Experimental Methods in Fluid Mechanics
Professor Dr. Pranab Kumar Mondal
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati
Lecture 42
Error Propagation: function of two variables, several variables

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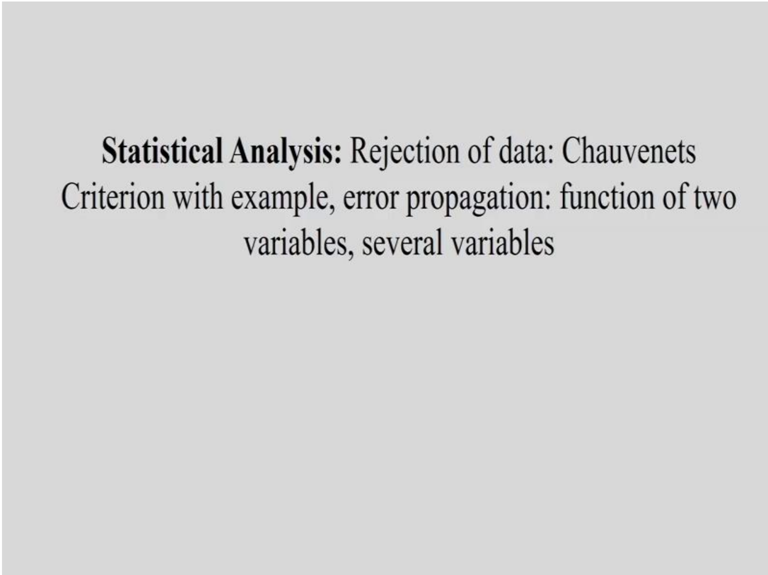

Good afternoon, we will discuss today about you know the rejection of data. In fact, this is in continuation of my last class, where we have you know discussed about the method that is the Chauvenets Criterion, in which is used to reject data that is experimental data.

And now what we will do, we will take up an example and then considering that example we will see that if we use the Chauvenets Criterion rather the steps which we have outlined in the last lecture, we will try to follow those steps to reject a particular data.

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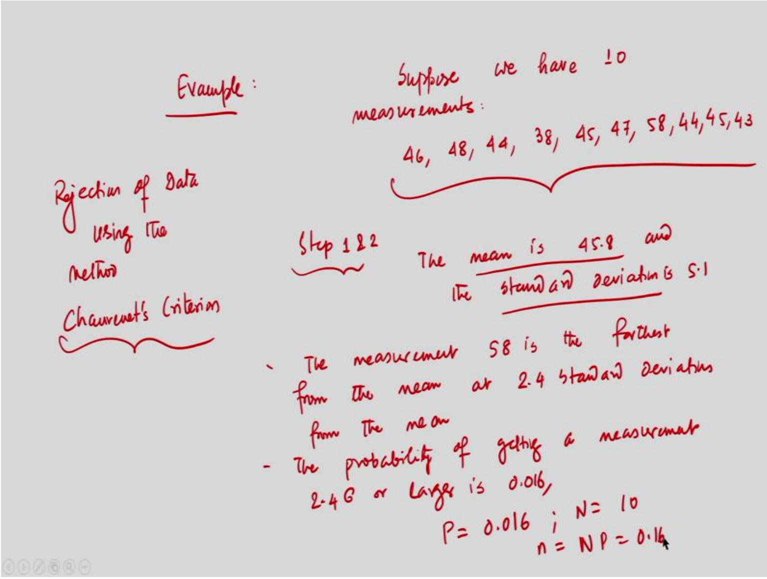

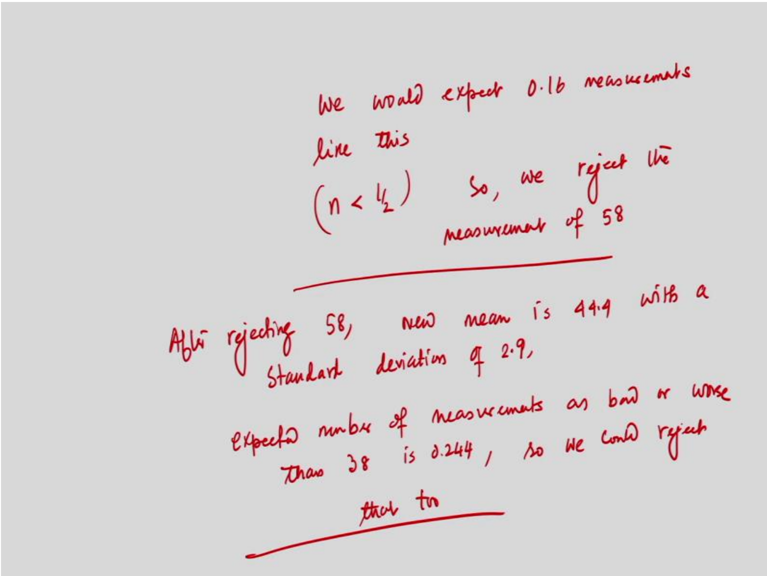

So, just we will take an example say, example we are considering that is the rejection of data using the method and that is known as, you know Chauvenets Criterion. So, example that means we will be using Chauvenets Criterion to reject experimental data.

Now, if we have you know set of experimental data. Suppose I am writing, suppose we have 10 measurements, that is say, if we just list down 46, 48, 44, 38, 45, 47, 58, 44, 45, and 43. So, if we just look at the set of data, we can really find that the data point 58 and 38, these two data points are not in the you know order, order in the sense, if we look at these 10 different data points, we find that 58 and 38 these two data points are not the maybe correct.

