

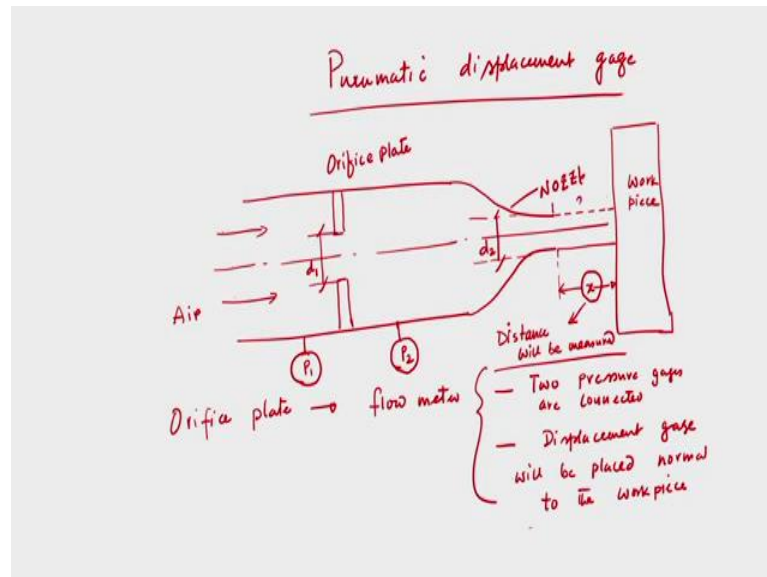
Experimental Method in Fluids Mechanics
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Lecture 08

Dimensional measurement Gauge blocks, the pneumatic displacement gauge
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Displacement and Area Measurement: Dimensional measurement Gauge blocks, The pneumatic displacement gauge.

Good Afternoon, I welcome you to this session of experimental methods in fluid mechanics. Today we will continue our discussion on, displacement and area measurement. In fact in my last lecture, I have discussed about the, you know, pneumatic displacement gauge. So, and we will continue with that. So, today again I need to draw that schematic of our pneumatic displacement gauge then we will see that how we can measure that small distance x that is what we can, we have seen in the last lecture.

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So, to start with again I will be, you know drawing that schematic of a pneumatic displacement gauge. So, pneumatic displacement gauge. So, this is basically used to measure displacement, to precise a very small displacement. So, again I am drawing the schematic. So, basically it is having you know, basically it is having a circular tube and it is this you know this circular tube containing and orifice plate.

This orifice plate is basically so, this is the orifice plate, and that air is flowing from left to right. Now, this orifice plate so, the basically the flow meter we have, in fact the you know in most of the undergraduate text book, this orifice plate or this is a flow meter is discussed you know is described in detail. So, orifice plate, orifice plate, it is a flow meter.

So, this flow meter is now placed inside the cylindrical set of body and now, this orifice flow meter is followed by a nozzle on a common axes. So, it will have a nozzle, like this and the area, I mean distance the diameter of the nozzle that is d_2 and diameter of this orifice plate, say this is d_1 . So, this is orifice plate and this is nozzle and we have object over here and that object, say we need to know the displacement, small displacement x , through a, through this displacement gauge, rather using this displacement gauge.

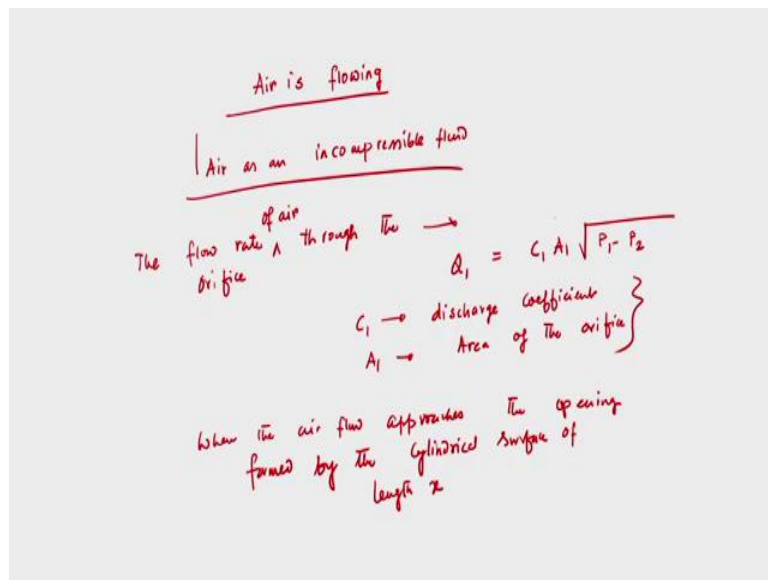
So, this is a work piece. Now, this is the, you know construction details of a displacement gauge. Now, two important things, one is that this we need to have, we need to you know, connect two special gauges, I mean one is upstream of the orifice plate and another is downstream of the

