Fundamentals of Compressible Flow
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Module - 04
Lecture - 12
Normal Shock Waves

Welcome you again to this course Fundamentals of Compressible Flow. We are in the Module 4 - Normal Shock Waves.

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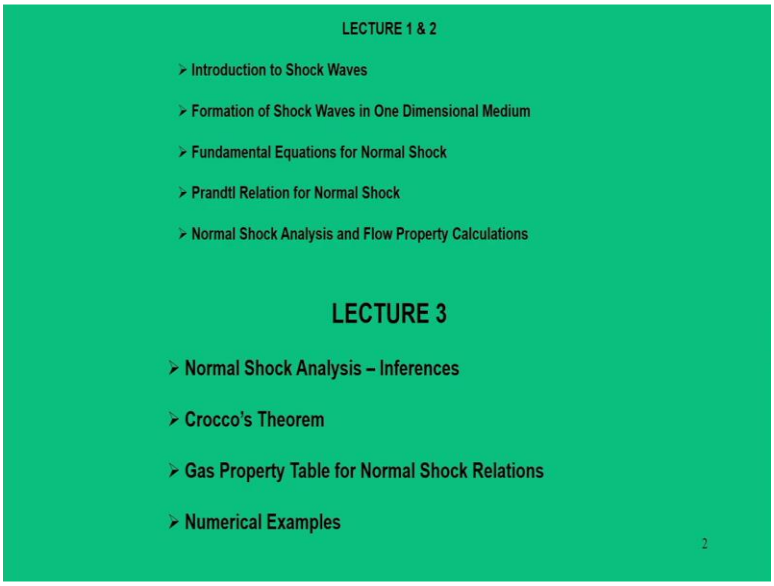

So, previously we have gone through two lectures in this module, where we discussed about the formation of shock waves, what is its importance, we discussed about the fundamental equations, and we derived one important relations known as Prandtl relations for the normal shock.

And based on these relations one can find out the flow property across a normal shock. So, we will move further from this contents and today in this lecture we are going to discussed some important inferences that we come across from this normal shock analysis.

Now, apart from that we are trying to introduce a theorem which is known as Crocco's theorem which is a very vital and important theorem that relates the fluid kinematics aspects with respect to thermodynamic relations that normally is encountered across a normal shock. Of course, this Crocco's theorem has nothing to do with normal shock phenomena. Crocco's theorem is independent in nature that correlates the fluid kinetic concepts to thermodynamic concepts. And in our study that is in the normal shock analysis, we will try to see how this theorem is useful to us to describe certain flow fields across a normal shock.

Now, having said this we will now move to introduce a gas property table for normal shock relations. So, previous lectures you derived the mathematical formulations of different parameters for a normal shock, but many a times these parameters are difficult to evaluate, and for which we have to refer to the property tables.

So, I will introduce this gas property table, how you are going to see the table to find out different values. Then we will try to see that how that gas table can be used to solve certain numerical examples. So, this is the broad outline for this lecture for today.

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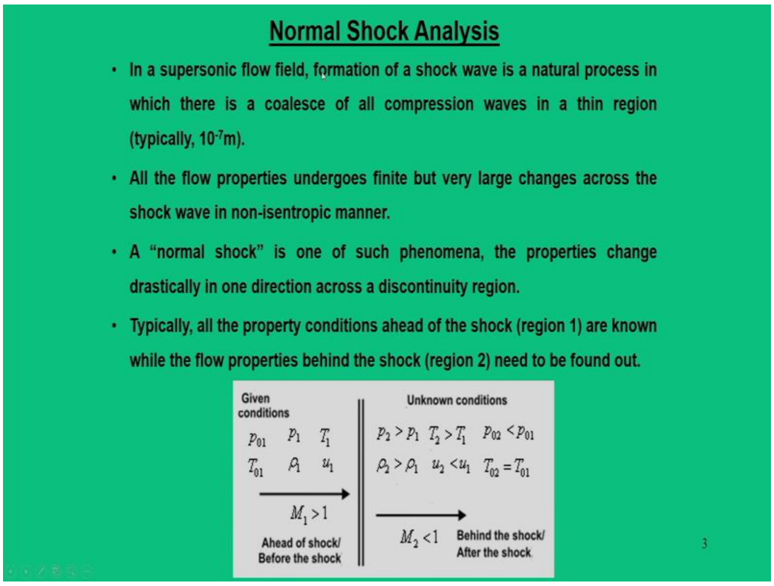

Just to give the brief insight what we have learnt in the previous class, I can say that a normal shock is a very natural phenomena for a supersonic flow field in which all the compression waves merge in a very thin region, and this thin region has a typical dimension of 10^{-7} m.

And across this thin region all the flow properties before the shock and after the shock needs to be correlated. So when you see this magnitude of this property, they jump in a very steep manner or they jump drastically in one direction and across this discontinuity region.

So, the given problem that we have is that we have a standing normal shock. So, the conditions that are upstream conditions are known to us such as pressure, temperature, density, velocity, Mach number, all stagnation properties they are known. And what are the unknown condition is what happens after the shockwave. So, in our previous study, we tried to derive the correlations between these properties value upstream and downstream across this shock wave.

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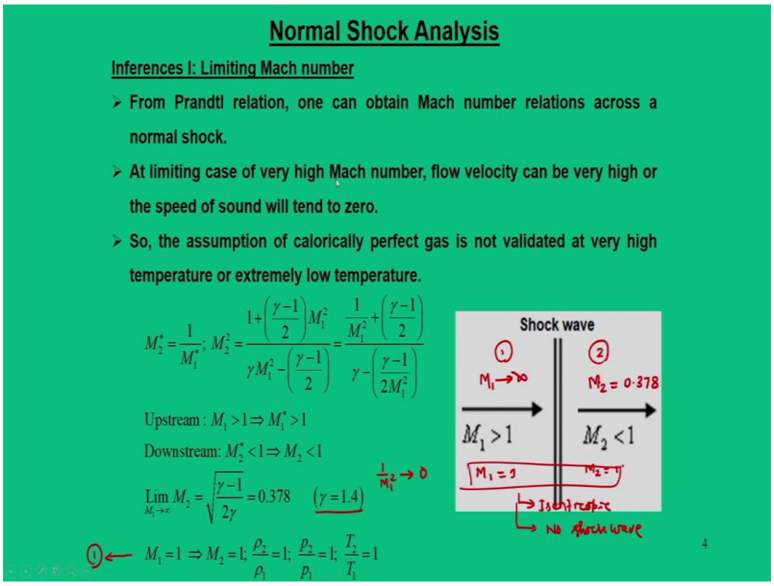

So, after doing so, we will try to find out; of course we have derived so many parameters across the normal shocks. So, we will try to find out some important consequences. So, with which I call as inferences, inferences number 1. So, first inferences that we can talk about is limiting Mach number.

