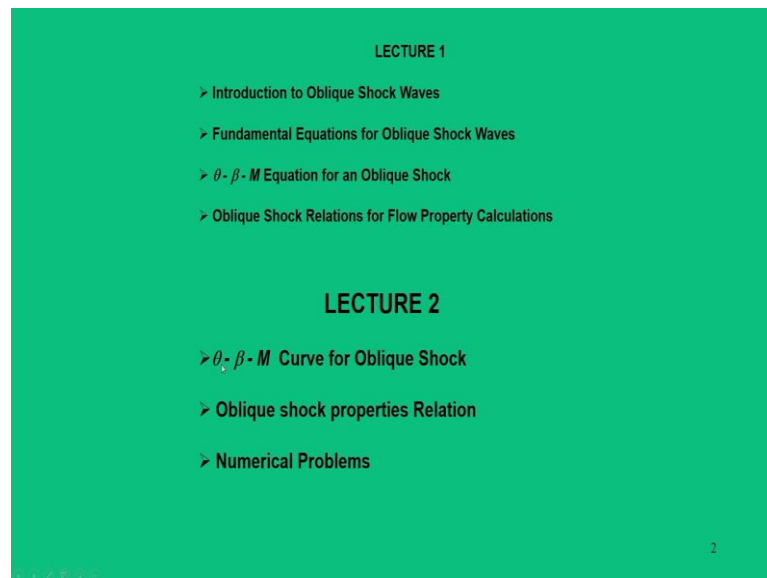


Fundamentals of Compressible Flow
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Module - 05
Expansion Waves and Oblique Shocks
Lecture – 15
Expansion Waves and Oblique Shocks- II

Welcome you to this course again, Fundamentals of Compressible Flow. We are in the Module 5; the topic of this module is Expansion Waves and Oblique Shocks.

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So, in the first lecture of this module, we discussed about the oblique shocks, its fundamental equations; one of the important relations known as θ - β - M equation. Then, we derived the property relations for the oblique shock.

So, now moving in this lecture 2, we will address again the property relations, but in a different context. And most importantly, we will discuss about a curve which is very common in oblique shock situations which is known as θ - β - M curve. After having that, we will try to solve some numerical problems. This is all about this content of this lecture.

Now, just to brief about what we have learnt from this last lecture of this module, though our main focus was on oblique shock.

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Normal Shock and Oblique Shock

- The normal shock waves a special case of oblique waves family in which the streamlines are perpendicular to the shock waves.
- A strong oblique shock is typically considered as normal shock while weakest possible oblique shock could be a Mach wave.
- The normal shock waves are straight and the direction of flow before and after the shock. The oblique shocks are straight but inclined at an angle to the upstream of the flow. This angle is always higher than the Mach angle.
- Across an oblique shock, a supersonic flow when the flow can turn towards main bulk of flow through certain flow deflection angle in a concave corner.

Given conditions				Unknown conditions			
M_1	p_1	T_1	ρ_1	M_2	p_2	T_2	ρ_2
$M_1 > 1$				$M_2 < 1$			
Ahead of shock/ Before the shock				Behind the shock/ After the shock			

Note: $\beta \rightarrow 90^\circ$ is normal shock

Now, when a oblique shock is formed? The first question arises that, when a supersonic flow encounters a compression corner and this particular compression corner what you can see in this figure is a type of concave corner which is inclined by an angle θ . So, I can say that this corner is oriented by an angle θ . So, when a supersonic flow sees this compression corner an oblique shock wave is formed.

Now, across this oblique shock when it crosses, the general tendency would be that the flow will turn by a certain angle. And, in this case since the corner is inclined at an angle θ , the flow will also turn by same angle θ so that the streamlines in the downstream will be parallel to this corner.

Then a shock wave angle is formed that is known as β . Now, across this oblique shock, we have shown that all the static properties in the downstream they increases, but the Mach number decreases. In fact, we also told that when the shock wave angle becomes 90° , that particular situation becomes a special case where we say it is a normal shock. So, oblique shock becomes a normal shock, when this shock wave angle β becomes 90° .

And this is the situation and when the shock wave becomes a normal shock, again the all the static properties increases; but, most importantly the Mach number becomes subsonic, that is M_2 is less than 1 for a given supersonic flow in the upstream condition.

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Oblique Shock Waves

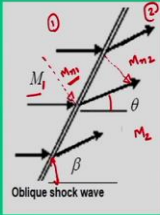
- All the governing equations for oblique shocks are identical to the normal shock relations when the velocities are treated as normal to the wave.
- Thus, the changes across an oblique shock is governed by normal component of the free stream velocity.

$$\tan \theta = 2 \cot \beta \left[\frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \right]$$

$\theta = \beta = M_1 \sin \beta$

$$M_{n1} = M_1 \sin \beta; \quad M_{n2}^2 = \frac{\left(\frac{M_1^2}{\sin^2 \beta} + \frac{2}{\gamma - 1} \right)}{\left(\frac{2\gamma}{\gamma - 1} \right) M_{n1}^2 - 1}; \quad M_2 = \frac{M_{n2}}{\sin(\beta - \theta)}$$

$$\frac{p_2}{p_1} = \frac{(\gamma + 1) M_{n1}^2}{2 + (\gamma - 1) M_{n1}^2}; \quad \frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma + 1} (M_{n1}^2 - 1); \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right) \left(\frac{\rho_1}{\rho_2} \right); \quad s_2 - s_1 = c_p \ln \left(\frac{T_2}{T_1} \right) - R \ln \left(\frac{p_2}{p_1} \right)$$



So, now with this oblique shock situations; like if your shock wave is angle is β , the flow deflection angle is θ and upstream Mach number is M_1 ; then, we derived in the last lecture some the fundamental equations and the summary of those fundamental equations are given here. So, what we have shown here is that, across an oblique shock, it is the normal component of the Mach number that becomes vital for flow property calculations.

So, in the upstream condition 1, if your Mach number is M_1 , the normal component of Mach number will be M_{n1} ; if your downstream condition 2, if your Mach number is M_2 , then the normal component of Mach number is M_{n2} . And, when I say these two are normal components; so this oblique shock can be effectively treated to be a normal shock for upstream Mach number of M_{n1} and downstream Mach number of M_{n2} .

When we say this two parameters are fixed, then all property ratios like density ratio, pressure ratio and temperature ratio, they can be evaluated based on the normal shock Mach number M_{n1} . And then these normal shock Mach numbers can be further related to main flow Mach number, like M_{n1} can be related to M_1 ; by this equation that is $M_{n1} = M_1 \sin \beta$ for which we require this shock wave angle β . Similarly for the downstream situation, the normal component of Mach number M_{n2} is related to M_2 ; that

is $M_2 = \frac{M_{n2}}{\sin(\beta - \theta)}$.

