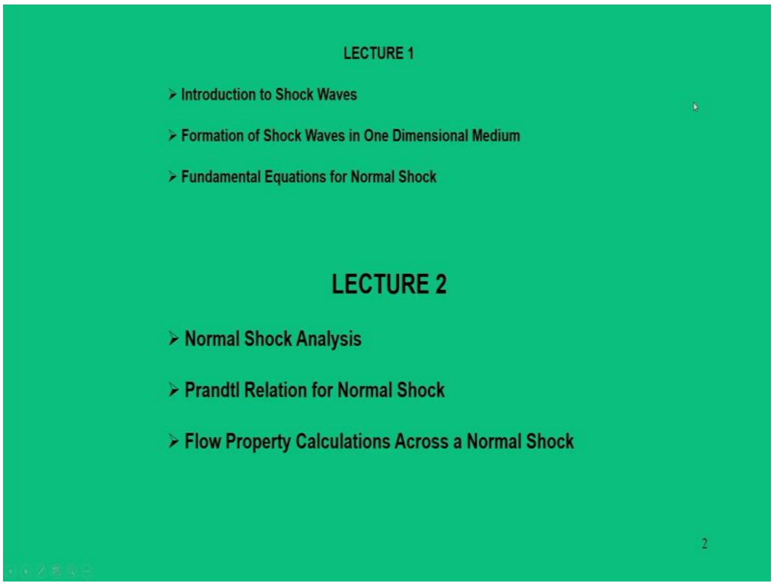
Fundamentals of Compressible Flow
Prof. Niranjan Sahoo
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Module - 04
Lecture - 11
Normal Shock Waves- II

Welcome you again for this course that is Fundamentals of Compressible Flow. We are in the 2nd lecture of the module 4 that is Normal Shock Waves.

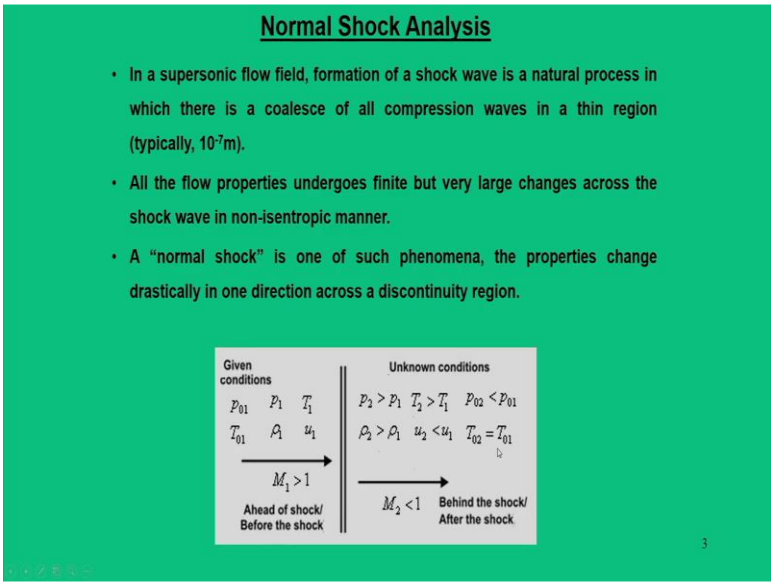
(Refer Slide Time: 00:39)



So, if you look at the first lecture at this module; what we have discussed is about the concept of shockwaves. Then in a one dimensional medium, we explained the formation of the shock waves and how it propagates in the flow and also we gave the correlation between a standing shock wave and a moving shock waves. Then, after that we framed the fundamental equations of normal shock that can be applied in a flow field.

Now, moving further in this lecture, we will again discuss more about this normal shock analysis and that starts from this fundamental equations. And, we try to derive a relations known as Prandtl relations for a normal shock. In fact, this relations forms the fundamental basics for all subsequent flow property calculations across the normal shock; so all these things will be covered in this lecture.

(Refer Slide Time: 02:12)

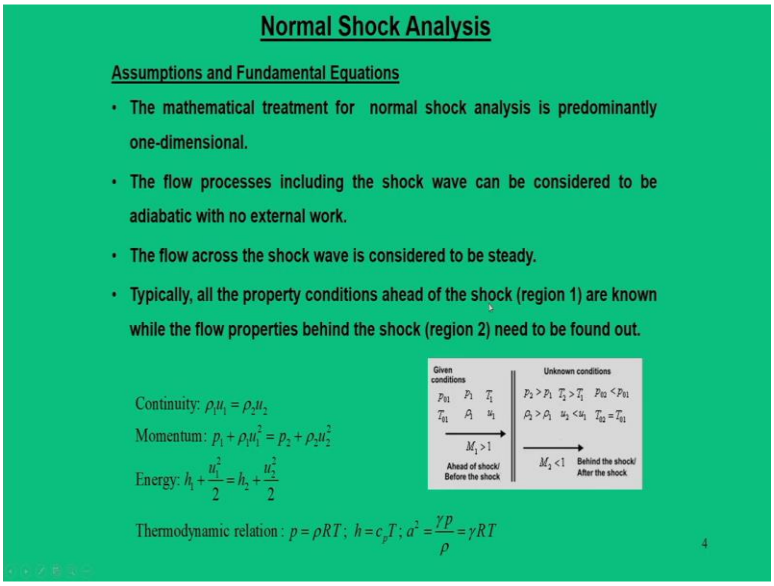


Now, just to brief about what we have framed the concept of normal shock. What we can say is that; normal shock is a inevitable phenomena in a supersonic flow field and, it occurs naturally by the merger of all compression waves in a very thin region. And, this thin region is typically measured to be a size of 10^{-7}m.

And, whatever property change that come across this thin region or the shock wave that happens to be non isentropic; which means, entire flow field involving a normal shock can be considered to be isentropic except this thin region.

So, in other words what it means to us; in our normal shock calculations, whenever we want to apply the property relations across this normal shock, we cannot assume the process to be isentropic. So, subsequently we say that this normal shock is a phenomena in which property change drastically in one directions across a discontinuity region.

