Experimental Methods in Fluid Mechanics

Professor Dr. Pranab Kumar Mondal, Assistant Professor
Department of Mechanical Engineering
Indian Institute of Technology, Guwahati

Lecture 39
Examples

Good afternoon, we will continue our discussion on experimental methods in fluid mechanics and today we will try to discuss about the response characteristics. In fact, we have discussed in the last class that we need to know a few important issues and these important issues rather the fundamental issues are important in the context of measuring process, measuring method, that means if we need to measure any parameter in the context of fluid mechanics, maybe it is pressure or sometimes we may need to measure temperature of a system.

And if we need to measure either temperature or pressure, then we will be using equipment, instruments, mechanical device. Sometimes we need to use electronic circuit to obtain the output characteristics, output parameter. Now, if we recall what we have discussed in the last class that there are the fundamental questions that need to be answered with respect to the any measuring instrument, equipment to be precise the measurement method.

So today we will discuss that if we would like to measure temperature, temperature of the fluid which is flowing through a fluidic channel or confinement then if the temperature is fluctuating, if the temperature is not steady or even if the temperature is steady then if we measure that steady temperature using any temperature measuring instrument, then we have seen from the last class that even if the input is steady, system or the instrument will take some finite time for the steady output.

So the fundamental questions that we need to consider when using any particular system or instrument in measuring any flow parameter are we have discussed in the last class.

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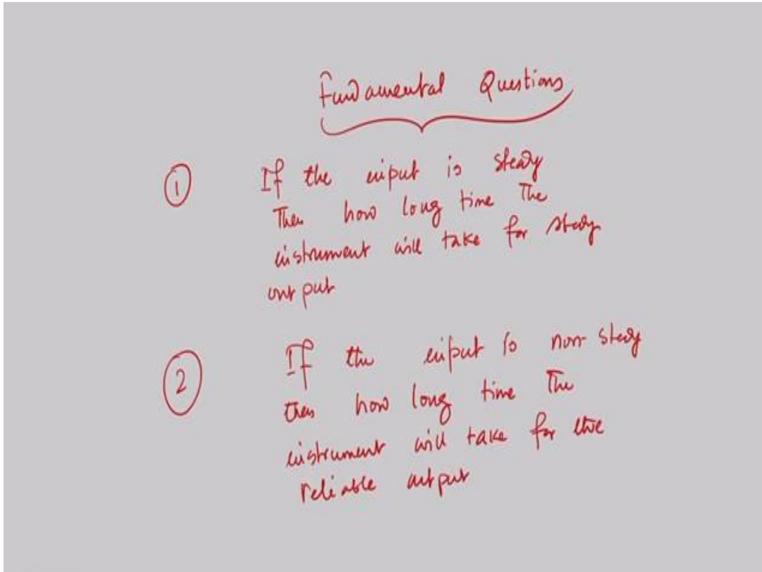

So I am writing the fundamental questions. What are the fundamental question? Number one, if the input is steady, then how long time the instrument will take for steady output? So this is important question that is what we have discussed in the last class that if the input is steady.

Now second question is which is very important, if the input is non-steady, then how long time the instrument will take for the reliable output? That means if the input is not steady, non-steady, then if we use any particular equipment in measuring that particular flow parameter maybe it is temperature or it is velocity or pressure, then how long time instrument will take for the reliable output?

So we can get output from the instrument or device but question will be whether that input, whether that output is correct or not. That means whether we are getting reliable output from that instrument and if we need to get the reliable output from that particular instrument, how long we need to wait? So that is the important question in this context.

So today we will discuss, in fact if we try to recall that, if we try to recall in one of the previous classes we have discussed about the transient response characteristics. So we will discuss that again today.