So, as you say see from this Prandtl relations which is the star conditions of M_1 and M_2 across a normal shock, they are inversely related. And from these relations, we can actually find out the Mach number relations across the normal shock that is M_2 and M_1.

So, this is the given by these equations. Now, having said this we can say that how M_2 will vary with respect to M_1. So, this says that if your M_1 is supersonic, M_2 has to be subsonic. So, it means that has to be subsonic, so that means, this is the first important relations what we get but why M_1 has to be supersonic that we will come back later when you do the entropy analysis.

But the for the time being let us say that M_1 is supersonic, now when M_1 is supersonic the minimum value of M_1 should be 1. So, when it is M_1 is 1, M_2 is 1 which means there is no shockwave. It is just a simple Mach wave or just a beginning of formation of shock wave. So, effectively in the flow is there is no shock wave, entire flow condition is isentropic.

So, this is one case that is what I can say the beginning condition of shock wave. So, we say that this is the first limiting conditions. Now, second limiting conditions that how long I can increase M_1. So, if I go on increasing the M_1, we will find there is a upper limit of M_2. So, M_2 cannot be more than certain value.

So, this we can see from these equations. The relation between M_2 and M_1 can be also written in this form that is $M_2^2 = \dfrac{\dfrac{1}{M_1^2} + \left(\dfrac{\gamma-1}{2}\right)}{\gamma - \left(\dfrac{\gamma-1}{2M_1^2}\right)}$.

Now, in this equation when I put the limiting condition, when M_1 goes to infinity, means this region, the upstream goes to infinity that means we are putting a limit that we keep on increasing the Mach number on the upstream side. So, what happens in the downstream if this particular term when at this condition we can say $\dfrac{1}{M_1^2}$ goes to 0.

So, when I say $\dfrac{1}{M_1^2}$ goes to 0 in the expression of this M_2, we can say these two particular term becomes 0. So, this leaves out the limiting case of M_2 will be equal to $\sqrt{\dfrac{\gamma-1}{2\gamma}}$. Now, when we put gamma is equal to 1.4 for air, so this value turns out to be 0.378. So, at this condition we say your M_2 will be fixed by 0.378.

So, in other words, the M_2 value cannot come below this number provided you are using air that is first consequence, and that is the upper limit. Lower limit M_1 can be 1, so M_2 will be 1. So, this particular situation will tell let you know that the flow is isentropic, and there is no shock wave. So, this is the first limiting case, this is the limiting case for Mach number.

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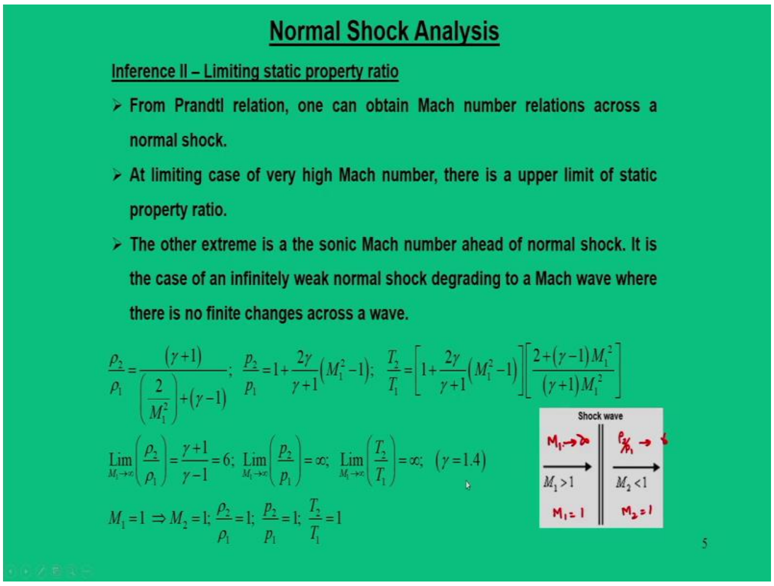

Then moving further, let us see what happens to static property ratio. By static property ratio I mean static pressure ratio $\frac{p_2}{p_1}$ static temperature ratio $\frac{T_2}{T_1}$, and static density ratio $\frac{\rho_2}{\rho_1}$. So, previously we have derived these relations across the normal shock, and all these relations are nothing but the functions of M_1. So, looking at this equation when what one can say that when M_1 goes to infinity, the $\frac{p_2}{p_1}$ and $\frac{T_2}{T_1}$ also goes to very high value. So, there is no limiting case because this equation does not restrict these numbers.

But whereas, interesting phenomena will happen for the density; so when M_1 goes to infinity, unfortunately this term $\frac{1}{M_1^2}$ goes to 0. So, in $\frac{1}{M_1^2}$ goes to 0, the density ratio $\frac{\rho_2}{\rho_1}$ turns out to be $\frac{\gamma+1}{\gamma-1}$. And for gamma is equal to 1.4, this ratio has to be 6.

So, here also I can say that M_1 is equal to 1, M_2 will be 1, but at the same time your $\frac{\rho_2}{\rho_1}$ goes to 6 when M_1 goes to infinity. So, irrespective of what value of pressure temperature, the density ratio cannot be more than 6 for air that is irrespective of Mach number.