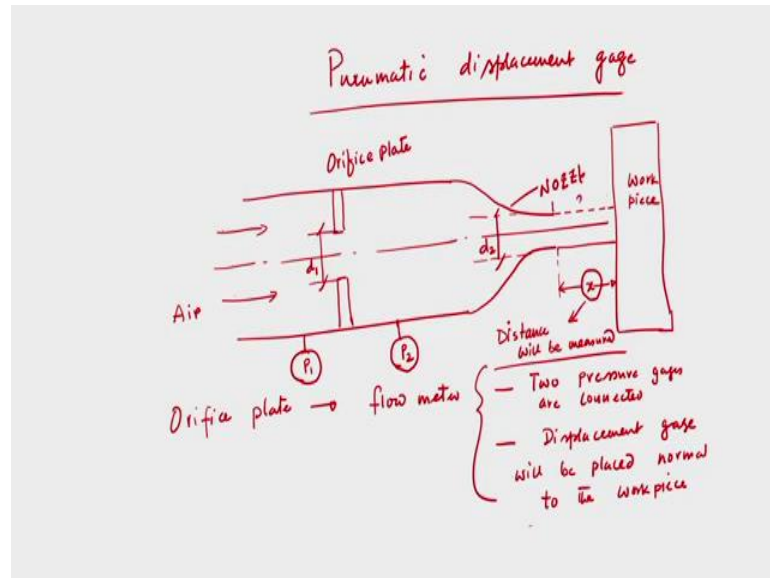
orifice plate, say this is P1 another is downstream of the orifice plate part it is in between orifice plate and nozzle so P2.

So, now this is there that two pressure gauge, two pressure gauge are connected and the, this you know orifice meter plus nozzle that these 2 parts constitute you know together to form a displacement gauge and this displacement gauge will be placed normal to the work piece or object. So, these are the two important things, otherwise we cannot measure the small displacement and I am again, you know marking this small distance that we would like to measure.

So, this is the distance will be measured. So, now this is what I have, you know drawn in last lecture but again I am, I have drawn it, essentially to have, you know convenient in our you know. Understanding that we need to now, derive the mathematical equations and while we are deriving the mathematical equations it would be easy if we have the schematic with us. So, to have it, to have it we again we have drawn the schematic.

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Now, question is air is flowing and perhaps I have you know mentioned this, that for this analysis we are considering that, air as an incompressible fluid. This is another important assumption, air as an incompressible fluid. That is what we need to consider for this analysis.

Now, the fluid if we refer to that, you know schematic, then the flow rate, the flow rate through the orifice, through the orifice because the orifice is essentially a flow meter. So, the flow rate to the orifice will be say, Q_1 and that will be called to $C_1 A_1 \sqrt{P_1 - P_2}$. So, because we have, if I go back to my previous slide where it is clearly seen that two pressure gauges are connected, one is at the upstream another is at the downstream of the orifice plate.

So, definitely the drop in pressure between whenever the, whenever air is passing through this orifice meter, will be measured by these two gauges. Now, this is the flow rate, expression of the flow rate of air through the orifice meter. So, basically this is flow rate air through this orifice meter.

Now, this is very common expression we know, because we have started this in fluid mechanics and, and C_1 is known as discharge coefficient. So, this is known as discharge coefficient and A_1 that is known as area of the orifice very important. Now, the flow rate through the opening rather flow rate through the nozzle, opening formed by the cylindrical surface that is what we can see from the schematic, say this flow rate again will go through this you know cylindrical body and the cylindrical circular tube and then ultimately it will approach the nozzle and then we have to again calculate what will be the flow rate.

Because, whenever the air flow approaching that you know nozzle part that means an opening formed by the cylindrical surface, circular tube definitely but that was from using a cylindrical surface. So, basically when the airflow approaches the opening formed by the cylindrical surface and see again I am going back to this, say this cylindrical length is x that is what we would like to measure now.

Here, this length is x that is what we would like to measure, again this is x. So, as if the opening area formed by cylindrical surface of length x, cylindrical surface of length x. So, ultimately, after passing through the orifice meter, this air will pass through the area, rather the opening formed by the cylindrical surface of length x. Now, since the flow rate must be same in both the cases and we have considered here as an incompressible fluid.

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Handwritten mathematical derivation showing flow rate through a nozzle and the relationship between flow rates through two different openings.

$$\begin{aligned} & \text{flow rate through nozzle} \\ & Q_2 = C_2 A_2 \sqrt{P_2 - P_a} \\ & \text{flow rate must be same} \\ & Q_1 = Q_2 \rightarrow C_1 A_1 \sqrt{P_1 - P_2} = C_2 A_2 \sqrt{P_2 - P_a} \\ & \Rightarrow \frac{P_2 - P_a}{P_1 - P_2} = \frac{C_1^2 A_1^2}{C_2^2 A_2^2} \\ & \Rightarrow \frac{P_2 - P_a}{P_1 - P_2 + P_2 - P_a} = \frac{C_1^2 A_1^2}{C_2^2 A_2^2 + C_1^2 A_1^2} \\ & \Rightarrow r = \left(\frac{P_2 - P_a}{P_1 - P_a} \right) = \frac{1}{1 + \frac{C_2^2 A_2^2}{C_1^2 A_1^2}} = \frac{1}{1 + \left(\frac{C_2 A_2}{C_1 A_1} \right)^2} \end{aligned}$$

So, we can write that again the flow rate while passing through that, you know cylindrical safe opening area, length is x, that means, flow rate through the nozzle, through nozzle, say Q2 that is C2 A2 into root P2 minus Pa, because that is atmospheric pressure.

