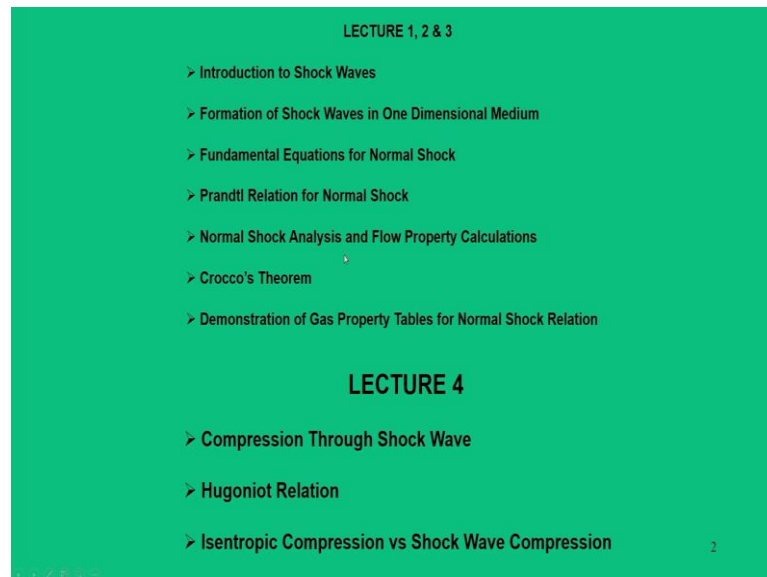


Fundamentals of Compressible Flow
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Module – 04
Lecture – 13
Normal Shock Waves- IV

Welcome to this course, Fundamentals of Compressible Flow; in the module 4 Normal Shock Wave.

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LECTURE 1, 2 & 3
➤ Introduction to Shock Waves
➤ Formation of Shock Waves in One Dimensional Medium
➤ Fundamental Equations for Normal Shock
➤ Prandtl Relation for Normal Shock
➤ Normal Shock Analysis and Flow Property Calculations
➤ Crocco's Theorem
➤ Demonstration of Gas Property Tables for Normal Shock Relation
LECTURE 4
➤ Compression Through Shock Wave
➤ Hugoniot Relation
➤ Isentropic Compression vs Shock Wave Compression

We covered in last 3 lectures the following topics. So, the topics of shock waves, its formation in one dimensional medium. Fundamentals equations have been derived and most importantly the relations that governs through Prandtl equation is also framed from these fundamental equations.

Then, we move to the normal shock analysis and flow property calculations across the normal shock, which uses this Prandtl equation. And, having said all this property calculations, we move to a very important theorem called as Crocco's Theorem. This theorem talks about the fluid kinetics aspects with respect to thermodynamic concepts, and that can be correlated for the normal shock.

Now, having said all these things, we tried to demonstrate the gas property tables. And, this gas property tables was derived from the fundamental equations of normal shock analysis that are used for flow property calculations. In fact, this gas property table can be considered as a database or data set to find out the flow property that are prevailing across a normal shock.

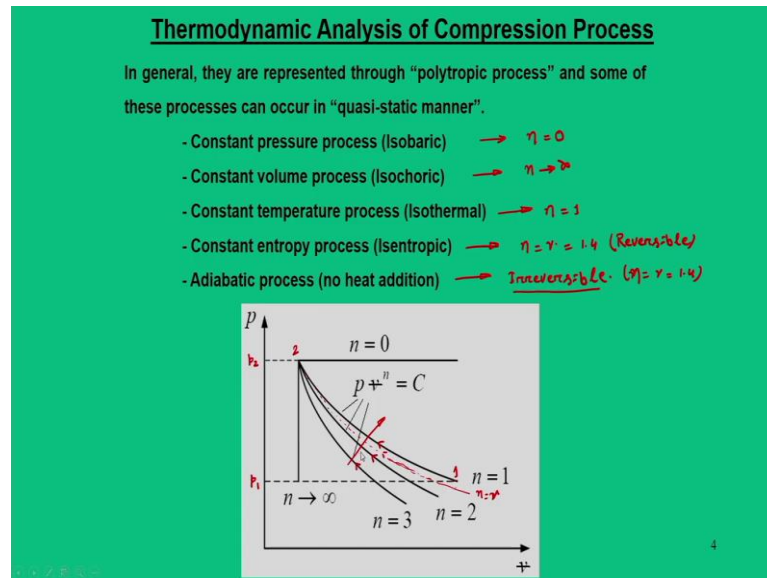
So, these are the very basic topics that were covered in this normal shock. Now, in this lecture we will try to justify certain important facts with respect to the shockwaves as a compression device. So, what we have seen here is that always there is a steep pressure raise across a normal shock. So, the question arises can you think of the shockwave as a mechanism to compress a gas? Is it will be effective?

So, all these things we will try to answer by considering the fundamental thermodynamic aspects that is with respect to isentropic compressions versus shock wave compressions. Now, when you deal with this isentropic compressions, normally there are standard thermodynamic relations that are available, but we do not know the thermodynamic background although we found out the flow property, but we do not know the thermodynamic background.

So, for that a relation which is derived which is called as Hugoniot relations and in fact, these forms the basics of thermodynamic concepts that happens the pressure rise across a shock wave. And, using this relation one can find out that how a shock wave compression mechanism can be understood thermodynamically.

So, before you move to this shock wave analysis of compression. So, let me give some very basic background of thermodynamic analysis of a compression process.