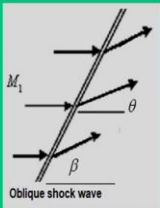
So, we must know the flow deflection angle θ and β for calculation of M_2 , M_{n2} , M_{n1} . So, to do these things, we have this trigonometric relations, which is known as θ - β - M equation. So, this equation will tell you that for a given flow deflection angle and upstream Mach number what will be the shock wave angle β .

$$\tan \theta = 2 \cot \beta \left[\frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \right]$$

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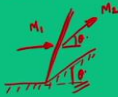
θ - β - M Curve

- When a supersonic flow has to undergo deflection across an oblique shock, the flow deflection angle is considered as a vital parameter and a trigonometric relation is obtained.
- The changes across an oblique shock depends on two parameters i.e. upstream Mach number and shock wave angle.
- But the shock wave angle depends on downstream Mach number and flow deflection angle.



$$\tan \theta = 2 \cot \beta \left[\frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \right]$$

$$M_{n1} = M_1 \sin \beta; M_{n2} = \frac{M_2}{\sin(\beta - \theta)}$$



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Now, let us talk about something more about this θ - β - M relations. So, now if you look at this equations, one can easily say that there are two approaches to solve this equations; the first one is that if you know M_1 , if you know β , then a direct relations can be used, so that you can calculate θ ; that is from this equations.

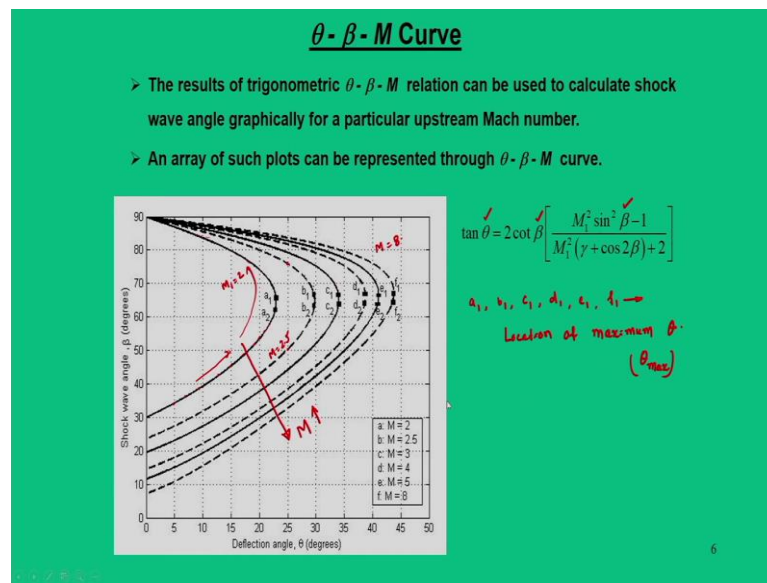
So, θ is nothing but a typically geometrical parameter which is a compression corner needs to have. For a example, if you have a concave corner, then θ is typically this angle. And in fact, this oblique shock which sits on here has to turn and the free stream flow when it crosses this oblique shock, it has to turn by an angle θ .

So, which is same as that of geometric parameters. So, the flow deflection angle is normally same as the geometric shape of this concave corner in which it is oriented. Thus, in many practical situations, the θ is a typically a known parameter and upstream flow Mach number is also a given parameter.

Now, if you say that, if you know M_1 and θ and look at that equations; it looks like an implicit nature, you require a trial and error approach to solve for β . In fact, this becomes a tedious approach.

So, what has been done is that, a series of curves that are obtained for different values of theta and different values of Mach number. So, different curves can be plotted; so, which you call this has a θ - β -M curve. So, what we can say is that the θ - β -M curve is nothing but the graphical representation of this θ - β -M equation. How that equation will look like?

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So, this equation will look like in this form.

$$\tan \theta = 2 \cot \beta \left[\frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \right]$$

So, if you look at this curve, x axis of the curve represents the flow deflection angle θ , while y axis of the curve represents the shock wave angle β . So, this is the θ and in this equation, right hand side of the equation contains β and left hand side of the equation contains θ . Now, what the relation has been drawn is that, for a given Mach number say, let us say M_1 . Let us say, in this case we have taken for the curve a, first curve M is equal to 2. So, M_1 is equal to 2; when you vary θ , then you can get the value of β . So, if you go along this curve M_1 , we can say that across the M_1 curve where M_1 is equal to 2, every point your θ value changes. For different values of θ , we can get series of values of β .

Similarly, we can do for other Mach numbers as well. So, in this plot we have plotted it up to Mach number 8; the last plot that is curve f is for M is equal to 8. And, the other curves like curve b is 2.5, then Mach number 3; if curve d is the Mach number 4 and then curve e is Mach number 5 and curve f is Mach number 8. So, if you go along these directions, effectively your Mach number increases.

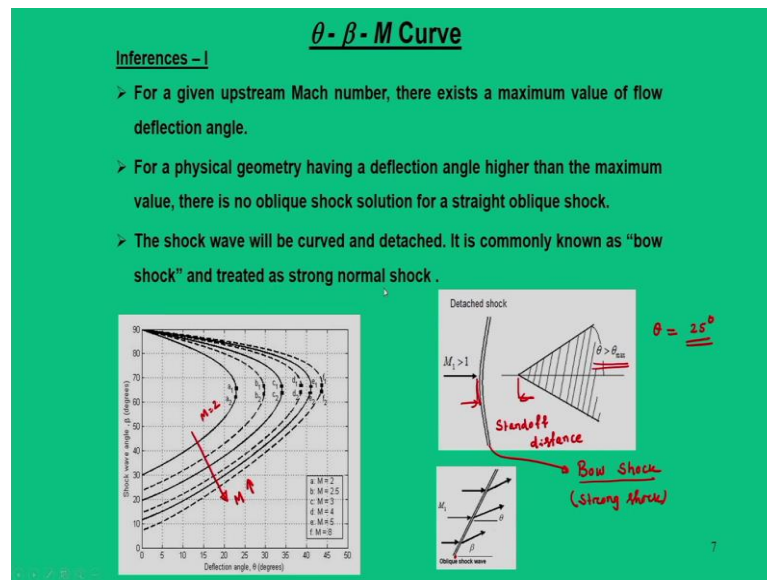
But, what interestingly you can see that, one interesting inference that you can get from here that for a given Mach number we cannot arbitrarily increase the θ values. For instance, if you do it, there will not be any solutions possible through this equations. So, in other words what we say that when your θ value goes beyond a maximum value, there is no oblique shock solution is possible.

For example, if I say that same curve for M_1 is equal to 2 and if I choose θ as 25° ; I would not find any value of β , because it has cross this maximum value of θ . But the same 25° angle will have a solutions for Mach number of 2.5.

So, if this curve is for M is equal to 2.5; but same 25° , we can say there are possible solutions of oblique shocks. But, for Mach number of 2, 25° deflection angle will not have an oblique shocks solutions; this one of the important inferences that we can get; or in other words, there is a maximum possible angle beyond which the oblique shock solution is not possible and those points are denoted for each Mach number as a_1 , b_1 , c_1 , d_1 , e_1 and f_1 .

So, these are the points of a_1 , b_1 , c_1 , d_1 , e_1 and f_1 represents the point of or location of maximum flow deflection angle, location of maximum θ known as θ_{\max} . So, this is one of the important inference that we can get. Now, when I say this; what its effectively mean to us?

