Experimental Methods in Fluid Mechanics

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Lecture 5 - Basic concept of dynamic measurements

Good afternoon! I welcome you to this session for Experimental Methods in Fluid Mechanics. We will continue our discussion on the basic concept of dynamic measurements, we have started this discussion in the last class and to continue with that, today we will take up one example and we will see what are the several aspects are there while you are measuring through that measurement system, in particular, dynamic measurement.

So, we have discussed that there is a difference between static and dynamic measurement. Static measurement is that when the flow variable is, flow variable that means we would like to measure anything, flow parameter, say pressure, temperature, velocity. So, when the flow variable that is constant with respect to time, that is static but the flow variables which are not constant rather which are function of time, which are changes with respect to time are the dynamic measurements.

Now we also have discussed that static measurements are relatively simple and straightforward and dynamic measurements on the other hand is very complex and this is what is very important because we will see that all the measurements are basically function of time, so accounting that aspect we should know how we can have dynamic measurements through that measurement system which we have discussed through a block diagram in the last class. Now while we are talking about dynamic measurements, the complexity is there not only that we need to consider several other factors. Now, today we will discuss about dynamic measurement.

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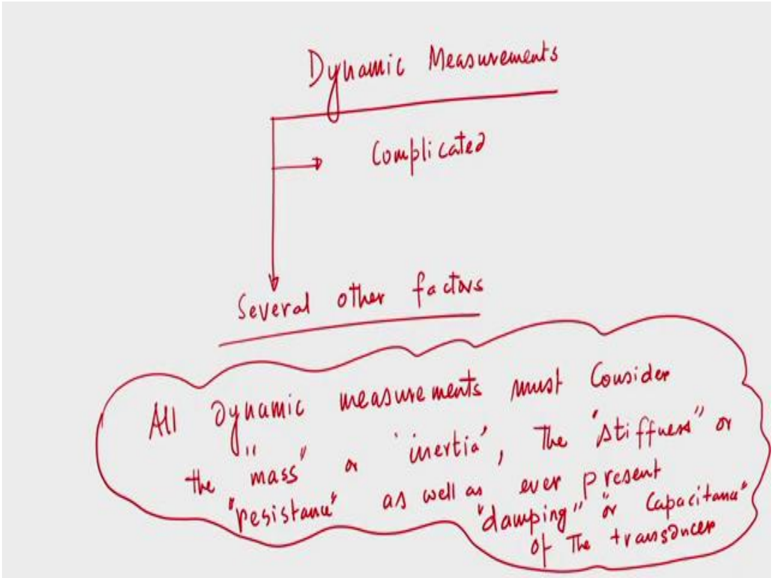

Dynamic measurements, this is very complex, I can say complicated if you compare this with respect to static measurements of course and it requires several other factors while we are having, rather we need to have measurement of any variable through dynamic consideration. Today, as I said, I will try to discuss this with an example and to begin with, at least we should know that all the dynamic measurements, all dynamic measurements are rather having other factors, rather I can say all dynamic measurements must consider the mass or inertia, the stiffness or resistance, as well as which is very important as well as the ever present damping, as well as ever present damping or capacitance. Of what? Of the transducer.

So, this is very important that all the dynamic measurements are complex and complexity is there and basically this would consider mass or inertia, stiffness or resistance as well as ever present damping or capacitance of the transducer. So, now we will move to see how we can proceed to have dynamic measurements through the mathematical modeling. So, normally we have seen that we have a detector transducer, if the detector transducer needs to measure variables or quantities which are changes or which are changing with respect to time then we usually model the response of the transducer using a simple second order ordinary differential equation in time.

So, I do repeat, we have seen that in the measurement system we have a detector transducer. Now if the detector transducer needs to measure a variable which is changing with respect to time then we usually model our transducer, rather response of our transducer using a simple second order ordinary different equation in time.