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We all know from the beginning of thermodynamics course, we used to have a very simple concept of using a figure considering a piston cylinder device in which gas is contained. and when the gas is supposed to be compressed, the piston has to move towards the dead end of the cylinder. If, the gas has to expand the piston has to move away from the dead end.

So, this is the way we can make this gas to compress or expand. Now, in fact, what we can say is that the gas which is there in this cylinder as change in the specific volume or density when the pistons moves in and out from the dead end of the cylinder.

So, we view this method to be compression or expansion. And, in fact, many a times we can also make this system as and while motion of the piston is regulated in the cylinder, we can say there is a work transfer in terms of PDV work.

And, other analysis we could perform that one can add heat into this gas and thereby see what happens to the piston movement. So, there are multiple ways one can handle this particular thermodynamic aspects of this piston motion inside the cylinder. So, such a process are generally given a names, those names we can call this as a constant pressure process or isobaric process, constant volume process (isochoric process), constant temperature process (isothermal process), constant entropy process (isentropic process).

And, all these four processes can be thought of occurring in a very steady state manner and in a very small or we can say in a very quasi static manner. So, that the process can be thought of happening in a reversible way, but other process that may not be a reversible, we call this as adiabatic process where there is no heat additions; that means, this cylinder does not encounter any heat interaction.

So, this is how we handle this motion of the piston inside the gas accordingly we assign different thermodynamic processes. Now, when all these thought processes we call them as a polytropic processes. So, when I say polytropic processes and if we try to plot the pressure volume diagram. So, normally we call this as a p-v diagram; pressure and specific volume diagram.

So, this process we say it is occurring in a polytropic manner, where $p\upsilon^n = C$. So, following these equations the pressure volume relation is considered. Now, let us see that what does this mean, now when n is equal to 1? So, when n is equal to 0. So, it is like p is pressure is equal to constant. So, there is no change in the pressure.

So, we cannot recognize this to be a compression process. But, apart from n is equal to 0 for all other values of n, we can say that when there is a decrease in the specific volume the pressure always increases.

So, now the value of n essentially decides, the how a process can occur in a very slowest possible manner? When we are occurring the process in the slowest possible manner, then it will be very quasi static in nature and the conditions of maintaining the reversible nature will be assured.

So, as and when we move n towards up the process occurs in a very slow manner. So, let us say when n is equal to a 1. So, this process is $p\upsilon^n$ is $p\upsilon = C$, and this relation is true for isothermal process and let us say in a constant pressure process, we can say it is n is equal to 0. In fact, it is no longer a compression process. And, in a constant volume process that is the other limit where this compression process happens to be vertical very steep manner.

So, it is a constant volume process. So, here n goes to infinity. But, any other value of n makes this curve to be steep. Now, let us say that when we are undertaking a compression process for a certain change in the pressures. So, if say p_1 initial state and p_2

and if at all I want to do this compression process, then if I start from n is equal to 1 means isothermal process.

So, if the initial state we can start from 1 and final state starts ends at 2. So, I can go in a slowest manner that is n in a isothermal process. Similarly, keeping increasing n further the curve becomes steep. But, what the basic condition is that? The work transfer which is essentially quantified that area under the diagram comes down if the curve becomes steep. Now, in particular when we have a constant entropy process, which is isentropic. So, there it is nothing but the reversible adiabatic process.

So, there n becomes γ that is specific heat ratio. So, somewhere for air this becomes 1.4, somewhere we can have another curve which is just close to n is equal to 1. So, this we can say n is equal to γ . So, normally we can say roughly an isentropic process will be somewhere in this manner. But, when we say another process which is called adiabatic? So, this need not be have to be reversible in nature.

So, although we can say n is equal to γ is equal to 1.4 for the process is irreversible whereas, this is a constant entropy process, this is reversible in nature, which means while going from 1 to 2, I can come along the same path while in return with very minimal loss. So, this is how we view this compression mechanism in a $p-v$ diagram.

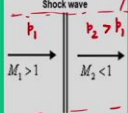
So, with this background let us see that, how I can incorporate a shockwave as a compression device. Does any of the relation suits to me or not?

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Compression Through a Shock Wave

- A supersonic flow encountering a normal shock undergoes steep gradient in its flow properties.
- Since the static pressure always increases, the shock wave can be visualized to be a “thermodynamic device for compression”.
- Based on second law, the entropy must increase across a normal shock for the flow to occur in upstream direction – (Non-isentropic preferably an Adiabatic Process)
- The static pressure rise has a high and steep gradient over a very short thickness (10^{-7}m) of the normal shock.

$$s_2 - s_1 = c_p \ln \left[\left(1 + \frac{2\gamma}{\gamma+1} (M_1^2 - 1) \right) \left(\frac{2 + (\gamma-1)M_1^2}{(\gamma+1)M_1^2} \right) \right] - R \ln \left\{ 1 + \frac{2\gamma}{\gamma+1} (M_1^2 - 1) \right\}$$

$$M_1 > 1 \Rightarrow s_2 - s_1 > 0 \Rightarrow s_2 > s_1$$


To do this analysis let us see that when we undertake a compression in a shock waves. So, we say there is a standing normal shock. So, across a shock wave the Mach number drops, but your static pressure p_2 always rises, but in a very steep manner. Now, one angle we can give to this particular problem and obviously this process cannot be considered as any of the quasi static process, because this jump is very instantaneous or very steep. And, this happens in a very shortest time.