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So, what it means to us? So, again we refer the same curve, so I will say that, the first inference that we are going to say is that; we are going across different Mach numbers ranging from 2 to 8, for a given upstream Mach number there is a maximum flow deflection angle that is a_1 , b_1 , c_1 , d_1 , e_1 and f_1 .

Now, this maximum flow deflection angle is decided by θ - β equations. But, in general not necessarily your physical compression corner or physical concave corner will have a deflection angle less than θ . Now, what will happen if the deflection angle is greater than θ_{\max} ? So, for example, if you say that Mach number of 2 and let us choose a value of our physical geometry theta or a compression corner which is 25° .

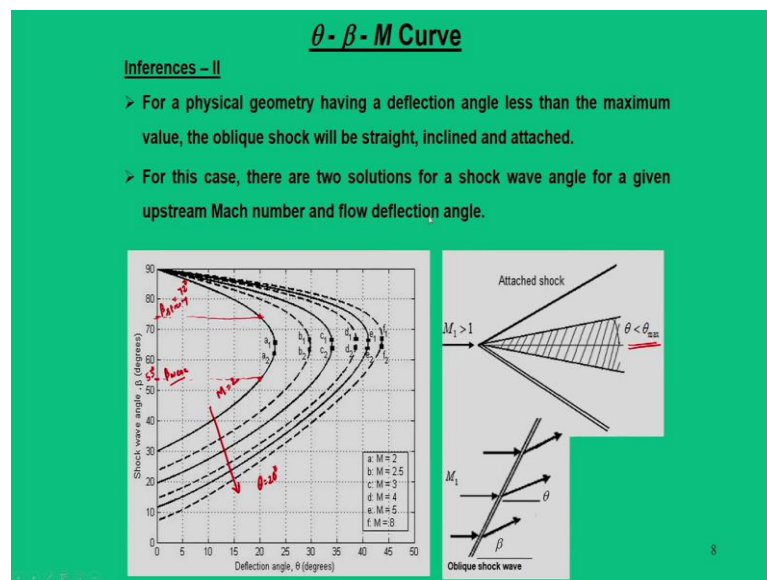
Now, if I look at this figure for this 25° at Mach 2, no solution is possible. So, when there is no oblique shock solution is possible; what will physically means to us that the oblique shock will not be attached to this surface, rather it will stand at certain distance and this particular distance is known as standoff distance. And in fact, this nature of the shock wave will be no longer a oblique shock, it is a shape of a bow shock.

So, this will be another characteristics of shock wave, which is neither a normal shock nor a oblique shock. So, it is a kind of a bow shape. So, it is a bow shock or in terms of oblique shock terminology, we call it as a strong shock; but, this characteristics is that it does not get attach to this surface. So, likewise if an oblique shock has to be there, then it

would have been in this shape and it would have been attach to this surface at this corner, which is not possible in this case.

So, in this case for Mach number of 2 will have a detach shock as shown in this figure. So, that is what I have written here for a physical geometry having a deflection angle higher than the maximum value, there is no oblique shock solution for a straight oblique shock. The shock wave will be curved. So, instead of straight, the shock wave will now be curved and it is detached. So, it is commonly known as a bow shock and treated as a strong normal shock. This is the first important inference we get.

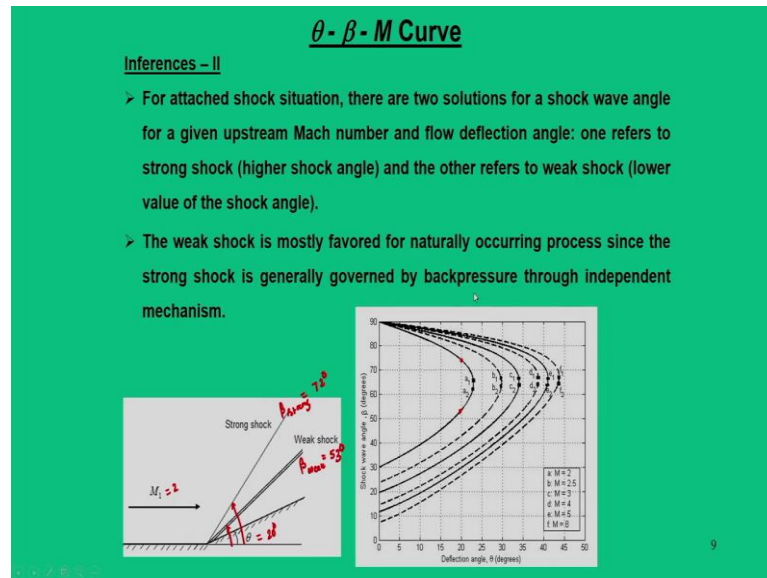
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Now, moving further, will the next important inference that, now if I say that if the shock wave is attached, so that means for if your θ is less than θ_{max} . So, instead of choosing 25° , if I choose a 20° for same Mach number of 2; I will have two shock points on this curve, θ - β -M curve. And, for each point I can find out one β value. I can say β_{max} or I can say β_{strong} and this value I will say β_{weak} .

And, for this 20° ; so correspondingly β will be across about may be 53° that is weak value and strong value could be about 72° . So, from this curve we can directly obtain. So, we say that there are two solutions of shock wave angle for a given upstream Mach number and flow deflection angle.

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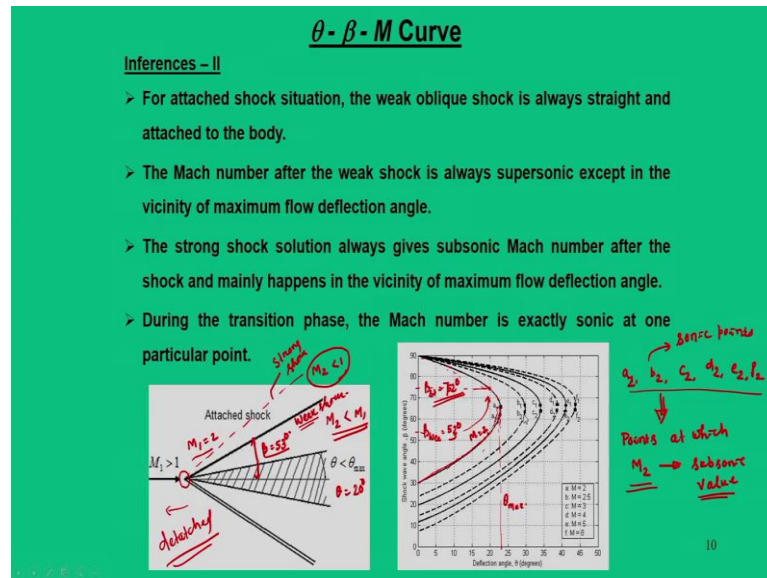


Now, if it is a attached shock situation, then there are two possibilities we can; let us say in this case, when your M_1 is 2, I found out theta as 20° and β_{weak} will be roughly about 53° and β_{strong} will be about 72° . So, this I can say, these are the values for β , strong and weak solution for Mach number of 2.

