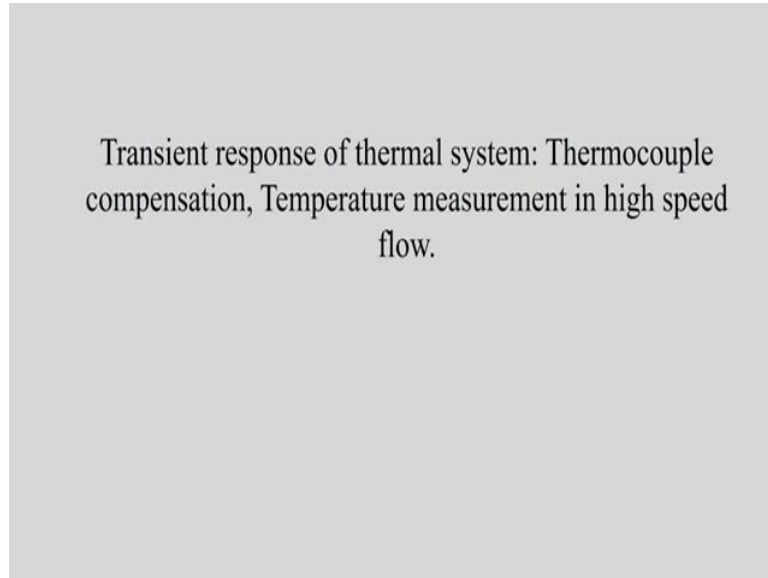


Experimental Methods in Fluid Mechanics
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Lecture 27

Transient response of thermal system, Thermocouple compensation, high speed flow

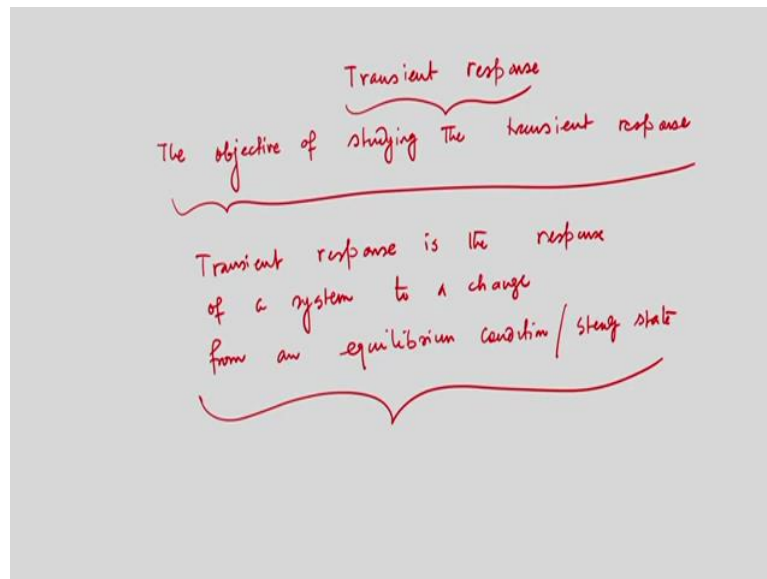
Good afternoon, we will discuss today the transient response of the thermal system.

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And we will discuss in detail about the transient response characteristics towards the end of this course, but at least today we will see what is the transient response and why it is important, of course from the perspective of experimental methods and then taking an example we will see that how can we obtain the response, response characteristics of a thermal system. So, to start with, we would like to know what is the, why this transient response characteristics is important.

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So, that is transient response. Why it is important? What is the meaning of that? That is what we should discuss today. So, the I can write the objective of studying the transient response of any system. That means, so what is tolerated response? Why we have to study this part? And this is very important in the context of experimental methods, experimental techniques, which are used for in different areas, but particularly in the area of thermal fluid system that we are discussing.

So, I am writing transient response, which is essentially the response of a system to a change, you know the project transient response is a response of a system to a change from an equilibrium condition or sometimes we, since we are using the word, transient, so it is the steady state. So, this is we should know that the transient response is nothing but the response of a system to a change from an equilibrium condition or steady state.