(Refer Slide Time: 03:46)



Now, while dealing with the mathematical analysis, we have made certain assumptions. So, the first assumption was treated to the fact that the normal shock we are that we are considering each one dimensional in nature; that means, in this flow field we can say that streamlines are perpendicular to the shockwave. Now, the flow process including the shockwave can be considered to be adiabatic with no external work.

So, obviously, there is no external work and only there is a flow work that is involved when it passes the shock wave and while talking about the adiabatic, what we can say that when you deal with this normal shock; we can say the region in which this normal shock wave is considered can be treated to be a duct consisting of large number of streamlines and these streamline are parallel to each other. And, while dealing with the streamlines you have to deal with the streamlines one before the shock also and also for the after the shock.

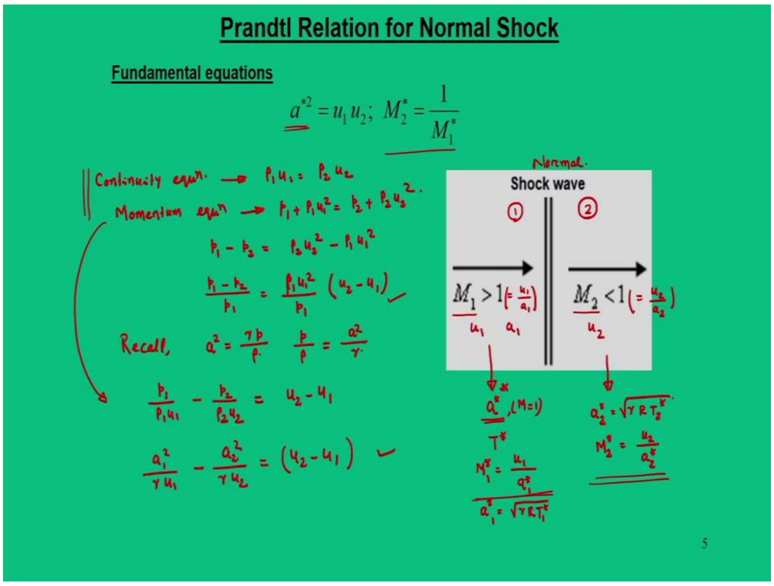
So, the very basic problem that we are going to address through this continuity momentum and energy equations are as follows like; what conditions we know is that the given conditions, which we say that upstream of the flow that is ahead of the shock or before the shock. So while dealing with what we do not know is the downstream conditions, which are considered to be unknown conditions.

And, these inequality signs I have just put just to show you that certain conditions like all static condition; static pressure, temperature, density, all these parameter increases, velocity drops, stagnation pressure drops, whereas the total temperature or stagnation temperature remains constant. In fact, Mach number becomes suddenly subsonic.

So, what it means entire kinetic energy of the flow which was there in the upstream side of this normal shock, all of them gets converted to internal energy, when this speed drops. So, thereby these conditions get prevalent in the downstream conditions.

In fact, all these inequality signs, we can essentially prove by considering this continuity momentum and energy equations. And, apart from these 3 equations you also have to consider a calorically perfect gas for which we can ideally make the conditions for enthalpy and we also know that equation of state for a gas. So, these relations we are going to use for all subsequent analysis.

(Refer Slide Time: 07:17)



So, the first important analysis that we are going to discuss is the Prandtl relations. In fact, this is one of the fundamental relations that forms the basics for all subsequent property calculations before and after the shock wave.

So, here just to what this relation talks about is $a^{*^2} = u_1 u_2$, or $M_2^* = \frac{1}{M_1^*}$. What it physically means that in a certain flow field with a standing normal shock, we say that in the upstream side of the flow, the Mach number is M_1 whereas in the downstream side of the flow the Mach number is M_2. In fact, later on we will say that M_1 is always greater than 1 and M_2 will be always less than 1, that can be proved from this; Prandtl relation as well.

Now, just to explain about this equation what it essentially means is as follows. What we are trying to say here, we introduce a parameter a*; a* means it is a condition that any arbitrary flow field can be taken to star conditions where its velocity becomes a*.

So, for example, in the upstream side of the flow if your velocity of the flow is u_1 and in the downstream side of the flow, the velocity of the flow is u_2. And, correspondingly the speed of sound in the medium 1 or that is the upstream region 1 and downstream region 2. So, we can say $M_1 = \frac{u_1}{a_1}$. Similarly, we can say $M_2 = \frac{u_2}{a_2}$.

Now, what we are trying to say that, since these are any arbitrary flow field situations. So, we can bring these conditions to star condition. So, when I say star conditions. So, here, the flow becomes a*; that means, the final velocity becomes Mach number 1 and correspondingly when you say a*, we assign a temperature condition T^*. So, we can define $M_1^* = \frac{u_1}{a_1^*}$

So, this is how we can define where $a_1^* = \sqrt{\gamma R T_1^*}$ and similarly for the downstream conditions, we can also write $a_2^* = \sqrt{\gamma R T_2^*}$ because the temperature conditions will be different. Similarly we say $M_2^* = \frac{u_2}{a_2^*}$. So, with this one can define the star conditions in this flow field.

Now, to start deriving this expressions what we can recall is that we start with continuity equation. So, in the continuity equation we can write $\rho_1 u_1 = \rho_2 u_2$ and in the momentum equation we can also write as $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$. So, we know both the equations and now what you try to do, you try to find out what is $p_1 - p_2$ that is nothing, but $\rho_2 u_2^2 - \rho_1 u_1^2$.