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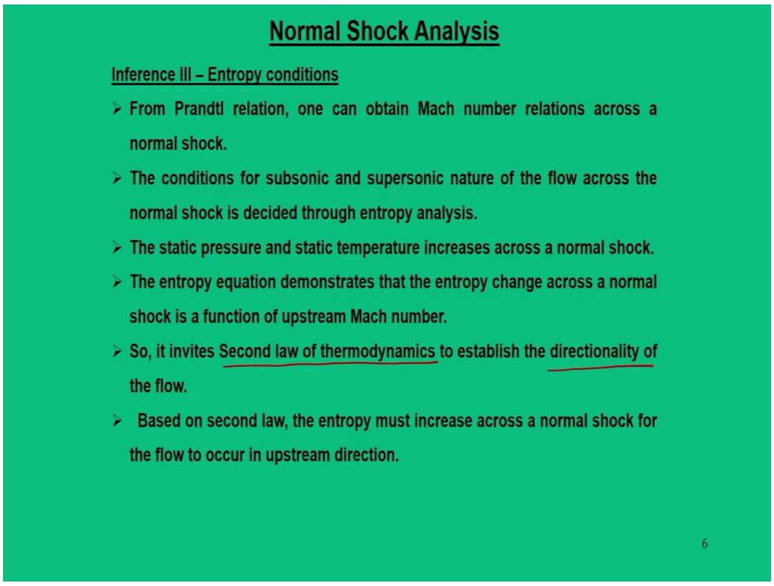

And next we will move to the third inferences that is for the entropy conditions. So, in the beginning, I told that we say that M_1 is supersonic, now we will and try to find out this answer why this M_1 should be supersonic. So, these particular things can be answered from this entropy analysis.

So, what we learnt so far that from Prandtl relation one can obtain Mach number relations across a normal shocks. So, the conditions for subsonic and supersonic nature of the flow across the normal shock is decided through entropy analysis. The static pressure static temperature increases across the normal shock.

So, the entropy equation will demonstrate that the entropy change across a normal shock is a function of Mach number. In fact, we derived this from our previous analysis. Then we will try to invoke that what the second law of thermodynamics tells us. So, the second law of thermodynamic tells us the important consequence that is directionality of a flow. So, this flow will proceed in a direction in which the thermodynamic property that is entropy should increase.

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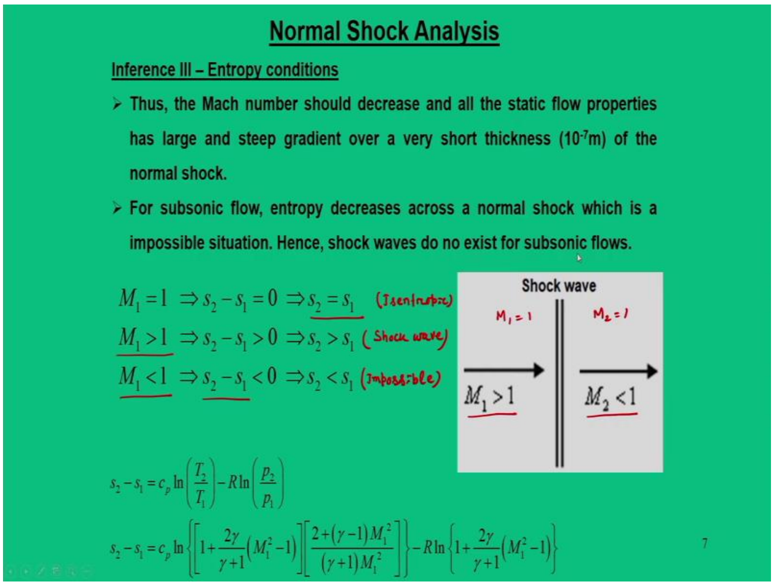

So, now to show that why M_1 cannot be subsonic, so first thing we have to see this particular important equation, this is the very thermodynamic fundamental equations how you want to calculate the entropy change between two states 1 and 2. Now, that is

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right).$$

Now, this is the broad analysis. Now, we try to use these equations for a shock wave. So, we know this temperature ratio; we know the pressure ratios. So, when you put those expressions which turns out to be this. Now, one thing important to be noted here that when M_1 is equal to 1, so $s_2 = s_1$. This is what we say that when M_1 is equal to 1, M_2 is equal to 1, so this is $s_2 = s_1$, then this is the entire isentropic as if there is no shock wave.

And in this equation when you put M_1 is greater than 1, we will find the $s_2 > s_1$. So, this is the conditions for shock wave that we have been analyzing so far. So, that for which we say that M_1 will be always greater than 1 for which M_2 should be less than 1. So, these two relations holds good.

But if you see the third relations, what we find out that if M_1 is less than 1, your $(s_2 - s_1) < 0$. So, this does not follow the second law that entropy decreases in the direction of the flow. So, this is a impossible situation as far as the second law of thermodynamics is concerned. So, always we consider the relation which is written in the middle that is M_1 must be greater than 1. And also we can say that for subsonic flow, entropy decreases across a normal shock and hence shockwaves do not exist in the subsonic flow.

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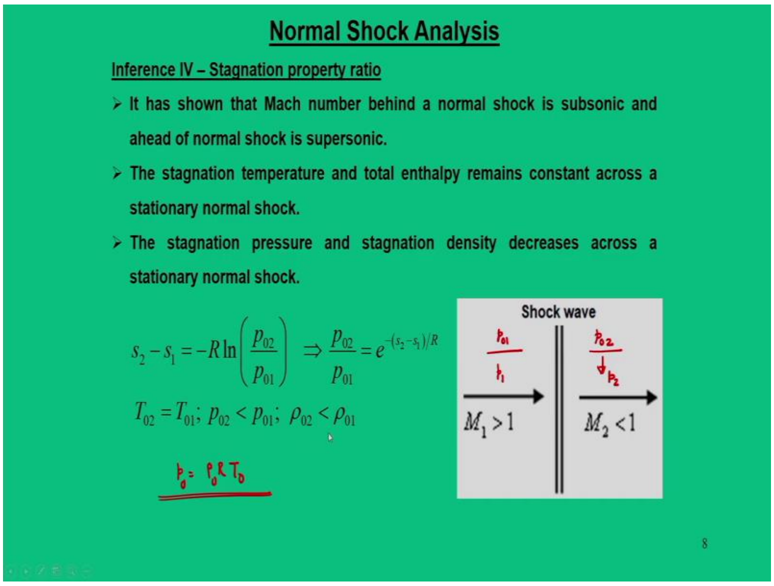

And again moving further from another inferences 4. So, here we will talk about what happens to the stagnation property ratio. So, one of the important relations from entropy change across a normal shock can be calculated as $s_2 - s_1 = -R\ln\left(\frac{p_{02}}{p_{01}}\right)$. So, p_{02} is nothing but the stagnation pressure after the shock, p_{01} is the stagnation pressure before the shock.