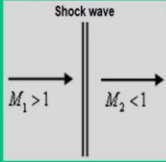
So, none of the quasi static process will follow. Then, even it cannot be also considered as an isentropic, because entropy always increases across the shock wave. But, one more realistic way of looking at this problem, because when you derive this normal shock relations, we can say there is an imaginary duct consisting of stream line, but that duct is adiabatic duct.

Adiabatic duct means there is no heat interaction into this medium. So, to some extent the process will be an adiabatic process. But, although process is adiabatic, whether we have to really dependent on the specific heat ratio gamma or not, that become still a question mark; so, all these analysis we are trying to see that if we can give a thermodynamic meaning to this compression process.

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Hugoniot Relation

- It relates the thermodynamic quantities across a normal shock and holds good for all types of gases (i.e. perfect gas, chemically reacting gas, real gases etc.)
- The changes across a normal shock can be independently assessed purely on thermodynamic aspects without involvement of any reference velocity/Mach number. Such a relation is known as "Hugoniot Equation".
- The "Hugoniot Equation" can be derived from basic governing equations for a one-dimensional flow prevailing in an adiabatic duct.
- It has a great resemblance to "First law of Thermodynamics for an Adiabatic Process".



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So, such a thermodynamic meaning we can keep through an relation what we call as Hugoniot relation. So, what it says is that till this point of time, we always talk about normal shock from its analysis, the relations, flow property calculations. But, the time being let us see only thermodynamic aspects of increase in the pressure and whether we can say that this increase in the pressure can have any thermodynamic link.

So, this is the entire motive of this analysis and so, what we say here is that changes across a normal shock can be independently assessed purely on thermodynamic aspects that is without involvement of any reference velocity or Mach number. Because, in all earlier analysis we used to say for m for different values of Mach number, these are the property ratio $\frac{p_2}{p_1}$, $\frac{T_2}{T_1}$ and so on.

But, here we will say that without involvement any velocity or Mach number how we can give a meaningful thought and that equation is known as Hugoniot Equations. And, these Hugoniot equations can be derived from the basic governing equations for one-dimensional flow prevailing in a adiabatic duct.

So, this equation has a great resemblance to the "First law of Thermodynamics for an Adiabatic Process". That is what the process or mechanism of shockwave; since, we followed to a some extent we say that it is a adiabatic duct because there is no heat transfer involved.

So, with these logics we can say the process is in adiabatic nature and when Hugoniot equation is formed we can say that it has a definite link to the first law of thermodynamics.

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Hugoniot Relation

Governing equations for Normal Shock

Continuity: $\rho_1 u_1 = \rho_2 u_2$

Momentum: $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$

Energy: $e_1 + \frac{p_1}{\rho_1} + \frac{u_1^2}{2} = e_2 + \frac{p_2}{\rho_2} + \frac{u_2^2}{2}$

Continuity: $u_2 = u_1 \left(\frac{\rho_1}{\rho_2} \right)$ or $u_1 = u_2 \left(\frac{\rho_2}{\rho_1} \right)$

Momentum: $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 \left(u_1 \frac{\rho_1}{\rho_2} \right)^2$

$p_1 u_1^2 - p_2 \frac{\rho_1^2 u_1^2}{\rho_2^2} = p_2 - p_1$

$\Rightarrow p_1 u_1^2 \left(1 - \frac{\rho_1}{\rho_2} \right) = p_2 - p_1$

$\Rightarrow u_1^2 = \left(\frac{p_2 - p_1}{p_2 - p_1} \right) \left(\frac{\rho_2}{\rho_1} \right)$

Shock wave

<p>①</p> <p>p_1, T_1</p> <p>$M_1 > 1$</p> <p>$e_1 = c_v T_1$</p>	<p>②</p> <p>$p_2 > p_1$</p> <p>$T_2 > T_1$</p> <p>$M_2 < 1$</p> <p>$e_2 = c_v T_2$</p>
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$u_2^2 = \left(\frac{p_2 - p_1}{p_2 - p_1} \right) \left(\frac{\rho_1}{\rho_2} \right)$

Substitute u_1^2 & u_2^2 in Energy eqn.

So, to do that to do this analysis; so, let us revisit the governing equations of normal shock, which is continuity, momentum and energy equations. So, continuity equations we say it is $\rho_1 u_1 = \rho_2 u_2$ momentum equations is $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$. And, energy

equation we are writing in this form that is $e_1 + \frac{p_1}{\rho_1} + \frac{u_1^2}{2} = e_2 + \frac{p_2}{\rho_2} + \frac{u_2^2}{2}$.

Why you bring this internal energy concept here? Because in the beginning of analysis of normal shock, we define that entire flow kinetic energy due to high speed nature immediately, because what we told that after this normal shock, the flow becomes suddenly subsonic from it is supersonic value. So, whatever kinetic energy of the flow it has in the upstream, all of them gets converted to internal energy thereby increasing it is temperatures.

So, what we say is for this condition 1 and 2; p_1 and T_1 and here $p_2 > p_1$ and $T_2 > T_1$; so, for internal energy e_1 and e_2 . So, normally this $e_1 = c_v T_1$ and $e_2 = c_v T_2$. So, there is a rise in the internal energy. So, that is what these equations are reframed in this manner.

So, now let us relook into these equations, but in a different context. So, from continuity equation one can rewrite $u_2 = u_1 \left(\frac{\rho_1}{\rho_2} \right)$ or $u_1 = u_2 \left(\frac{\rho_2}{\rho_1} \right)$. So, now we take only this expressions and substitute in momentum equation. So, these we get from continuity and we substitute that thing in momentum equation.