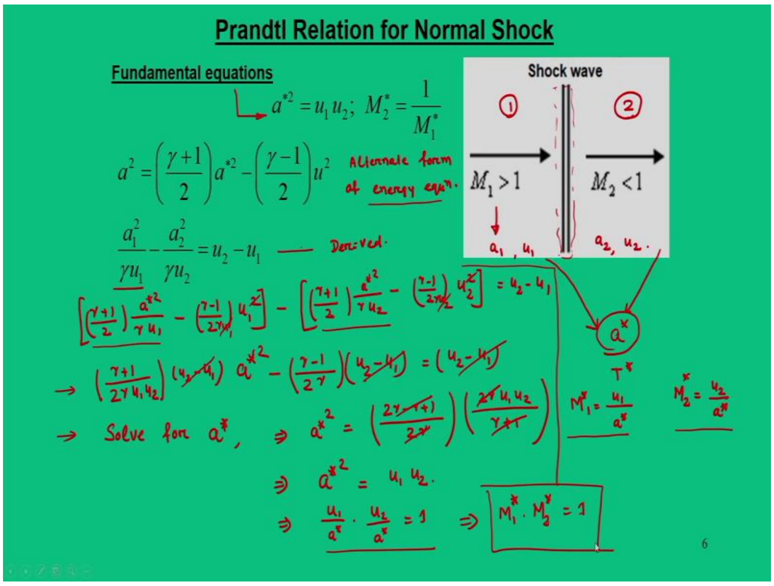
So, now, divide both sides by p_1. So, we can write $\frac{p_1 - p_2}{p_1} = \frac{\rho_1 u_1^2}{p_1}\left(u_2 - u_1\right)$.

Then, you have to recall $a^2 = \frac{\gamma p}{\rho}$. So, we can say $\frac{p}{\rho} = \frac{a^2}{\gamma}$. So, I can write in this equation, this is one way. And, again the other form of equation what we can find from this momentum equation, we can also rewrite the fact that $\frac{p_1}{\rho_1 u_1} - \frac{p_2}{\rho_2 u_2} = (u_2 - u_1)$.

How do you get it? So, in the momentum equations the left hand side you divide by $\rho_1 u_1$ and the right hand side you divide by $\rho_2 u_2$. This is another expression that you get out of it. So, we get one important equation this; other equation we can get out of this as like $\frac{p_1}{\rho} = \frac{a_1^2}{\gamma}$. So, we can say $\frac{a_1^2}{\gamma u_1} - \frac{a_2^2}{\gamma u_2} = u_2 - u_1$. So, these are the 2 equations we get in the combined form of continuity and momentum equations.

So, we are now going to move further with these 2 equations.

(Refer Slide Time: 15:32)



So, this particular equation we derived and we also want to use the another form of equation, that is alternate form of energy equation. So, if you recall this alternate form of equation where did you get? In our isentropic flow situations, we derived the equation in which we involved the speed of sound, the velocity of the flow and star conditions in one form of these equations.

$$a^2 = \left(\frac{\gamma+1}{2}\right)a^{*2} - \left(\frac{\gamma-1}{2}\right)u^2$$

Now, these 2 equations we are going to derive these relations. So, we know what is a and this condition 1, this is the condition 2. So, this condition 1 for which we have speed of sound a_1 and velocity u_1 and for condition 2 the speed of sound is a_2 and velocity u_2.

So, when I write these 2 equations for a_1 and a_2 what I can write is that.

$$\left[\left(\frac{\gamma+1}{2}\right)\frac{a^{*2}}{\gamma u_1} - \left(\frac{\gamma-1}{2\gamma u_1}\right)u_1^{\ 2}\right] - \left[\left(\frac{\gamma+1}{2}\right)\frac{a^{*2}}{\gamma u_2} - \left(\frac{\gamma-1}{2\gamma u_2}\right)u_2^{\ 2}\right] = u_2 - u_1$$

So, this equation can be simplified,

$$\left(\frac{\gamma+1}{2\gamma u_1 u_2}\right)(u_2 - u_1)\, a^{*2} - \left(\frac{\gamma-1}{2\gamma}\right)(u_2 - u_1) = (u_2 - u_1)$$

So, overall we can say next expression this will get cancelled and after simplification we can solve for a*. So, this will be

equal to $a^{*2} = \left(\frac{2\gamma-\gamma+1}{2\gamma}\right)\left(\frac{2\gamma u_1 u_2}{\gamma+1}\right)$.

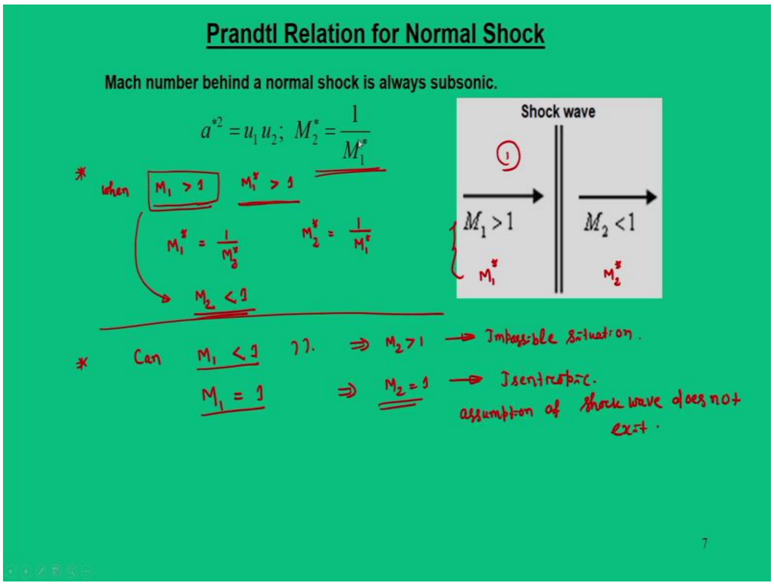
So, this expression you can do by simplifying this. So, ultimately we get land off having an expression $a^{*2} = u_1 u_2$.

Now, when I say this we can rewrite this equation as this will also implies $\frac{u_1}{a^*}\frac{u_2}{a^*} = 1$. What it physically means; that, when we say a* here, we are defining these conditions, bringing the flow from upstream and downstream both to a common condition a* and when you define this a*, correspondingly we define T^*, and when you say T^* and Mach number M^*, we say $\frac{u_1}{a^*}$ and Mach number in the other side we can say $\frac{u_2}{a^*}$.

So, what we try to do is that across this thin region, we try to correlate the star conditions of M_1 and M_2 and this star condition essentially reflect what are the upstream conditions for actual flow Mach number M_1 and the what is the downstream condition of actual flow Mach number M_2.