So, higher shock angle represents strong shock, the lower value of shock angle represents the weak shock. But, in fact in a natural occurring processes, a weak shock solution is mostly preferred; because the strong shock solution is not is essentially driven by some back pressure mechanism and it is not natural occurring process.

So, for example, what do we mean by natural occurring process? That means, when some object is flying at supersonic velocity and if an oblique shock solution needs to be formed, then it is suppose to be weak in nature. The strong shock is not a general occurring phenomena, rather it is driven through some back pressure mechanism. So, this will be covered subsequently, under what circumstances we can have a strong shock.

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So, when it is a weak shock and it is an attach shock. So, let us say in the previous case when your theta was 20° , Mach number M_1 is 2 and we had the weak shock angle will be about 53° .

So, this is how a typical example what I can give for Mach number of 2. So, what we can say; for this situation if it is a weak shock, now if I want to calculate M_2 is less than M_1 . When I say M_2 is less than M_1 , then I am in the lower half of the curve. So, if I say that is θ_{\max} , point a_1 refers to be θ_{\max} and when you take this 52° , we are basically in the lower half of this particular $\theta - \beta - M$ curve for M is equal to 2.

And, when we were in the lower half of this curve, we say you have M_2 is less than M_1 ; but had you been in the upper half of the curve where for same M_1 , we have $\beta_{\text{weak}} 53^\circ$ and β_{strong} about 72° degree. So, if you choose a solution of β_{strong} , then had this been a strong shock; downstream Mach number would have been less than 1.

So, when it is a weak shock, we say a weak shock solution for which β is 53° ; when it is a strong shock, it is β is 72° . And, in this case your Mach number would be subsonic. So, this is how we say that, strong shock solution will always give a subsonic Mach number after the shock. And, what will happen that in a given curve, if I move from strong shock solution moving right towards maximum theta deflection angle and when I am moving from weak shock region and moving towards θ_{\max} , when I am in the strong shock region and moving towards θ_{\max} .

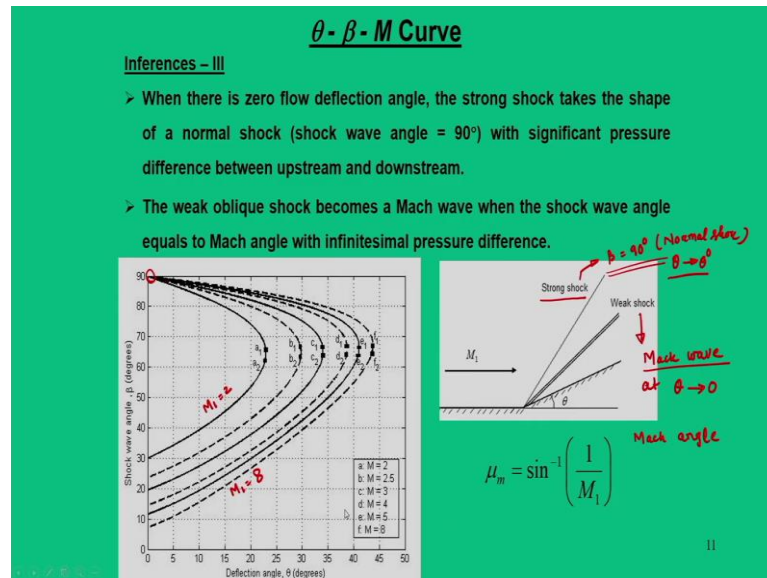
Trying to say that, the weak shock solution will give you a move towards the stronger and strong shock solution will give toward this weaker. But, in most of the situations, in a naturally occurring process, when a weak shock solution becomes stronger and stronger; so, it moves towards the strong shock solutions.

In other words what I can say, the attached shock which is a weak in nature will try to get detached from this point. So, that means the Mach number which was supposed to be supersonic tries to become subsonic. So, that means when you are moving towards right, I am essentially trying to say that I am moving towards strong shock region. And, while moving towards strong shock solution, the downstream Mach number tries to become subsonic.

So, so there is a intermediate point which is known as transition phase, that is point a_2 , b_2 , c_2 , d_2 , e_2 and f_2 ; these points represents the points at which the downstream Mach number M_2 approaches to subsonic value; that means, if I chose any point beyond a_2 for M is equal to 2, then I will land off a subsonic region. And, almost at one particular point, when I am moving from a low supersonic Mach number to subsonic Mach number, at one particular point the flow tries to be sonic.

So, this points a_2 , b_2 , c_2 all these things are the sonic points. So, this is what it has been written here. During transition phase, the Mach number is exactly sonic at one particular point and this one particular point will be different for different Mach number. So, in this case it is a_2 ; the second Mach number for point 2.5, M_1 is equal to 2.5, it will be point b_2 ; likewise for other Mach number.

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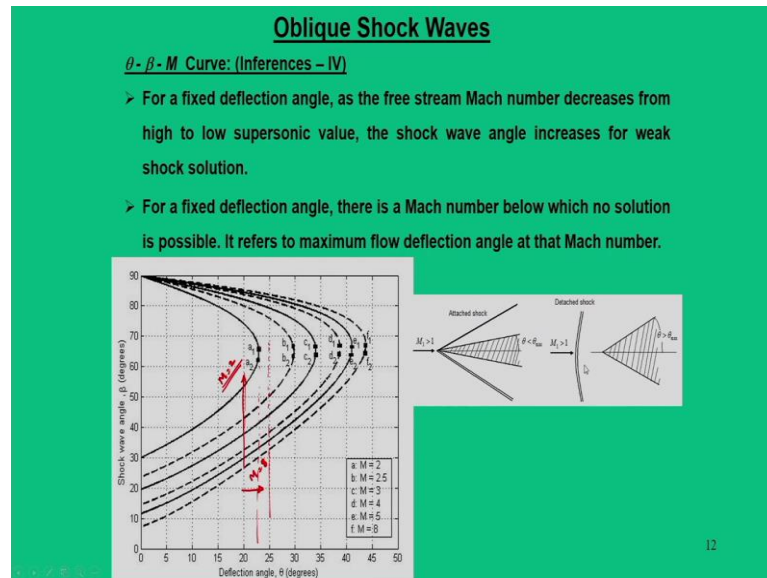
So this is how we say that what will happen when we are in this domain. But, let us see that for a given Mach number, what are the upper limits and lower limits? So, first upper limit I can say, when we say it is a strong shock; the strong shock becomes a normal shock when β is equal to 90° , we say it is a normal shock.

So, if we look at this particular plot, what will see that, all the curves merge to 90° point at this, all the points tries to merge or converge to a particular point; that means, at this point all the curves merge to a normal shock. But, this is not so in the lower limit of those curves, if we look at the lower limit of this curve.

So, if this is for M_1 is equal to 2, and this is for M_1 is equal to 8; if you say lower limit of this curve, the shock wave angle is about 30° for M_1 is equal to 2. Whereas, the lower limit curve of M_1 as 8 is close to about 8° ; how do you get it? Because these are nothing but that a weak shock becomes a Mach wave at θ goes to 0.

