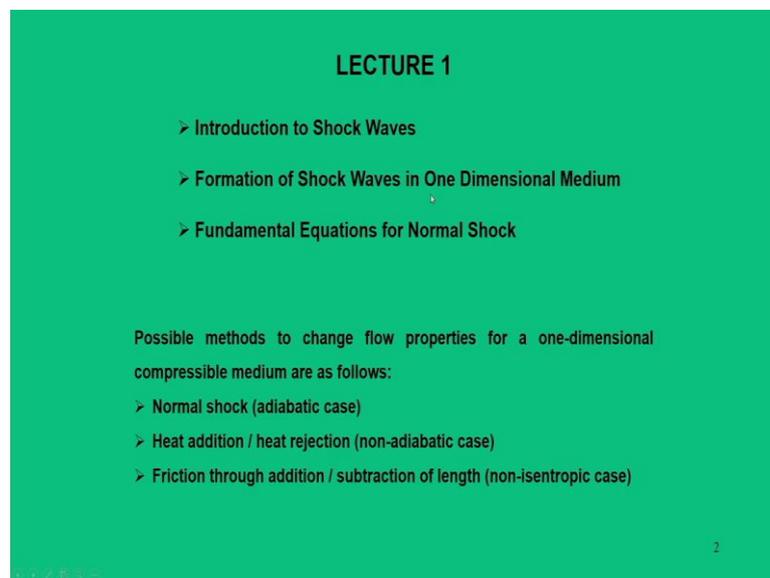


Fundamentals of Compressible Flow
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Module - 04
Lecture - 10
Normal Shock Waves-I

Welcome you again for this course Fundamentals of Compressible Flow; we are in a new module that is module 4 that is Normal Shock Waves. So, prior to this module we were mainly concentrated on one dimensional flow and in particular the flow is isentropic.

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So, we analysed how we are going to evaluate if a complete flow field is isentropic and in this isentropic flow field we try to evaluate all the parameters, such as stagnation pressure, static pressures, static temperatures and all of them are function of the flow velocity or Mach number.

So, the next aspect that we are going to decide is the property change in a non-isentropic medium. In fact, when we say isentropic flow its a kind of a ideal situations where we want to find out the stagnation conditions from its corresponding static values. So, this stagnation condition could be a pressure, could be temperature, could be density, could be enthalpy. So, for any arbitrary flow field these stagnation properties are essentially the characteristics when the flow is moved isentropically to rest.

So, now moving ahead if the medium is non-isentropic, what are the possible situations? So, the possible methods to change the flow characteristics in one dimensional compression medium are the formation of a normal shock. So, this is a adiabatic case, but not isentropic.

Another way also in a one dimensional medium we can change the flow properties by adding heat or taking away heat from the system. Again since heat is added or subtracted then it becomes a non-adiabatic case.

Now, when we introduce a friction in a medium, how do you introduce a friction? We can introduce a length. Length means when a flow is taking place in a one dimensional duct; so, you can increase its length or decrease its length. So, that is what we say addition or subtraction of length. Since we are talking about friction, this is also a non-isentropic case.

So, in all subsequent situations we will analyse all these cases one by one and the first analysis what we are going to discussed in this module is a normal shock situation. So, prior to this lectures we are aware of the fact that how a sound wave is formed in a medium. So, it is a very small pressure disturbance wave, these are basically very small pressure waves.

Now, when the medium becomes stronger and stronger, how do you do? Then when we increase the flow velocity or we increase the Mach number. So that means, you are moving towards the supersonic speed. When we are moving towards the supersonic speed at the close to the Mach 1 that means, we are the initiation of supersonic situation. So, we can say that a Mach wave is generated. So, once and when the body moves still at higher velocity, these Mach waves become stronger and stronger.

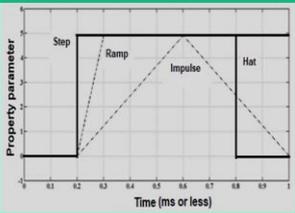
So, we start with this, when this Mach waves becomes stronger and stronger we define this as a shockwave. So, this is the basic definition of a shockwave how it is formed in a medium. So, in this lecture we will discuss about the shockwaves; how it is formed in a one dimensional medium and then moving ahead we will talk about the fundamental equations that is for normal shock.

Why we are saying it is a normal shock? Because under this situations we can say this medium can be treated to be a one dimensional medium. So, we will talk about those things one by one.

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Introduction to Shock Waves

- A shock wave represents abrupt change in fluid properties (such as pressure, temperature, density etc.) over a finite thickness comparable with mean free path of gas molecules.
- It is a very thin region (typically, 10^{-7} m) across which the flow properties instantaneously change.
- Mathematically, it is represented discontinuity in a medium through a "step/impulse/ramp/hat" function.



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Now, let us see, what is a shock wave. So, physically the shock wave represents the abrupt change in the flow properties such as pressure, temperature, density over a finite thickness comparable with mean free path. So, if you see the mean free path of gas molecules is very small and when we assume that within that amount of thickness if in a flow field the properties are going to change, then it has to be a shock wave because these are the instantaneous change.

And this shock wave is typically a thin region and you can visualise is to be about 10^{-7} m thickness. And, across this thickness we encounter instantaneous change in the flow properties.