Now, these two flow rates are equal, because mass flow rate must be same. So, basically this flow rate must be same. So, Q1 equal to Q2. So, from this expression we can write C1 A1 into P1 minus P2, that equal to C2 A2 root P2 minus Pa where P1, P2 are the upstream and downstream pressure of air, I mean upstream, the you know gauge pressure measured at the

upstream of the orifice plate and P2 is the pressure measured at the downstream of the orifice plate.

So, these two are equal so, from these expression we need to do, you know some algebraic manipulation, then what we can write that. Now, basically P1, P2 minus P1, sorry, P2 minus Pa divided by P1 minus P2, that is, that is C1 square A1 square by C2 square A2 square. So, this is what we are getting and if would like to write this expression in a bit different form that P2 minus Pa by P1 minus P2 because, see our target is to obtain.

See, Pa we know because atmospheric pressure we know P2 and P1 that we need to measure using the gauge, you know that pressure gauges that, pressure gauge that those are placed. Now, what we will do? We can write that P2 minus Pa divided by P1 minus P2 plus P2 minus Pa, that C1 square A1 square by C2 square A2 square plus C1 square A1 square.

That means we can write P2 minus Pa divided by P1 minus Pa and this expression we are giving a name say, this is call r, and this is equal to 1 by 1 plus C2 square A2 square by C1 square A1 square. Very simple straightforward equation. So, we can write it that, 1 minus, sorry, I can write 1 step further that 1 minus, 1 plus C2 A2, C2 by C1 A2 by A1, you know, square, square.

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The image shows handwritten mathematical derivations. At the top, it states $P \approx 1 - \left(\frac{C_2}{C_1}\right) \left(\frac{A_2}{A_1}\right)$. Below this, a red arrow points to the equation $\frac{P_2 - P_a}{P_1 - P_a} \approx 1 - \left(\frac{C_2}{C_1}\right) \left(\frac{A_2}{A_1}\right)$. Two boxed equations define the areas: $A_1 = \frac{\pi d_1^2}{4}$ and $A_2 = \pi d_2^2$. The next line shows the derivation of the pressure ratio: $\frac{P_2 - P_a}{P_1 - P_a} = r \approx 1 - \left(\frac{C_2}{C_1}\right) \left(\frac{A_2}{A_1}\right)$. This is followed by $r \approx 1 - \left(\frac{C_2}{C_1}\right) \frac{4 \pi d_2^2}{\pi d_1^2}$ and finally $r \approx 1 - \left(\frac{C_2}{C_1}\right) \frac{4 d_2^2}{d_1^2}$.

So, what I can write again that r equal to 1 minus C2 by C1 into A2 by A1. So, this is almost equal to, from the previous expression I can write this is almost equal to. Now, why we did this? If we look at the expression, you know then what we can see that this equation, only we only had

to measure gauge pressure at only one point P2, because r is equal to $P2 - P1$ by $P1 - P_a$ equal to that is what we are writing, $1 - C2$ by $C1$ into $A2$ by $A1$.

So, from this expression it is clear that we need to measure pressure, rather we only have to measure pressure, gauge pressure at 1 point that is P2. As the supply pressure P1 can be maintained at constant value because, that that is pressure upstream, pressure at upstream location of the you know orifice plate, it can be maintained at constant value the supply pressure.

Now, only one pressure we need to measure that is P2 through a, that that gauge pressure need to measure. Now, areas because A2 and A1 that is very important because areas we know, whenever we are designing orifice plate, we know what is A1 and also A2 we know. Now, A1 that area of the orifice plate that is essentially from the geometry, you know from the dimensions given in that schematic, $\pi d1^2$ by 4 and A2 that is $\pi d2^2$ into x because it is cylindrical shape nozzle, so the nozzle shape is cylindrical.

So, now if we know A1 and from the geometry of there, you know of the nozzle and orifice, then that we can write r , that r equal to that is we can write $P2 - P1$ by $P1 - P_a$ that equal to r , sorry, that equal to r equal to $1 - C2$ by $C1$ into $A2$ by $A1$. So, again I am telling that we did this manipulation only because from this expression it is clear that, only we need to measure at, you know location, that is you know at a given location that is downstream of you know that orifice plate P2 through only 1 gauge pressure and from there we can get the, you know a relation between r and the area received and this is constant you know distance coefficient C2 and C1.

Now, this is, this can be, this is you know true because the pressure P1 can be maintained a constant pressure. Now, question is that r , if I write r that is very important because r we can measure, we know P_a , we know P1 that will be constant but P2 will change definitely. Now, if we measure P2 then I can have the numerical value of r through this expression.