So, when I substitute this in momentum equations, that is u_2 in the right hand side. So, we get $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 \left(u_1 \frac{\rho_1}{\rho_2} \right)^2$. So, we simplify these equations and solve for u_1^2 . So, how do you do? So, you can bring this particular term to the other side like $\rho_1 u_1^2 - \rho_2 \frac{u_1^2 \rho_1^2}{\rho_2^2} = p_2 - p_1$.

So, $\rho_1 u_1^2 \left(1 - \frac{\rho_1}{\rho_2} \right) = p_2 - p_1$. So, ultimately we can write this equation as $u_1^2 = \left(\frac{p_2 - p_1}{\rho_2 - \rho_1} \right) \left(\frac{\rho_2}{\rho_1} \right)$. So, this is what we get from momentum equation.

Similarly, when you put this u_1 in the momentum equations left hand side, then we can also get u_2 , $u_2^2 = \left(\frac{p_2 - p_1}{\rho_2 - \rho_1} \right) \left(\frac{\rho_1}{\rho_2} \right)$. So, now we have 2 expression u_1^2 and u_2^2 . So, now, you substitute u_1^2 and u_2^2 in energy equation.

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Hugoniot Relation

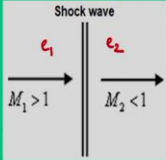
- The "Hugoniot equation" relates the thermodynamic quantities across a normal shock that holds good for all types of gases (i.e. perfect gas, chemically reacting gas, real gases etc.)
- It has a great resemblance to "First law of Thermodynamics for an Adiabatic Process" i.e. change in internal energy is equal to change in specific volume multiplied by mean pressure across the shock wave.

Change in internal energy $e_2 - e_1 = \left(\frac{p_2 + p_1}{2} \right) (\vartheta_1 - \vartheta_2) = \left(\frac{p_2 + p_1}{2} \right) \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right)$

Step change in I.E. $\Delta e = -p_{avg} \Delta \vartheta$

Differential form $de = \delta q - p d\vartheta$

Shock wave



So, when you do that we arrive at these particular expressions. And, we do and simplify then we arrive at this particular expressions.

$$e_2 - e_1 = \left(\frac{p_2 + p_1}{2} \right) (\vartheta_1 - \vartheta_2) = \left(\frac{p_2 + p_1}{2} \right) \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right)$$

What it says is that the left hand side of this expression it is $e_2 - e_1$, which we say is that change in internal energy. So, change in the internal energy across the shock wave. That is equal to the $\left(\frac{p_2 + p_1}{2} \right) (\vartheta_1 - \vartheta_2)$. Now, expressing this specific volume into density one can write $\frac{1}{\rho_1} - \frac{1}{\rho_2}$. So, this is how we say we can write this particular expression?

Now, if you give a little bit of thought assuming that in normal sequence, when you calculate the average pressure normally you add up all the pressures and divided by the total numbers of terms we are going to add. So, here there are two pressures. So, we represent this as $-p_{avg} \Delta \vartheta$, $-\Delta \vartheta$ because the density increases, the specific volume drops that is why minus. So, across this the shock wave we say there is a step change in internal energy.

Now, this is what we see across the shock wave. Now, let us say see the First law of Thermodynamics. So, in a first law of thermodynamics in a differential form we see we

write $de = dq - pd\psi$. Now, in a situation when the process is an adiabatic process this term we can neglect. So, that we can see $de = -pd\psi$.

Now, if you look at these two equations. So, this is for a differential form of internal energy. And, we see that there is a great resemblance between these two terms. Of course, the only difference that we have here is that here the change is a very finite, but this change we represent in a differential form.

So, that is what it is written here that it has a great resemblance for first law of thermodynamics for an adiabatic process that is the change in internal energy is equal to change in this specific volume multiplied by mean pressure across the shock wave.

In fact, this relation is true for all the thermodynamic processes and that holds good across a normal shock. Since we did all these things from these basic fundamental equation, it is true for all types of gases, real gases, chemical reacting gas, perfect gas as well.

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Hugoniot Relation

Hugoniot Equation

$$e_2 - e_1 = \left(\frac{p_2 + p_1}{2} \right) (\psi_1 - \psi_2) = \left(\frac{p_2 + p_1}{2} \right) \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right)$$

$e = c_v T$; $c_v = \frac{R}{\gamma - 1}$ and $T = \frac{p\psi}{R}$ (Perfect gas)

$$\frac{p_2}{p_1} = \frac{\left(\frac{\gamma + 1}{\gamma - 1} \right) \frac{\psi_1}{\psi_2} - 1}{\left(\frac{\gamma + 1}{\gamma - 1} \right) - \frac{\psi_1}{\psi_2}}$$

Pressure-Volume relation for a Normal Shock

Shock wave

<p>①</p> <p>p_1</p> <p>$M_1 > 1$</p> <p>$\psi_1 = \frac{1}{\rho_1}$</p>	<p>→</p>	<p>②</p> <p>p_2</p> <p>$M_2 < 1$</p> <p>$\psi_2 = \frac{1}{\rho_2}$</p>
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So, this is again another way of interpreting this Hugoniot Equations. So, basically we framed this equations from our analysis that is change in internal energy and it is represented through this Hugoniot equations.