So, many mathematicians represent this kind of phenomena of shock wave formation as a discontinuity in the medium. So, a function is continuous everywhere, but except at a point. So, something of that sorts we say that such a phenomena we can represent as a step or impulse or ramp or hat functions.

So, these are the terms that are used by mathematicians to talk about some kind of mathematical background of shockwaves. So, in our philosophy what does this mean is

that let us look at this figure; so, what you are trying to see that, any parameter which is going to change with respect to time. So, this time can be of day, it can be of hour, it can be of second or minute and what we are looking at? We are looking at this time to be very small as far as the human view is concerned.

So, if I talk a timescale of a millisecond or less. So, if any event is going to happen in this small instant of time then; obviously within this timescale if there are drastic change in the medium, then we call this as a formation of a instantaneous change in the medium.

And if you look at any property parameter which is changing with respect to this kind of time step, then we can call this either as a step. So, a step means it is a instantaneous change. By instantaneous change I mean looking at this figure, let us say I am talking about a 0.2 milliseconds. So, at 0.2 milliseconds the property parameter happens to be 0, but again at maybe very small instantaneous time, these property again suits up to a value of magnitude of 5, but with a very short span of time and it retains that properties continuously till the end of the event and in this case let us the event ends at 1 millisecond.

So, if this is the case we call this as a step response. So, instantly property change and it remains with that number 5 as that for 1 seconds. Now, if I want to close that event at some particular time like 0.8 milliseconds then that property starting from that coordinates that is from 5 it has to come down instantaneously.

So, when I say come down instantaneously, this means the way it has risen its magnitude; it has come down again to the same 0 value and again it remains as it is. So, such a function it looks like a hat. So, it is a hat function. Now, let us say that although we are going to that magnitude of 5 instantaneously, but in our case let us allow that to reach that magnitude the phenomena takes certain amount of time. So, we call this as a impulse.

So, for this event we are taking this response as a magnitude. So, it is responding to that event as a slope; that means, we reached maybe to the same magnitude of property 5, but we take about a timescale from 0.2 to 0.6 millisecond difference that is 0.4 millisecond to reach that magnitude. And again with same way the value drops; that means, the number that does not remain at that value with respect to rest of the time, but rather it falls. So, we call this as a impulse. So, it is a like of a impulse.

Another way of looking at the event is that the event while rising it may take at some fashion of rising let like in this case that is in the case of ramp. So, we say that we reached with same magnitude parameter of 5, but within a time span of 0.1 milliseconds, but not necessarily that with same time it will fall back, it may immediately fall back or it may take a bit longer time. So, this particular event we call this as a ramp.

So, likewise the way we define the functions as step, ramp or impulse or or hat. So, typically the phenomena of shockwave events are similar to of this nature. So, in the bottom line we can say that during a shockwave phenomena the flow property changes instantaneously and when they change instantaneously that is a drastic change in the magnitude. So, since there is a drastic change in the magnitude, so entire event that has to happen in a very small milliseconds.

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Introduction to Shock Waves

Subsonic vs Supersonic Events

- When a body moves at subsonic speed, the flow information is shared everywhere in the medium because the pressure disturbances moving at the speed of sound always warns the medium about the presence of the body. In this way, a subsonic flow can adjust the gradual change in flow properties.
- On the other hand, the pressure disturbances moving at speed of sound in a supersonic flow can never warn the medium about the presence of the body. The sound waves can no longer propagate upstream, instead, they tend to coalesce to a short distance ahead of the body forming a thin shock wave.

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So, now with this background let us see that why that phenomena is important to our case. So, now, will talk about something called as super subsonic or supersonic events. So, as we know earlier situations that in a subsonic flow field, the pressure disturbance in the form of sound waves propagates everywhere in the mediums. And even the body is moving, it is moving in such a manner that it can never overtake the path travelled by the sound wave. So, that is the reason entire information about the presence of the body is felt everywhere in the medium.

But when you move to the supersonic situations; that means, when your body is moving at very high speed more than speed of sound then we will land off in a situation that the information will not be shared into the medium. So, that is what we say that supersonic flow never warns the medium about the presence of the body, because the sound waves can no longer propagate upstream and because at any point of time the velocity that is V is always greater than a .

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Introduction to Shock Waves

Characteristics of Supersonic Events

- Nature establishes “shock formation” through which supersonic flows can adjust the presence of the body.
- Shock wave is a mechanism of instantaneous compression of a gas within a thin region.
- Instantaneous rupture of a diaphragm, detonation waves, explosion, motion of missile/space shuttles, normally encounter shock wave phenomena.

Shock wave

T_1		$T_2 > T_1$
P_1		$P_2 > P_1$
$M_1 > 1$		$M_2 < 1$
Ahead of shock/ Before the shock		Behind the shock/ After the shock

But then if that is the case then how one has to know about the presence of the body. So, for that situation what we say as a characteristics of a supersonic event. So, what we say if such a situation arises; so, nature establishes a phenomena what we call as “shock formation” process through which the supersonic flows adjust about the presence of the body. So, this adjustment has to be done in a very small region and that is nothing but the shockwave.

And hence the shockwave is a mechanism of instantaneous compression of the gas within the thin regions. Why this is a compression? Because the body is moving at a very high speed, all of a sudden this velocity has to be come down to a subsonic value, because this event is only possible or formation of shockwave is only possible when a body is moving in supersonic speed and when such a things happens, the Mach number of the flow has to be subsonic all of a sudden. When these things happens, entire gas gets squeezed in a very thin regions that is why the pressure suits up. And, when it is

squeezed in these small thin regions and thereby the entire kinetic energy of the flow gets converted into internal energy thereby temperature also rises.