Now, if I can somehow relate r with x then we can measure the small displacement. Now, what is r ? r is equal to $1 - C2$ by $C1$, these constants are again (18:12) because we can get it from the, what is the value of these such coefficient. Typically their values varies from 0.85, 0.88 like this, because that is available in most of the, you know undergraduate textbooks but otherwise again we have to calculate to experimental observation, experimental calibration.

Now, this I can write A2 and A1 in terms of you know, their relation that is what we have written above that is $\pi d_2 x$ divided by πd_1^2 by into 4. Now so, that is r equal to $1 - \frac{C_2}{C_1}$, this again constant 4, $d_2 x$ by d_1^2 . Now, it is seen from this that, r can be related with x so that the displacement, small displacement that is what our objective is to measure using this gauge, can be calculated, can be measured through this gauge.

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$0.4 < r < 0.9$

$$r \approx \frac{P_2 - P_1}{P_1 - P_2} = 1.1 - 0.5 \left(\frac{A_2}{A_1} \right) = 1.1 - 2 \left(\frac{d_2}{d_1} \right)^2 x$$

Supply Pressure $P_1 = P_2$
 then Pneumatic displacement gage pressure $\rightarrow (P_2 - P_1) = P_r$

We can obtain a linear function between x & P_r

$$x = \left(\frac{d_1^2}{2 d_2 P_2} \right) \left[1.1 P_2 - P_r \right]$$

$\left(\frac{d_1}{d_2} \right) \frac{d_1}{2 P_2}$

$$\frac{x}{d_1} = \left(\frac{d_1}{d_2} \right) \frac{1}{2 P_2} [1.1 P_2 - P_r]$$

Pneumatic displacement gage sensitivity

Pneum

The sensitivity of this gage is indirectly affected by the supply pressure as we will only be able to reliably measure pressure above a given value depending on the manometer we are using

Now, experimentally we need to know what is a value of r , I mean experimentally it is, we find that if the value of r it becomes less than 0.9 and greater than 0.4, that was experimentally

observed by the Avers and his colleagues and that the value typically, the value of r is typically less than 0.9, greater than 0.4. Now, it varies between these 2 values.

Now so, this r basically can be retained, r equal to P_2 minus P_a by P_1 minus P_a equal to 1.1 minus $0.5 A_2$ by A_1 is equal to 1.1 minus $2 d_2$ by d_1 square into x . So, better approximation that was given. If the value r varies between in this range then a better approximation for r is given by this. Now, from this it is, it can be seen that we can now measure x by how again?

So, if we define the supply gauge pressure that is you know, P_1 . So, basically we have supply pressure, supply pressure P_1 , if this is P_s and then the pneumatic displacement gauge pressure, then, then pneumatic displacement gauge pressure, displacement gauge pressure that is equal to P_2 minus P_a . So, that equal to say, P_p if I write then we can have, then we can obtain a linear function between x and P_p .

As then x will be equal to d_1 square by $2 d_2$ into P_s into $1.1 P_s$ minus P_p . So, this is the linear function between x and P_p . So, if we maintain supply pressure at supply gauge pressure rather, if I maintain supply gauge and you know supply gauge pressure that is P_s and then the pneumatic gauge pressure that will measure that is P_p and then we can have this linear function, you know between x and P_p .

So, now measuring P_p we can have, because we know d_1 . Now, measuring P_p we can obtain the displacement x of that object and it is used normally to measure a very small displacement. Now, from this expression I would like to say that it can be again you know formed in a different way, that, I am writing. So, this expression say this term can be written that d_1 by d_2 , into d_1 , by twice P_s , twice P_s . Now, the pneumatic displacement sensitivity, that is very important that x by d_1 .

So, if I write this expression again that I can write x is equal to, x is equal to, rather x by d_1 is equal to x by d_1 is equal to d_1 by d_2 , d_1 by d_2 into rather d_1 1 by $2 P_s$ and 1 minus rather $1.1 P_s$ minus P_p , $1.1 P_s$ minus P_p . So, this x by d_1 is basically known as pneumatic displacement gauge sensitivity.

So, this is pneumatic displacement gauge sensitivity and this pneumatic gauge pneumatic displacement gauge sensitivity depends upon diameter ratio d_1 by d_2 . Because it is clearly seen from this expression that it is a function of d_1 by d_2 and of course P_s because P_p that we will

measure. So, basically upstream pressure rather supply pressure P_s and the diameter ratio d_1 by d_2 .