So, here it becomes a strong shock when θ goes to 0. So, at $\theta=0$, there are two possible of strong solutions; one is β 90 degree that refers to normal shock and for other cases the limit is the θ 0, it is the a limit which is a Mach wave and that angle is known as Mach angle. And, this is we know from this expression, that is $\mu_m = \sin^{-1}\left(\frac{1}{M_1}\right)$. So, oblique shock becomes a Mach wave.

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The next important inference that we can say is that, for a fixed deflection angle as the free stream Mach number decreases from high to low supersonic value. So, for a given deflection angle θ let us say 20° ; if I am moving from high supersonic, that is if I am moving from M is equal to 8 to M is equal to 2, what we see in the weak shock region, your shock wave angle increases. So, in the weak shock region, the shock wave angle increases.

But, if I am moving towards the maximum θ deflection angle for this M_1 is equal to 2 and beyond which there is no solutions possible for M_1 is equal to 2. But, it is for the rest of the Mach numbers there are possible oblique shock situations.

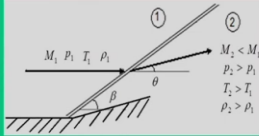
For a fixed deflection angle there is a Mach number below which no solution is possible, it refers the maximum θ deflection angles. And, when we are in the domain of θ less than θ_{\max} will have attach shock; when we are above that θ greater than θ_{\max} will have a detach shock.

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θ - β - M Curve

Controlling parameters

- When the normal component of Mach number ahead of shock wave increases, the strength of shock wave also increases.
- The normal component of Mach number can be increased by increasing shock wave angle (through higher flow deflection angle).
- The other possibility of increasing the normal component of Mach number can be achieved by increasing upstream flow Mach number.
- As a rule of thumb, as flow deflection angle increases (keeping upstream Mach number constant), the shock wave angle increases.
- As a rule of thumb, upstream Mach number increases (keeping flow deflection angle constant), the shock wave angle decreases.



$$\tan \theta = 2 \cot \beta \left[\frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \right]$$

$$M_{n1} = M_1 \sin \beta; M_{n2} = \frac{M_{n1}}{\sin(\beta - \theta)}$$

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So, in other words what I have shown here is that from this θ - β - M curve, we can say using this relation, what parameters that controls? Whether a oblique shock will be a Mach wave or oblique shock will have a strong shock solution or oblique shock will have a weak shock solution that is mostly decided the parameter which is normal component of the Mach number.

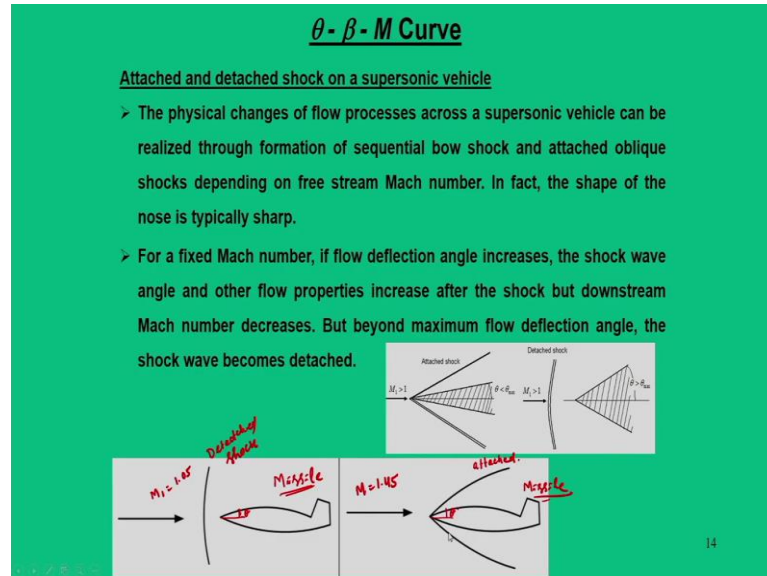
So, looking at this equation what we can say that, the strength of an oblique shock is higher if the normal shock Mach number strength is more. When the normal shock Mach number is more; then obviously, the downstream Mach number M_{n2} will be all become subsonic.

When it becomes subsonic, so it is a strong shock; when it is supersonic, it need not be a strong shock, it will be a weak shock. And, M_{n1} can be increased by two ways; one is by increasing the M_1 or increasing the β , increasing the β we do not have control, but to increase β we have to go for higher flow deflection angle.

So, whatever I discussed, it has been summarized here; but, as a rule of thumb what you can say, as the flow deflection angle increases keeping upstream Mach number constant, the shock wave angle increases. If the upstream Mach number increases keeping the flow deflection angle constant, then shock wave angle decreases. In one case we are keeping Mach number constant, in other case we are keeping flow deflection angle constant. And

accordingly we can say that, we can vary the shock wave angle, whether you can increase or decrease.

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So, this is the another example what I am trying to give that, whatever I have summarized that in most of this supersonic objects or you can say missile of this particular shape; so you can say it is a missile, missile shape object. Now, when this missile shape object travels at a high altitude, at different altitudes it moves in different flow regimes.

So, in fact for this particular shape, there is a certain deflection angle θ . And, when it is moving at different velocities, what may have is at some situations, some flow conditions upstream and downstream the shock wave may be a detached one or shock wave may be a attached one.

For example, I can say that, let us say it we starts with Mach number of M_1 is equal to 1.05; and under those conditions and for this θ a we can have a detach shock. But, not necessarily that this shock wave will always be detached; but, if I increase this Mach number to be 1.45, it tries to get attach to the body.

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$\theta - \beta - M$ Curve

Attached and detached shock on a supersonic

- For a fixed deflection angle, if upstream/free stream Mach number increases, the shock wave is first detached and becomes attached when the free stream Mach number becomes to a value that corresponds to the maximum flow deflection angle.
- Further increase in Mach number will always leads to attached shock with decrease in shock wave angle while all other flow properties will increase after the shock.

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So, likewise initially another situation may be, it may be initially attached, then move to a detached one; which means, that not necessarily that always will have only attach shock or only detach shock in a flow changing environment. By controlling the Mach number, we can say that we can either move from attach shock solution to a detach shock solution or we can go from detach shock solution to a attach shock solution, just by controlling the Mach number and θ . So, this observations of oblique shock properties gives a very vital understanding about the flow behavior in a supersonic objects.