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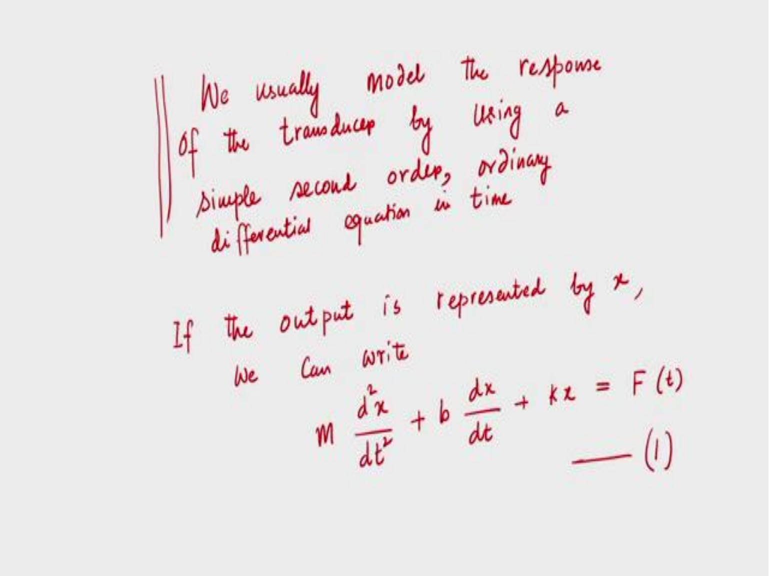

So, I am writing it that we usually model the response of the transducer by using a simple second order ordinary differential equation, simple second order ordinary differential equation in time. So, that is the important thing that our transducer that is there in the measurement system and if you would like to measure any flow variable which is transient rather the flow variables which are changes with respect to time, we need to go for modeling and we usually model the response of the transducer using a simple second order ordinary differential equation in time.

Today, we will discuss one example, but we will discuss more typical second order systems in detail. Later in this course when we will discuss the thermocouple compensation and also we will discuss at least one or two examples showing the dynamic response modeling. So, now what I can say that for the time being at least we will consider simple second order ordinary differential equation which is equation in time and we will try to see, I mean, what are the critical issues, critical aspects we need to know before I go to measure that, those variables which are function of time.

So, if the output, say we would like to measure using a transducer and output is represented by x, then we can write the modeling equation that is what we have written using a second order ordinary differential equation, that is m d 2 x, d t 2 plus b dx dt plus kx is equal to function what is t, that is the function f is essentially forcing function. So, we would like to measure x that is the output, forcing function which is of course a function of time, now we can represent, rather we can model that measurement using a simple second order differential, ordinary differential equation, that is what I have written this equation number one, say this is equation number one.

Now in this equation it is, you can see that m is the mass or inertia that is what we wrote in the last slide, b is the damping and k is the spring constant and F t, right hand side which is the forcing function.

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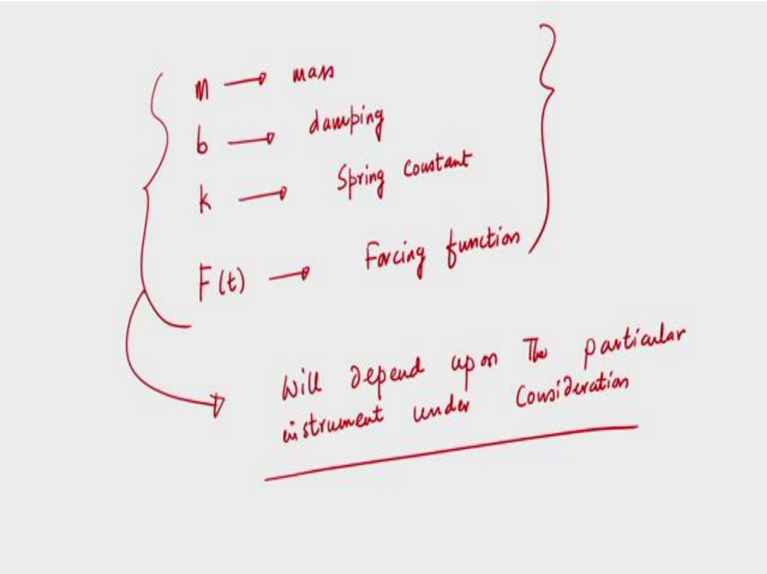

So, I am writing for the sake of completeness, that m is equal to mass or inertia, b is damping, k is spring constant. You can see from the equation that this equation represents a modeling equation through which I would like to measure the displacement of a spring when the spring is, you know disturbed by a forcing function F t and F t is the forcing function.