Because remaining other 8 data's, I mean 46, 48, 44, 45, 47, and 45, 44, 43 these remaining 8 data, they are having almost similar magnitude, but 38 and 58 these two data are not you know falling in this range. So, if we now consider the steps, that is we can see from this data set that 58 should be rejected and 38 also should be rejected from this set of data.

Now, we will see if we use Chauvenets Criterion to eliminate data that is 38 and 58 then we will use the steps that we have you know understood in the last lecture. So, the first step is mean. So, if we now try to calculate, step 1, so the mean is 45.8 and the standard deviation is 5.1. So, the standard deviation is 5.1 and mean is 45.8.

So, we have 10 data points and mean and standard deviation we have calculated. So, that is step 1 and step 2, if we follow step 1 and step 2, step 1 and 2 then we can calculate this. Now, what we can see, the measurement 58 is the furthest, furthest from the mean. So, the measurement 58 is the furthest from the mean at 2.4 standard deviation from the mean.

So, from the mean. Now, the probability getting measurement 2.4 sigma or larger is 0.016 and. So, if I now go to the next slide, so the probability. So we can see that if we have calculated standard deviation and mean and the measurement 58 is the furthest from the mean and at a standard deviation 2.4, at 2.4 standard deviation from the mean that is what we have calculated.

Now, the probability of getting a measurement 2.4 sigma or larger is 0.016, so that is what we have calculated probability. Now, if we show here, if we follow the steps then P is 0.016 following the Chauvenets Criterion, we have calculated mean and standard deviation. We have calculated a rate, that is Ti for each data, then we can find 58 is the farthest.

So, that is TS is maximum, so TS that is call Ti, the 5th number, if I calculate now error for each measurement and then we will find 58 is the furthest and error is maximum and if we call it TS, then 58 can, I mean we can expect that 58 should be rejected.

Now, this 58 is the furthest from the mean and standard deviation at 2.4 standard deviation from the mean and the probability of getting 2.4 sigma is 0.016 using that probability density function you can calculate. If P is 0.016 and N is 10 that is 10 data points are there.

So, now small n that will be 0.016, small n that is capital N into P that is 0.16. So, if we go to the next slide, then we can write that you know, we would expect 0.16 measurement like this. And this small n is less than half. So, we reject the measurement of 58. So, we can reject 58.

If we reject 58, then now if we go back to the previous slide. So, if we reject 58 then we will have 9 other measurements and again we have to calculate the new standard deviation. So, if we calculate the new mean, so after rejecting 58, new mean is 44.4 that is very simple, if we calculate with a standard deviation of 2.9.

So, if we follow the similar steps then expected number of measurement as bad or worse than 38 is 0.244 and so we could reject that too. So, I am writing this, that means after rejecting 58, to reject 58, 58 we have followed the steps, which are either using Chauvenets Criterion and we have seen that 58 can be rejected.

After rejecting 58 we have calculated new mean, new standard deviation and if we follow the steps that we have outlined in the last lecture and we will see the 38 also can be rejected. So, the expected number on measurements as bad or worse than 38 is 0.244 which is also less than half and we can, we could reject that too.

So, this is what is called Chauvenets Criterion for the rejecting data. So, this is not very difficult, we can reject data using this method. And, so this example I have taken only to considering a few points, but in real experiments, if we record, if we collect data, then we will have 50 or 100 data points, from there obviously a few data, a few points will be there, which we can reject and we can, we can reject data using this method.

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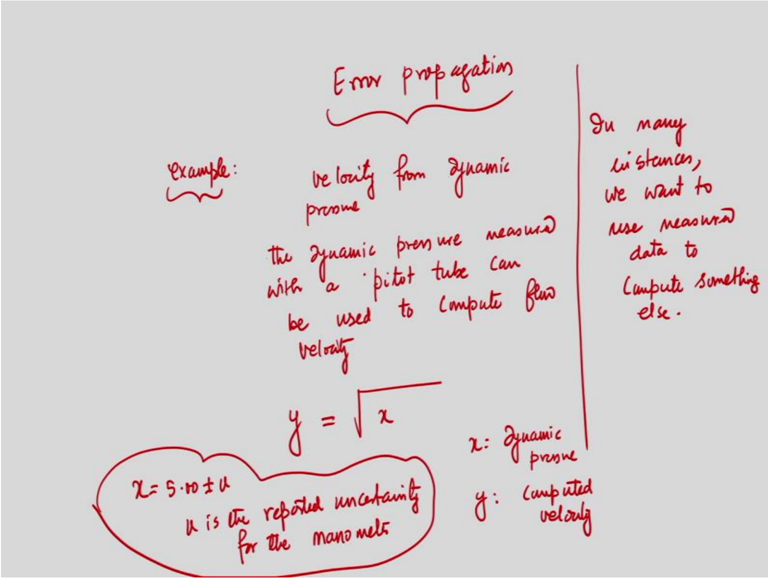

So, next we will be discussing about the Error Propagation. What is error propagation and why do we need to study this aspect is also, rather we should know about this. And there are a few cases. In fact, in many instances, we would like to use measured data to calculate or to predict something else. Say, for example, we would like to calculate velocity using the measured value of pressure.

So, if we use, rather if we measure the pressure using nanometer and by measuring the pressure we can calculate, we can predict the flow velocity. So, if you now take this example, say, so what I have written is that in many instances, we want to use measured data to compute something else.

So, if we now take the example that velocity from dynamic pressure. Velocity from dynamic pressure, this is the example, that means the pressure, the dynamic pressure measured with a pitot tube can be used to compute flow velocity. So, if we say velocity is y, then y is equal to square root of dynamic pressure say x, where x is the dynamic pressure. And y is computed velocity.