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Gas Property Tables for Normal Shock Relations

M_1	$\frac{P_2}{P_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{P_{02}}{P_{01}}$	$\frac{P_{02}}{P_1}$	M_2
0.1000 + 01	0.1000 + 01	0.1000 + 01	0.1000 + 01	0.1000 + 01	0.1893 + 01	0.1000 + 01
0.1020 + 01	0.1047 + 01	0.1033 + 01	0.1013 + 01	0.1000 + 01	0.1938 + 01	0.9805 + 00
0.1040 + 01	0.1095 + 01	0.1067 + 01	0.1026 + 01	0.9999 + 00	0.1984 + 01	0.9620 + 00
0.1060 + 01	0.1144 + 01	0.1101 + 01	0.1039 + 01	0.9998 + 00	0.2032 + 01	0.9444 + 00
0.1080 + 01	0.1194 + 01	0.1135 + 01	0.1052 + 01	0.9994 + 00	0.2082 + 01	0.9277 + 00
0.1100 + 01	0.1245 + 01	0.1169 + 01	0.1065 + 01	0.9989 + 00	0.2133 + 01	0.9118 + 00
0.1120 + 01	0.1297 + 01	0.1203 + 01	0.1078 + 01	0.9982 + 00	0.2185 + 01	0.8966 + 00
0.1140 + 01	0.1350 + 01	0.1238 + 01	0.1090 + 01	0.9973 + 00	0.2239 + 01	0.8820 + 00
0.1160 + 01	0.1403 + 01	0.1272 + 01	0.1103 + 01	0.9961 + 00	0.2294 + 01	0.8682 + 00
0.1180 + 01	0.1458 + 01	0.1307 + 01	0.1115 + 01	0.9946 + 00	0.2350 + 01	0.8549 + 00
0.4000 + 01	0.1850 + 02	0.4571 + 01	0.4047 + 01	0.1388 + 00	0.2107 + 02	0.4350 + 00
0.4050 + 01	0.1897 + 02	0.4598 + 01	0.4125 + 01	0.1330 + 00	0.2159 + 02	0.4336 + 00
0.4100 + 01	0.1944 + 02	0.4624 + 01	0.4205 + 01	0.1276 + 00	0.2211 + 02	0.4324 + 00
0.4150 + 01	0.1993 + 02	0.4650 + 01	0.4285 + 01	0.1223 + 00	0.2264 + 02	0.4311 + 00
0.4200 + 01	0.2041 + 02	0.4675 + 01	0.4363 + 01	0.1173 + 00	0.2318 + 02	0.4299 + 00
0.4250 + 01	0.2091 + 02	0.4699 + 01	0.4440 + 01	0.1126 + 00	0.2372 + 02	0.4288 + 00
0.4300 + 01	0.2140 + 02	0.4723 + 01	0.4512 + 01	0.1080 + 00	0.2427 + 02	0.4277 + 00
0.4350 + 01	0.2191 + 02	0.4746 + 01	0.4586 + 01	0.1036 + 00	0.2483 + 02	0.4266 + 00
0.4400 + 01	0.2242 + 02	0.4768 + 01	0.4661 + 01	0.9948 - 01	0.2539 + 02	0.4255 + 00
0.4450 + 01	0.2294 + 02	0.4790 + 01	0.4738 + 01	0.9950 - 01	0.2596 + 02	0.4245 + 00
0.8000 + 01	0.7209 + 02	0.5565 + 01	0.7209 + 02	0.8488 - 02	0.8207 + 02	0.7929 + 00
0.9000 + 01	0.9433 + 02	0.6551 + 01	0.6699 + 02	0.4964 - 02	0.1048 + 03	0.3898 + 00
0.1000 + 02	0.1165 + 03	0.5714 + 01	0.2039 + 02	0.3045 - 02	0.1292 + 03	0.3876 + 00
0.1100 + 02	0.1410 + 03	0.5762 + 01	0.2447 + 02	0.1945 - 02	0.1563 + 03	0.3859 + 00
0.1200 + 02	0.1678 + 03	0.5799 + 01	0.2894 + 02	0.1287 - 02	0.1859 + 03	0.3847 + 00
0.1300 + 02	0.1970 + 03	0.5826 + 01	0.3380 + 02	0.8771 - 03	0.2181 + 03	0.3837 + 00
0.1400 + 02	0.2285 + 03	0.5851 + 01	0.3905 + 02	0.6138 - 03	0.2528 + 03	0.3829 + 00
0.1500 + 02	0.2623 + 03	0.5870 + 01	0.4460 + 02	0.4395 - 03	0.2902 + 03	0.3823 + 00
0.1600 + 02	0.2985 + 03	0.5885 + 01	0.5072 + 02	0.3212 - 03	0.3301 + 03	0.3817 + 00
0.1700 + 02	0.3370 + 03	0.5898 + 01	0.5714 + 02	0.2390 - 03	0.3726 + 03	0.3813 + 00

$\tan \theta = 2 \cot \beta \left[\frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2} \right]$
 $M_{n1} = M_1 \sin \beta; M_{n2} = \frac{M_{n1}}{\sin(\beta - \theta)}$

Oblique shock wave

16

Reference: John D. Anderson Jr (1990), Modern Compressible Flow with Historical Perspective, McGraw-Hill, Singapore

Now, I will try to demonstrate the gas shock property tables that are to be used for oblique shock relations. So, here the property table that you see here is for a normal shock relations. But, as I mentioned while looking at this particular table, you should look at this particular M_1 ; instead of looking at the Mach number M_1 , we have to look at this as M_{n1} . So, as I said that, it is the normal shock Mach number that is important to see the property table.

First thing is that, for a given upstream Mach number M_1 and flow deflection θ ; we can find out value of β using this particular θ - β curve. Once you know β , then we can find out M_{n1} and then you refer this table M_{n1} , where you will land off. And correspondingly, the property values ratios can be denoted and here also instead of M_2 , we have to say the effective number will be M_{n2} .

Now, when I say M_{n2} ; so for a given M_1 when I find M_{n2} , I know β and θ . So, I can also calculate M_2 . So, this is how your approach should be how to combinely look into the θ - β - M curve and this normal shock property table to find out the oblique shock property relations for static pressure, temperature, density, and stagnation pressures.

(Refer Slide Time: 40:25)

Numerical Problems

Q1. A supersonic stream with Mach number 3 and pressure 0.9 bar and 20°C is deflected through a compression corner by an angle of 20°. Calculate the shock wave angle, pressure, temperature, Mach number, stagnation temperature and stagnation pressure after the deflection.

Soln

$\theta = 20^\circ$, $M_1 = 3 \Rightarrow \beta = 37^\circ$, $M_{n1} = M_1 \sin \beta = 1.82$