Now, moving further from this analysis this can be related as $M_1^* M_2^* = 1$. So, this fundamental equation's now is called as Prandtl relations. So, this is one of the very fundamental basics that how a Prandtl's relation is derived.

(Refer Slide Time: 24:10)



Now, we will move further; what is the essential significance of this Prandtl relations? So, the very basic philosophy that happens here is the fact that, we say here M_1^*, we say M_2^*. Now, from the Mach number relations we knew that when M_1 greater than 1, M_1^* also greater than 1; means, when Mach number is supersonic the corresponding star Mach number is also supersonic.

Now, from this Prandtl relation we say that, there are two possibilities one can have one is we can write $M_1^* = \frac{1}{M_2^*}$, or $M_2^* = \frac{1}{M_1^*}$. But, the very important question here is that M_1 should be always greater than 1.

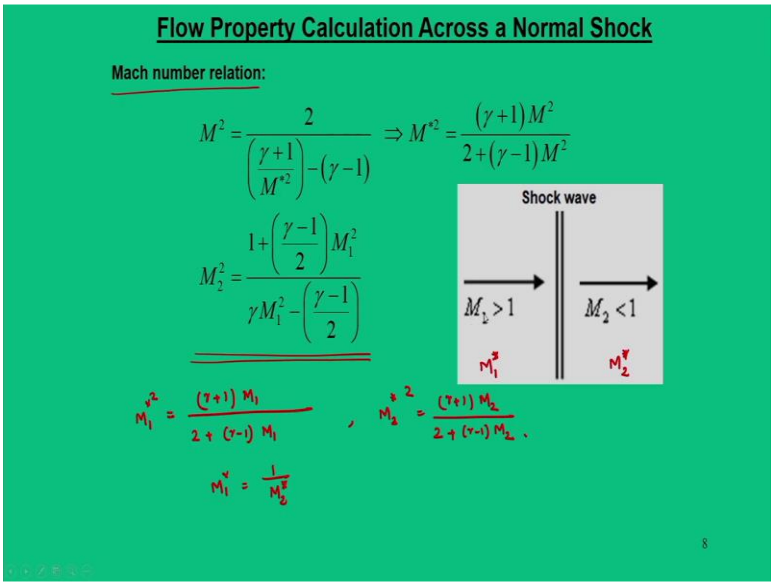
So, when M_1 is greater than 1, this will also implies M_2 should be less than 1. So, what it says is that Mach number behind the normal shock is always subsonic. But, in this Prandtl relation it does not talk about why the Mach number behind the normal shock is always subsonic. But, question remains that can M_1 less than 1 or can M_1 is equal to 1.

There are two other possibilities, when M_1 less than 1; so this will also implies M_2 should be greater than 1. But, the question remains this? In down the line will prove that this will be an impossible situation. So, it cannot happen for some other thermodynamic regions, which we will prove later.

The other possibilities could be M_1 can be 1; so this also implies M_2 will be also equal to 1. So, this is a situation that there is no change in the Mach number across the shock wave. So, as if the shock wave does not exist. When such a thing is there, this will reflect a situation that flow field is isentropic. So, this assumption of shock wave does not exist.

But, the very basic philosophy that I want to address here, we will say that M_1 is always greater than 1 and M_2 is always less than 1 that is Mach number beyond a normal shock is always subsonic.

(Refer Slide Time: 28:02)



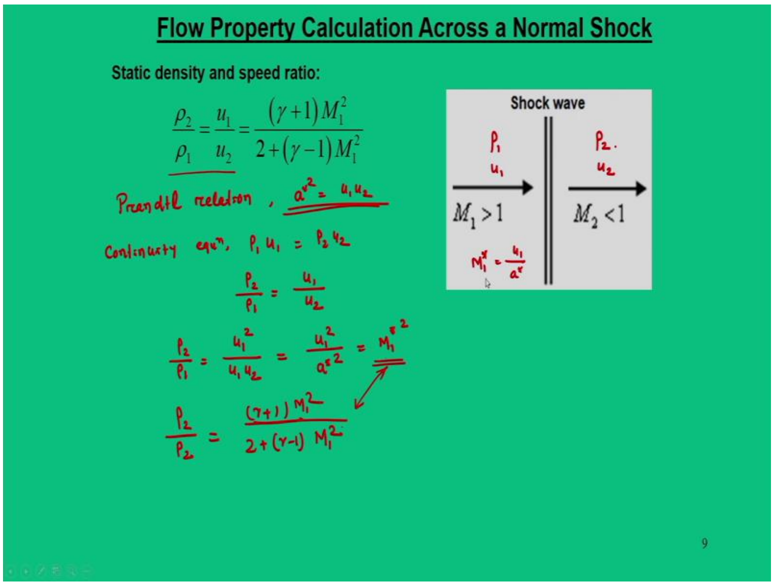
So, in the previous relations the Prandtl number talks about the relation between M_1^* and M_2^*. But, it does not talk about what is relation between M_1 and M_2. But, we know that any arbitrary Mach number can be correspondingly connected to its star condition of Mach number, that is through this expressions.

So, we can effectively write that $M^{*2} = \frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}$. Now, if this relation we are going to apply for condition M_1. So, we can write that $M_1^{*2} = \frac{(\gamma+1)M_1^{\,2}}{2+(\gamma-1)M_1^{\,2}}$. Similarly, we can say $M_2^{*2} = \frac{(\gamma+1)M_2^{\,2}}{2+(\gamma-1)M_2^{\,2}}$.