Now, in this equation we will try to see that these relations when you do it for perfect gas. When you recall that for a perfect gas and calorically perfect gas we can write

$$e = c_v T, \quad c_v = \frac{R}{\gamma - 1} \quad \text{and} \quad T = \frac{p \vartheta}{R}.$$

Now, when you put all these relations, the Hugoniot equation turns out to be a pressure volume relations between two thermodynamic states.

$$\frac{p_2}{p_1} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right) \frac{\vartheta_1}{\vartheta_2} - 1}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\vartheta_1}{\vartheta_2}}$$

So, here are two thermodynamic states are 1 and 2, like the pressure ratio across this we are now representing specific volume, like here if I say p_1 , pressure is p_2 , corresponding specific volume is ϑ_1 and in fact that is nothing but $\frac{1}{\rho_1}$. And, its corresponding specific

volume after this normal shock would be $\frac{1}{\rho_2}$.

So, what it says is that, the pressure ratio are now expressed in terms of specific volumes for a calorically perfect gas. So, that is what I can say that Hugoniot equation represented as pressure volume relation for a normal shock. More specific it will be pressure specific volume relation across a normal shock.

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Hugoniot Relation

Inferences

- At equilibrium condition, a property value at any thermodynamic state can be expressed as a function of other state variables.
- For a given upstream conditions of pressure, specific volume and velocity, Hugoniot equation gives the relation for downstream parameters of pressure and specific volume. The plot of this relation is known as "Hugoniot curve".
- This curve is the locus of all possible pressure and specific volume conditions behind a normal shock of various strengths for one specific upstream values.

$$e_2 - e_1 = \left(\frac{p_2 + p_1}{2} \right) (\vartheta_1 - \vartheta_2)$$

$$e = e(p, \vartheta) \Rightarrow p_2 = f(p_1, \vartheta_1, \vartheta_2, u_1)$$

$$u_1^2 = \left(\frac{p_2 - p_1}{\frac{1}{\vartheta_2} - \frac{1}{\vartheta_1}} \right) \left(\frac{\vartheta_1}{\vartheta_2} \right) \Rightarrow p_2 = p_1 - (\vartheta_2 - \vartheta_1) \left(\frac{u_1}{\vartheta_1} \right)^2$$

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So, now let us see how this Hugoniot relations has a great significance for our understanding? So, for that we will now talk about its inferences. What does it signifies to us? So, if we recall this particular equation in a very generic way of thermodynamic analysis one property can be represented to any other independent parameters.

For example, I can say internal energy as a function of pressure and specific volume. So, this is as per the thumb rule of any thermodynamic property can be represented in other two independent parameters. And, in this case your change in the internal energy is related to pressure and specific volume, through this Hugoniot relation.

So, in other words I can say that from using this equation I can express this $p_2 = f(p_1, \upsilon_1, \upsilon_2, u_1)$. And, the how this functional relations forms; one of the functional relation is this u_1^2 expressed into as a function of p_1 and p_2 . So, this functional relation can be exactly defined by this Hugoniot equations.

So, from this one can find out what is the pressure $p_2 = f(p_1, \upsilon_1, \upsilon_2, u_1)$. So, what it physically means to us like that suppose you have the given conditions or known condition are p_1 , υ_1 and u_1 . This is normally known, that is upstream condition and downstream condition, we can say that we can calculate p_2 , but what condition you require? If you know one of the particular conditions υ_2 , then in fact, we will just for the time being we will not talk about the Mach number.

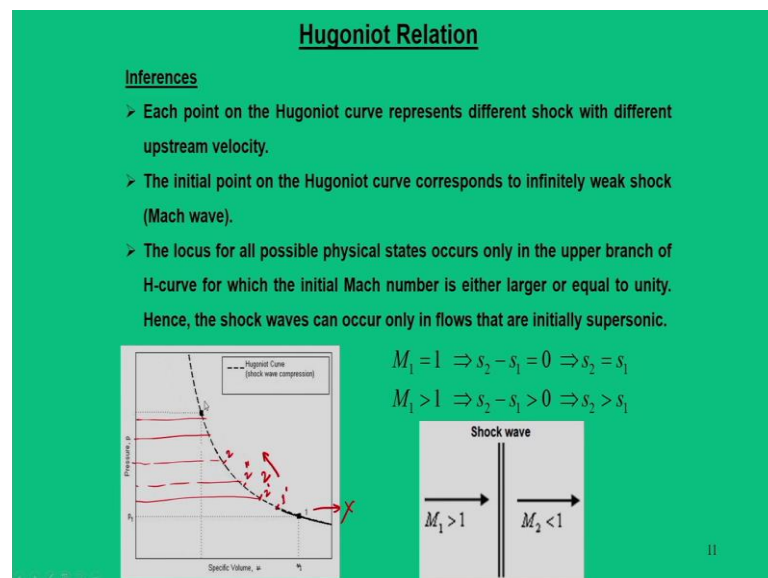
So, instead of Mach number if you know only the specific volume in the downstream condition, then one can correlate p_2 with the knowledge of $p_1, \upsilon_1, \upsilon_2, u_1$. So, this is also a similar context; in earlier situation we were talking in terms of Mach number, here we are not talking in terms of Mach number rather we talk about with another known parameter and in this case is this specific volume.

So, this gives a curve which is known as Hugoniot curve. So, what it means is that at equilibrium conditions the property value of thermodynamic state can be expressed as a function of other state variables. And, in this case for a given upstream conditions for a normal shock that of pressure, specific volume, velocity. The Hugoniot equation that is this gives the relation for downstream parameters of pressure and specific volume. So, downstream parameter of pressure and specific volume can be found out.