$M_{n2} = 0.4752$

$\frac{p_2}{p_1} = 3.698$

$\frac{T_2}{T_1} = 1.647$

$\frac{\rho_2}{\rho_1} = 0.8438$

$\frac{p_{02}}{p_{01}} = 0.4752$

$\frac{T_{02}}{T_{01}} = 0.7209$

$M_2 = \frac{M_{n2}}{\sin(\beta - \theta)} = 2.1$

$p_2 = 3.32 \text{ bar}$

$T_2 = 48.3 \text{ K}$

$p_{02} = 2.65 \text{ bar}$

$T_{02} = 820 \text{ K}$

$T_{02} = T_{01} = 820 \text{ K}$

Normal shock table

M_1	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{02}}{p_{01}}$	$\frac{T_{02}}{T_{01}}$	M_2
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.2	1.0692	1.0692	1.0692	0.9933	0.9933	0.7845
1.4	1.1592	1.1592	1.1592	0.9791	0.9791	0.6724
1.6	1.2680	1.2680	1.2680	0.9670	0.9670	0.5817
1.8	1.3969	1.3969	1.3969	0.9568	0.9568	0.5142
2.0	1.5438	1.5438	1.5438	0.9482	0.9482	0.4551
2.2	1.7145	1.7145	1.7145	0.9410	0.9410	0.4034
2.4	1.9133	1.9133	1.9133	0.9351	0.9351	0.3580
2.6	2.1448	2.1448	2.1448	0.9304	0.9304	0.3183
2.8	2.4138	2.4138	2.4138	0.9268	0.9268	0.2835
3.0	2.7232	2.7232	2.7232	0.9242	0.9242	0.2527
3.2	3.0767	3.0767	3.0767	0.9225	0.9225	0.2254
3.4	3.4780	3.4780	3.4780	0.9216	0.9216	0.2011
3.6	3.9301	3.9301	3.9301	0.9214	0.9214	0.1792
3.8	4.4361	4.4361	4.4361	0.9218	0.9218	0.1591
4.0	5.0000	5.0000	5.0000	0.9227	0.9227	0.1408
4.2	5.6267	5.6267	5.6267	0.9240	0.9240	0.1241
4.4	6.3199	6.3199	6.3199	0.9257	0.9257	0.1088
4.6	7.0834	7.0834	7.0834	0.9277	0.9277	0.0948
4.8	7.9221	7.9221	7.9221	0.9300	0.9300	0.0820
5.0	8.8406	8.8406	8.8406	0.9325	0.9325	0.0703
5.2	9.8437	9.8437	9.8437	0.9352	0.9352	0.0596
5.4	10.9362	10.9362	10.9362	0.9381	0.9381	0.0498
5.6	12.1231	12.1231	12.1231	0.9411	0.9411	0.0408
5.8	13.4103	13.4103	13.4103	0.9442	0.9442	0.0325
6.0	14.8037	14.8037	14.8037	0.9474	0.9474	0.0248
6.2	16.3093	16.3093	16.3093	0.9507	0.9507	0.0176
6.4	17.9321	17.9321	17.9321	0.9541	0.9541	0.0108
6.6	19.6781	19.6781	19.6781	0.9576	0.9576	0.0044
6.8	21.5533	21.5533	21.5533	0.9612	0.9612	0.0000
7.0	23.5637	23.5637	23.5637	0.9649	0.9649	0.0000
7.2	25.7153	25.7153	25.7153	0.9687	0.9687	0.0000
7.4	28.0239	28.0239	28.0239	0.9726	0.9726	0.0000
7.6	30.4953	30.4953	30.4953	0.9766	0.9766	0.0000
7.8	33.1353	33.1353	33.1353	0.9807	0.9807	0.0000
8.0	35.9497	35.9497	35.9497	0.9849	0.9849	0.0000
8.2	38.9435	38.9435	38.9435	0.9892	0.9892	0.0000
8.4	42.1327	42.1327	42.1327	0.9936	0.9936	0.0000
8.6	45.5333	45.5333	45.5333	0.9981	0.9981	0.0000
8.8	49.1613	49.1613	49.1613	1.0027	1.0027	0.0000
9.0	53.0337	53.0337	53.0337	1.0074	1.0074	0.0000
9.2	57.1675	57.1675	57.1675	1.0122	1.0122	0.0000
9.4	61.5707	61.5707	61.5707	1.0171	1.0171	0.0000
9.6	66.2513	66.2513	66.2513	1.0221	1.0221	0.0000
9.8	71.2173	71.2173	71.2173	1.0272	1.0272	0.0000
10.0	76.4777	76.4777	76.4777	1.0324	1.0324	0.0000

Now, with this will try to solve some numerical problems. So, whatever I have analyzed this far. So, here just to say these things; so, first problem it is talks about a supersonic stream of Mach number 3 pressure 0.9 bar and 20°C.

So, first solution for this would be, you have to draw the physical geometry. So, you when you draw this physical geometry, first see that it is a compression corner, a supersonic flow has to turn a shape. So, the angle is given as 20° . So, you know M_1 , we do not know M_2 ; we do not we need to find out V_2 , T_2 , ρ_2 , P_{02} and so on, T_{02} . All upstream numbers are known P_1 0.9 bar, T_1 20°C that is 293K.

So, first thing we need to find out is that, obviously since you know this; we also require stagnation properties for this Mach number. And so, Mach number is given as 3; for given Mach number and p_1 and T_1 , we can use the isentropic property table to find out p_{01} ; p_{01} as 33 bar and T_{01} as 820K.

So, instead of using isentropic property table, we can also find out from this relation

$$\frac{p_{01}}{p_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}. \text{ And similarly, for } \frac{T_{01}}{T_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right).$$

So, either you use isentropic property table or use this relations, we can estimate this. So, by this we can say all upstream parameters for region 1 is known. So, we need to find out the downstream numbers. To approach would be something like that. So, first thing we need to know, we have θ 20° , we have M_1 3.

So, M_1 3 means, you have to go for this curve c and θ 20° . So, this will give you β as close to 37° from this curve; once I say β 37° , I can find out M_{n1} that is $M_1 \sin \beta$.

So, M_{n1} would be 1.82. Now, when I know M_{n1} , you take this normal shock table. So, this is some extract data I have taken from normal shock table. So, close to 1.82, we can note down the properties values; the properties values would be like M_{n2} 0.6121; then,

$$\frac{p_2}{p_1} \text{ is } 3.698, \frac{T_2}{T_1} \text{ } 1.547, \frac{p_{02}}{p_{01}} \text{ } 0.8038. \text{ So, once we know this ratios then we can easily}$$

compute as p_2 as 3.32 bar, T_2 would be 453K. So, you take this ratio, you know upstream number and p_{02} 2.65 bar.

Now, what is remaining is M_2 . So, M_2 can be calculate as $\frac{M_{n2}}{\sin(\beta-\theta)}$. So, β is known, θ

is known, M_{n2} is known; so, you can find out M_2 as 2.1. So, what you see is that, initial M_1 was 3; so Mach number is still supersonic across this oblique shock.

(Refer Slide Time: 46:15)

Numerical Problems

Q2. With the data referring to Q1, if the deflection angle is increased to 30° , calculate the shock wave angle, pressure, temperature, Mach number, stagnation temperature and stagnation pressure after the deflection. Compare the results with Q1.

Q1 data
 $p_1 = 0.9 \text{ bar}$
 $T_1 = 293 \text{ K}$
 $T_{01} = 820 \text{ K}$
 $p_{01} = 33 \text{ bar}$

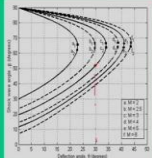
$\beta \rightarrow 30^\circ \rightarrow M_1 = 3 \Rightarrow \beta = 52^\circ$

Repeat same process. $M_{n1} = M_1 \sin \beta = 2.36$

$\frac{p_2}{p_1} = 6.276$, $\frac{T_2}{T_1} = 1.993$, $\frac{p_{02}}{p_{01}} = 0.5615$, $M_{n2} = 0.5286$

$M_2 = \frac{M_{n2}}{\sin(\beta - \theta)} = 1.41$

$p_2 = 5.65 \text{ bar}$
 $T_2 = 584 \text{ K}$
 $p_{02} = 18.53 \text{ bar}$
 $T_{02} = T_{01} = 820 \text{ K}$



M_1	β	θ	$\frac{p_2}{p_1}$	$\frac{T_2}{T_1}$	$\frac{p_{02}}{p_{01}}$	M_2
1.2	78.1	0	1.04	1.00	1.00	1.20
1.5	65.9	0	1.34	1.06	0.929	1.50
2.0	47.5	0	2.18	1.33	0.721	2.00
2.5	35.7	0	3.48	1.69	0.499	2.50
3.0	27.7	0	5.76	2.20	0.328	3.00
4.0	16.2	0	16.9	4.75	0.129	4.00
5.0	10.0	0	45.0	12.9	0.048	5.00
6.0	7.0	0	129	42.0	0.023	6.00
7.0	5.5	0	300	129	0.013	7.00
8.0	4.5	0	720	300	0.008	8.00
9.0	3.8	0	1700	720	0.005	9.00
10.0	3.3	0	4000	1700	0.003	10.00

So, in the next problem is that, in the same question 1, if the deflection angle is increase to 30° , we have to calculate the same properties value. When θ goes to 30° and your M_1 is 3. So, this will turn out to be β from this curve. So, here we have to refer 30° and M_1 3; curve c.