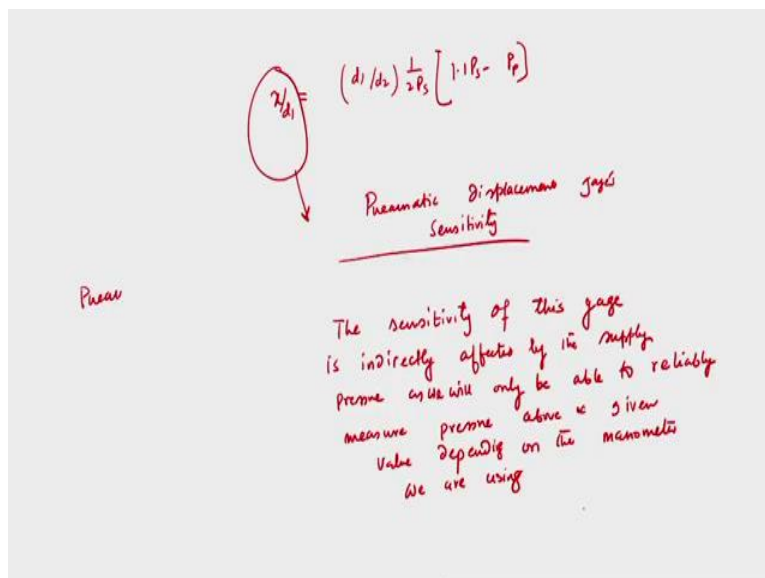
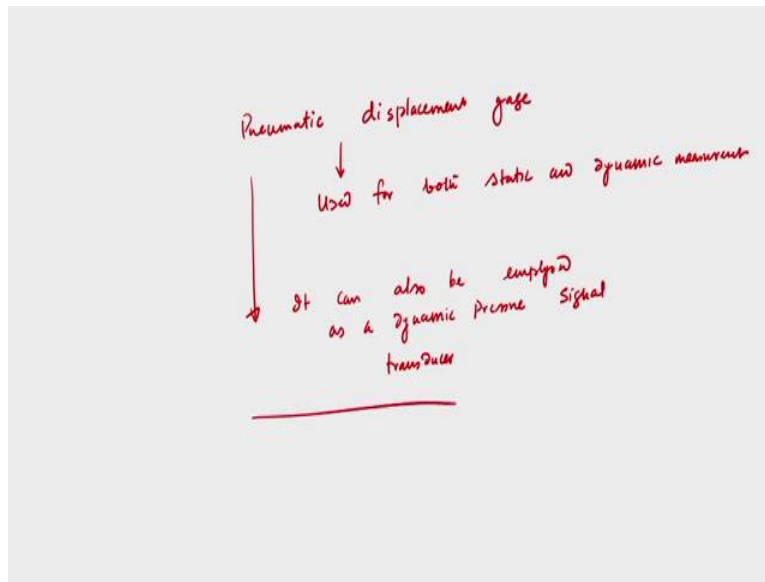
These two important, you know factors are responsible for the, for measuring or for dictating the $(())(25:03)$ in pneumatic displacement gauge sensitivity. So, another important thing that the sensitivity of this gauge is indirectly affected it is not directly affected rather indirectly affected by the supply pressure as will only be able to reliably measure pressure above given value, above the given value depending on the manometer we are using.

So, this very important, I am writing this again that the sensitivity of this gauge, the sensitivity of this gauge, of this gauge is indirectly you know affected by the supply pressure, supply pressure as will only, as we will only be able to measure pressure above a given value, given value depending on the manometer we are using, manometer we are using.

So, as I said you that the sensitivity depends upon diameter ratio as well as P_s but the sensitivity of this gauge is indirectly affected by the gauge, supply gauge pressure P_s as we will only be able to measure pressure rather we will only be able to reliably measure, that is very important word, that reliably measure pressure above the given value depending upon the manometer we are using.

So, this another important aspect and this pneumatic displacement gauge is used for both the steady and dynamic measurements and it can also be employed as a dynamic pressure signal transducer.

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So, this pneumatic displacement gauge rather I am writing in a next slide that this pneumatic displacement gauge it is used, you know it is used for both static and dynamic measurements, for both static and dynamic measurements and this pneumatic displacement gauge can also be employed as a dynamic measure, dynamic pressure sensor transducer.

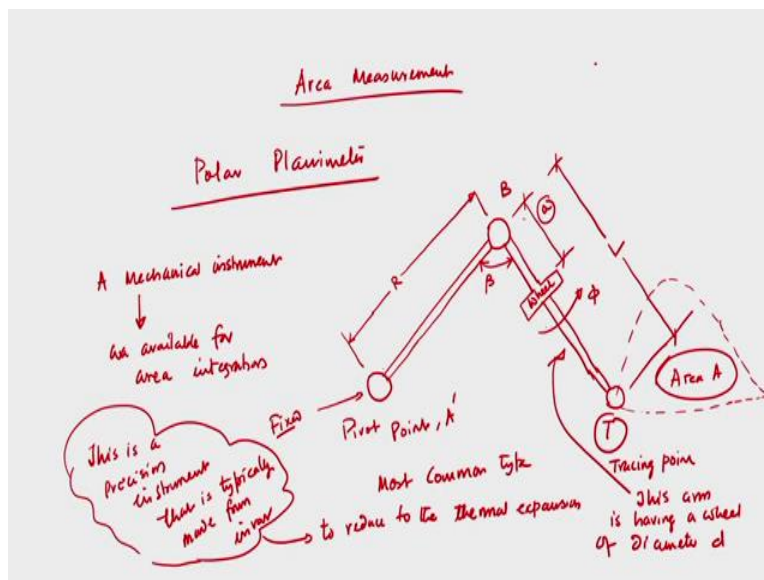
So, it also it you know can also be employed as a you know dynamic pressure signal transducer. So, from this discussion we can say that essentially would like to measure pressure, it is an, if I go to my previous slide we have as pneumatic displacement gauge sensitivity, that depends upon

the diameter ratio as well as upstream pressure P_s , but again we have seen that this is indirectly affected by it.