So, that means, when temperature rises, your T_2 will also be greater than T_1 , if this is the temperature T_1 before the shock. So, this is how the event of shockwave that takes in a supersonic medium. So, typical examples in our day to day life we can say that in a philosophical sense we can say accident is a kind of situation if there is a instantaneous accident where there is a large amount of failure. So, you can say it is a philosophical aspect of a shock wave.

And in fact, in our day to day life we also create explosion like burning firecracker and in a many aerodynamic facilities that try to rupture a diaphragm instantaneously or we can create a detonation waves. So, these are the day to day life we can create these facilities in the laboratories and in fact, in coming to the real situations the motion of missiles and space shuttles normally encounter this shockwave phenomena.

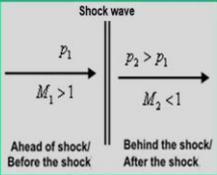
So, the shockwave is a very natural occurring event when anybody that travels at supersonic speed.

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Introduction to Shock Waves

Characteristics of Supersonic Events

- The energy for compression of the gas is derived from the kinetic energy possessed by the upstream flow. It leads to increase in internal energy and thereby increase in temperature.
- While flowing through a shock wave, the gas experiences decrease in its available energy. It leads to increase in entropy. Hence, compression through a shock wave is considered as highly irreversible and non-isentropic process.



The diagram illustrates a shock wave as a vertical line. To the left of the line, the flow is supersonic with Mach number $M_1 > 1$ and pressure P_1 . To the right of the line, the flow is subsonic with Mach number $M_2 < 1$ and pressure $P_2 > P_1$. Below the diagram, the regions are labeled: 'Ahead of shock/ Before the shock' on the left and 'Behind the shock/ After the shock' on the right.

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So, this is how I have already explained that with respect to these characteristics of shockwave event. It is the instantaneous compression and with this instantaneous compression, the energy for the compression is derived through kinetic energy possessed

by the upstream of the flow and this leads to the internal energy increase and temperature increase in the medium.

So, again what happens to entropy? So, while flowing through a shock wave, the gas experiences a decrease in its available energy. So, thereby it leads to increase in the entropy. So, it is a non - isentropic situations and the phenomena across a shock wave is highly irreversible and non-isentropic.

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Introduction to Shock Waves

- The shock processes can be classified as follows:
 - Stationary shock
 - Moving shock
 - Normal shock – One dimensional flow
 - Oblique shock – Two dimensional flow
- A normal shock is one phenomena where the properties change drastically in one direction.

Shock wave

P_1 $P_2 > P_1$

$M_1 > 1$ $M_2 < 1$

Ahead of shock/ Before the shock Behind the shock/ After the shock

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Now, in our lectures, our sequence of view will be something like this. So, we can say that this particular shockwave whenever it generates in a medium, it can be a stationary shock.

So, with a stationary shock means I am saying that there is a shock wave sitting in a medium, some flow is entering at some velocity and it is leaving at some velocity. This is how we view by keeping the shockwave as stationary.

So, we call this as a moving shock. Now, when the shock wave is moving; that means, your medium is stationary, but the wave is moving in the medium. So, accordingly we use the terminology like ahead of the shock or before the shock, behind the shock or after the shocks. So, when we say it is a stationary shock we normally refer what happens before the shock and what happens after the shock.

Now, when we say it is a moving shock when the shock wave is moving, this particular gas is stationary then we will say that this particular region is behind the shock, because shock has not cross this region. And this region which is the left hand side of the shock wave we call this as a ahead of the shock because shock wave has already cross this region. So, this is how we do this stationary shock and moving shock.

Now, when you say it is a normal shock. So, normal shock is a situation where we say it is a one dimensional flow why we say, let us say when you are looking at a stationary shock; that means, shockwave is standing in this way. So, what happens in a normal shock the streamlines that comes are essentially perpendicular to this shock wave. So that means, any point of time the streamlines form 90° with respect to shock wave.

So, what we can say we can imagine this to be a duct or imaginary duct where consisting of large number of streamlines and all the streamlines are perpendicular to this shock wave. So, this we call this as a normal shock and obviously, it is a one dimensional phenomena

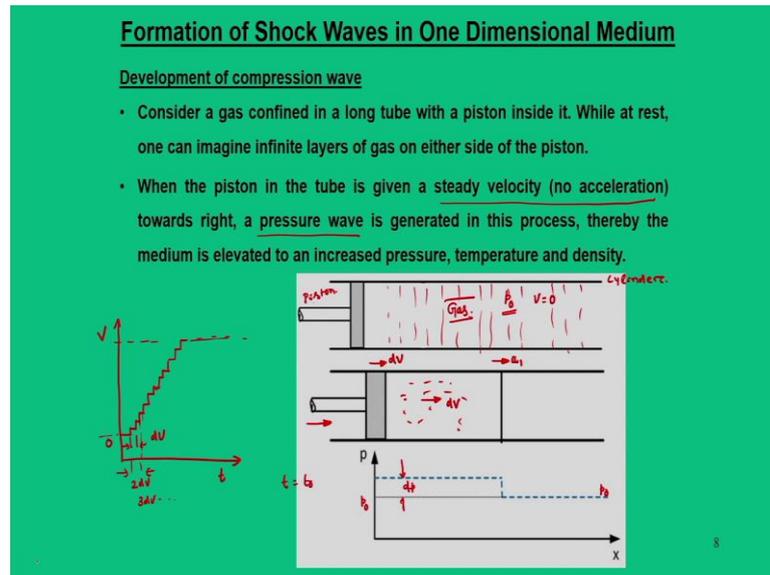
Now, in subsequent analysis will talk about a oblique shock, which is a two dimensional phenomena. Now, you can imagine this shockwave can be inclined like I can say that I can incline the shockwave when the streamline crosses this wave it tries to deflect. So, such a things we call this as a oblique shock. So that means, when the shock wave is normal to this streamlines it is a normal shock and it is the strongest one. As and when it becomes inclined, so, it becomes a oblique shock. So, it is a two dimensional flow phenomenon. So, we will discuss this in the all subsequent lectures.