Now, if the pressure, which is measured using pitot tube or using manometer is found say, if I go to the next slide, say what here I am writing x is 5.0 plus minus u. That means, the pressure which is measured using manometer is found to be 5.0 plus minus u, where u is the reported uncertainty of the manometer, where u is the reported uncertainty for the manometer.

So, that is what I was talking about in the last class that any system, instrument or device which is produced in a manufacturing, in manufacturing unit should be certified with the uncertainty, that means that is what I took an example that if we, if a product, you know production unit is, you know manufacturing gas tank.

Then, the internal gas pressure that will be, you know that should be considered, the design value of internal gas pressure for which cylinder, other gas, gas tank can withstand without any mechanical failure, it should be certified that means, it is not the case that the gas tank will be always safe at the design value, rather we will have plus minus you know range of a compulsion that means if the design value is x then, x plus minus should be the range for which the gas tank will be safe for the operation.

Similarly, when we are using a, using manometer in you know for the measurement of the pressure, then the reading which we are getting from the manometer should be something plus minus uncertainty range. Now, if the measurement is x, measurement x for this particular case that is the pressure that we are measuring using manometer is 5.0 plus minus u, where u is the reported uncertainty of the manometer.

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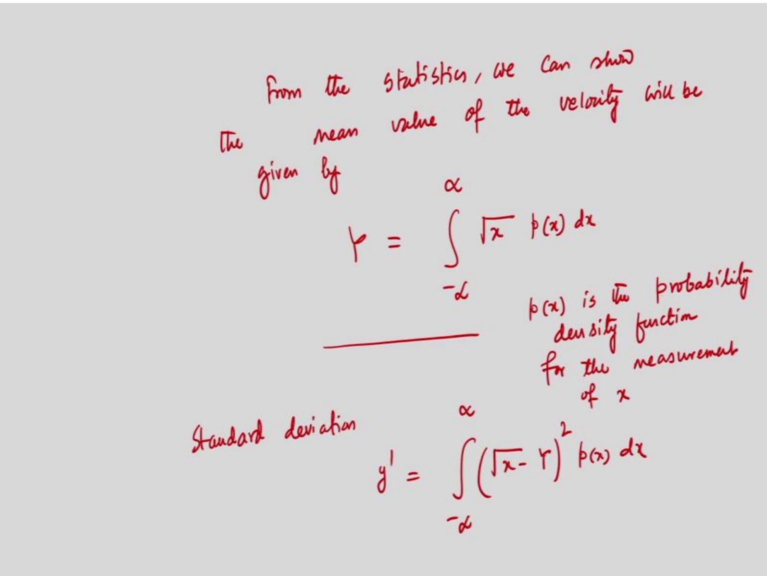

Then, from the statistics, I am writing next slide that is from the statistics. From the statistics, we can show that the mean value, the mean value of the velocity, from the statistics, we can show that the mean value of the velocity will be given by Y that equal to minus infinity to infinity, root of x, this probability density function dx, where p x is the probability density function for the measurement of x, and the standard deviation.

So, this is, I am not going to discuss that part, we have, we can show from the statistics that the mean value of velocity will be given by this expression and the standard derivation that y prime is minus infinity to infinity, root of x minus mean square p x, dx. So, this is the standard deviation.

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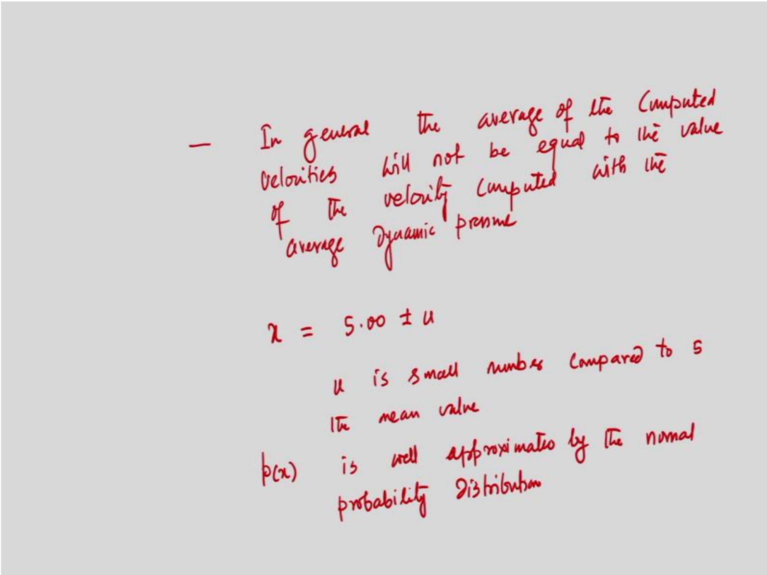

So, from this, we can conclude one important thing that, in general the average of the computed velocity, velocities will not be equal to the value of the velocity computed with the average dynamic pressure. That means, the average of the computed velocity will not be equal to the value of velocity, computed with the average dynamic pressure.

So, this is in general, so this, another important thing if we now go back to the previous slide, then this p x, probability density function is, you know if we need to calculate the standard deviation and mean value of the you know, velocity then we have to know the probability density function explicitly and these integrals could be very difficult to evaluate.

So, this is the case. Now the x, what, I mean we have written that is 5.0 plus minus u. That is what we have you know written in the last slide. Now, if we assume that u is small number compared to 5 the mean value. So if the, uncertainty is very small, and then we also can assume the probability density function p x, is normal probability distribution, is well approximated by the normal probability distribution.

So, if we go to the previous slide, what I said that if we need to calculate the velocity, average velocity, the mean value of the velocity, then we have to know explicitly the probability density function and this, you know this integral could be very difficult to evaluate.