So, one can generate a plot known as Hugoniot curve and this curve is nothing but the locus of all possible pressure and specific conditions across a normal shock. So, means if you know these known conditions one can generate a lot of data by just imposing some numbers to this. And, all of them will decide about the possible flow conditions across a normal shock. And, in fact, the depending on the strength of the normal shocks these numbers will vary.

So; that means, this locus of all possible pressure and specific conditions of normal shock various strength can be found out for one specific upstream values.

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So, to start this, one can say that one can draw an Hugoniot curve of like this in a pressure specific volume diagram. So, for different values of p_1, v_1 one can find out all possible numbers. Now, when I say so, next question arises, where is this initial point?

So, if you look at the initial points, the initial point should be corresponds to 0 entropy, as if the flow was isentropic. So, s_2 was is equal to s_1 ; that means, initially if you start from this particular point; so, initial point p_1 and p_2 as if compression process is just initiated.

Now, if I want to think about bring the non isentropic nature through a shock wave, then I can say the point 2 will proceed along this curve in this directions. Now, when I move

So, all these point is nothing but all possible conditions of a normal shocks. In fact, we can start a point somewhere here let us say $1'$. So, correspondingly this point can be a $2'$ or this point can be $2''$. So, likewise all possible conditions one can generate. So, point 2 will move towards left. Why left because the pressure should increase and corresponding y axis will talk about what pressure we are supposed to get.

So, this direction of movement is not possible because it will lead to an expansion not a compression. So, it will be impossible situation and shock wave will not occur. So, that means, in shockwaves flow can occur that are initially supersonic. So, all these points will lie in the supersonic region.

Hugoniot Relation

Inferences

- The LHS of the equation represents the slope of the “Hugoniot curve” passing through the points ‘1 & 2’ while the RHS of the equation represents the mass flux (mass flow per unit area) upstream of the flow.
- For one particular shock situation (point ‘1’), all the upstream conditions are known. With known mass flux (i.e. given slope) a straight line can be drawn at point ‘1’ that will intersect “Hugoniot curve” at the some point ‘2’ which is the down stream condition of the shock wave.

$$\frac{P_2 - P_1}{v_2 - v_1} = - \left(\frac{u_1}{v_1} \right)^2$$

$$\left(\frac{u_1}{v_1} \right)$$

Mass flux / unit area.

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So, let us revisit this equations of Hugoniot relation that is $\frac{p_2 - p_1}{\vartheta_2 - \vartheta_1} = - \left(\frac{u_1}{\vartheta_1} \right)^2$. So, if you

look at this normal shockwave, what does this normally mean u_1 and ρ_1 is the nothing but the velocity and density that is nothing, but $\frac{1}{\vartheta_1}$.

So, it is a mass flux. I can say ρu is equal to constant that is $\rho_1 u_1 = C$. So, I can say this as mass flux per unit area. Because, we can say it is a per unit area. So, which means that or in other words I can write this is nothing but $\frac{u_1}{\vartheta_1}$.

So, this is also represented in this as mass flux per unit area. So, now this gives a logical meaning that the right hand side of this equation is the nothing but the mass flux per unit area which is normally known for a normal shock. And, left hand side of this equation is nothing but slope along this curve.

So, means that if you know the mass flux per unit area, then you also know the slope. So, one can calculate the slope and this negative slope will take you from point 1 to 2. So, in other words what it means, for a given condition 1, which is located here if I know this particular number $\left(\frac{u_1}{\vartheta_1}\right)^2$ and by taking this negative slope, if I draw a line then it will cut this point at point 2.

So, this particular slope is nothing but $\left(\frac{u_1}{\vartheta_1}\right)^2$. So, I have to put minus because we are moving towards the negative specific volume directions. And, if for another situation of $\frac{u_1}{\vartheta_1}$, if I can draw another line from this point which may take me to another point of 2` for different values of u_1 and ϑ_1 . So, likewise any number of states that is possible for given state one.

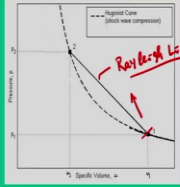
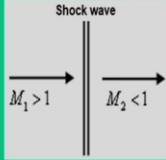
So, for a given upstream conditions different states of 2, 2`, 2`` is possible. So that means, the straight line can be drawn that will intersect the Hugoniot at some point 2.

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Hugoniot Relation

Inferences

- The straight line joining the points '1 & 2' is known as "Rayleigh line". It is always drawn upward that leads to decrease in specific volume after the normal shock.
- Downward slope of Rayleigh line (i.e. increase in specific volume after the normal shock) will create an impossible situation violating "second law of thermodynamics".
- In this way, all the thermodynamic states across a shock wave can be located graphically on "Hugoniot curve".

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So, now when I join this line 1 and 2 in this Hugoniot curve. So, this line is known as Rayleigh line. Because, it is drawn for a particular mass flux. And, in fact, I told that the downward slope of the Rayleigh line is not possible, because we cannot go below this line.

So; that means, if I start from the point 1 I should proceed in this directions, upward directions; downward slope is not possible. In this way we can say all thermodynamic states across a shockwave can be located graphically on a Hugoniot curve.