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Formation of Shock Waves in One Dimensional Medium

Development of compression wave

- Consider a gas confined in a long tube with a piston inside it. While at rest, one can imagine infinite layers of gas on either side of the piston.
- When the piston in the tube is given a steady velocity (no acceleration) towards right, a pressure wave is generated in this process, thereby the medium is elevated to an increased pressure, temperature and density.



So, now let us see the next topic of this lecture that is formation of shockwave in one dimensional medium. So, in previous to this, I elaborately told that how a sound wave is generated in a medium. So, that point of time we had a very conceptual thermodynamic systems like a piston cylinder device in which one side of the cylinder is open. So, we will revisit the same example again, but in a different context.

So, in this context what we can say in the first figure what we see, we can say there is a piston. So, this is the piston and this is the cylinder; its a long cylinder, but the end is open. So, what we can say there is a gas in this medium any gas; so, since the gas is a stagnant situations and is not moving. So, we say its condition as p_0 and its gas velocity is 0 and all the conditions let us say its temperature and velocity is 0.

Now, what you do is. So, you can say this gas may have different layers of gas; we can imagine this gas to be arranged in layers that is there are infinite layers of the gas, I mean it can be on either side of the piston. Now, what we do is, now in the next event that is this is the initial situation; then once after that, the first initial time the piston of the tube is given a steady velocity that is with no acceleration; that means towards right.

So, by looking at the steady velocity and no acceleration what I mean, if I try to plot these velocity of the time plot for the piston. What I can view that initially it is 0. So, I give this stepwise very small magnitude of discreet manner. So, if I want to change the

velocity from 0 to certain finite value then I am looking at this to be happening in very small steps.

So, what I do is, I give a very small and these steps we can say as small dV and at the second instance I say it becomes $2dV$ and third instance it becomes $3dV$ and so on. So, that means, now let us say when we are giving the first steady velocity dV to the piston. So, obviously, when I give this velocity dV , the pistons tries to move at certain velocity.

So, as a results since it is a very small pressure wave disturbance, we can say it will generate a sound wave or in this case let us put as a compression waves or very weak compression wave and that is nothing but the pressure wave and that is generated in the process and it moves a speed a_1 .

So, since this sound wave has already crossed this medium, this gas has also started moving which is contained in this region. They started moving with same velocity dV and when we have the same velocity, then the initial gas situation which is at p_0 and which was the at the initial state, now it has risen to a magnitude dp .

So, with this first wave the medium gets a pressure disturbance dp . So, this is how the weakest wave that is going to have in the medium. Now, since its weakest wave we call this as a sound wave that is quite obvious, but with same time instant if I just increase this magnitude to another dV and we are looking at the next sequence of events.

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Formation of Shock Waves in One Dimensional Medium

Development of compression wave

- A second additional increment in velocity to piston would generate the second pressure wave that propagates into the medium which is already at elevated pressure and temperature.
- So, the speed of second pressure wave is always higher than first one i.e. second pressure wave moves faster than the first one and eventually gets closure at subsequent time instance.

$a = \sqrt{\gamma R T}$
 $a_2 > a_1$ $l_2 > l_1$

So, what do we do is here, the same piston we are looking at another time $t = t_1$. So, that previously that time instant we say it is at $t = t_0$ this is the time instant.

Now, what we are looking at this time instant as $t = t_1$. So, here the piston was given the next subsequent velocity, piston is allowed to move with an additional increment of velocity $2dV$. So, by this time the first sound wave would have reached at this location. If you look at the pressure versus x diagram, so, it would have reached some locations beyond that we have velocity of the gas 0 and pressure is p_0 .

So, in this line we will say this line refers to p_0 and this one, the gas has velocity dV and the pressure has p_1 . And the second subsequent increment will make this gas velocity to be $2dV$. So, this region will see the next level of pressure increment and we say the absolute pressure to be p_2 . So, what I can say that the jump in pressure from p_0 to p_2 has happened in two steps, due to two step increment in the velocity to the piston.

And now same event when you look at different time instant $t = t_2$; obviously, we will see that the second wave is generated here that is a_2 and this a_2 will have higher velocity than a_1 , why? Because when second wave is generated, by the time the first wave has cross the medium, the temperature would have increased from its t_1 .

So, in other words what we say that second wave which is generated in the medium it is moving to a elevated temperature. So, since the speed of the sound is dependent on the temperature; that means, since $a = \sqrt{\gamma RT}$ and T_2 greater than T_1 . So, we say a_2 will be higher than a_1 .