And for that we are considering that we are taking, we are assuming that u is very small, that is uncertainty is very small and the probability density the p x is well approximated by the normal probability distribution.

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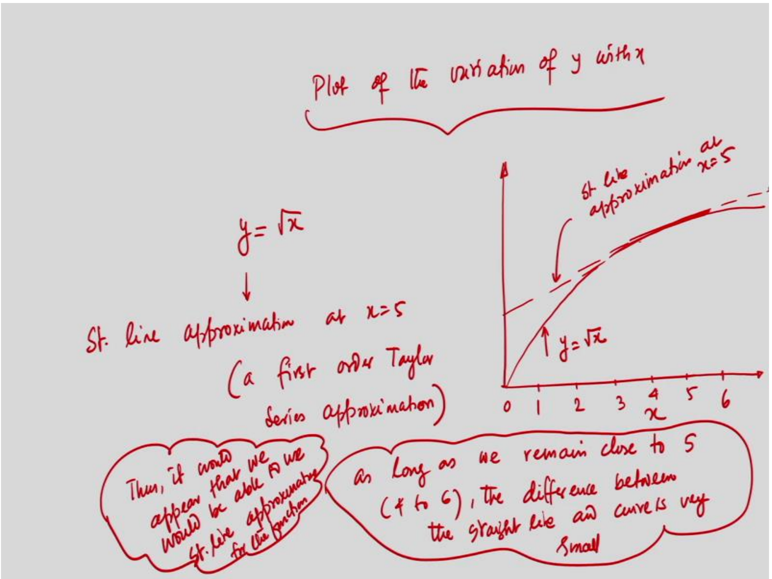

In that case, if we try to know the, plot the plot of the variation of y with x. So, if we try to plot it, then this is x this is 0, 1, 2, 3,you know 4, 5, 6 and we will get, now, so that is, this is y equal to root x and this is straight line approximation at x equal to 5. So, we can, at x equal to 5, we can assume the straight line approximation. So, what we can see is that, that means, we can see y equal to root x and the approximation, approximation rather a straight line approximation at x equal to 5 that is our first order Taylor Series expansion.

So, y is equal to root x and if we you know consider straight line approximation, then that is at x equal to 5 that is, our first order you know Taylor Series approximation at x equal to 5, what you can see from this figure you know is that very important that you know as long as we remain close to 5, that is 4 to 6, so this is 1, 2, 3, 4, 5 then this is 6. So, as long as we remain close to 5, 4 to 6, the difference between straight line and curve is very small.

And if we consider this, so as long as we remain close to 5 then difference between straight line and curve is very small and thus, thus it would appear that we would be able to use straight line approximation for the function. So, we would be able to use straight line approximation for the function. Now, this is what we have discussed.

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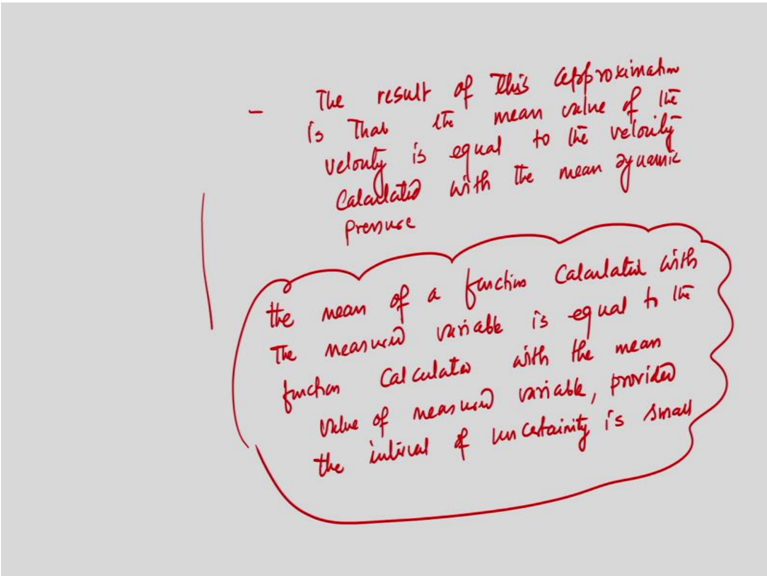

Now, the if we you know approximate by the straight line, then the result of this approximation is that the mean value of the velocity is equal to the velocity calculated with the mean dynamic pressure. That means, from this exercise, we would like to draw an important conclusion is that the mean of a function calculated with the measured variable is equal to the function, calculated with the mean value of measured variable, measure variable, provided uncertainty is very small.

So, that is why I was you know doing this exercise that from this typical example, we have been able to, you know so that the mean of a function calculated with the measured variable is equal to the function calculated the mean value of measured variable provided the interval of uncertainty is small. So, this is an important conclusion, which we should know at least who are doing experiments in the area of fluid dynamics.