So, this is clear. That means, we have a spring model and we would like to measure the displacement of a spring when the spring is disturbed or spring is actuated by a external forcing function which is F t. So, we would like to measure because displacement obviously is the function of temperature. Now we will discuss the nature of the forcing function which we may consider, and if we consider what will be the implications, and then we will try to find out the solution and then the solution that we will obtain from there we will try to draw few insights. I mean how that solution, that measurement will change with the change in frequency, amplitude and the damping coefficient.

So, this is what we have understood that we would like to measure the displacement of a spring which is externally subjected to a forcing function which is F t. Now the m, b, k, t, whatever we have written, these four different terms, this will depend upon the particular instrument under consideration. So this is not the case that all these parameters will be constant always. So this will depend upon the particular instrument under consideration.

So, it depends upon the instrument, a particular instrument under consideration those are not universal.

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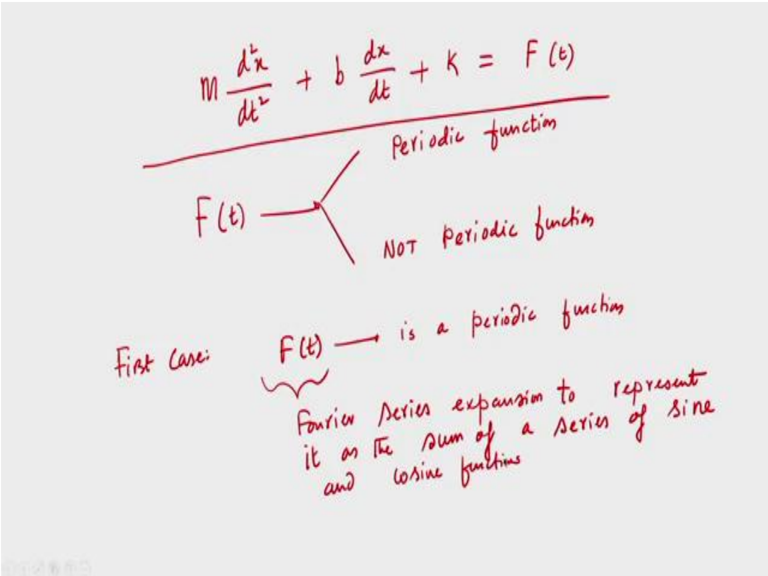

Now if we write the equation again that m d 2 x d t 2 plus b dx dt plus k is equal to F t, so m d 2 x d t 2 plus b dx dt plus k is equal to F t and where m, b, k and f are defined in the previous slides, these parameters depend on the particular instrument under consideration. Now we will discuss about the nature of the forcing function that is actuating the spring.

Now F t, so this is forcing function, now this forcing function it may be two option. It be a periodic function, so a periodic function or not periodic function and it may not be a periodic function. If F t is periodic function then what will be the result and if it is not then how we can solve this equation, that is what we will discuss. Say, first case I am considering that F t is a periodic function, so is a periodic function and if it is a periodic function to have a solution of this equation what can we do? We can now decompose F t using a Fourier series expansion.

So, we can write Fourier series expansion to represent it, this function as the sum of a series of sine and cosine function, as the sum of a series of sine and cosine function and cosine functions. So, if F t is periodic, first step is we can write F t, you can represent F t using a

Fourier series of expansion so that the function can be represented by a series of, rather as the sum of series of sine and cosine function.

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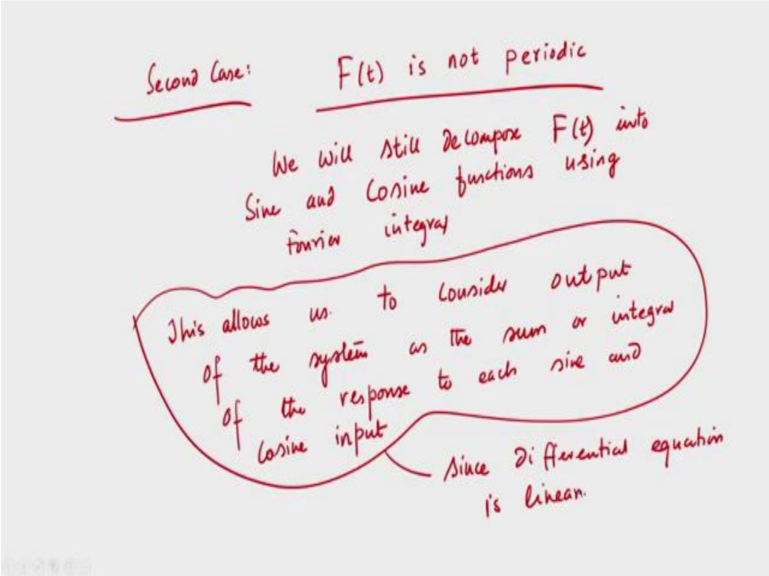

Now we may have another case, that is second case, so as a second case say F t is not periodic then again you need to have solution of that equation and in that case what we will do? If it is not periodic, we can still decompose F t, so we will still decompose F t into sine and cosine function using Fourier integral not a Fourier series function. So, into sine and cosine functions using Fourier integral.