So, in other words what I am trying to say that when same two waves we are looking at different time instant, we will see that the second wave will come closer to the first wave, because it will try to overtake. And we will see this same event that is getting squeezed in a small region.

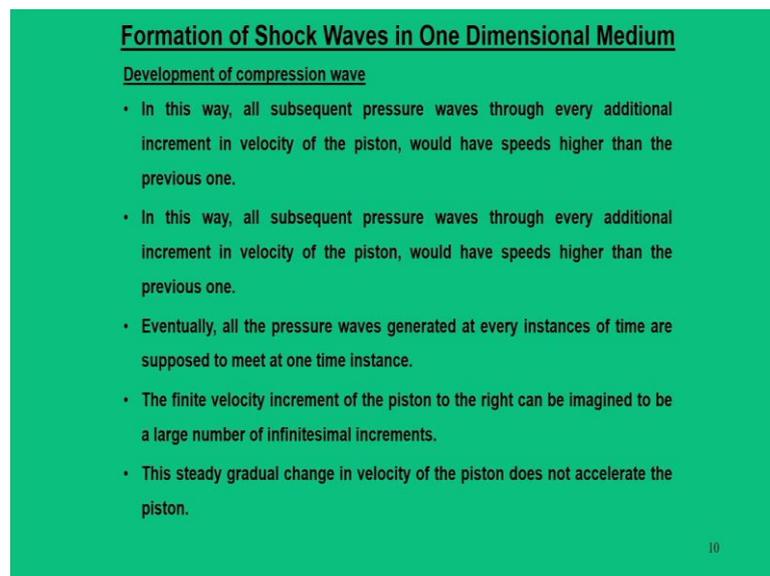
So, we get this p_2 gets squeezed in a small region. So, this is the first wave, this is the second wave and here this side is V is equal to 0, it is p_0 and this is T_1 and this is dV and this is $2dV$. This is how you see. So, all these things, the second wave we will try to overtake.

So, likewise every in additional increment of velocity will lead to different sound waves and all of them will try to move faster than the first wave. So, this essentially means, now we will not use the word sound wave here because when you say the first wave it could be a sound wave, but all subsequent waves will not use as the sound wave will call this as a pressure wave because they are not weak. So, all this pressure waves their moves faster than speed of sound. So, we call this as a pressure wave.

And very bottom line is that as you say there are any number of increments if you say there are infinite number of increments of this velocity, we can say there are infinite number of such pressure waves getting generated in the medium and every pressure wave will have higher velocity than the first one. And with lapse of time they try to merge in a very small region. So, this is the philosophy that happens.

Now, what we see is when you view this velocity increment in very small manner.

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Formation of Shock Waves in One Dimensional Medium

Development of compression wave

- In this way, all subsequent pressure waves through every additional increment in velocity of the piston, would have speeds higher than the previous one.
- In this way, all subsequent pressure waves through every additional increment in velocity of the piston, would have speeds higher than the previous one.
- Eventually, all the pressure waves generated at every instances of time are supposed to meet at one time instance.
- The finite velocity increment of the piston to the right can be imagined to be a large number of infinitesimal increments.
- This steady gradual change in velocity of the piston does not accelerate the piston.

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So, this is how the development of compression wave. So, in this way we call this as a compression waves, because we are not using the word sound wave now, except for the first one which is the first sound wave.

So, in this way in fact, whatever I discuss so far if you summarise them we can say that all subsequent pressure waves through every additional increment of the velocity of the piston, would have velocity higher than the previous ones.

So, all the subsequent pressure waves through additional increment of the piston would have speeds higher than the previous one. Eventually, all the pressure waves generated at every instances of time are supposed to meet at one point. And the finite increment of the piston to the right can be imagined to be large number of infinitesimal increments.

This steady gradual change in the velocity of the piston does not accelerate the piston, because it is steady change in the velocity. So, change in the velocity between two successive time step is very small. So, it essentially does not accelerate the piston.

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Formation of Shock Waves in One Dimensional Medium

Merger of compression waves

- Now, the piston can be imagined to accelerate from rest to a finite magnitude of velocity to the right. So, there will be series of compression waves generated through this process each moving with infinitesimal small velocity.
- The first wave of the series is called as 'head' while the last one is termed as 'tail' of the wave.

Now, here most important things happens that when these waves will try to merge. So, here instead of looking at this event we are now look at the situation in such a way that we are going to accelerate the piston. So, when I am going to accelerate the piston; so, if I plot the velocity and time; so, instead of giving this velocity increment in steps what I am trying to give, I am giving a sharp shoot.

In other words entire steps now becomes a sharp shoot; that means, I squeeze this time t tends to 0, so that I can get the same rise in velocity, but in a very short span of time. So, in this case I will say instead of dV , it is a ΔV . So, what I say here that instead of dV I am giving ΔV to the piston; that means, piston is given a finite value of velocity instantly.

So, when such a things happens we can say that whatever we discuss so far that there will be series of waves that get generated subsequently. So, that means, these small small waves now they keep generating in that small time instance.

So, I can say that, we will have a these pressure waves, we can have a first wave which is getting generated. This is the first wave and this is the nth wave, this is the last wave. So, there are n number of waves that tries to gets generated. Now, with this n number of waves, if you look at the pressure in the medium so, this region we have gas stagnant gas that is V is equal to 0 because the wave has not reached this region. And, all these regions this gas has experienced the finite value of dV and it has seen the pressure p_n .