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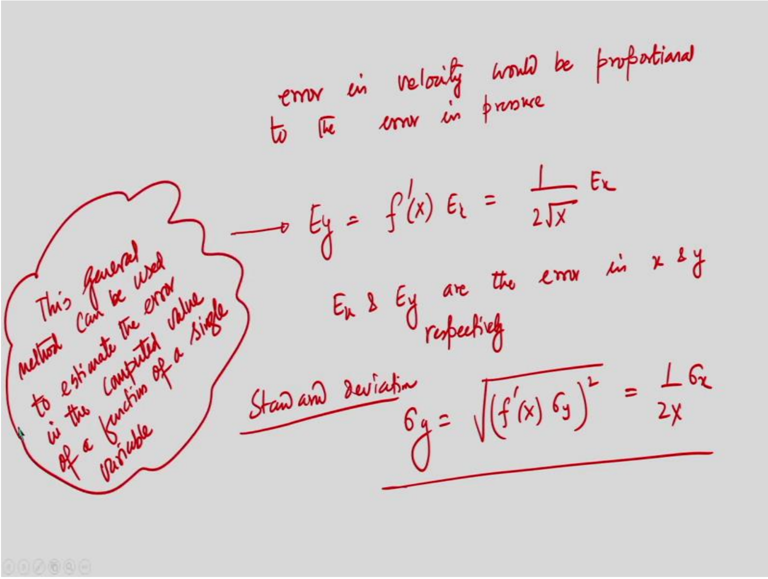

So, this is what we have discussed and also would like to see that error in velocity would be proportional to the error in pressure that is what we have, we can see that means Ey that will be f dotx Ex that is 1 upon 2 root x Ex, where Ex and Ey are the error in x and y respectively. And standard deviation of y that is sigma y will be equal to f dash x into sigma y square that is 1 upon 2x into standard deviation x, that is sigma x. So standard deviation of y can be also related to the standard deviation of x with this relationship.

Now, this general method, which we have discussed can be use to estimate the error in the computed value of a function of a single variable. So, till now we have discussed, rather taking an example, we have discussed that the error, this method can be used to calculate, to estimate the error, for error in the computed value of a function in of a single variable.

That means, computed value is y which is function of single variable that is only x and we can calculate Ey error. So, that is the important, so we can calculate, so I am writing this general method can be used to estimate the error in the computed value of a function of a single variable.

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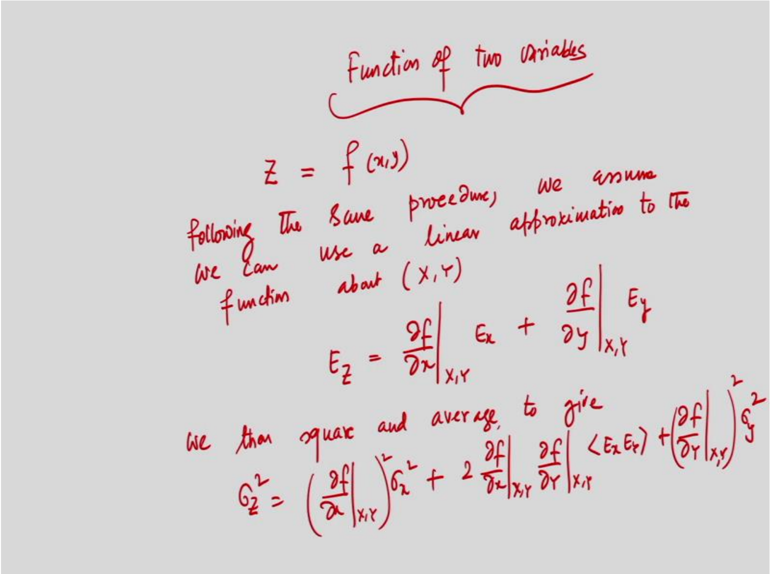

So, till now we have discussed a case where we have calculated y which is a function of a single variable, but if we now have, we will see that if we have function of two variables, say we have a function, we have a function Z which is f xy with similar notation as above. So, that we have a function Z which is you know function of two variables that is x and y with similar notation that we have discussed till now.

So, following the same procedure, we can, so following the same procedure, we assume that we can use a linear approximation to the function about x comma y that and given by Ez that is del f del x, xy, Ex plus del f del y, xy, Ey, and what we do? We then square and average to give sigma Z square equal to del f, del x, xy, square sigma x square plus 2 del f, del x, xy, del f del y, xy, Ex, Ey plus del f del y, xy, square sigma y square.

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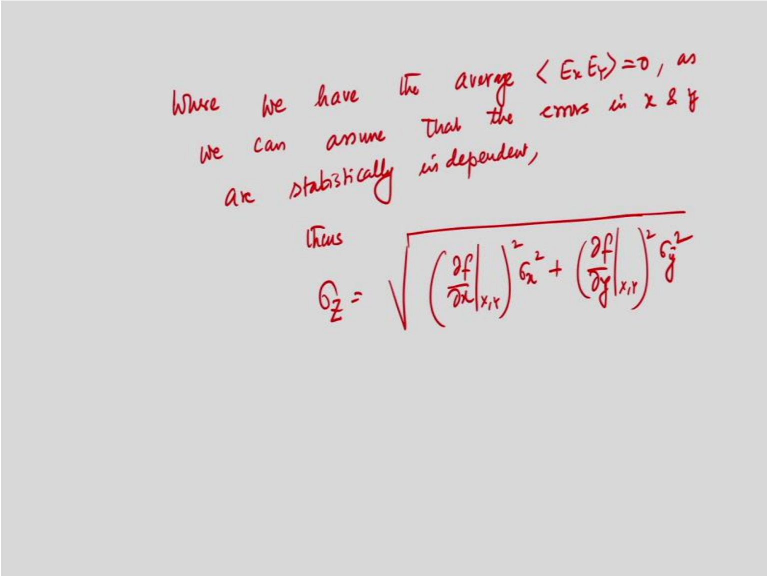

So, here we can see if we now go to the next slide just I am doing this mathematical analysis, so following the same procedure, we assume we can use a linear approximation to the function x and y and then we can calculate error in Z and we can then square and average to obtain the sigma z square that is the standard deviation and then what we can see from this equation is that we have the average Ex, Ey, that equal to 0.