Since, we say $M_1^* = \frac{1}{M_2^*}$. So, by putting these expressions here, we will arrive at the relation between the Mach number between these. So, if you see that this relation will be only true when your Mach number is supersonic for M_1.

$$M_2^2 = \frac{1+\left(\frac{\gamma-1}{2}\right)M_1^2}{\gamma M_1^2 - \left(\frac{\gamma-1}{2}\right)}$$

(Refer Slide Time: 30:14)



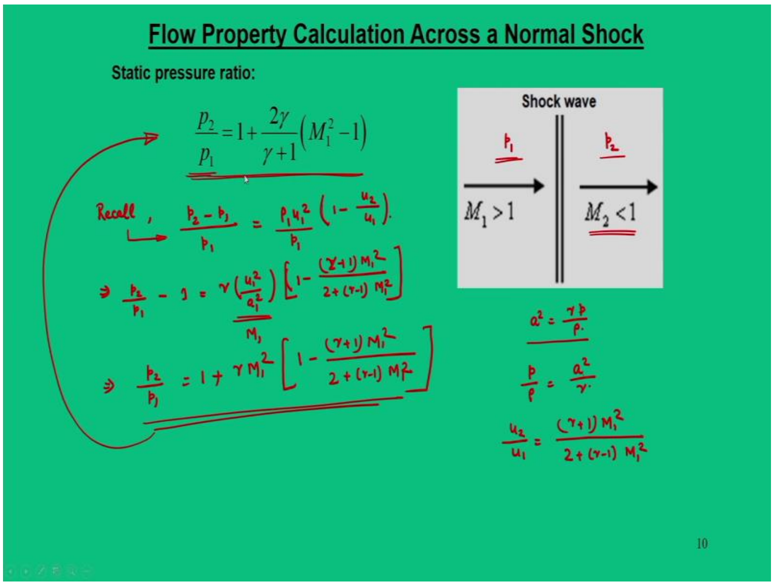
The next expression that we are going to talk about is the static density and speed ratio. So, now we are moving with the fact that, we are going to calculate the flow property across a normal shock. So, for this flow property calculations the very basic relations that you have to remember is the Prandtl relation. So, these Prandtl relations will say that $a^{*2} = u_1 u_2$. So, this relation will be very vital for these calculations.

And, in this case what we are trying to say that what is going to happen to the density ratios. So, if I put the density terms as ρ_1 and ρ_2 and velocities are u_1 and u_2. So, from continuity equation, we can write $\rho_1 u_1 = \rho_2 u_2$. So, we can say that is $\frac{u_1}{u_2} = \frac{\rho_2}{\rho_1}$; so this relation first holds good.

Now, let us see that what is u_1 and u_2. So, we can write this equation as $\frac{\rho_2}{\rho_1} = \frac{u_1^2}{u_1 u_2}$. So, I multiplied u_1 in the numerator and denominator. So, this is written as $\frac{\rho_2}{\rho_1} = \frac{u_1^2}{a^{*2}}$. So, we say this is M_1^{*2}. So, we say $M_1^* = \frac{u_1}{a^*}$. So, we also know that; when I say M_1^* what is the relation between M_1^* and M.

So, we can rewrite this equation as $\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$. So, this relation we get from M_1 and M. So, between M_1 and M, this is the relation exists. So, we can write this equation. So, when I say $\frac{\rho_2}{\rho_1}$, then I also can calculate what is $\frac{u_1}{u_2}$. So, this is what we do for density ratio and speed ratio across this normal shock.

(Refer Slide Time: 33:50)



Now, we will move to the conditions for static pressures. So, very basic things that we want to say the static pressures are defined as p_1 and p_2, that is before the shock and after the shock. So, our basic philosophies that we want to find out the ratio of $\frac{p_2}{p_1}$. So, this $\frac{p_2}{p_1}$ we are trying to express in the form of M_1, because you do not know the M_2, but we know the flow Mach number and the upstream.

So, how we get this relations; to do that we have to recall these momentum equations that we have already derived in one of the slides that is $\frac{p_2 - p_1}{p_1} = \frac{\rho_1 u_1^2}{p_1}\left(1 - \frac{u_2}{u_1}\right)$. So, in the last slide we derived this equations from the momentum equation and continuity equations.

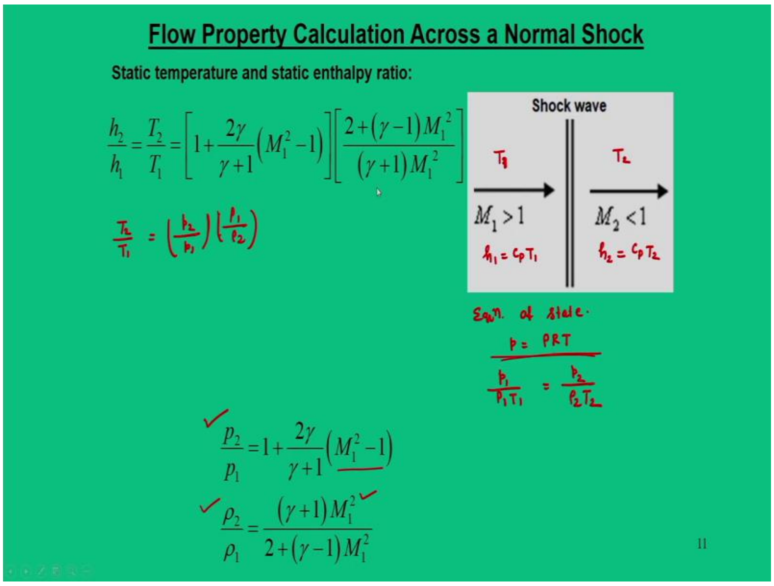
So, when I say this; so again I also told that we know the other relations like $a^2 = \frac{\gamma p}{\rho}$. So, we can say $\frac{a^2}{\gamma} = \frac{p}{\rho}$.

Now, then what you do? We simplify this; so we say $\frac{p_2}{p_1} - 1 = \gamma\left(\frac{u_1^2}{a_1^2}\right)\left(1 - \frac{u_2}{u_1}\right)$. So, in earlier expressions we know $\frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$. So, in the last slide we derived this relation between the speed ratio as a function of Mach number.

So, when I put this here, I can write $\frac{p_2}{p_1} - 1 = \gamma\left(\frac{u_1^2}{a_1^2}\right)\left(1 - \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$. So, here $M_1 = \frac{u_1}{a_1}$ and, finally, we will land up an expression like $\frac{p_2}{p_1} = 1 + \gamma M_1^2\left(1 - \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)$. So, this expression can be further simplified to reach this fundamental relations between the static pressure ratio $\frac{p_2}{p_1}$. So, this is how the relations are framed for static pressure.