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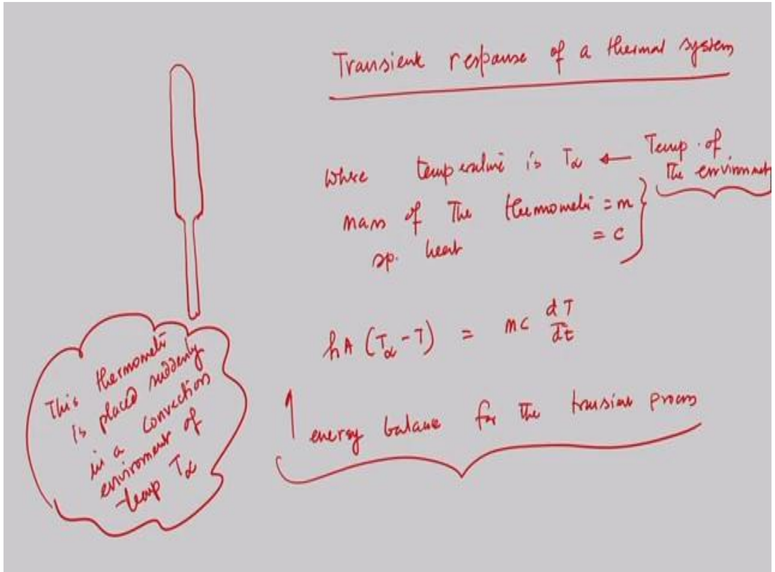

So first we will try to discuss about the transient which is very important to know, that is transient response of a thermal system. So if we are using any temperature measuring instrument, for example if we consider a thermometer, then if we use thermometer in measuring temperature in a situation where the input temperature is continuously fluctuating. That is the input temperature is not steady. In that case, if we use that thermometer in measuring the temperature, temperature will continuously, thermometer will give us continuously oscillatory output.

Now to know whether that oscillatory output is correct one or not or if we need to obtain the reliable output then, do we need to consider a few important aspects of the measuring device or system and this, for this we need to consider the transient response of a thermal system.

So the transient response of a thermal system is important only when the input of the input parameter is not steady. So if we try to recall that, say if we have a, this is a thermometer. Now if we use this thermometer in a condition, in a situation where temperature is T infinity and mass, so T infinity temperature of the ambience, temperature of the environment, temperature of the environment and mass of this thermometer is m, mass of the m and specific heat is c. If this is the case and what will happen?

So, if we place this temperature in a place where temperature is T infinity and if mass is the m and specific heat is c, then just simply an energy balance, from this energy balance we can write hA into T infinity minus T so heat will be now convected from the ambience and that is what we would like to record, we would like to measure and that is nothing but m into c into dT by dt, small t.

So this is the equation we have discussed in one of the previous classes. And this is the energy balance for the transient processes, so why we are calling it as transient processes? That means this thermometer is placed in a suddenly, so I am writing this thermometer is placed suddenly in a convection environment of temperature T infinity.

So this is the case. So we are placing suddenly in a convection environment of temperature T infinity, then this is the energy balance equation and this is important to study that is the transient response of this system is very important to know that we are placing this thermometer suddenly then it will take some time. So initially temperature of the thermometer is say T and now it is placed at in a situation T infinity then how much or how long time the system will take to give a reliable output. That is very important and for that we need to know the transient response characteristics system.

So in the last class we have discussed about the dynamic response consideration in the context of pressure measuring using in the context of the pressure proof, that means if we use pressure proof in measuring the pressure of the total pressure or the pressure in a, of a fluid in a fluidic environment, then we have seen that there are two different ways by how we can measure the pressure.

In the first case, we can mount up precise sophisticated small sized transducer pressure transducer at the sight, where we would like to measure the pressure. Now we have seen that this, those pressure transducer which are very sophisticated and small size those are not readily available and they are very costly.

So this case is not used in most of the practical applications and that is why we have seen that what is done normally, we can take a small pressure tap and the small pressure tap is now connected to the pressure transducer via Tygon type tubing and in that case if we would like to measure pressure, then we have seen that dynamic response consideration plays an important role. So that is the case. And that will definitely leads to a one important equation and that is what we have said in the last class.