Where we have the average, where we have the average Ex, Ey 0 as we can assume that the errors in x and y are statistically independent. Thus, sigma Z that we can write del f del x, sigma x square plus del f sigma y square. So, this is we have calculated, if we go back to the previous slide, that means we have considered today 1 case where the function is a single variable function.

We have discussed the steps in detail and we have calculated the error and the standard deviation. If the function, if you consider function of two variable, then following the same procedure we can, we assume that we can use linear approximation to the function and we have calculated the error and just with an square and average to obtain the standard deviation, but here that we have the average Ex, Ey that equal to 0 and we can assume that the error in x and y are statistically independent and we have calculated.

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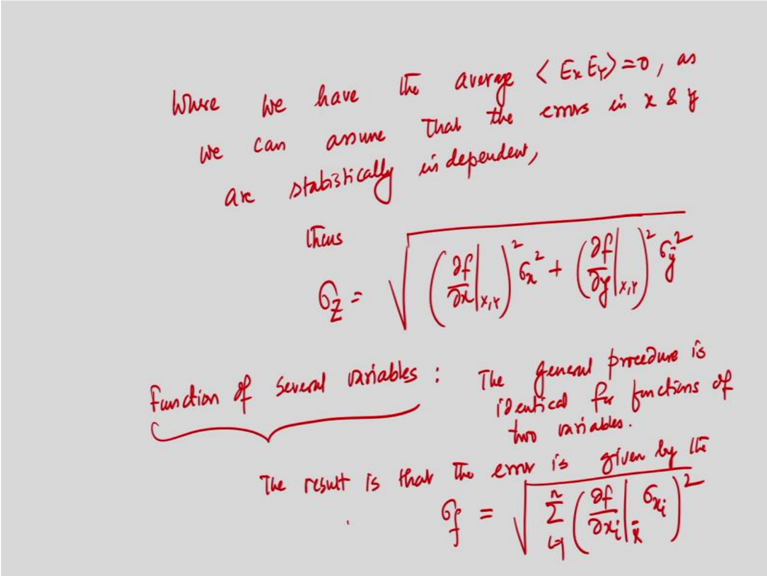

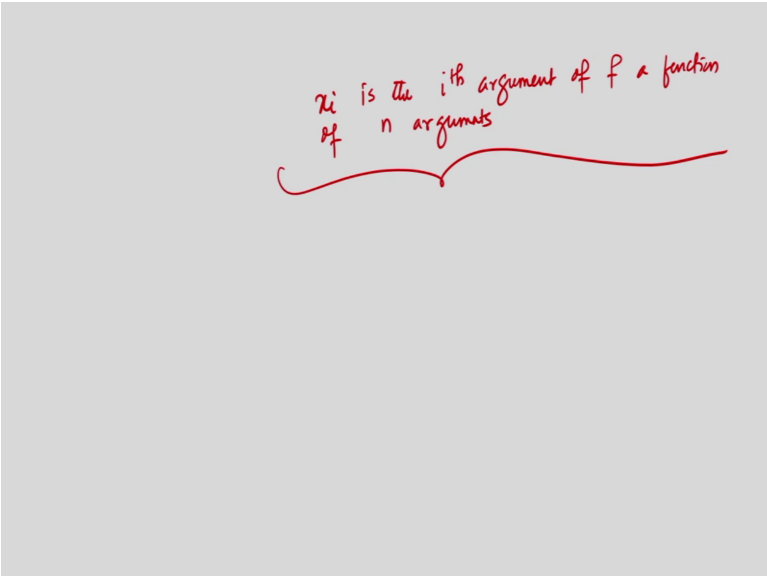

Now, if we consider function of several variables, general you know that means, if, now writing, so that means, if we now consider function of several variables. So, we need to know the error propagation that is what we have taken 1 example, but if the function is having several variables, then the general procedure is identical for functions of two variables.

And the result is that the error is, the result is that the error is given by the general formula, sigma f that will be, so I am writing in the next, okay I am writing here, given by sigma f is equal to under root, i equal to 1 to n, del f del xi, x bar sigma xi square, where xi is the i th argument of f, a function of n arguments.

So, this is the case. Now, so this is the our case if you consider the, you know several variables.

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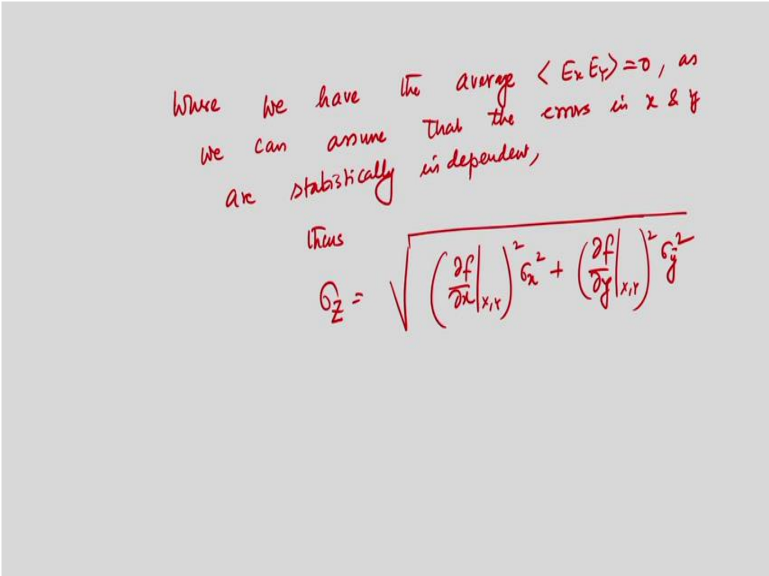

So, what we have discussed is that, if we have the single variable function then we have discussed about the error propagation, what we what do we mean by that and we need to know that.