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)$$

(Refer Slide Time: 38:09)



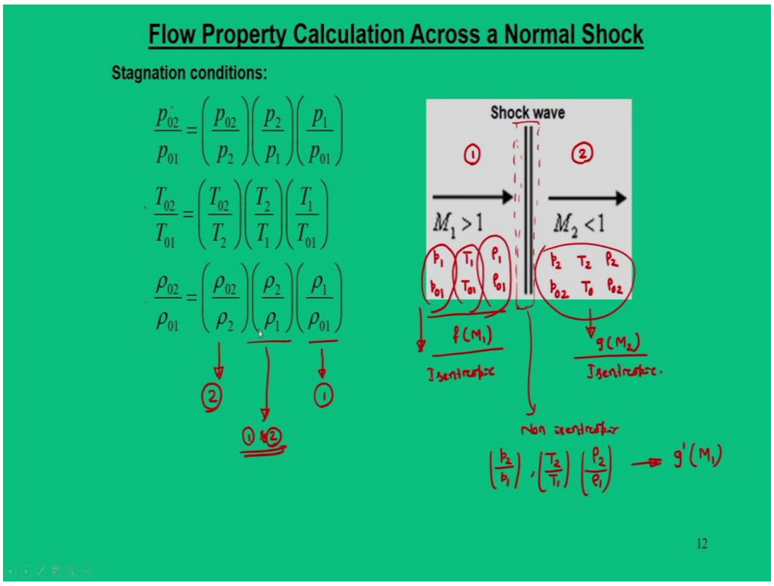
Now, then next point we are going to address is the static temperature and enthalpy. So, here if I define this static temperature as T_1 and T_2 that is one is before the shock, other is after the shock and static enthalpy $h_1 = c_p T_1$ and static enthalpy h_2, after the shock will be $c_p T_2$. Also we have to write equation of state, that is we can write $p = \rho RT$.

So, we can write $\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2}$, by satisfying this equation of state, one can write this relation $\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2}$. Then, we can find out $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{\rho_1}{\rho_2}\right)$; so we get this equation.

Now, fortunately in the previous slides, we know what is the static pressure ratio $\frac{p_2}{p_1}$ as a function of M_1, we also know static density ratio as a function of M_1. So, when I put this equation here in this relation, we arrived at the static temperature ratios, which is same as the static enthalpy ratio. And, again this is the function of upstream Mach number M_1

$$\frac{h_2}{h_1} = \frac{T_2}{T_1} = \left[1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)\right]\left[\frac{2 + (\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right]$$

(Refer Slide Time: 40:34)



The next important condition that we are going to talk about is the stagnation conditions. So, for stagnation conditions what I can say this is 1, this is 2. So, I define the stagnation conditions like stagnation pressure ratio, stagnation temperature ratio, stagnation density ratio.

So, when I say stagnation conditions then corresponding static pressure will be p_1, temperature will be T_1 density will be ρ_1. Stagnation pressure will be defined as p_{01} T_{01} and ρ_{01}. So, how do I calculate this ratio? These ratios are function of M_1, so we already know. We get it from isentropic relation right.

Now, correspondingly in similar way we say p_2 T_2 ρ_2 that is after the shock, stagnation values will be p_{02}, T_{02}, ρ_{02} and all these terms are another function of M_2 and, this relation we will get it from through isentropic.

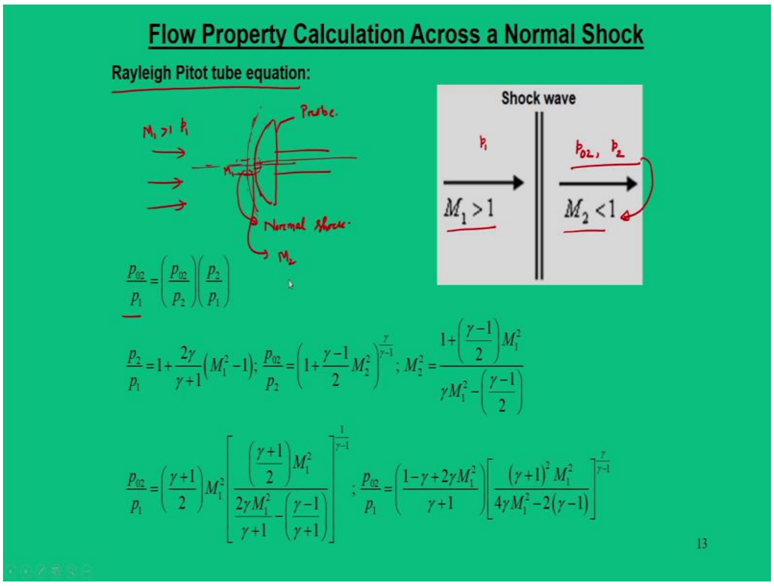
So, these has nothing to do with the shockwave and as I mentioned within the shockwave the flow process is non isentropic. But, still we are able to manage this static pressure ratio $\frac{p_2}{p_1}$, $\frac{T_2}{T_1}$ and $\frac{\rho_2}{\rho_1}$. We derived this as some function of M_1, these relations we already derived in the earlier slides.

So, now if you look at this stagnation pressure ratio $\frac{p_{02}}{p_{01}}$, I can write this as $\left(\frac{p_{02}}{p_2}\right)\left(\frac{p_2}{p_1}\right)\left(\frac{p_1}{p_{01}}\right)$. So, all these number will get cancelled finally, I will land up in this stagnation pressure ratio. Same concept applies for stagnation temperatures as well as stagnation density.

Now, the expression will be very big here, because first term in this refers to the isentropic conditions that is condition 2. The last term refers to condition 1 only. And, in between the middle term refers to the condition 1 and 2 and they are related through function of M_1.

So, all these things are known to us so we can essentially calculate this stagnation pressure ratio after the shock.