So, we have seen two different cases, one case if it is periodic then we need to express F t using a Fourier series of expansion so that the function can be represented as the sum of a series of sine and cosine function; otherwise if it is not a function then still we can decompose into sine and cosine function using a Fourier integral, whatever may be the case that depends upon the excitation of the spring using external forcing factor now.

So now, why we are going to decompose either following Fourier series of expansion or Fourier integral method into sine and cosine function that this decomposition allows us to consider output of the system as the sum or integral of the response to each sine and cosine input. So that means external forcing factor F t if we can somehow decompose that forcing factor into sine and cosine as input then the output of the system we can consider as the sum of sum or the integral of the response to each sine and cosine inputs.

So this is very important and this, see note it, this we can do since our differential equation is linear. So this we can do since differential equation is linear.

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$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + k = F(t)$$

$$F(t) = F_0 \sin(\omega t)$$

$$x = \frac{(F_0/k)\cos(\omega t + \phi)}{\left\{\left[1-(\omega/\omega_n)\right]^2 + \left[2(b/b_n)(\omega/\omega_n)\right]^2\right\}^{1/2}}$$

$$\text{where } \phi = \text{phase shift} = \tan^{-1}\left[\frac{2(b/b_n)(\omega/\omega_n)}{1-(\omega/\omega_n)^2}\right]$$

$$\omega_n = \sqrt{k/m} \text{ is the natural frequency}$$

So, now we will quickly do that, say again I am writing this equation, m d 2 x, d t 2 plus b dx dt plus k is equal to F t where forcing factor F t is equal to F not sine omega t, so if we examine the response of the system by considering a forcing function which is F not sine omega t then I am not going to write the intermediate step, steps of the solution procedure, rather straightaway I can write the solution that is easily available in many textbooks that solid mechanics textbooks, vibration and springs and vibration.

So, if our response of the system is, I would like to examine rather and the system has been modeled using this equation where system is excited using a external forcing factor F not sine omega t, in that case our response that is solution x can be written that is what I am writing, f not by k cos omega t plus phi, this is standard and easily available in many textbooks. This is 1 minus Omega by Omega n Square plus twice b by b n into omega by omega n square to the power half, total to the power half. Where phi that is the phase shift, this is the phase shift, phase lag whatever, tan inverse 2 b by b n, omega by omega n, 1 minus omega by omega n square. Omega n is the root of k by m is the natural frequency of the system. Phi is the phase shift, omega n is the natural frequency of the system.

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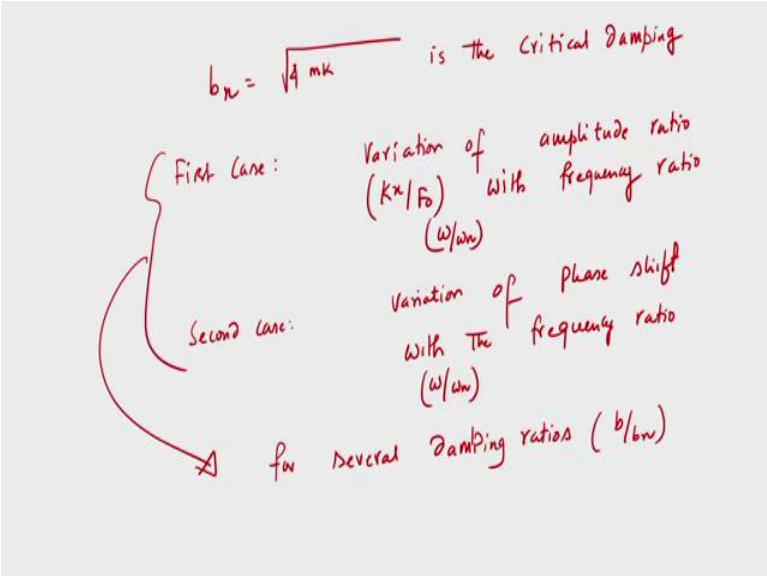