That is the sensitivity is indirectly affected by the supply pressure because we are having manometer so basically we will be able to reliably measure pressure above the given value depending upon the manometer we are using, that is one important thing and from this expression that is what I have discussed that if I know the diameter ratio and if I calculate P_p that is of course we need to have one pressure gauge that is manometer we have to, you know, insert and from that we will get the reading of P_p and if I know the P_s that is supply gauge pressure that will be, you know maintained at you know at its constant value.

So, from there we can calculate x . Now, x that is basically the function we are used that this r it varies between 0.4 to 0.9 then perhaps we can write like this, but that is again experimentally observed value that it varies almost, you know very linearly. Now, now, now, from this we can obtain the, you know x , what will be is minimum x . So, this is basically what do we do, we measure pressure difference that pressure difference in a way gives us an indication about the displacement x of that object from the nozzle. So, now this is all about the pneumatic pressure gauge.

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Now, we will discuss about area measurements, I will briefly discuss about the area measurements. See area measurement is remarkably difficult task for a general reason. It is not

so easy and simplest method we know that if we have a boundary, if you would like to, area of a boundary and then we have to you know plot the boundary on a graph paper which has a ruled square, there are ruled grid and count the number of square completely within the region and number of square which are on the boundary and then we can calculate area, but that is again it is not so easy and accuracy of this method is you know dependent on the size of the square.

So, basically, this method is used for the rough estimate, I mean for the rough estimate, for rough estimate and as a check on the integration scheme, on other, you know integration scheme. So, basically what I am telling that area measurements is not so easy, one of the simplest method is that we have to plot in our particular area on a graph paper and then if we can calculate the, of course the graph paper will have ruled you know grid and if we measure the number of square present within the region and number of square present on the boundary, from there we can calculate, but this is again a rough estimate and accuracy of this method is not very good, rather it depends upon the size of the square.

Rather, today will discuss about the another one meter through which we can measure area and that is known as planimeter, the polar planimeter. So, again I will try to draw the schematic of that, you know, meter, that is known as polar planimeter. So, now, I will draw the schematic and then I will explain by how it, it measures area.

So, say I am drawing that polar planimeter we have, actually it is having 2 arms and say this is one arms and like this pivoted point. So, basically this is pivot point and this is another pivot point, say B and this is pivot point say A. Now, and distance of this arm is say R, so, distance is R between these 2 pivot points.

Now, again we have another arm which is again connected with one wheel, this wheel and again it will have extended portion and another pivoted point like this and this is wheel wh double e l, that is wheel and distance of this wheel from these pivot points B is say a, and this angle is beta and, and this wheel actually can be you know rotated. So, say this is phi and distance of this arm is say L.

So, this is the schematic and maybe we would like to measure one area so, say like this, we would like to measure area like this and this is area A that is what we would like to measure, area here. So, the planimeter actually you know plot polar planimeter is most common type

mechanical instrument, this is a mechanical instrument of course, mechanical instrument and this is available for the area integration.

Basically we, we measure area through area integration so this is available, this is available for area integration and will discuss about a few things but before that we need to know what are the, you have drawn the schematic but we need to know that how it is functioning. So, this is the most common type, this is the most common type and it is constructed from 2 arms that is, that is clearly seen from the schematic that are pivoted at point B and one arm of length R has a pivot point O that can be fixed.

So, I mean one arm R that is a pivot point say A prime that is fixed. So, this is fixed, this point is fixed and other arm that is having length L has a tracing point T, say this is point, tracing point T. So, this point T is tracing point. So, this is a mechanical instrument. This is a precision instrument that is typically made from, very precision instrument. So, this is I am writing a precision instrument that is typically made from ember, typically made from ember.

Function, because to it is made from ember essentially to reduce the thermal expansion, because it is used to measure area integration, available for area integration. So, very important is that it should not have, the arms should not have the expansion, due to you know, thermal gradient or temperature change.

So, to reduce that we, this you know, instrument is typically made from ember and these 2 arms are you know pivoted at point B, you know and that is the an arm one first you know the arm are, I mean one arm which is having length R and we can say the arm R which is fixed you know at point, pivoted point A prime, while the arm L which is having another tracing point T and this point is used to stress the, you know, reason we would like to measure.

Now, the arm L rather the length of the, you know arm is L second arm, which is having one wheel of diameter d. So, basically this wheel the, this arm or second arm, this arm, this arm is having a wheel of diameter d and so, this wheel can be rotated and, and that is, that is what I have given the phi so, this is free to rotate about the axes of the arm and the distance between the wheel and pivot I mean, B.