So, we can say that this p_{02} is related is calculated from p_2 and M_2 and p_{01} is calculated from p_1 and M_1. So, we can say that the entropy change is given by these relations. So, this says that $p_{02} < p_{01}$ that means stagnation pressure always drops. Why it always drops? Because in earlier analysis we have said that energy equation does not change. So, total temperature or stagnation temperature remains constant.

So, having said this, and if you say the equation of state $p = \rho RT$, and for this case if you say $p_0 = \rho RT_0$. So, if this equation has to be satisfied, then these two conditions must valid. So, as this is equal; stagnation pressure should drop. So, stagnation density also should drop.

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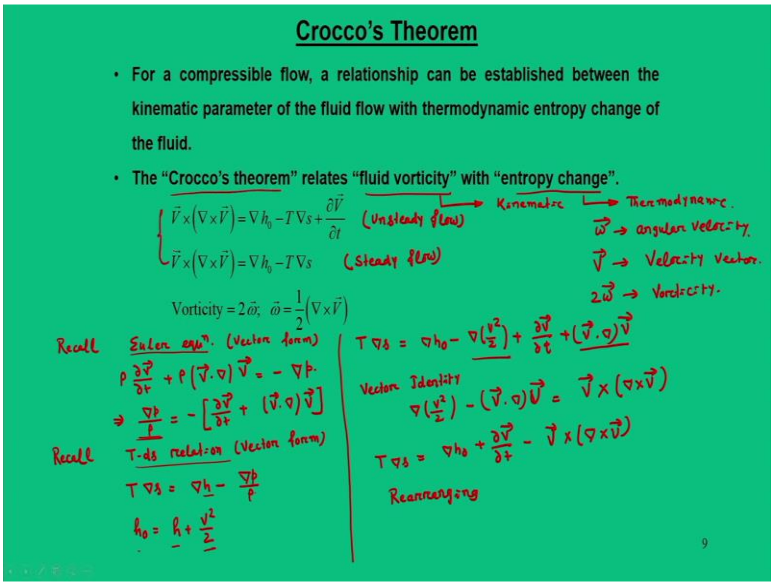

So, this is all about what we have studied from this analysis of normal shock and its inferences. Now, we will move to a theorem which is known as Crocco's theorem. Just to start this. So, let us forget that we are dealing with a normal shock analysis, but what we are dealing with just a fundamental theorem and that is applicable for all situations that involves fluid kinematics and thermodynamics. And in particular we are trying to apply this Crocco's theorem for our compressible flow theory.

So, what does this theorem tells; it tells about the relation between a fluid vorticity with respect to entropy change. So, when I say fluid vorticity, its a kinematic parameter and when I say entropy its a thermodynamic parameter. Now, what it tells is that, so when I say fluid kinematic parameters the common parameter that I should know is about this angular velocity $\vec{\omega}$, then velocity vector and another parameter what you will say vorticity vector. So, vorticity vector is nothing but twice omega vector.

So, now we know that from fluid kinematics analysis, we can say that $\vec{\omega}=\frac{1}{2}\left(\nabla\times\vec{V}\right)$. So, this is how these fluid kinematic parameters is defined. So, what we are trying to see here is that Crocco's theorem which is given by this particular expression, $\vec{V}\times\left(\nabla\times\vec{V}\right)=\nabla h_0-T\nabla s+\frac{\partial\vec{V}}{\partial t}$. So, this is a vector relation.

And to derive this equations, so let us start with the equations that we can recall this equation called as Euler equation, but in vector form. So, when I write this Euler equation in vector form, what I can write $\rho\frac{\partial\vec{V}}{\partial t}+\rho\left(\vec{V}.\nabla\right)\vec{V}=-\nabla p$.

So, what I can write is, we can find out a term $\frac{\nabla p}{\rho}=-\left[\frac{\partial\vec{V}}{\partial t}+\left(\vec{V}.\nabla\right)\vec{V}\right]$. Then also we have to recall a Tds relation that is the in vector form. So, one thing I need to emphasize this Euler equation is a fluid parameter term, Tds relation is for the thermodynamic term.

So, what I can write $T\nabla s=\nabla h-\frac{\nabla p}{\rho}$. So, here we also know that $\frac{\nabla p}{\rho}$ from this Euler equations. Also we know $h_0=h+\frac{V^2}{2}$ that is static enthalpy plus $\frac{V^2}{2}$ is stagnation enthalpy. So, from this we can find out what is ∇h.

So, once you do that, so you have to do some mathematical jugglery. So, we put this $\frac{\nabla p}{\rho}$ in this, Tds relations. So, what I can write $T\nabla s=\nabla h_0-\nabla\left(\frac{V^2}{2}\right)+\frac{\partial\vec{V}}{\partial t}+\left(\vec{V}.\nabla\right)\vec{V}$. So, here you have to know certain mathematical relation what we call as vector identity that relates between these two parameters.

So, what do we write is $\nabla\left(\frac{V^2}{2}\right)-\left(\vec{V}.\nabla\right)\vec{V}=\vec{V}\times\left(\nabla\times\vec{V}\right)$. So, this is a mathematically derived term. Now, when I put this vector identity equation for these two terms in this equation, then we write this $T\nabla s=\nabla h_0+\frac{\partial\vec{V}}{\partial t}-\vec{V}\times\left(\nabla\times\vec{V}\right)$.