So, through this process I can say that we can say this pressure p_n that gets generated with n number of waves and if you say the first wave and this is the nth wave and this one we call as 'head' of the wave and this is what we say that is last wave we say 'tail' of the wave. So, that means, all subsequent wave number 2, 3, 4 likewise there are nth waves and all subsequent waves will have speed higher than the first one. That means, nth wave will have speed obviously higher than the n-1th wave.

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Formation of Shock Waves in One Dimensional Medium

Merger of compression waves

- In these series of compression wave, the 'tail' of the wave has higher speed than the 'head' of the wave.
- With the lapse of time, all the compression waves generated through this process will eventually tend to overtake at some point downstream of the tube.
- Though the propagation of the compression waves, the flow property information are sheared to the medium.

So, likewise the event you look is one time instance t_1 , when you see this event in the next time instant, we can see the same phenomena; first wave and nth wave and they tried to squeeze in a small region, I will say this as a Δx .

So, this entire pressure from p_0 to we say p_n that happens in a very small thickness Δx and we are looking this at an another time instance maybe I say t is equal to t_2 . So, with the lapse of time, all the compression wave generated will try to eventually take over at some point downstream. And through this propagation of compression waves, the flow property information are shared. As and when these compression waves moves into the medium the properties are shared, shared I mean that the pressure and temperature in the medium gets changed.

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Formation of Shock Waves in One Dimensional Medium

Normal shock

- At this stage, the isentropic assumption is no longer valid because the viscous and heat transfer effects can not be neglected.
- The waves would try to 'topple' with one another. Physically, it means different values of flow property information at same time at a given location which is an absurd situation in the medium.
- Hence, the compression waves tend to overtake, the phenomena may be viewed as merger of all compression waves instantly at a given location and the process becomes infinitely 'steep'.

So, this is how we see the sequence of event. So, in the sequence of event when I say $t = t_1$, $t = t_2$ and I will say this t is equal to at one particular important situation t_n , where location x is equal to x_n . So, here I say this finite length Δx_1 and this is a Δx_2 and we say here Δx tends to 0.

So, what happens here, wave number n eventually with respect to time and time t is equal to t_n , what will happen the n th wave will try to overtake. So, when the n th wave try to overtake, it means n th wave has merged with $n-1$ th wave then subsequent $n-2$ waves and finally, second wave gets merged and finally, and in the end it will merge with the first wave.

So, when such a things happens it may happen that the n th wave will try to overtake. So, when we tries to overtake we can represent such a phenomena as a dotted line. So, that

nth wave try to overtake the first wave so; that means, that will be this wave is at the top and this first wave is getting lagged.

So, physically what it means that, in a given location of the flow field x_n will have a different properties which is not true. So, if you look at this particular phenomena physically that does not true because at a given location and given time, the gas has different values of properties. By different value properties means the first wave will have assigned has one properties and nth wave will have another properties.

So, this is not true or not physical in nature. So, such a event will try to avoid such things, because natural occurring phenomena will give a solution to it as a merger of shock wave. So, this merger of shock wave and in this case it becomes a normal shock. So, in other word it means there is no compression of waves as such now.

So, in these two situation there are infinite number of compression waves, but in this situation there is no compression waves, because they all these compression waves has merged to a shock wave. And since, it is a one dimensional medium we call this particular shock wave as a normal shock.

So, this is how I tried to explain this particular figure and whatever has been written we can say that at this stage, when there is a normal shock wave is formed its isentropic assumption is no longer valid because viscous heat transfer effects cannot be neglected. All the waves will try to 'topple' with one another, it means it will assign different values of flow property at same time at a given location which is absurd situation in the medium.

Now, hence the compression waves tend to overtake, this phenomena may viewed as a merger of all compression waves instantly at a given location and the process becomes infinitely 'steep'. So, instead of the property change what that has supposed to happen gradual manner, this gradual manner has now happens in a very steep manner; that means, change is instantaneous and change in a very small thickness domain.

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Formation of Shock Waves in One Dimensional Medium

Normal Shock

- A naturally occurring process would render the merger of all compression waves through a single wave at a particular time instance and known as “shock wave” having a very small thickness (typically, 10^{-7} m).
- Mathematically, it is represented instantaneous step change and discontinuity in the medium with a limiting case when time tends to zero.
- Physically, all the flow properties undergoes finite but large changes across the shock wave in non-isentropic manner.
- It may be noted that the weakest compression wave is a “Sound wave” whereas a “normal shock wave” is a strongest compression wave. A “Mach wave” is the initiation of shock wave.

Normal Shock

Given conditions	Unknown conditions
p_{01} p_1 T_1 T_{01} ρ_1 u_1 $M_1 > 1$	$p_2 > p_1$ $T_2 > T_1$ $p_{02} < p_{01}$ $\rho_2 > \rho_1$ $u_2 < u_1$ $T_{02} = T_{01}$ $M_2 < 1$
Ahead of shock/ Before the shock	Behind the shock/ After the shock

So, this is how we say a shock wave is formed and the simplest form of a shock wave is a normal shock, but it is very strong. So, coming back to our physical phenomena we say a natural occurring process will tell us that a merger of all compression waves will lead to a single wave at particular instant of time and we call this as a “shock wave” and physically it has a thickness of in the order of 10^{-7} m which is almost is very less. And mathematically it is represented as a, this is instantaneous change or discontinuity in the medium with a limiting case when time tends to 0.