And also we need to write b n is equal to root of twice m, root of 4 m k is the critical damping. So now it would be nice if we try to discuss this by plotting the variation of the amplitude ratio, now what should I do? We will try to see the solution that is what we have obtained from the equation when that equation represents, that equation models a system, a second order system where the variable which we would like to measure as a function of time and the system is excited by a forcing factor which is the form or nature of the forcing function is given F not sine omega t and the solution of this equation we will give like this, x is equal to f not by k cos omega t plus phi, where phi is the phase shift and omega n is the natural frequency and also b n that is the critical damping.

So, now what should I do, I will try to draw the variation of amplitude ratio, so two case, so first case what I will do? Variation of amplitude ratio and from there we will see that what are the different critical aspects we need to know of this dynamic measurement. Variation of amplitude ratio kx by f not and with frequency ratio, omega by omega n. And second case also, we will try to obtain that variation of phase shift with the frequency ratio omega by omega n and for these two cases we will try to see our deserved variations for different damping ratios.

So, we will see these two cases through a graphical representation for several damping ratio that is b by b n.

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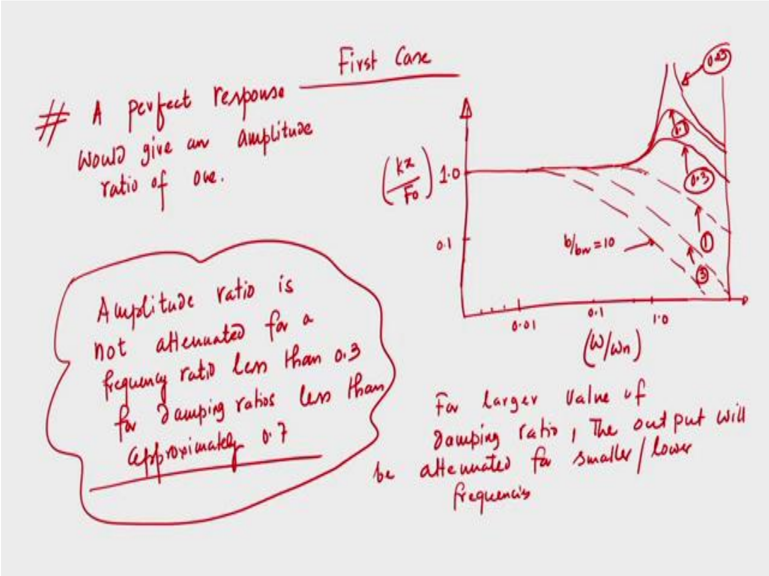

So now I will try to plot the first one, what will be, the first one is that variation of graphical representation of the variation of amplitude ratio with frequency ratio and we will obtain this variation for different damping ratios. So the first case I will try to now draw and I am drawing only to extract the physical insights, those we need to know while we are measuring any variable which is function of time through dynamic measurement system.

So, now I am trying to draw say, this is x axis or we are taking in a frequency ratio and y axis that is kx by f not, that is basically amplitude ratio. Now this is say 1 and we have if we plot in log log, if we depict the variation in log log plot, say this is 0.01, this is 0.1 and this is 1. Similarly, this is say, this is 0.1 and this is 1, so what we will see? I will write whatever we would like to conclude from this, from our observation which we will see from this plot.

Now I am drawing say several, at least I can show like this, so we can write like this, they are not lying on like this. So say, these plots we have obtained for different damping ratios, say the first one that we have obtained is say 10, so this is let us say for b by b n is equal to 10, then say this is for 0.3, 0.03, this is for0.1, this is for 0.3, this is for 1 and this is for 3. So this is for 3, this is for 1, this is for 0.03, this is for 0.1, and this is for 0.03 for different b by b n ratio.

Now see, I would like to draw a few conclusions from this, what I can write that a perfect, rather I am writing over here. A perfect response would give an amplitude ratio of 1. So this is very important, so we will try to obtain response from our transducer, a perfect response will be given amplitude ratio of 1. Now from this figure what we can see that amplitude ratio is not attenuated for a frequency ratio less than 0.3, for damping ratios less than approximately, this is not at per scale, approximately say, this will be 0.7.