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Isentropic Compression vs Shock Wave Compression

- The static pressure always rises across a normal shock while specific volume decreases across a normal shock. So, it can be considered as a compression device.
- All the thermodynamic states across a shock wave can be represented graphically on "Hugoniot curve".
- The effectiveness for shock wave as compression device can be judged by comparing it with isentropic compression.

$$p_2 = f(p_1, v_1, v_2) \Rightarrow p_2 = p_1 - (v_2 - v_1) \left(\frac{u_1}{v_1} \right)^2 \rightarrow \text{H. eqn.}$$

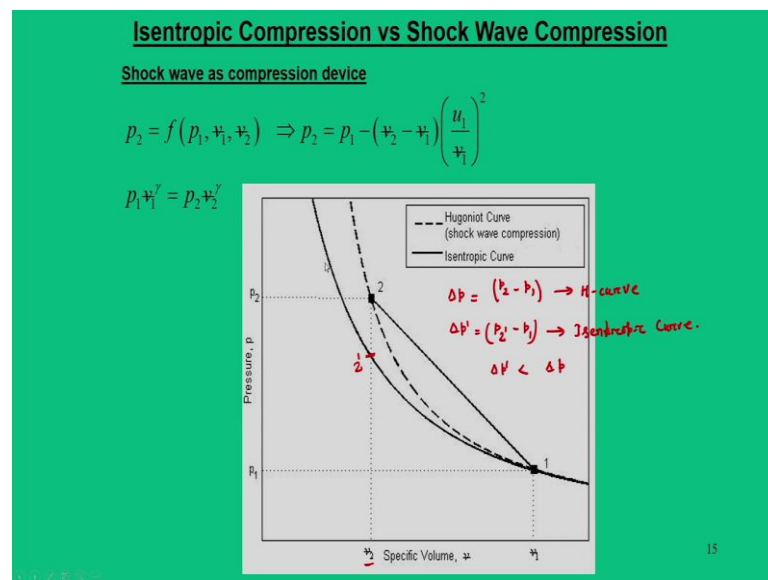
$$p_1 v_1^\gamma = p_2 v_2^\gamma \quad (\text{Reversible adiabatic})$$

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So, having giving a name as Hugoniot curve or simply H curve, we are now able to see that whether I can compare this particular shock based compression to an isentropic compression. So, for an isentropic compression I can say this process is reversible adiabatic. So, we can say it is we can say $p\upsilon^\gamma = C$ or $p_1\upsilon_1^\gamma = p_2\upsilon_2^\gamma$.

So, I say it is a reversible adiabatic process. And, this is nothing but we say H Hugoniot equation. So, this is what we say, an isentropic process is governed quantitatively and the Hugoniot equation governs the shock wave compression quantitatively. So, when I compare both these two, these two equations, I can judge the effectiveness of shock wave compression, with respect to isentropic compression.

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So, to do that one interesting thing what we can say that we can bring a concept as shock wave as a compression device. So, what I did is in the same Hugoniot curve, I superimpose this isentropic curve; that means, when I am starting from same point 1 and I try to go on a Hugoniot curve that is dotted line, and from same point I also can start my compression journey in the isentropic curve.

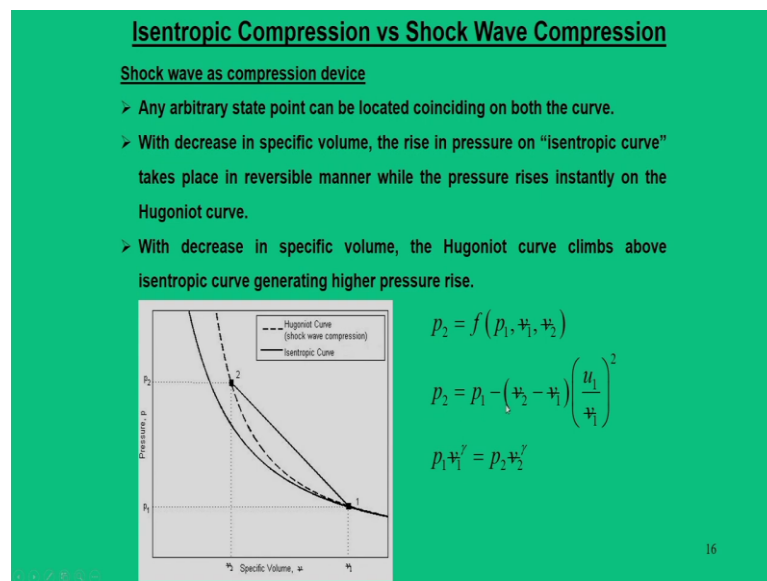
So, let us see how I can do that? So, what you see is that then the initial phase, you will see that both the dotted line and solid line they are more or less same. Now, once you proceed further I mean once the specific volume decreases at a faster rate, then we see that the gap is becomes higher and higher. So, for instance, if at all my point 2 lies here; I would have reached a pressure p_2 , if I go on a Hugoniot curve.

So, compression that means, Δp on a Hugoniot curve would have been $p_2 - p_1$; that means, pressure rise on Hugoniot curve will be that is on the H curve would have been $p_2 - p_1$. But, for same specific volume, if I would rather choose an isentropic curve, then I would have landed up at point 2'.

So, my $\Delta p'$ would have been $p_2' - p_1$. So, this is how we say it is an isentropic curve. Obviously, we say that $\Delta p' < \Delta p$, which means that isentropic curve gives a less compression for a same specific volume change than the Hugoniot curve.

In fact, this gap will become more and more if you climb on a Hugoniot curve, or in other words at higher value of specific volume rise, the Hugoniot curve climbs over the isentropic curve. So, this is the key point of the Hugoniot curve analysis that says that a shockwave can be a thought of a compression device.