So, β would be 52° . When I say β is 52° , we have to repeat the same process. But the other ratios we can have, like will have p_1 as same as 0.9 bar, T_1 as which was this 293K. So, same as for question 1 data, then T_{01} as 820K, p_{01} as 33 bar. So, in these oblique shock, here this angle is 30° , this β is now 52° and this region 1 and we want to find out region 2.

So, having said this; so, we have to repeat the same process means, first we have to find out M_{n1} is $M_1 \sin \beta$ that is 2.36. So, you have to use per normal shock table may be closed to a value of 2.35, I can note down this number.

So, you can say $\frac{p_2}{p_1}$ as 6.276, $\frac{T_2}{T_1}$ 1.993, $\frac{p_{02}}{p_{01}}$ 0.5615; then, this will lead you and you will

have M_{n2} 0.5286. So, thus you can find out M_2 to be $\frac{M_{n2}}{\sin(\beta - \theta)}$. So, M_2 would be 1.41;

likewise, we can say p_2 would be 5.65 bar, we know this T_2 would be 1.993×293 , so 584K, p_{02} would be 0.5615×33 will be about 18.53 bar. So, likewise we can solve this particular problems.

So, in this question what we have shown is that, flow deflection angle increases but, your Mach number now becomes 1.41. So, in previous question the Mach number M_2 was 2.1; so, Mach number gets decreased if the flow deflection angle increases.

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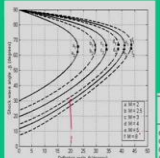
Numerical Problems

Q3. With the data referring to Q1, if the free stream Mach number is increased to 5, calculate the shock wave angle, pressure, temperature, Mach number, stagnation temperature and stagnation pressure after the deflection. Compare the results with Q1.

Ans. $M_1 = 5$, $\theta = 20^\circ \Rightarrow \beta = 30^\circ$
 $M_{n1} = M_1 \sin \beta = 2.5$ $M_{n2} = 0.513$
 $\frac{p_2}{p_1} = 7.125$ $\frac{T_2}{T_1} = 2.137$ $\frac{p_{02}}{p_{01}} = 0.499$
 $M_2 = \frac{M_{n2}}{\sin(\beta - \theta)} = 2.95$
 $p_2 = 6.41 \text{ bar}$
 $T_2 = 626 \text{ K}$
 $p_{02} = 237.5 \text{ bar}$

From Q1: $M_1 = 3$, $\theta = 20^\circ$
 $\beta = 17^\circ$
 $p_1 = 0.9 \text{ bar}$
 $T_1 = 293 \text{ K}$
 $p_{01} = 33 \text{ bar}$
 $T_{01} = 1758 \text{ K}$

Comparison: $T_{01} = T_{02} = 1758 \text{ K}$



M_1	$\frac{p_2}{p_1}$	$\frac{T_2}{T_1}$	$\frac{p_{02}}{p_{01}}$	M_2
1.0	1.000	1.000	1.000	1.000
1.2	1.042	1.069	0.993	0.721
1.4	1.094	1.143	0.971	0.577
1.6	1.157	1.235	0.933	0.475
1.8	1.231	1.347	0.882	0.403
2.0	1.316	1.485	0.721	0.328
2.2	1.413	1.650	0.578	0.271
2.4	1.523	1.847	0.455	0.229
2.6	1.647	2.080	0.348	0.197
2.8	1.786	2.359	0.266	0.173
3.0	1.941	2.684	0.200	0.154
3.2	2.113	3.057	0.147	0.141
3.4	2.304	3.482	0.105	0.132
3.6	2.516	3.962	0.073	0.126
3.8	2.751	4.500	0.050	0.122
4.0	3.011	5.103	0.033	0.119
4.2	3.297	5.784	0.022	0.117
4.4	3.611	6.546	0.014	0.115
4.6	3.954	7.393	0.009	0.113
4.8	4.328	8.330	0.005	0.111
5.0	4.735	9.360	0.003	0.110

The next question is that, we are keeping this in the question 1; if the free stream Mach number is increased to 5. So, the question remains same, what you have, the question 1 remains same with a only difference that is there; instead of Mach number 3, I have increase this to 5, but theta becomes same angle as 20° .

So, when I change the, we have to find the condition 2. Now, for same p_1 was 0.9 bar, T_1 was 293K; since, the Mach number was increased, so p_{01} will also be increased. So, similarly way we can calculate p_{01} is 476 bar. So, you just imagine that from 33 bar, it has to increase to 476 bar when Mach number is increased; stagnation temperature becomes 1758K.

So, this is the property conditions in the region 1. So, we require to find out β . So, solution process goes in similar manner. So, here we have to see for curve b that is Mach number 5 and θ as 20° . So, somewhere you will have landed off β .

So, M_1 is 5, θ is 20° ; this will give you β value from this θ - β -M curve is as 30° . So, when I know β , I can find M_{n1} is $M_1 \sin \beta$, that number would be 2.5. And, for this 2.5 M_{n1} ; if

I look for the normal shock table, I will say M_{n2} as 0.513 and $\frac{p_2}{p_1}$ becomes 7.125,

$\frac{T_2}{T_1}$ becomes 2.137 and $\frac{p_{02}}{p_{01}}$ would be 0.499. Then, once I put this, then we have to find

out M_2 is $\frac{M_{n2}}{\sin(\beta - \theta)}$.

So, this turns out to be M_2 2.95. What we see is that, when your M_1 is increased, your downstream Mach number also increases. So, we all know upstream parameters of this value, we know this ratios; then, you can say p_2 as 6.41 bar, T_2 as 626K and p_{02} as 237.5 bar. But, what we see here, what remains again same in all these three problems; your T_{01} would be T_{02} ; in fact, this is the conditions that remain constant.

In fact, all the previous problem also T_{02} would be T_{01} 820K; in this case it was 820. And, in the first question also T_{01} was also 820 Kelvin. So, total temperature does not change as it is in the case of normal shock. So, this is how you have to refer this normal shock table as well as θ - β - M curve to calculate the property for oblique shocks. So, with this I will conclude the topic of oblique shock.

Thank you for your attention.