So, amplitude ratio is not attenuated for frequency ratio less than 0.3, I mean if we considered the frequency ratio of 0.3, so say this will be 0.3 and for the amplitude ratio that will be not less, equal to 1 but it will be less than 1, I mean 0.7. Now this is one and for larger value of damping ratio the output what we can see, the output will be attenuated for smaller or lower frequencies. So larger value of damping ratio, that means if we go 1, 10 and 3, what we can see, the output will be attenuated for smaller frequency that is even less than 0.1 and 0.2.

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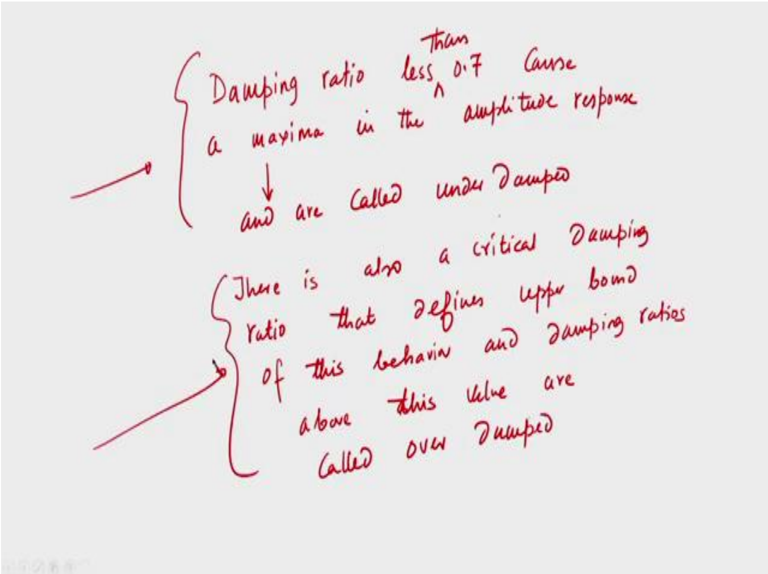

Now another two important observation that I would like to draw from this figure is that damping ratio of less than 0.7 cause a maxima in the amplitude response, and are called under damped, and are called under damped. So, if I go back to my, so what we have seen? Damping ratio less than 0.7, damping ratio of less than 0.7 cause a maxima in the amplitude response and they are called under damped. So, damping ratio less than 0.7 and we have seen those are under damped, so that will amplify the, that is clearly observable from the figure we have in this slide. So these are called under damped, not only that this is important. Not only that, we also can see there is also a critical damping ratio that defines upper bound of this behavior and damping ratios above these critical values, damping ratios above this critical value, above this value are called over damped, so this is another important point.

So, this two is important that there will be a critical damping ratio that defines the upper bound of this behavior and damping ratios above this critical value are the over dumped, this is very important. From there we can say that we have a good representative measurements if the highest frequency in the signal occurs at a frequency less than the critical value. Otherwise it will be over damped and that will be at the critical damping ratio. So what is the conclusion? That we would have good representative measurement if the highest frequency in the signal occurs at a frequency less than some critical value at the critical damping ratio.

But we could still get good response if the system is under damped provided the highest frequency is sufficiently below the natural frequency of the system. Now if the frequency are too large then we could have another problem that will make the system unstable, so we will have a instability problem. Finally, a very good rather an important conclusion is that over damped systems can also be used. So we have seen that we may have over damped system and that is why we can, a good representative measurement is that the signals that will occur, that occur at a frequency which is less than some critical value at the critical damping ratio.

Not only that we can still have good response if the system is under damped provided the highest frequency is sufficiently below the natural frequency of the system and if the frequencies are not sufficiently below the natural frequency then we will have another problem of instability. Finally, over damped system can also be used, however the maximum frequency must be well below the natural frequency.

So, these are the conclusion we have drawn from this typical example that when we are going to have a dynamic measurement then how we can measure and what are the several effects of the frequency ratio of the signal that we would like to process and perhaps from this understanding we can now have information so that whenever we are going to measure any variable which is function of time then if you would like to model then what are the things we need to take into account while modeling that system that we should consider.

So, with this I stop my discussion today and we will continue our discussion in the next class and in the next class I will try to discuss about the phase shift diagram with the frequency ratios and then we will go to the next topic. Thank you very much.