So, now bringing all this mathematic and physical nature we can now for our compressible flow field what we can say, the flow properties undergoes a finite change. There is a finite change obvious, but it is large, across a shockwave in a non- isentropic manner. So, what we can view is a normal shock, the property change are finite, but very large. For instant what you can say that for a standing normal shock like this, if I assign the properties as p_1 , T_1 , ρ_1 , u_1 , stagnation pressure p_{01} , stagnation temperature T_{01} , then what I am going to see.

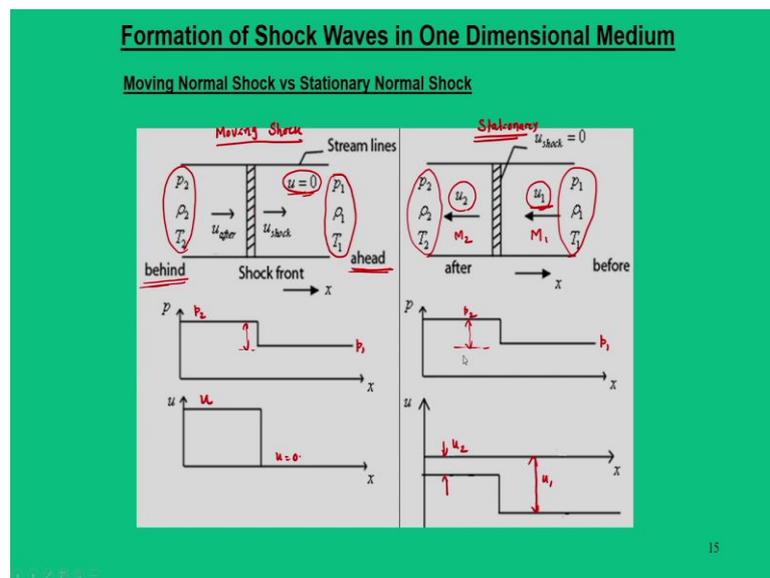
So, this is a given condition the whatever I am going to see is that, pressure is going to suit up; that means, static pressure is tremendously high, total pressure will drop likewise and Mach number will become all of a sudden subsonic.

So, this says that these change takes place and in this situation the process is happens in a non-isentropic manner; that means, s_2 must be greater than s_1 . If your entropy are calculated we can say entropy it is a non-isentropic process entropy greater than s_1 .

So, as I discussed all these things that the weakest compression wave is a “sound wave” that is what in the beginning I said that when you give a push to the piston, a sound wave is generated. Now, when increment in the push is more and more, we try to increase more changes in the medium.

And, at one point at time it becomes a strongest compression wave and that is we call this as a shock wave. But since in a supersonic event, shockwave is a natural occurring process, but its initiation; that means, when Mach number just tries to become slightly higher than 1, it is seen as a Mach wave. So, “Mach wave” is initiation of the shock wave.

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Now, here we will talk about something on moving shock and stationary shock. I also imagine that a shock front can move or shock front can be view as stagnant. So, in the first figure what we are seeing in a one dimensional medium consisting of large number of streamlines, we are saying that a shock front is moving, then when the shock front is moving; so, your initial gas has no velocity here.

So, we can say that when the shock front is moving in one direction, initial condition of the medium is p_1, ρ_1, T_1 and final condition of the medium is p_2, ρ_2, T_2 . And as I said since in this medium the shock wave has not crossed; so, it's ahead of the shock. And in this case the shockwave is already crossed; so, we say behind the shock.

So, likewise the similar way we can say pressure p , initial pressure let us say p_1 and initial pressure which becomes p_2 . So, this is how we view that property change across the shock wave. Now, in this case the shock wave has not crossed. So, gas velocity u is equal to 0 and here it has a finite value of u . So, this is a scenario when we have a moving shock.

Now, in the same scenario we bring this shock stationary; we say it is a stationary medium. So, why we do this, because let us say if at all we want to come across similar pressure change across a normal shock. What do you do here is arrest the shock to stationary; so, that means, there is no shock velocity; you allow the gas to pass through the shock wave.

So, in other words what we are trying to say to come across similar jump that we had in the moving shock, if you want to have in the same change in the stationary shock; then, what would have been our scenario.

So, here the terminology that we use as before the shock and after the shock. So, before the shock our case condition was this p_1, ρ_1, T_1 , but you allow the gas to move at u_1 and finally, we arrived this final velocity of the gas u_2 . So, this corresponds to a Mach number M_1 and this corresponds to a Mach number M_2 . So, this is after the shock.

Now, if same thing, if you want to see here, the pressure signal change will remain the same, but what changes is the gas velocity. So, here what we have to do, since it is a relative velocity, we were looking here in a laboratory frame for a moving shock, but when we are looking in a laboratory frame as a stationary shock what we see here is gas is moving. So, your initial velocity will now become u_1 which is this one that corresponds to Mach number M_1 and the final velocity is u_2 . This is how the change that happens here.

So, in our scenario we will only talk about a stationary shock situations and will try to see what are the conditions before the shock and after the shock. So, this particular

correlation was done so that in many practical problems we will try to see that moving shock situations, but if you can bring them into a stationary shock situations the analysis will become simpler.