Now, after rearranging, one can obtain this particular relation written in the first equation. So, if you look at this equation, it contains all these terms that relates one side of this equation contains the kinematic parameter that is velocity vector; other side of the equation contains the entropy term, temperature, and enthalpy term. Also there is another parameter which is a unsteady parameter $\frac{\partial\vec{V}}{\partial t}$. So, this equation I can say it is for unsteady flow. And when for a steady flow this term this term will vanish. So, this becomes steady flow. So, this is how the Crocco's theorem equation are defined by these two expression.

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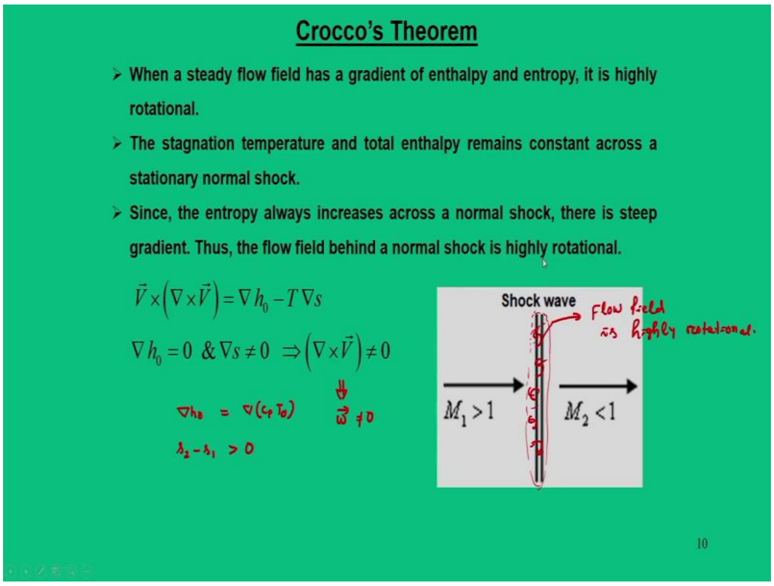

Now, let us see how this Crocco's theorem is useful to us. So, now, let us say if we are talking about a shock wave, and we are trying to correlate the fluid parameter with respect to thermodynamic parameters. So, here we are talking about the h_0 that is ∇h_0, and that is nothing but $\nabla\left(c_p T_0\right)$. So, that does not change because total temperature across a shock wave is remains constant. So, ∇h_0 is 0.

Now, what happens to ∇s? $s_2 - s_1$ always greater than 0, so that means, entropy gradient of entropy cannot be equal to 0. So, this says that your $\left(\nabla\times\vec{V}\right)$ cannot be equal to 0. When $\left(\nabla\times\vec{V}\right)$ cannot be equal to 0, this turns out to be $\vec{\omega}$ cannot be equal to 0 which means that when I say thin region of the shock wave, the flow field in this thin region is highly rotational, which means if you visualize the flow field we will find this fluid elements try to have rotational vector in this thin region. So, across a normal shock, the flow is highly rotational. This is one of the important inference that a shockwave analysis gives to us.

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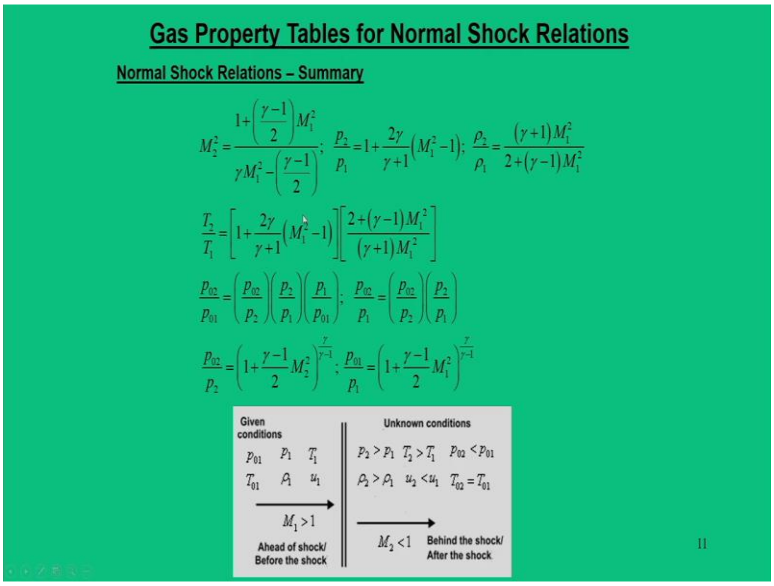

Now, whatever we have learnt so far we if we want to summarize normal shock relations, we said about the Prandtl relations that relates between the shock Mach number before the shock and after the shock. So, very basic bottom line is that if we have a standing normal shock the conditions that are known to us is the upstream conditions, all the flow parameters are known; the conditions that are not known to us as the all the downstream parameters.

But what we know simply is the relation between Mach numbers M_1 and M_2 through these Prandtl relations. Now, using these relations, one can find out all the static pressure ratio, static density ratio, static temperature ratio. And if you see here, all this number will increase. So, in fact, we also can have stagnation pressure ratio that drops; total temperature is equal. So, in this way we can calculate.

So, one way of looking at this equation if you see, all these ratios are the functions of Mach number and which is in the upstream regions, which is known to us. And every time if we know the Mach number, every time it is almost a time consuming task to use these equations to know the flow parameters.

So, one convenient way of looking at this approach would be create a database in which we will vary M_1, and try to find out how the ratios will be. So, in this way a table can be prepared for a normal shock and we call this as a gas property table for all the normal shock relations. So, by that table, we will avoid the time consuming task of using calculating the flow parameter using these equations.