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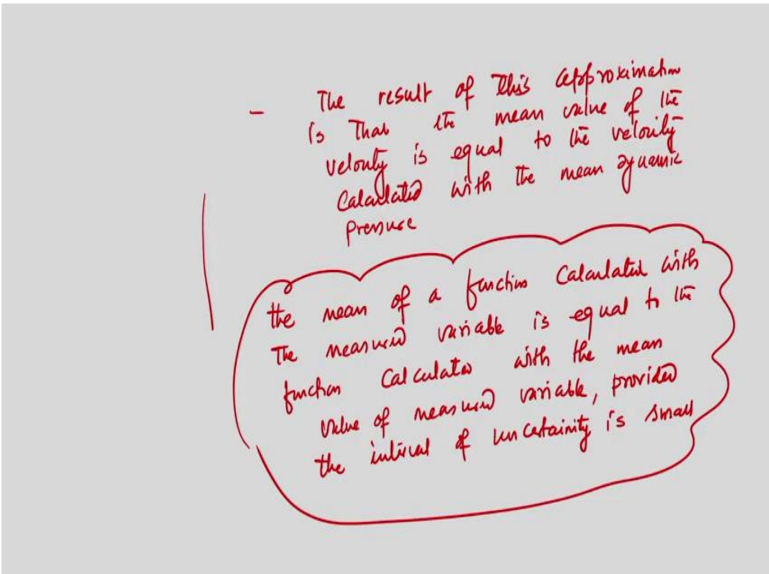

And finally, we have come up with an important conclusion that the mean of a function calculated with the measure variable is equal to the function calculated with the mean value of measure variable provided the interval upon certainties is very small that is what we have you know seen today.

And finally, we will work out 1 example that which is, we have discussed about the error, today we have discussed about the you know different types of error and now we have seen that the errors, measurement errors can be classified into 3 different categories that is the blunders, fixed errors and random error.

Random error cannot be eliminated, can never be eliminated rather and blunders can be eliminated, I mean if we are careful in measuring data and when we are using any instrument to measure data, and fixed error, the fixed error can be, you know we have given 1 example and from there we have seen that the fixed error can be eliminated provided if we considered the correction factor.

So, now what we will do, we work out 1 example, which is related to the error. Now fixed error, I can say. So, we have seen that the random error cannot be eliminated, blunders can be eliminated if we carefully you know use instrument or device in measuring data while fixed error can be eliminated, I mean, we can eliminate rather if we are, if we consider the correction factor.

So, as I said that the errors cannot be completely eliminated, but it can be minimized. So, if we are careful in measuring, when we are measuring any variable using any particular instrument then you can reduce the probability of the error. Now I am considering 1 example of fixed error.

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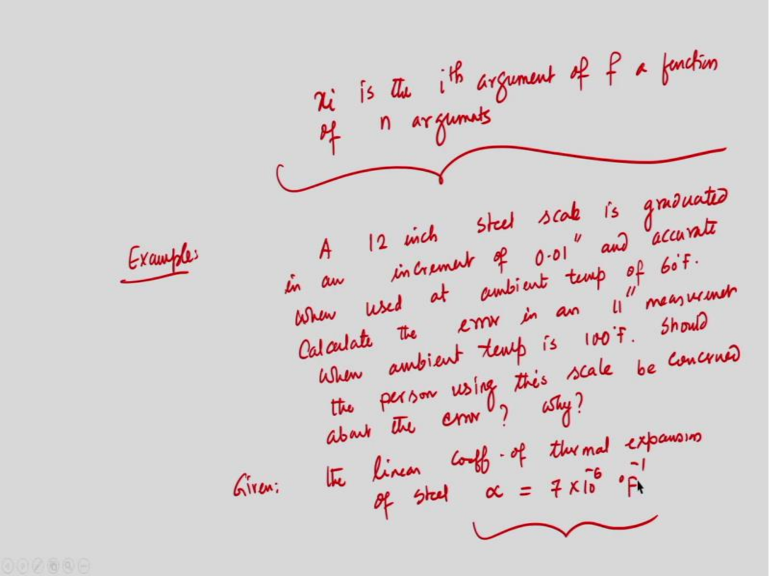

So, example, that are 12 inch steel scale is graduated in an increment of 0.01 inch and accurate when used at ambient temperature of 60 degree Fahrenheit. Calculate the error in an 11 inch measurement when ambient temperature is 100 degree Fahrenheit. Second question, should the person using the scale, using the scale be concerned about the error and why?

So, this is the example, we have to calculate that a 12 inch steel graduate, steel scale that is a metal tape, metal scale is graduated in an increment of 0.01 inch and accurate when used at ambient temperature of 60 degree Fahrenheit. We need to calculate the error that we will get, definitely get with an, or in an 11 inch measurement, when ambient temperature is 100 degree Fahrenheit.

Second question is the person who is using the, who will be using this scale be concerned, the person who is using or who will be using this scale should be concerned about the error if yes, then why? So, now 1 hint is given that the, given that the linear coefficient of thermal expansion of steel that alpha is 7 into 10 power minus 6 degree Fahrenheit inverse. So, this is given.