So today we will discuss, today we are discussing about the transient response of a thermal system and we have seen that if we place the thermometer the initial temperature at T is equal to 0, temperature of the thermometer say T naught and now if we are placing the thermometer in a situation where temperature is suddenly or suddenly it is immersed in a place where temperature is T infinity, then this is energy variance equation we need to, we have seen.

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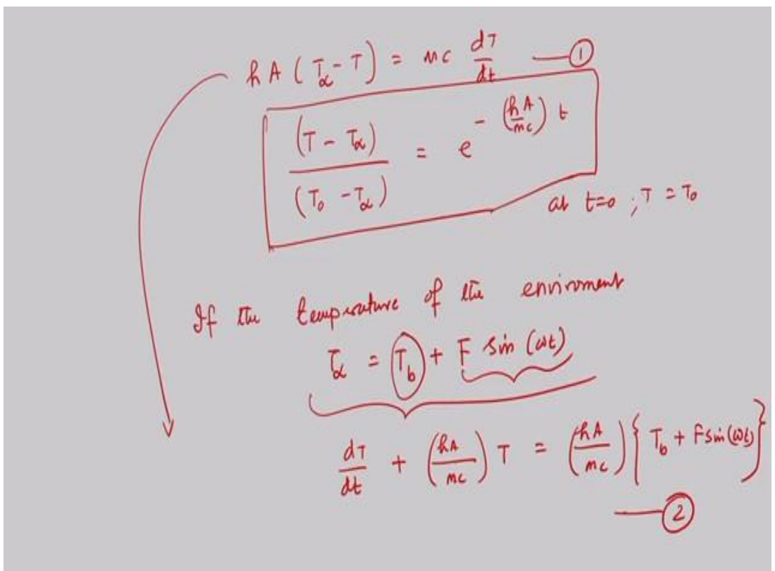

Now question is if we solve this equation we will get that T minus T infinity, so I am writing this equation once again that h into A T minus T infinity, sorry T infinity minus T, that is nothing but mc into dT by small dt. Now h is a convective witness by coefficient and A is the area. So now if I solve this equation and then ultimately this is the important equation, energy variance equation that is what I have written in the last class that energy variance is for the transient process.

Now, we will get what, the solution T minus T infinity and that is nothing but T naught minus T infinity, so that will be equal to e power minus mc by, sorry hA by mc into t. So this is the solution. And in this solution we are assuming that at t equal to 0, temperature T is equal to T naught, so initial temperature is T naught and that is the equation. So we can calculate what will be the time required to reach the steady state temperature. So this is that is what we have seen.

Now question is we have discussed this, now so this is the case where we have placed but one thing we have to consider that for this case we have assumed that the temperature T infinity that is constant, that is not fluctuating. But if the situation is like that, like this that if we that now if the temperature of the environment is T infinity that is T say b plus F sin omega t, so instead of a constant temperature if the temperature of the environment where we have placed the thermometer suddenly is varying and that is given by this equation, then the transient response is very, very important to know the, about the output characteristics of the thermometer.

So if we now solve equation one that is what we have written on the top, considering the fact that the place where thermometer is immersed having temperature which is having mean component as well as the fluctuating component. So this fellow is the mean and this is the fluctuating part.

So that means the environment temperature is not constant rather environment temperature is a temperature having a mean component and the fluctuating part, in that case what can I write that if we write the, if we start from this equation and we can write that dT by d small t plus ha by mc into T that will be equal to hA by mc into Tb plus F sin omega t.

So that means just what I have done, I have just put the value of T infinity in equation one and ultimately we are getting this equation and this equation is say equation number 2. So this is the equation which will govern the transient response characteristics of the thermal system. To be precise, transient response characteristics of the thermometer which we have used in measuring the temperature of a place where temperature is not constant rather temperature is having mean part as well as the fluctuating part.