(Refer Slide Time: 44:33)



There is another kind of relation that we are trying to get here, which is typically called as Rayleigh Pitot tube equations. What is the very basic background; maybe some lectures down the line we will come to know during the measurements in supersonic flow.

We will show that a pitot tube is generally used to measure the stagnation pressure in a flow field. So, when some pitot probe is placed in a supersonic flow field, we will have a bow shock sitting on the surface. So, this probe is typically a blunt shape nose; so we say it is a probe.

So, there will be bow shock sitting on this, but what we look at along this axis; in this domain if you put a sensor somewhere here and try to capture the flow information. So, in this domain it is always assumed to have a normal shock. So, when I say normal shock, I know the inflow condition is M_1 as if a condition M_2 happens to be here, in this situation what we measure is after the shock.

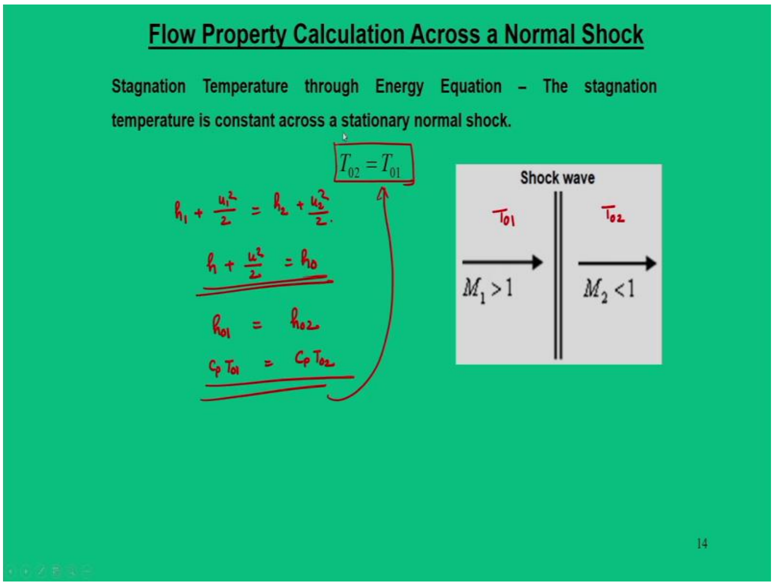
So, what you typically measure downstream is p_{02}, that is stagnation pressure because entire flow field comes to rest at this nose point. But, many a times we are known conditions are p_1 that is in the static flow field the conditions are known as p_1. Now, the a relation that is derived is $\frac{p_{02}}{p_1}$ and that is equal to $\left(\frac{p_{02}}{p_2}\right)\left(\frac{p_2}{p_1}\right)$. So, all these conditions again if I say p_2 and p_{02}, then correspondingly it is related to Mach number M_2.

So, when I say Mach number M_2; so we know this particular relation is only function of Mach number; this particular relation that is $\frac{p_2}{p_1}$ function of Mach number in the upstream. Whereas, $\frac{p_{02}}{p_2}$ is a function of Mach number in the downstream. And, moreover we know the, what is M_1 and M_2 through this relations.

And, ultimately we land up in a big expression like this that talks about ratio of stagnation pressure after the shock to the static pressure before the shock. So, this is a very vital as far as the experimental point of view, so we call this as a Rayleigh pitot tube equation.

$$\frac{p_{02}}{p_1} = \left(\frac{\gamma+1}{2}\right) M_1^2 \left[\frac{\left(\frac{\gamma+1}{2}\right) M_1^2}{\frac{2\gamma M_1^2}{\gamma+1} - \left(\frac{\gamma-1}{\gamma+1}\right)}\right]^{\frac{1}{\gamma-1}} ; \quad \frac{p_{02}}{p_1} = \left(\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right)\left[\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}}$$

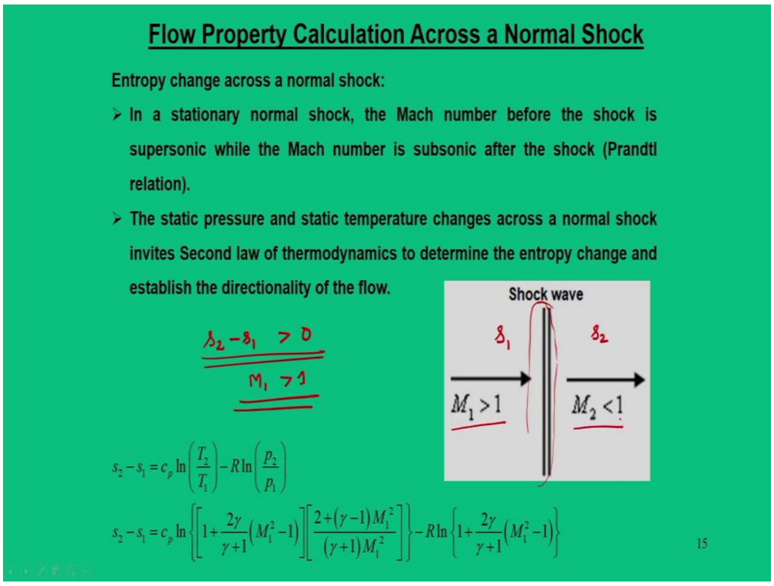
(Refer Slide Time: 47:54)



Another important relation that we are going to talk about the stagnation temperatures. So, we say the before the shock it is T_{01}, after the shock it is T_{02}. So, as I say that energy equation does not change; so I can write $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$ and we also know $h + \frac{u^2}{2} = h_0$.

So, we can write $h_{02} = h_{01}$, we can say $c_p T_{02} = c_p T_{01}$. Then this total temperature relation we can say as $T_{02} = T_{01}$, which means stagnation temperature does not change or is constant across a stationary normal shock.