So, basically these 2 arms are connected at point B and wheel is a distance small a distance apart. So, this is a small a distance which is a distance apart from the pivot, pivot B. Now, basically the

area, this is the schematic of the area in a palni in a polar planimeter, that is used to measure area. So, the tracing point which is connected through wheel I mean and the wheel can freely rotate about the axes of the about its axes or rather about the axes of the arm and the trace, tracing point move and to trace the you know, region for which we would like to measure the area.

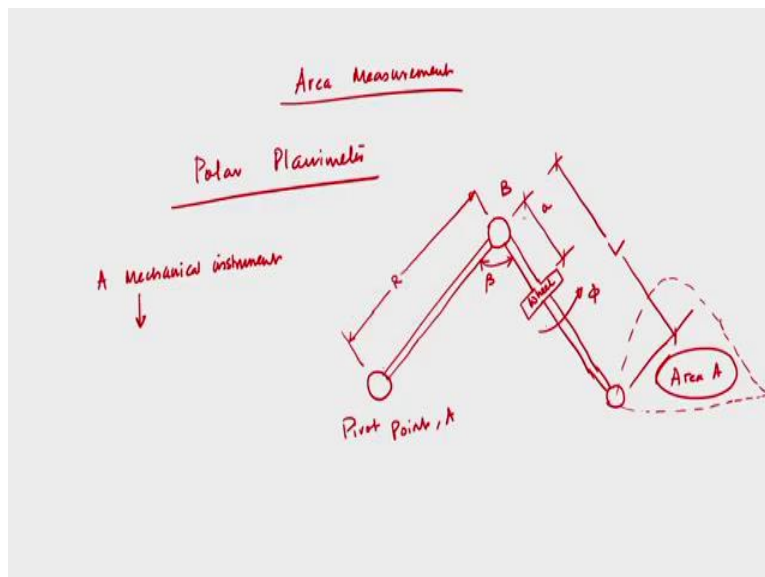
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The area is obtained from a counter which is connected to the wheel using the following formula

$$\text{Pole outside area} = \pi dLn$$

$$\text{Pole inside area} = \pi dLn + (R^2 + L^2 - 2aL)\pi$$

$n \rightarrow$ is the number of revolutions of the wheel



Now, now, now, the area is obtained from a counter that is connected to the wheel using the following formula. So, basically now I am writing that the area is obtained, very important the

area is obtained, area is obtained from a counter, from a counter which is placed or that is connected, or which is connected, connected to the wheel, using the following formulas.

So, this is by how we measure areas because the tracer point will trace. Now, this is connected with the wheel, wheel can rotate freely. Now, as it is rotating freely, then counter is there which is connected to the wheel and that will read area and which is, you know, measured based on the following, following 2 formulas. So, basically pole outside area, very important, pole outside area is equal to $\pi d L n$, then d , then L , then n and pole inside area, that equal to $\pi d L n$ plus you know R^2 minus, R^2 plus L^2 , L I have used capital L .

So, this will be capital L , capital L , so this is $\pi d L n$ you know, πd capital $L n$ plus L^2 minus twice into a L into n , into π , sorry this is into π because wheel can rotate. Now, n is the number of evolution sorry, number of revolution of the wheel. So, the counter is there which is connected to the wheel and wheel can rotate freely about the axes of the arm and it is having trace, tracer point it will trace the region.

Now, the counter will read the areas following these 2 formulas pole outside area $\pi d L n$, pole inside area $\pi d L n$ plus R^2 plus L^2 minus twice a into L , into π because all these are the $L n R L$ are defined. So, d is basically that is, what is important d diameter of the wheel that is I have written. So, now so, d is diameter of the wheel and n is the number of revolution of the wheel.

Now, the based on these 2 formula the, this meter polar planimeter will read the area. The accuracy of this instrument as I said that, you know this is you know very precision instrument that is typically made from ember essentially to reduce the thermal expansion and area is determined by tracing around the boundary of a carefully made drawing.

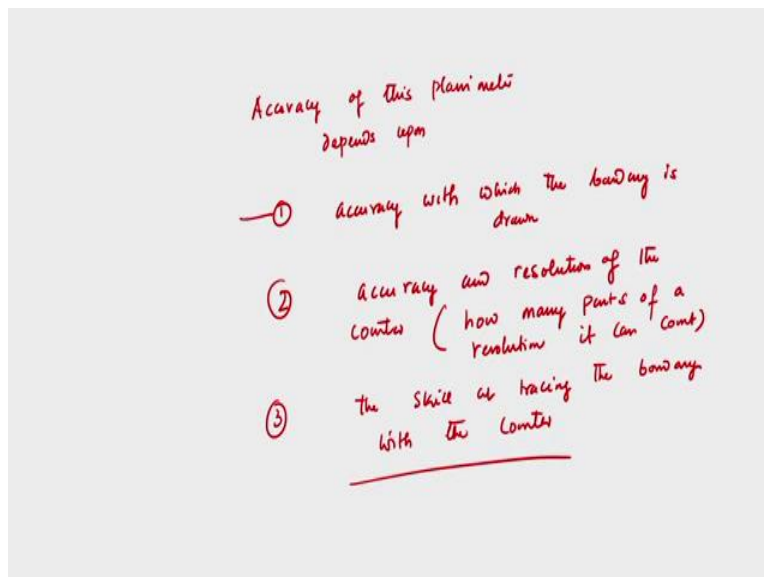
So, basically you have to made the drawing carefully and we can, we can trace the, trace that area boundary, so that we can measure the area, I mean if we know that boundary of the area, we trace the boundary that is what I was telling that if you trace the region of area we would like to measure.