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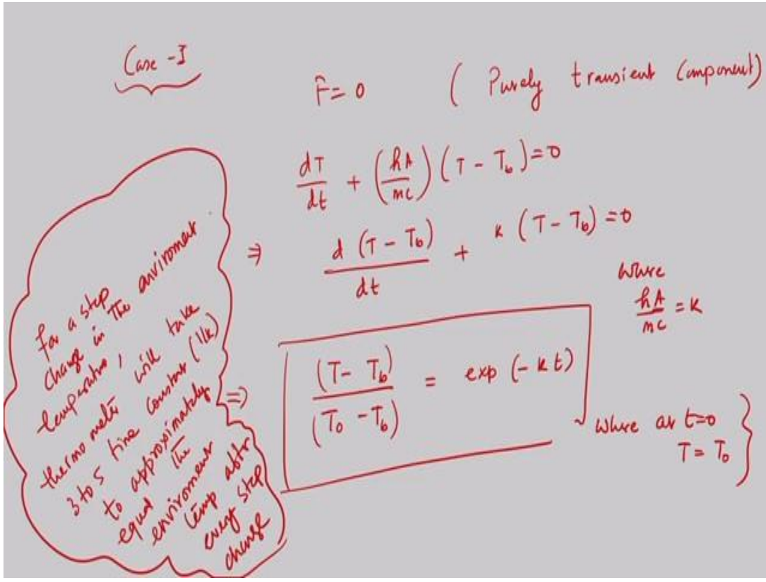

So now this equation if we go back to the previous slide, this equation can be solved using the IF that is integration factor, but so this spot I have solved but again I am solving today. So if we consider that F is equal to 0, so case one and if this is the case and if you are considering F is equal to 0 that is purely transient component, purely transient component.

In that case we can write dT by small dt plus hA upon mc into T minus Tb that will be called to 0 or I can write d of T minus Tb, T minus Tb divided by dt plus k into T minus Tb equal to 0 and where hA by mc that is k. And we can write that T minus Tb by T naught minus Tb will be equal to exponential minus k into t.

So this is the case where at t equal to 0 T equal to T naught. So this is the solution, that is what we have seen from the last class but again today we are doing it to understand. Now if it is, if the situation is purely transient that means there is no fluctuating part, then what will happen? We will get the solution like this and this is the solution.

So what we can see that we are writing here that for a steps change, for a step change in the environmental temperature, environment temperature, so if we purely transient if we change the temperature of the environment stepwise, then our objective is to find out the time required, the time that will be taken by the system to give a steady output and that is what is very important in the context of the measurement process. So, for a step change in environment temperature, so it is transient. Now what will happen? I mean the system or the thermometer, I am writing thermometer will take 3 to 5 time constants 1 upon k to, thermometer will take 3 to 5 time constant 1 upon k to approximately equal the, to the, equal the bath temperature, equal the environment temperature.

So this is very important which we are getting from this exercise is that if it is purely transient component, no fluctuating part, then if we use this thermometer in measuring that, in measuring temperature of the plus where there is a step changing temperature, then after every step, so temperature I am writing after every step change, that is important.

So after every step change the thermometer will take 3 to 5 time constant to approximately equal to the environment temperature after every step change. So if it is step change environment, no fluctuating part, in that case if we change every step then thermometer will take time which will depend upon the constant k and the constant k, the time that will be taken by the 3 to 5 times for the constant. So it will take constant for those equal to both temperature, so this is the case.