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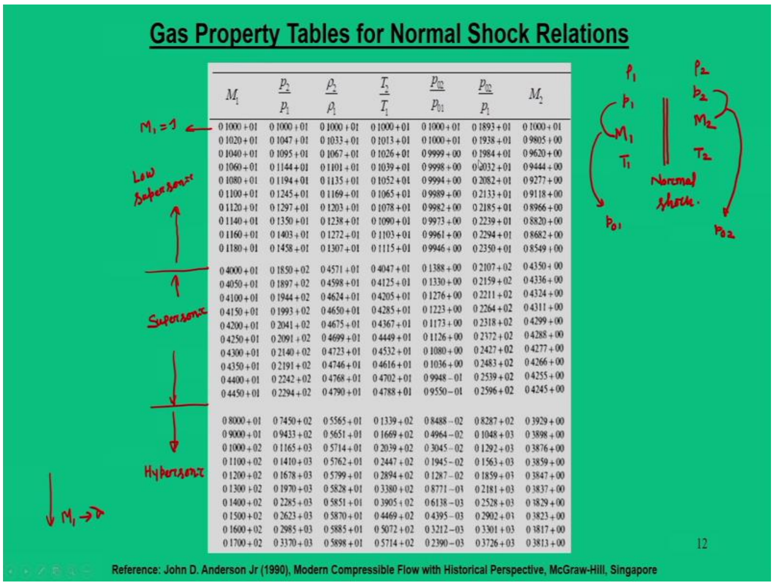

Now, let us see how this table is the framed. So, for this table, I have taken this extract from this reference book John D Anderson Modern Compressible Flow that is McGraw Hill Publications. So, this is one source of extract what I have just taken. What it says is that this has about 7 column of data. The first column belongs to the Mach number which is upstream.

So, for the normal shock, the first column that talks about is M_1 and the last column is M_2. So, across a normal shock, I know what is my value of M_1, and correspondingly the last column will talk about what will happen to the M_2 conditions. Similarly, pressure ratio, if I want to calculate if it is p_1 and if it is p_2. So, second column will give the pressure ratio across the normal shock.

If your density is ρ_1 and ρ_2, the third column will talk about the density ratio which is the function of Mach number. Fourth column will talk about the temperature ratio, T_1 and T_2. Fifth column will talk about the stagnation pressure ratio. So, stagnation pressure ratio, that means, for corresponding p_1 and M_1, we can define a stagnation pressure p_{01} and for M_2 and p_2, we can define the stagnation pressure p_{02}. So, that ratio is given by this $\frac{p_{02}}{p_{01}}$.

And also there is another relations once you have this p_{01}, and we can also correlate with p_1; this is another important relations we call this as Rayleigh-Pitot equations. So, many a times this ratio is also very important which we can directly find out from the gas table. So, what I can we can see that there are three regime is just I have written. First regime is you can say the first value that stands as M_1 is 1 that is what we say and this M_1 can goes to infinity.

So, here it has been plotted about 17, Mach number of 17. And till now this is the realistic number what we can say. So, the first part of this is can say it is a low supersonic, middle one is in the range of maybe supersonic, the bottom one is very high supersonic and we call this as hypersonic flow.

So, there are any number of intermediate data that can also be generated. So, this is really just to give you a glimpses that how a gas property table looks like. Now, knowing on this, we can just find out all the parameters.

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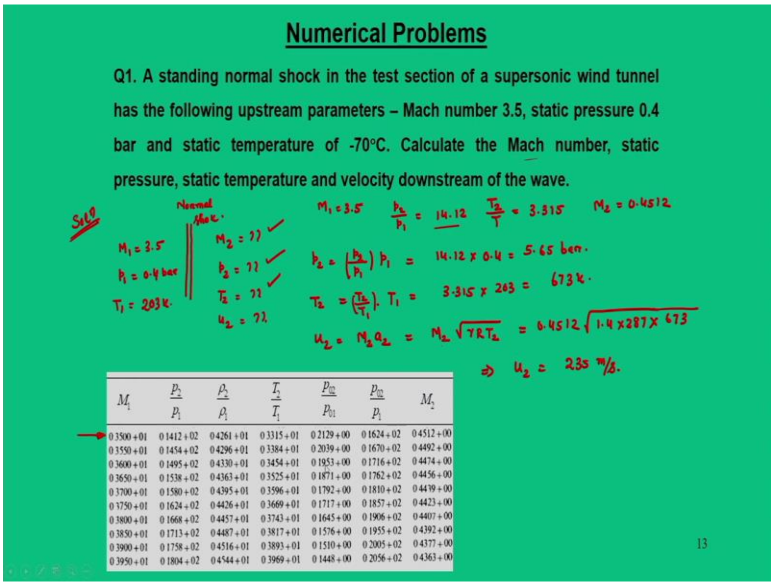

So, with this, let me see demonstrate that how one is supposed to refer this gas property tables. So, for that let us solve some numerical problems. The problem that is given to us that we have a standing normal shock in a in the test section of a supersonic wind tunnel, it has the parameter that is upstream for which the Mach number is 3.5, static pressure ratio is 0.4 bar and static temperature is -70°C. So, we have to calculate the Mach number, static pressure, static temperature, velocity downstream of the wave.

So, if you want to solve this problem, so first thing we have to draw a very schematic sketch that briefs about the problems. So, if you have a normal shock, what we know is upstream which is M_1 that is 3.5, static pressure p_1 0.4 bar, static temperature is T_1 that is -70°C or we can say 203K.

What you do not know is M_2, required to calculate what is p_2, and T_2 and u_2. So, one way to look at the problem is that you take all the standard relations, use your calculator and calculate all these parameters $\frac{M_2}{M_1}$, $\frac{p_2}{p_1}$, because M_2 will be a function of M_1, $\frac{p_2}{p_1}$ will be a function of M_1, all these numbers are known to us.

But what I am trying to say is that how to use the gas dynamics table. This is one such extracts from these gas dynamic table from the book, where I can get the information about Mach number of 3.5. So, if you look at this the first row on these things says that Mach number of 3.5, it talks about all the parameters. So, I can write this when M_1 is equal to 3.5, then I can say $\frac{p_2}{p_1}$ is 14.12. $\frac{T_2}{T_1}$ we say 3.315. We also require Mach number. So, we can say M_2 will be 0.4512.