Now, accuracy of this instrument depends upon the accuracy with which the boundary is drawn, so, if the boundary is very complex and the accuracy is not so high then accuracy of this measurement will not be, in a very good.

So, accuracy depends, rather accuracy of this instrument depends upon the accuracy of the boundary about, accuracy at which the boundary is drawn. The accuracy and resolution of the counter that means the counter is giving us the reading of the area that is measured by polar planimeter. So, how many parts of a revolution it can count, that is very important and the skill at tracing the boundary with the counter.

So, depending upon these three important things, we can say the accuracy, if these three important things, if we need if we take care these 3 things and then we can say that area measured by this polar planimeter will be accurate.

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So, basically I can say that area of this polar planimeter, I mean accuracy, accuracy of this planimeter depends upon. Number 1, that accuracy with which the boundary is drawn, accuracy with which the boundary is drawn. Essentially we need to trace the boundary of the area, traced the, you know region of the area that would like to measure. So, it depends upon the accuracy with which the boundary is drawn.

Number 2, that accuracy and resolution of the counter, because we will get reading from the counter that is connected to the wheel. So, accuracy and resolution of the counter, basically it, it indicates that how many parts of a revolution, how many parts of a revolution it can count, that is the revolution and accuracy of the counter.

How many parts of the revolution it can count and finally the skill at tracing the boundary, that is also important with the counter. So, we have counter so, the skill at which we are tracing the boundary of the, boundary with the counter that is also important, you know aspect to, dictate the accuracy of the instrument and if we can ensure that the boundaries drawn so accurately and accuracy of the counter, resolution of the counter is also very good and of course the skill, a skilled person has drawn the boundary and he has you know, also traced the boundary then the measured area by this planimeter will be accurate.

In this context I can say that there are several numerical methods available for finding the area under a curve, such as Trapezoidal rule, Simpsons rule, Gaussian quadrature and so on. There are many numerical methods are available. That Simpson's rules and Trapezoidal rules are mostly you know used or commonly employed to for integration of experimentally determined functions.

So, basically these two are used very importantly I mean very commonly and not only that there are many other you know numerical integrations scheme can be found in numerical analysis, I mean for revolution of multiple integrals. So, they range from extensive of the methods used to find the area under a curve to multi-curve methods. So, the numerical evolution of multiple integrals over general regions is well beyond this (50:00) course.

So, basically it is not that we will be discussing about the numerical methods rather numerical evaluation of multiple integrals of (50:13) what, general regions and so that we will calculate the area. So, that is available in most of the numerical you know in analysis literature, numerical analysis books. So, the books focusing on a numerical analysis there these methods are easily available. So, but this is not that part is beyond scope of this course.

So, to summaries today's discussion. We have seen that we can have pneumatic displacement gauge through which we can measure very small displacement, of course for that the pneumatic displacement gauge, you know is sensitivity depends upon the diameter ratio and as well as the you know that P_s that is not directly but indirectly on P_s that is the you know pressure at the upstream side of orifice plate.

And if we you know, can because diameter ratio of course it depends upon but diameter ratio, based on diameter ratio we can, we have seen that we can have this linear functional relationship

through which we can calculate, we can measure the x and then we have discussed about the area measurement using instrument, mechanical instrument that is polar planimeter.

We have seen different parts, schematically of polar planimeter and also we have seen how it is functioning to measure the area. So, it is having essentially two arms, they are connected at common point B. Second arm is having one wheel, one counter is there and the wheel can you know, this wheel can fully rotate about the axes of the arm and the tracer point we have seen that will trace the boundary which is you know bounding the area that would like to measure and then we have seen that the counter will give or counter will read the area following the two following two formulas, that is what we have written.

Now, accuracy of this planimeter we have discussed that it depends upon three important factors and if we can ensure that the accuracy with which the boundary is drawn is good and also accuracy and you know resolution of the counter is also good and the skill of, you know tracing the boundary is also you know, very accurate. Then perhaps the area measured by this polar planimeter will be the accurate one.

And finally we have discussed that there are many numerical methods available like you know, to fit area such as trapezoidal rule, Simpson's rule, Gauss quadrature, Gaussian quadrature so on and these methods are available in detail in many numerical analysis literature, numerical analysis textbooks and so and also that is, that aspect is beyond the scope this analysis and we have not discussed. So, with this I stop my discussion today and we will continue our discussion in the next class. Thank you.