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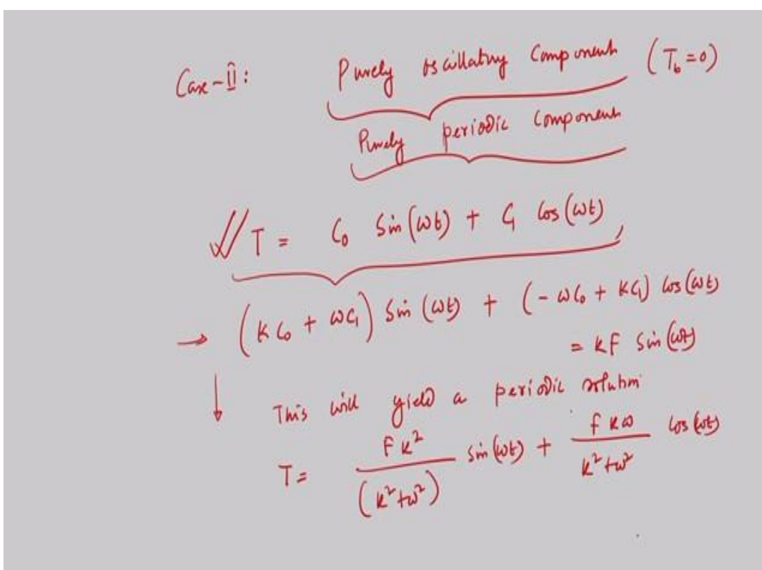

Now another case if we solve that is the case two, that is purely oscillatory component, purely oscillatory component that is Tb is equal to 0, that is Tb equal to 0 or sometimes it is known as purely periodic component. Then what we can consider, we can consider T is equal to C naught sin omega t plus C1 cos omega t.

So we can assume this is the oscillatory component, so temperature of the system is varying like this, C naught sin omega t plus C1 cos omega t. And if we that means our, we can write that if we consider this, then we can write K C naught plus omega into C1 into sin omega t plus minus omega into C naught plus K into C1 cos omega t is equal to kF into sin omega t.

So if we try to find out the solution, so if we put the value, if we consider that t and if we use the equation, that is we have written over here and then we will get the solution hA by mc that is nothing but k and if we, Tb is equal to 0. So if we consider this is the temperature profile, so the temperature is purely periodic and if we plug in the value of t in equation 1, then finally we will get the equation which is written in the second line.

Now so if we consider this, this will in the periodic solution so this will yield a periodic solution, so we are assuming the temperature of the environment is periodic and the periodic the form which is given over here and if we plug in the value of this in equation 1 we will finally get the equation which is given in the second line that is over here.

Now this will yield a periodic solution and that is nothing but T is equal to, I can write Fk square divided by omega square sin omega t plus Fk omega divided by k square plus omega square cos omega T.

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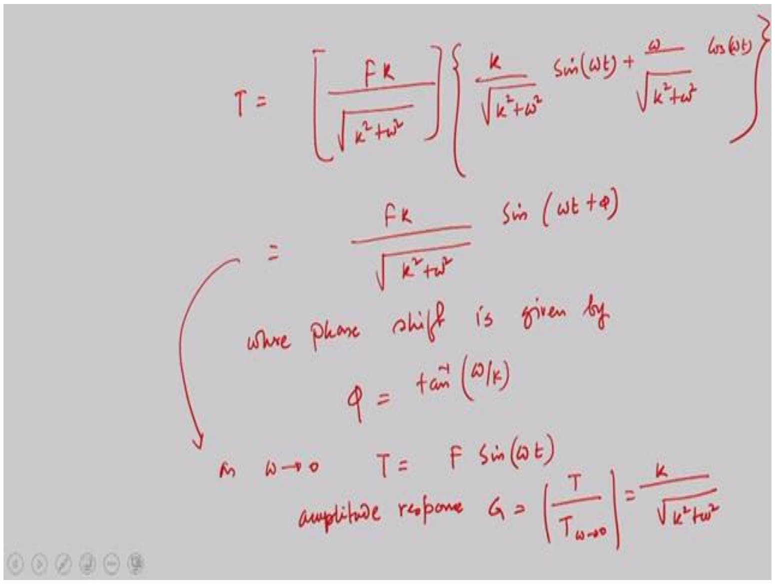

And if we do one more step, then we will get T is equal to, I am writing just, Fk divided by root of k square plus omega square into k by root of k square plus omega square sin omega t plus omega by root by k square plus omega square cos omega t. And we will get Fk divided by k square plus omega square into sin omega t plus phi where phase shift is given by phi is equal to tan inverse omega by k.