That means, a system will be, if a system, today we will try to discuss this taking an example of a thermal system. So, if the thermal system is disturbed by an external factor and the disturbances I mean if the system is getting disturbed by many sources, so the steady state or the equilibrium condition of the system will be perturbed, will be disturbed. So, that means, if a system if we now today we will discuss about it from the perspective of a thermal system.

So, a thermal system is now getting disturbed by a source and as a result of which the equilibrium condition or the steady state condition of the system is getting you know perturbed. Now, if you would like to measure say if you would like to measure the temperature of the fluid which is kept in a bath and if we insert a thermometer we can

measure the temperature. Now, if the temperature of the fluid is getting changed by an external agent and because of what the temperature of the fluid will be changed.

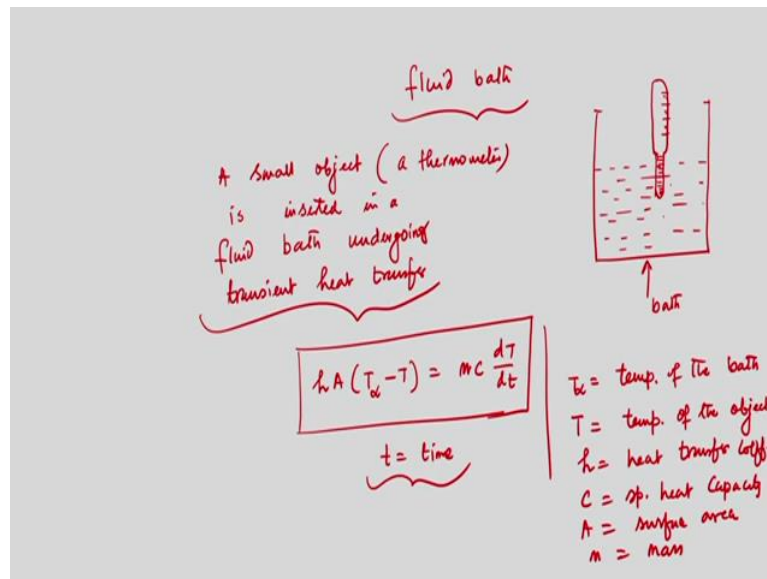
That means, when the fluid is in the steady condition, steady state fluid temperature is the steady state, I mean, temperature everywhere in the fluid is remaining same, we can measure the temperature using a thermometer. And if we insert the thermometer we will get the reading.

Now, temperature reading. Now, if the temperature of the fluid is disturbed by a source and as a result of which there will be temperature fluctuation, so we need to know the response characteristics, how quickly we can, you know I mean the temperature will be disturbed, but if you would like to measure the temperature instantaneously, so how quickly that thermometer can give the correct result.

And also, we will see today that our objectives would be the you know design a system, our objective should be record, measure the temperature of a particular thermal system, that is what I was talking about, that the disturbance, perturbations, the disturbances, perturbations which are there in the system externally that should be recorded by a thermometer and the time gap, that should be very small.

So, that means instantaneously we can measure the disturbance and that is there in the system. So, that means the response of a system to a change from the steady state, the system will response to a change and the change is from what, I mean changes from the steady state. So, now we will take an example.

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Say we have a fluid bath and this is filled up with 1 fluid and if we use a thermometer, so we can measure the temperature of the liquid of the fluid which is kept in the bath. So, this is bath, so now a small object or thermometer is inserted in a fluid bath undergoing transient heat transfer. That means, if we insert the thermometer, a small object in the fluid bath, then we would like to measure the temperature.

So, there will be heat transfer, but if the heat transfer, I mean the system is going you know, transient heat transfer, then we need to know the transient response characteristics, that is important. So, we need to model. So, today we will discuss about the modelling of the transient response of a system. And then, as I said towards the end of this course, we will discuss in detail about the response characteristics.

So, if this is the system and what will be the heat transfer? That means heat transfer, if we try to model then we can write that you know hA into T infinity minus T_a , that is mc into dT by dt where T infinity is the temperature of the temperature of the bath or you know of the fluid bath, T is the temperature of the object, H is the heat transfer coefficient. So, it is not necessary that always we have to insert a thermometer, it may be any object.

So, we are putting an object and heat transfer coefficient and C is the specific heat capacity and A is the surface area and m is the mass. So, that means, mass of the object that is what we have inserted there in the system, C the specific heat capacity, A is the surface area through which heat is being transferred from fluid into the object. Now, this is the equation which will govern the transient response characteristics.