(Refer Slide Time: 49:06)



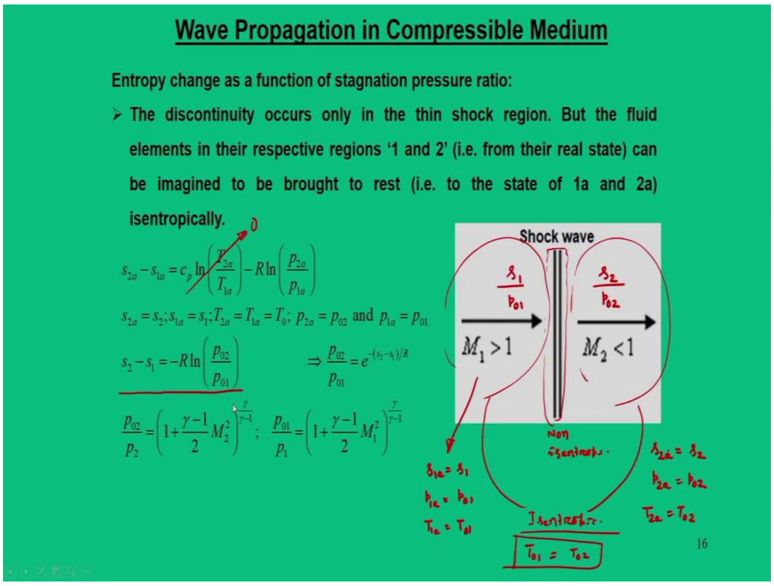
Then we are going to move a very important calculations that is entropy in the flow field. So, as you know that across the shock wave, the entire flow field is non isentropic. So, if I say absolute entropy before the shock is s_1, if I say absolute entropy after the shock is s_2 that is per unit mass. So, we can easily calculate what is happening $s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)$.

So, all the static property relations we know; so, we can directly calculate the entropy change across the shock. And from this relation what we will see that because the entropies related through second law of thermodynamics to define the directionality of a system or of a thermodynamic process.

So, in this context what we will show here that, this relation will be only true when M_1 is more than 1. This will not be true when M_1 is less than 1 because this entropy change will be negative. So, as per the second law of thermodynamics, this entropy must increase across this normal shock. So, $s_2 - s_1$ should always be greater than 0. So, this will be only true when M_1 is greater than 1.

Hence, we say that Mach number is always supersonic before the shock and it has to be subsonic after the shock.

(Refer Slide Time: 51:17)



So, this is one working formula for entropy calculations, in many a times in normal shock problems, normally this entropy change also calculated through another expressions in terms of stagnation pressure. So, we define the stagnation pressure p_{01} 1 and p_{02} and entropies as s_1 and s_2.

So, what we are trying to say is that, if you want to recall the same expression. We say that only within this shock wave the process is non isotropic and all other regions the flow field is isentropic. If, this is the case then what I can bring this particular condition 1 isentropically to rest.

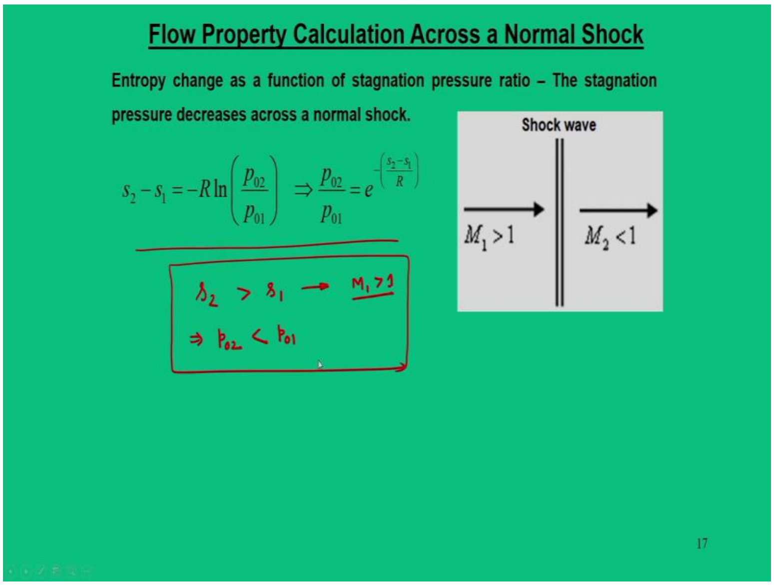
So that I can define for the condition 1, this s_{1a} which is nothing but s_1 and corresponding pressure p_{1a} will be p_{01}, T_{1a} will be T_{01} and, for this downstream side I can say s_{2a} will be s_2, p_{2a} will be p_{02}, T_{2a} will be T_{02}.

Now, if I want to calculate the entropy change that is $s_{2a} - s_{1a}$ which is same as $s_2 - s_1$, but what we see in the right hand side of this expression. So, what I write is $c_p \ln\left(\frac{T_{2a}}{T_{1a}}\right) - R\ln\left(\frac{p_{2a}}{p_{1a}}\right)$.

So, if you see here, in the previous expression we say $T_{02} = T_{01}$, we prove that this particular total temperature does not change across a normal shock. So, having said this, we can say first term of the right hand side expression will be 0.

So, the remaining term that the right hand side will contain the stagnation pressure ratio. So, ultimately we can write the $s_2 - s_1 = -R\ln\left(\frac{p_{02}}{p_{01}}\right)$. So, this is another expression and it is also convenient to calculate the entropy change across the shock wave. So, instead of calculating the entropy change from the static pressure values, we can directly calculate it from the stagnation pressure values.

(Refer Slide Time: 54:58)



So, this is what I just want to conclude in saying that, the fundamental expression from this we say that; entropy change $s_2 - s_1 = -R\ln\left(\frac{p_{02}}{p_{01}}\right)$ and, it is a function of stagnation pressure ratio. Since s_2 is always greater than s_1; so this will also imply p_{02} must decrease p_{01}. So, this is true when M_1 is greater than 1.

Hence, this is very basic philosophy of Prandtl relation that Prandtl relation always holds good and it has to be correlated in line with entropy change across the shock wave and that entropy must increase across the normal shock. If such a situation is there, M_1 must be greater than 1 and the stagnation pressure will drop across a normal shock.

So, with this I will conclude this lecture for today. In the subsequent lectures we will move further in other analysis of normal shock.

Thank you for your attention.