So this is the temperature solution. So the periodic solution that is given by, so if we start using equation 1 that is the, that the equation which we have drived in, drived that is equation number 1 which governs the transient response characteristics of the system. System that means we have considered a thermometer and which is used to measure the temperature of the, of a place where the temperature is T infinity.

If the period T infinity is having two different components that is what we have consider, one is the mean part another is the oscillatory part. So the purely mean component that is what we have considered that case 1 that purely transient component we have solved, again for the purely oscillatory or periodic component we have considered the profile of the temperature is given by c naught into sin omega t plus C1 cos omega t.

So we need to know what is the value of C naught and C1 and for that if we have used the equation one and finally we can write the temperature T naught, T is equal to Fk, temperature Fk by k square plus omega square into sin omega T plus phi where phi is the phase shift.

Now as omega tends to 0, T will be equal to F sin as omega tends to 0, sin omega T and amplitude response and amplitude response G that is T by T at omega tends to 0 and if we write we will get K divided by root of K square plus omega square. So as omega tends to 0, omega not equal to 0, in that case T is equal to F sin omega T that is what we are getting from this equation 1, from this equation. And amplitude response that is represented by G, we are writing like this.

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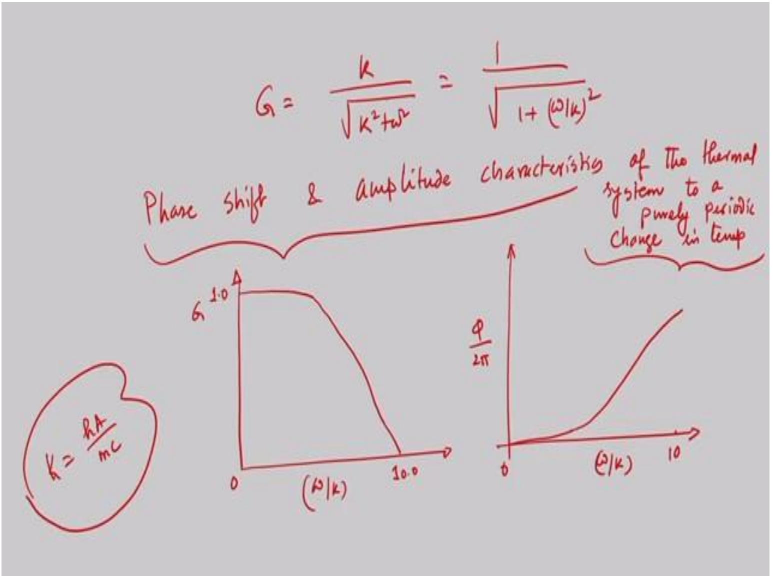

So we can write that G will be equal to k by root of k square plus omega square and that is nothing but 1 upon root of 1 plus omega by k square. Mind it k is equal to hA by mc that is what we have considered. Now, so this is the amplitude response and phase shift that is phi that is nothing but tan inverse omega by k.

Now if we try to obtain the phase shift and amplitude characteristics, then if we try to draw this we will get profile, so if we plot omega by k this is omega by k and this is G so profile will be like this and this is phi by 2 pi, so if we consider this is, say 10, this is 0 so this is 10 and this is 0, then this is 1 and this is 0 and finally it will be. So just these are the qualitative picture.

Now what we can see that this is the phase shift and amplitude characteristics of a thermal system, of the thermal system to a purely periodic change in temperature. So that is in the thermal system to a purely periodic change in temperature then this toward the qualitative profiles. Now what we can say that this k is hA by mc, k is equal to hA by mc.

Now our objective, our objective will be to have as small as a time constant as possible, that means if k is large. So if we go to the previous slide, then we will see that hA by mc is equal to k and we have written 1 by k, so our objective should be as small as small time constant.

That means if we place a thermometer in a place, if we place or if we insert a thermometer in a situation or if we put a thermometer in a place where temperature is having either periodic component or the mean component or the temperature can be, can have two components that is one is the mean other is the periodic component.