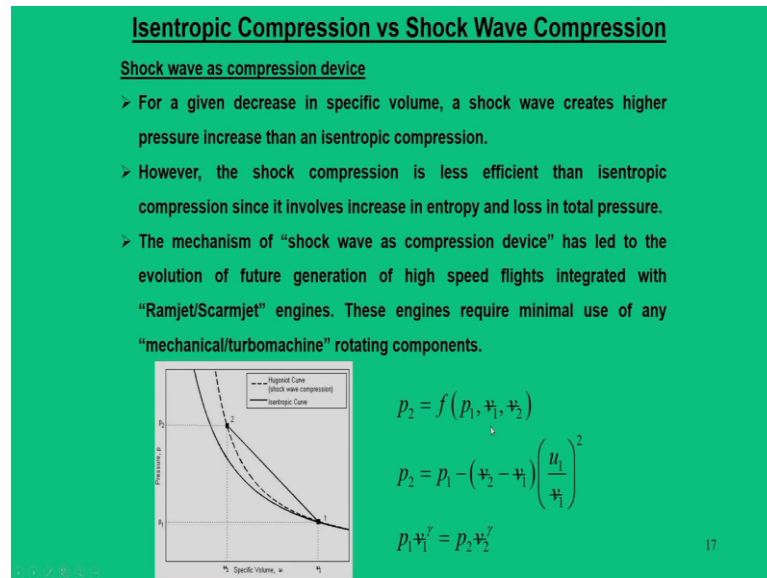
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So, this is how I have explained so far that how a shockwave as a compression device can be interpreted? I mentioned that with decrease in the specific volume rise in pressure in isentropic curve takes place in a reversible manner where pressure rises instantly on a Hugoniot curve. With decrease in the specific volume the Hugoniot curve climbs above the isentropic curve; that means, it generate a high pressure.

The same thing I have explained here. And, this is how the isentropic curve is governed $p\vartheta^\gamma = C$ whereas, the pressure rise across a shockwave is governed through Hugoniot equation.

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So, one can make summary of this analysis that why a shock wave compression is a effective method. So, one of the key point is that for a given decrease in this specific volume, that is $v_2 - v_1$, the shock wave creates higher pressure rise increase than the isentropic compression.

So, this is one of the catch point, but there are some side effect is that the shock wave compression is less efficient than isentropic compression, because we have seen that it is a non isentropic process. So, it involves increase in the entropy on loss of the total pressures, which we already view that across a shock wave total pressure drops.

So, this is also a negative side of the shockwave compression, but many a times the quantum of jump that we get through shockwave compression is such a significant that it gives a new thought as a mechanism of compression device.

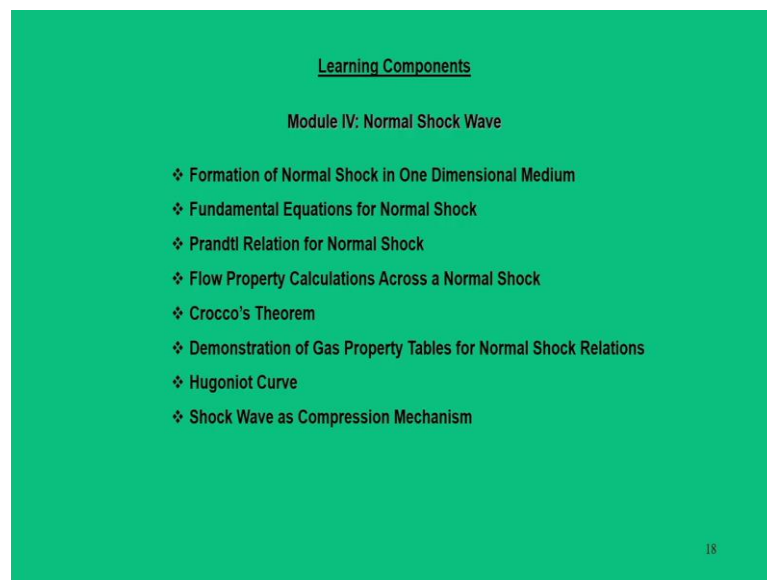
In fact, this particular concept has created the evolution of next generation of high speed flights, which we normally call them as ramjet or scramjet engines. And, these engines in fact, have very minimal use of rotating or mechanical components.

So, entire compression process in this engine is mostly governed through shock based compressions. And, these shock based compression does not require any mechanical or turbo machine or rotating components.

Many a times what happens that, when you travel faster, the structural limit of mechanical components, the thermodynamic limit of mechanical component does not allow to move faster. But, here it is not that aspect rather the engines can be thought of to be controlled through shock based compression process.

So, this is how the shock based compression has a great resemblance or significance for new generation evolution of flights.

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So, with this I conclude this module that is on normal shockwaves. So, in this module the some of these learning components are listed here. So, at the end of the model one should understands the phenomenal effect of normal shock, its fundamental equations. The relations that, governs the normal shock, how the property flow properties calculated across normal shock?

Apart from this we bring out two important concepts in this normal shock analysis that is Crocco's Theorem that talks about fluid kinematics that means fluid flow across a normal shock is highly rotational. And, also this Hugoniot curve that talks about that shock wave as a effective compression mechanism.

Now, apart from that we demonstrated a gas property table for normal shock relations. So, at the end of this module one should understand all these learning components and brush up his knowledge whether all these components are learnt properly or not. So, with this I will conclude for this lecture.

Thank you for your attention.