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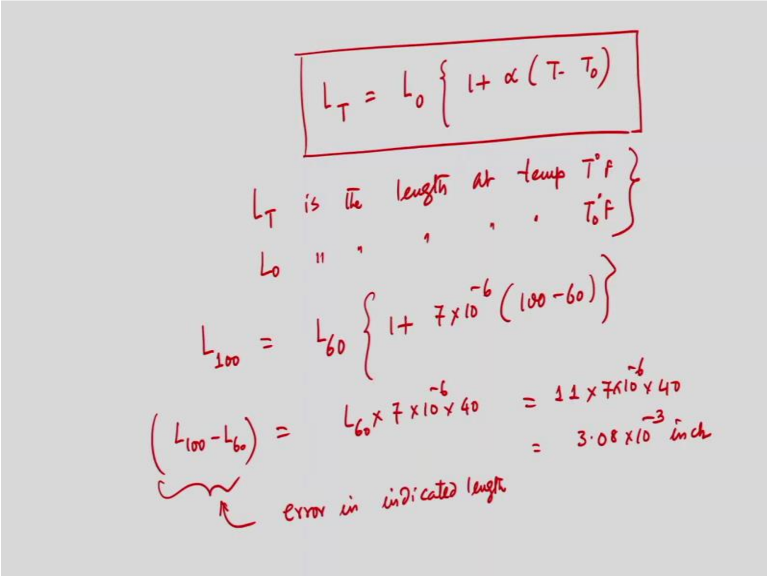

So, if you work out this problem, then we can write that, we have studied from our 10 plus 2 physics that LT will be equal L naught into 1 plus alpha into T minus T naught.

So, this is what we have studied in our 10 plus 2 level physics. So, the, you know with a change in temperature the, we have the linear coefficient of thermal expansion then what will be the change in length, with the change in the temperature. So, this LT is the length at temperature T degree Fahrenheit, L naught is the length at temperature 60 degree Fahrenheit.

So, our objective will be to calculate at 100 degrees Fahrenheit temperature that is L, so we can write that T naught degree Fahrenheit, so this is T not degree Fahrenheit. So, L 60 into 1 plus, 1 plus 7 into 10 power minus 6 into 100 minus 60. So, that is what we know and this is very straight simple, very straight forward, plus you know 8th level algebra. Now, we can calculate L 100 minus L 60 that is L 60 into 7 into 10 power minus 6 into 40.

So, this is the error in indicated length. So, this quantity is error in length and that will be equal to that 11 into 7 into 10 power minus 6 into 40. So, if we have, if we now go back to the previous slide where we have written that means, if you would like to calculate, if you would like to measure 11 inch using the same scale we will get error that is 3.08 into 10 power minus 3 inch. That is the error we will get. So, the first part of the question is calculate the error in an 11 inch measurement when ambient temperature is 100 degree Fahrenheit, so we will have error that is 3.08 into 10 power minus 3 inch. Now second question is should the person using the scale be concerned with this error, if he or she is concerned then why? And if they are not concerned with this measurement then also why they are not concerned?

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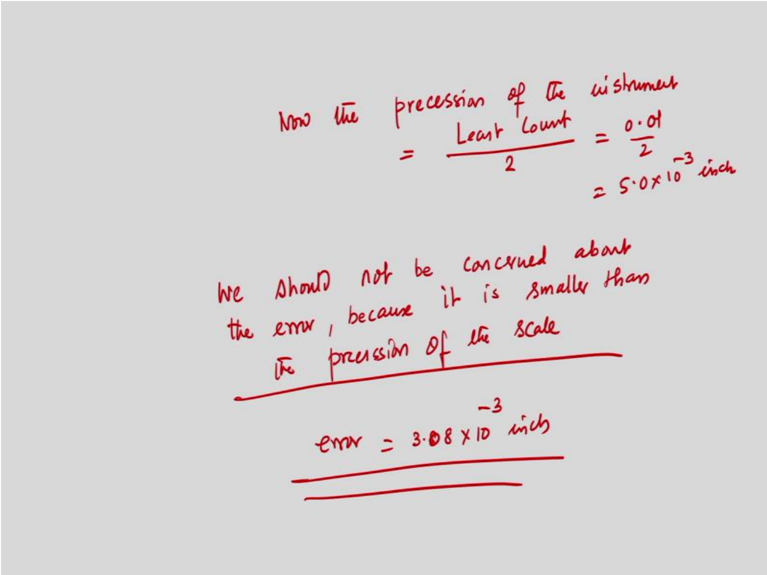

Precision, now the, precision of the instrument is list count by 2 that is 0.01 by 2, that 0, that is we can write 5.0 into 10 power minus 3 inch.

So, what we can see the precision of the instrument is 5 into 10 power minus 3, where error is 3.08 into 10 power minus 3, that means if we use this you know, scale to measure 11 inch in a condition at ambient temperature is 100 degree Fahrenheit, then the person who will be using this scale should not be concerned about the error because list count, so we should not, I am writing, we should not be concerned about the error, because error is, it is smaller than the precision of the scale.

So, this is the you know first answer and error, second answer error is 3.08 into 10 power minus 3 inch. So, this is the second answer. So, the person who will be using the scale should not be concern about the error because precision of the error is smaller than precision of the scale.

So, to summarize today's discussion, we have discussed about, we have taken 1 example to see rather to, you know follow the steps using the Chauvenets Criterion for rejecting data and then we have discussed about the error propagation and considering a single variable function, we have worked out the, you know we have analyzed the error propagation using statistical method and then we have discussed that if the function is two variable function, you know the function is having two variables or several variables then what will be the error and the standard deviation.

Finally, we have worked out 1 example to calculate the error associated with the measurement or distance using a you know metal scale and the problem which we have worked out is largely related to the fixed error. And we have seen that the, you know error which is coming out to this measurement, with this measurement.

And finally, we have discussed about the precision of the instrument and see, we have seen that the error which we have calculated, which is less than the, smaller than the, you know precision and that is why the person who will be using the scale in measuring that small distant should not be concern about the error. So with this, I stop my discussion today and we will continue our discussion in the next class. Thank you.