So even for the simple case we have seen that if it is purely transient component there is no oscillatory part. And in that case we have seen that 1 by k that is the time constant which will be, so for every, what we can see that every, after every step change the thermometer will have 3 to 5 time constant 1 upon k to approximately equal, I mean we will take time so to approximately equal to the temperature of the environment temperature.

So our objective should be the, will be to have as small a time constant or as small a time constant as possible which is equal to which is equivalent to k is large. So I am writing that, so that after every step change we can quickly measure the temperature, that is our objective should be. So we need not to wait for a long time. So objective should be the thermal response, so we can predict, we can estimate from the equation by, from the solution of the equation that we have solved today that the thermal system which we have considered should have as small as a time constant.

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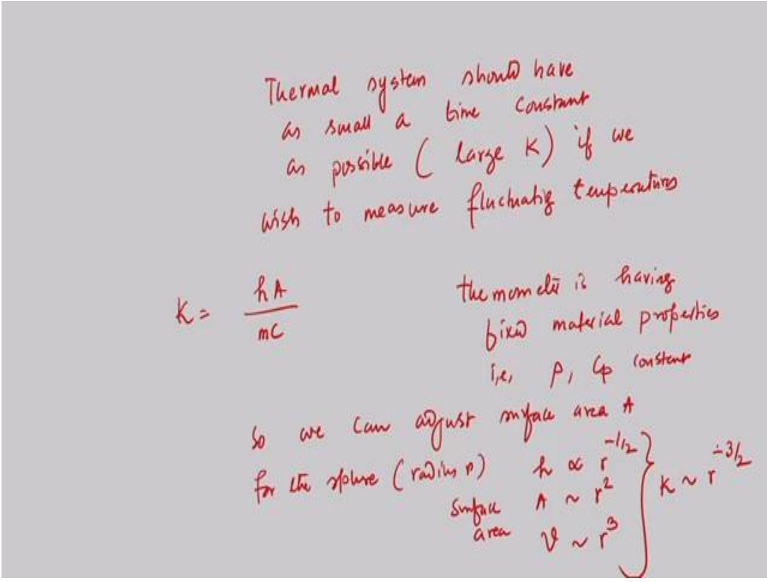

I mean I am writing that the thermal system should have as small a time constant as possible which is equivalent to large K. So 1 upon K is the time constant so the case would be large, if we wish to measure fluctuating temperatures. So we have seen from the previous note that if it is purely transient component it should have 3 to 5 time constant to approximately equal to the environment temperature after every step change. Now, if our object is to measure the fluctuating temperature, then the thermal system should have as small a time constant as possible and which is equivalent to larger K.

Now, as I said that K that equal to hA by mc. Now, if we talk about thermometer, then thermometer is having fixed material properties that is rho and h constant. So it cannot change rho, if we cannot change rho then mass cannot be changed, h cannot be changed, specific heat is, sorry rho and Cp, rho and Cp constant.

So mass cannot be changed, specific heat cannot be changed because fixed material. Convective heat and mass transfer, heat transfer coefficient, that is we cannot change, so that will depend that the environment so that we cannot change. Now, what we change that, we can increase the surface area. So aim we cannot alter, c we cannot alter, (())(40:21) is fixed, so we can alter only A.

So we can adjust surface area A, now for this fear, I am writing for the sphere if radius is r, so if we adjust a surface area that means we can make it sphere, we can cylindrical, it may, we can make it different other steps.

If it is sphere, radius R, then heat transfer coefficient h is proportional to r power minus half. Surface area is proportional to, so this is surface area proportional to r square and volume, v is proportional to r cube. If it is the case then k will be proportional to r power 3 by 2 minus 3 by 2.

So heat transfer coefficient is proportional to r to a minus half and surface area proportional to r square, then k will be proportional to r power minus 3. So what we can see that if we do the, if we plug in the values in the equation in k then we will get k is proportional to r power minus 3 by 2 which indicates.