So, that is what this judgement is made that a moving shock analysis how it is frame to a stationary shock reference and with that respect how the change is going to happen. So, with this logic we will move further only towards the stationary shock analysis.

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Fundamental Equations for Normal Shock

Assumptions

- A “normal shock” is one across which, the properties change drastically in one direction.
- A normal shock wave is perpendicular to the streamlines. Hence, it is modelled as flow in a “stream tube” in which the streamlines are parallel to each other.
- The analysis of a stationary normal shock wave is done by considering an one-dimensional flow across a constant cross-sectional area stream tube.

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To have this stationary shock analysis we will now talk about some fundamental equations of normal shock in the similar way what we did earlier for an isentropic flow in a one dimensional medium.

So, what we say, a “normal shock” is the one in which properties change drastically in one direction. So, here we view this normal shockwave as a stream tube; as I mention its a imaginary duct, we can assume to be a one dimensional duct which is imaginary duct which means as if there is no boundary.

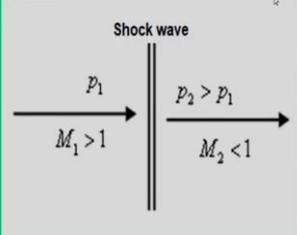
Although there is no physical boundary, but we can assume to be imaginary boundary that enclose this normal shock and the streamlines that comes there are perpendicular to this shock wave. So, obviously we call this as a “stream tube” and the stream lines are parallel to each other, but they are normal to the shock wave. And this analysis was consider in one dimensional flow framework with constant cross sectional area.

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Fundamental Equations for Normal Shock

Assumptions

- The frictional effects are negligible because the shear stress present within the small boundary layer is very small. In other words, the boundaries of the stream tube (imaginary duct) can be removed so that effects of boundary layer, wall friction and shear stresses can be neglected.
- Thus, the configuration can be treated as constant area duct.
- The shock thickness is very small compared to dimensions of the stream tube.



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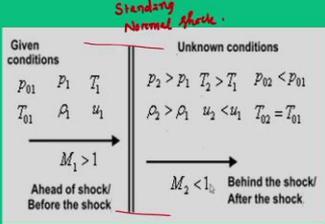
Another kind of assumption that we make is that since we say there is no physical boundaries. So, it is a imaginary duct. So, another assumption we can make there is no frictional effect, because shear stress are absent and there is no boundary layer and the configuration of the duct can be treated as a constant area duct. The shock thickness is very small compared to the dimension of the stream tube; that means shock thickness is very small with respect to stream tube dimension.

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Fundamental Equations for Normal Shock

Assumptions

- The flow processes including the shock wave can be considered to be adiabatic with no external work.
- The flow across the shock wave is considered to be steady.
- Typically, all the property conditions ahead of the shock (region 1) are known while the flow properties behind the shock (region 2) need to be found out.



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And the flow process includes to be adiabatic; that means, if you say it is imaginary duct there is no heat addition into the duct. And the flow process in the shock wave is assumed to be steady, there is a steady flow and the all the properties conditions in the region 1 and region 2 needs to be mapped.

So, basically this is the given problem that we have a standing normal shock and before the shock we have the given conditions of static pressure and static conditions and stagnation conditions, flow velocity. These are the given conditions and we were supposed to find out the unknown conditions that what happens after this shock.

So, after the end of the analysis we will try to see that what is the relation between the static pressure ratio before the shock and after the shock, static temperature ratio before the shock and after the shock. And, that we are trying to correlate with information in the velocity or flow Mach number that is before the shock and after the shock.

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Fundamental Equations for Normal Shock

Continuity: $\rho_1 u_1 = \rho_2 u_2$

Momentum: $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$

Energy: $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$

Thermodynamic relation: $p = \rho R T$; $h = c_p T$; $a^2 = \frac{\gamma p}{\rho} = \gamma R T$

Given conditions			Unknown conditions		
P_{01}	P_1	T_1	$P_2 > P_1$	$T_2 > T_1$	$P_{02} < P_{01}$
T_{01}	ρ_1	u_1	$\rho_2 > \rho_1$	$u_2 < u_1$	$T_{02} = T_{01}$
$M_1 > 1$			$M_2 < 1$		
Ahead of shock/ Before the shock			Behind the shock/ After the shock		

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So, this is the entire philosophy of our study and finally, we will use the same one dimensional framework equations that is continuity equation $\rho_1 u_1 = \rho_2 u_2$, momentum equation $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$ and since it is a constant area; so, there is no area term here in this equation.

And in fact, for the energy equation we use the same equation what we did for isentropic flow that is total temperature for region 1 and region 2 and here we have to assume that q to be 0, because it is an adiabatic duct.

But here our philosophy is that we do not know the conditions p_2 , but we also do not know what are the conditions M_2 . In isentropic flow with known Mach number we can correlate the property information at different locations in the flow field.

So, its adiabatic situation, there is no heat addition and apart from this we will use these other relations that is $p = \rho RT$, $h = c_p T$. This is the calorically perfect gas, this is the ideal gas equations and also use this term $a^2 = \frac{\gamma p}{\rho} = \gamma RT$.

So, this is the speed of sound; obviously speed of sound is very important here, because the Mach number is going to change. And in fact, we have to say except this thin shockwave region all the flow fields are isentropic. So, with this we will conclude for this lecture today.

Thank you for your attention.