So, we require p_2. So, p_2 can be written as $\left(\frac{p_2}{p_1}\right)p_1$. So, we know p_1, we know $\frac{p_2}{p_1}$. So, p_2 can be calculated as 14.12 x 0.4. So, it is about 5.65 bar. Similarly, T_2 will be $\left(\frac{T_2}{T_1}\right)T_1$. So, this will be if you calculate 3.315 x 203 = 673 Kelvin.

So, what is left out? We know M_2, we got p_2, we got T_2, we want u_2. So, $u_2 = M_2 a_2$. So, $u_2 = M_2\sqrt{\gamma R T_2}$. So, $u_2 = 0.4512\sqrt{1.4\times 287\times 673}$ $u_2 = 235 m/s$. So, this is how we look at this problem. So, we get all the parameters by using this table data.

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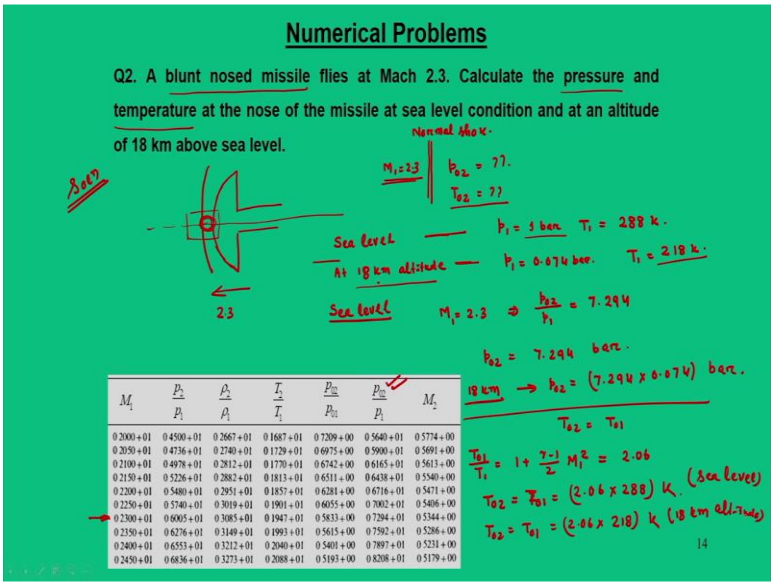

M_1	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{02}}{p_{01}}$	$\frac{p_{02}}{p_1}$	M_2
0.2000+01	0.4500+01	0.2667+01	0.1687+01	0.7209+00	0.5640+01	0.5774+00
0.2050+01	0.4736+01	0.2740+01	0.1729+01	0.6975+00	0.5900+01	0.5691+00
0.2100+01	0.4978+01	0.2812+01	0.1770+01	0.6742+00	0.6165+01	0.5613+00
0.2150+01	0.5226+01	0.2882+01	0.1813+01	0.6511+00	0.6438+01	0.5540+00
0.2200+01	0.5480+01	0.2951+01	0.1857+01	0.6281+00	0.6716+01	0.5471+00
0.2250+01	0.5740+01	0.3019+01	0.1901+01	0.6055+00	0.7002+01	0.5406+00
0.2300+01	0.6005+01	0.3085+01	0.1947+01	0.5833+00	0.7294+01	0.5344+00
0.2350+01	0.6276+01	0.3149+01	0.1993+01	0.5615+00	0.7592+01	0.5286+00
0.2400+01	0.6553+01	0.3212+01	0.2040+01	0.5401+00	0.7897+01	0.5231+00
0.2450+01	0.6836+01	0.3273+01	0.2088+01	0.5193+00	0.8208+01	0.5179+00

The second problem we will try to see in another way of looking at the problem. A blunt nosed missiles flies at Mach 2.3, calculate the pressure and temperature at the nose of the missile. So, in one of analysis is a blunt nosed missile. So, a blunt nosed missile is something is represented in this manner, nose is a blunt. So, for a blunt nosed missile, when it is flying at 2.3 Mach which will have a shock wave sitting onto this body at the nose.

So, what we want to see is that are the nose of this, we can zoom this version we can assume it to be a normal shock. So, when it is a normal shock, so upstream condition we can say that as if a gas is moving or moving from upstream to downstream for which your M_1 will be 2.3. And if there are two situations it is asked; when for this M_1 is equal to 2.3, what we require is pressure and temperature at the nose.

So, at the nose of this, what we see here the flow happens to be almost remain stagnant because if these flow comes at the this point, flow is almost stagnant, so that part will say that on a blunt nose body at the nose of the missile will encounter a normal shock, and whatever pressure and temperature that will be nothing but your stagnation pressure. So, for that what we want to find out what is p_{02}, what is T_{02}.

So, here the confusion may arise what pressure we should find, static pressure or stagnation pressure? But here your answer should be stagnation pressure because a blunt nose missile always encounters stagnation pressure at the nose, because the flow tries to comes down to almost rest at that point. And second assumption what we are doing at the nose portion, the shock wave will be close to a normal shock.

Now, this is the things we require. But what the condition that is given there are two situations. So, we have to find out one at sea level conditions, other is at 18 kilometre altitude. So, means that if a missile is flying at sea level what will be the p_{02} and T_{02}? If the missile is flying at 18 kilometre altitude, what will be this value?

So, for that will require we will require a sea level pressure; sea level pressure is normally represented in a static value. So, I can say p_1 to be 1 bar and T_1 to be maybe 15°C or 288K. Similarly, from the data table, if you look at the property data altitude versus pressure temperature, then we can find out that at an 18 kilometre altitude, the pressure will be very low, 0.074 bar and temperature is about 218K.

So, there is a drop in static temperatures with altitude. So, to do that, at sea level situation if you want to calculate, then what we can find out, the first relation that is we require we know p_1, we want p_{02}. So, basically we have to refer this particular column of this table. So, for this particular column and Mach 2.3, I have to refer this particular row.