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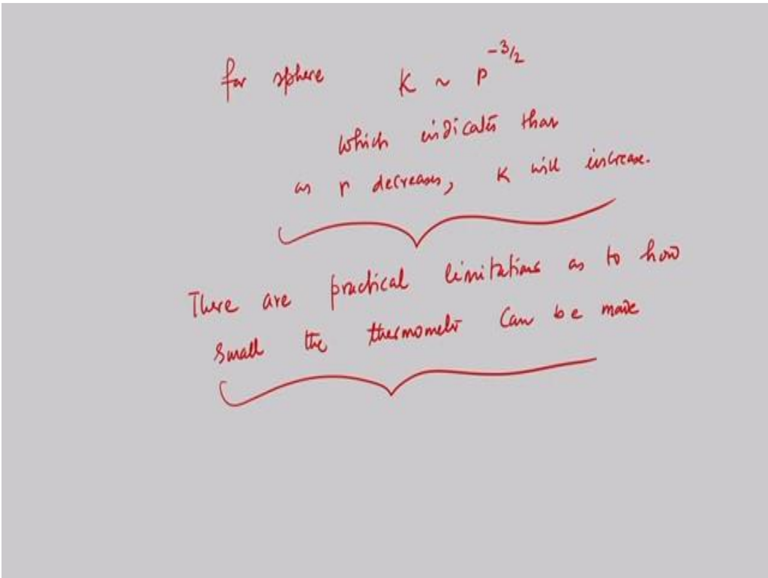

So for sphere, if we consider sphere, k is proportional to r power minus 3 by 2 which indicates that means which indicates that as r decreases k will increase. So our objective should be to have larger k, time constant will be very small that is what I have written in the last slide that as small as time constant as possible, larger k. If we would plan, if we are thinking to have larger k, that means r should be decreases, r should be reduced. Now r decreases that means the volume will decrease. So there are practical limitation, so we cannot reduce r drastically, so there are practical limitation as to how small the thermometer can be made.

So what we have understood from today's class is that our, we have taken an example, we have understood the importance of dynamic response consideration in the last class. In the context of measuring pressure using a pressure transducer we have discussed about the fundamental issues, fundamental aspects of this dynamic response consideration.

We have seen that there are two different methods available in measuring pressure, but in one method in the first method if we just mount the pressure transducer at the sight, then we can obtain the pressure with and only we need to wait for a time that is there is no more another problems.

But in the context of, in the second method, that means if we, we cannot use that, we cannot add up the first method always, the limitation we have discussed following the second method if we would like to measure pressure then we have seen that there are reasons which we need to consider for which we need to consider the dynamic response consideration.

Today, we have discussed about the measurement of temperature and in this context we have again seen that the dynamic response consideration plays an important role and which needs to be considered in measuring temperature. Considering or taking an example today, we have discussed the measurement of temperature using a thermometer or using a thermal system and if the thermal system is used to measure temperature where, of the environment where temperature is steady, only the transient response if the temperature is purely steady.

If the temperature is not steady but the temperature will have only transient component and if the temperature is non-steady or unsteady but the temperature is having fluctuating components and for that we have solved the differential, solved the equation. And from the equation we have come up with the conclusion that the temperature, I mean from this mathematical analysis we have come up with the conclusion that the r should be decreased so that the surface area plays an important role for the fix material because for the fix material we cannot alter the rho and the specific heat. So only what we can change, the change in the surface area that means we can alter, we can tune, we can adjust the surface area essentially to satisfy the objectives of a thermal system. And we have seen that the r, the radius if we model that thermometer by a sphere and then r should be reduced but we cannot reduce drastically the radius.

But I mean there are practical limitation but we have understood that if we would like to measure temperature that are fluctuating components, then case would be large and if we need to ensure that the case would be large, radius will be very small. That means only a smallest possible thermometer can give us the result which is used to measure temperature in the fluctuating environment.

So with this, I stop my discussion today and we will continue our discussion in the next class. Thank you.