So, I can get this number to be we can say for M_1 is equal to 2.3 will implies $\frac{p_{02}}{p_1}$ would be 7.294. So, from these things, one can evaluate what is at sea level we can say p_{02} would be 7.294 bar. And at 18 kilometer altitude, we can say p_{02} will be 7.294 x 0.074, so this much bar. So, this is how we get pressure.

Now, to find temperature, what you require because what we can say temperature across a normal shock do not change. So, I can say $T_{02} = T_{01}$. But $T_{01,}$ I can find out that $\frac{T_{01}}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$, this we get from isentropic relations.

So, for M_1 is equal to 2.3, this ratio turns out to be 2.06. So, we can say T_{02} at sea level would be or T_{01} would be 2.06 x 288 K that is at sea level. And at 18 kilometre altitude, we can say T_{02} will be T_{01} is equal to 2.06 x 218 Kelvin. So, this analysis tells us that when you go with altitude, the total the pressure drops.

So, what you can imagine that that at sea level if your pressure is close to 7.3 bar, but it drastically reduced to a very low pressure at that altitude. Since your drag will be less and that is the reason one can fly at very high speed high Mach number in higher altitude. In fact, temperature will also be less. This is all about the problem number 2.

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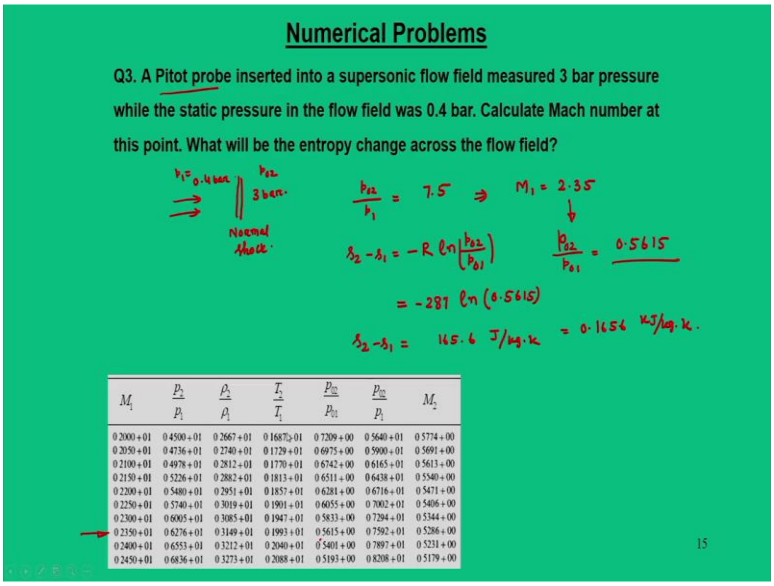

M_1	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{02}}{p_{01}}$	$\frac{p_{02}}{p_1}$	M_2
0.2000+01	0.4500+01	0.2667+01	0.1687+01	0.7209+00	0.5640+01	0.5774+00
0.2050+01	0.4736+01	0.2740+01	0.1729+01	0.6975+00	0.5900+01	0.5691+00
0.2100+01	0.4978+01	0.2812+01	0.1770+01	0.6742+00	0.6165+01	0.5613+00
0.2150+01	0.5226+01	0.2882+01	0.1813+01	0.6511+00	0.6438+01	0.5540+00
0.2200+01	0.5480+01	0.2951+01	0.1857+01	0.6281+00	0.6716+01	0.5471+00
0.2250+01	0.5740+01	0.3019+01	0.1901+01	0.6055+00	0.7002+01	0.5406+00
0.2300+01	0.6005+01	0.3085+01	0.1947+01	0.5833+00	0.7294+01	0.5344+00
0.2350+01	0.6276+01	0.3149+01	0.1993+01	0.5615+00	0.7592+01	0.5286+00
0.2400+01	0.6553+01	0.3212+01	0.2040+01	0.5401+00	0.7897+01	0.5231+00
0.2450+01	0.6836+01	0.3273+01	0.2088+01	0.5193+00	0.8208+01	0.5179+00

And problem number 3, this is again a similar problem which he says that Pitot tube is inserted in a supersonic flow and is measures 3 bar pressure while static pressure is 0.4 bar. We require Mach number and entropy change. Here the entire idea is mentioned that Pitot pressure, the Pitot probe measures stagnation pressure.

So, across a Pitot pressure, there will be a close to a very normal shock. So, across the normal shock the pressure is measured to be 3 bar that is p_{02}, and the static pressure in the flow field is 0.4 bar.

So, one should not confuse that this pressure is a static pressure that is 0.4 bar and when I use the word Pitot probe, then I must use this as a stagnation pressure now which stagnation pressure. So, this has to be stagnation pressure after this normal shock. So, when I say after the normal shock, then it must be p_{02}. So, when I say this, when I infer the data, then I can say I can easily calculate the ratio $\frac{p_{02}}{p_1}$.

This ratio is about 7.5. So, to do that I have to refer this data, because I do not know the Mach number, but Mach number you want to find out. So, from this table, we have to find out in this column of $\frac{p_{02}}{p_1}$ which data is close to 7.5. So, take the extract from that then this number turns out to be M_1 is equal to point 2.35. So, this will tell M_1 is 2.35.

So, when M_1 is 2.35, what you require entropy change. So, entropy change will be $s_2 - s_1$, one can use simple relation $s_2 - s_1 = -R\ln\left(\frac{p_{02}}{p_{01}}\right)$. And for this Mach number of 2.35, this ratio is 0.5615. $s_2 - s_1 = -287\ln(0.5615)$. $s_2 - s_1 = 165.6\,\text{J/kgK}$

So, this is how we can find out the Mach number if Mach number is not given. This is the essence. This is the most advantage part of the using data table, even though the Mach number is not given, but still we can given the data of property data, we can also calculate the Mach number. This is another way of visualizing the advantage of graphical or a data table for property calculations.

So, in this way I have just given some sample example how to use the data table for a normal shock applications and in fact one need not have to remember the big expressions of flow equations, but one has to understand that such equation exists. And in fact, all these data tables are derived from those relations. So, with this, I will conclude my lectures for today.

Thank you very much for your attention.