Now, that means this small t is the time. Now, when T infinity is constant throughout the, I mean if we now consider the fluid bath and if the temperature is uniform, that the bath temperature is uniform, T infinity, then this is the equation we can model. So, this is obvious that if we place an object inside a fluid bath, that may be a thermometer as well, if we need to measure the temperature of the fluid which is there in the bath, this is the equation.

That means there will be heat transfer and of course, the transfer of heat will take certain amount of time. So, that is hA into T infinity minus T into mc into dT by dt . So, the time, there will be finite amount of time after which the temperature of the bath, the temperature of the object will be equal to the temperature of the bath and the time required by the object will depends upon the specific capacity, mass as well as the surface area.

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$$hA(T_{\infty} - T) = mc \frac{dT}{dt}$$

If we consider

$$T_{\infty} = T_m + F \sin(\omega t)$$

↑
bath temperature

↓
is composed of the mean and periodic components

Then equation which will govern the object temperature

$$\frac{dT}{dt} + \frac{hA}{mc} T = \frac{hA}{mc} \{ T_m + F \sin(\omega t) \}$$

Now, if we so this is the equation. If we consider that T infinity is T mean plus F sine omega t . That is, this is bath temperature and this bath temperature is composed of the mean and periodic component. That means, if the T infinity is T_m plus F sine omega t , that is, it is having the mean component as well as the periodic component, then that equation which will govern the object temperature will be, so we had hA T infinity minus T , that is mc dT by small dt .

So, this is the equation. Now, we are considering, the T infinity, that is the bath temperature, the temperature of the fluid in the bath, which is having two components, one is the mean and another one is the periodic. Now, then the equation which will govern the object temperature will take different form and that is nothing but dT dt d capital T by d small t plus m , I can

write hA by mc into T that will be equal to hA by mc into T infinity, that is nothing but T_m plus $F \sin \omega t$.

So, this will be the equation. That means the, if the system is now having the temperature, temperature of the bath which is having two different components, one is mean and another is the periodic component. Now, then this is the governing equation, if we solve this equation, we will get the time versus temperature and we will see that what will be the time required to reach at this steady state condition.

Now, this equation we can solve using integrating factor, but it would be more convenient to consider first the solution to the corresponding purely transient component. So, this is the equation, this equation can be solved using integrating factor, but it will be more convenient, that means there are two different components of the bath temperature now. One is mean other is the another one is the periodic component. So, it would be more convenient to consider first.

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Handwritten notes on a slide:

First Case: Consider only the purely transient component (i.e. $F=0$)

$$\frac{dT}{dt} + \frac{hA}{mc} (T - T_m) = 0$$

or, $\frac{dT}{dt} + k (T - T_m) = 0$ [where $\frac{hA}{mc} = k$]

$$\frac{(T - T_m)}{(T_0 - T_m)} = e^{-kt}$$

[where T_0 is the initial temp.]

for a step change in bath temp, it will take 3 to 5 time constant ($1/k$) for the object to approximately equal to the bath temp

So, as the first case, the first case consider only the purely transient component. That is F equal to 0. Then we can write the equation d capital T by d small t by mc T minus T_m equal to 0 or if we solve this equation I can write dT by d small t plus some constant k into T minus T_m will be equal to 0 where hA by mc is k . Then T minus T_m divided by T naught minus T_m .

If we solve this equation we will get, that is nothing but e power minus kt where T naught is the initial temperature, where T naught is the initial temperature. So, this is the solution we

are getting for the purely transient component. That means there is no oscillating component. Now, if this is the case what we can say that you know for a step change in bath temperature, it will take 3 to 5 time constants, that is $1/\omega$ for the object to approximately equal to the bath temperature (21:09).

So, that means for a step change in bath temperature, it will take 3 to 5 time constant, $1/\omega$ for the object to be equal to the bath temperature after the step change. So, that is what, first case. So, our objective was to see that we have placed an object, if the temperature of the object is less than the temperature of the fluid which is kept inside the bath, then it will take some amount some time to reach the bath temperature which is equal to the temperature of the object rather which is equal to the bath temperature.

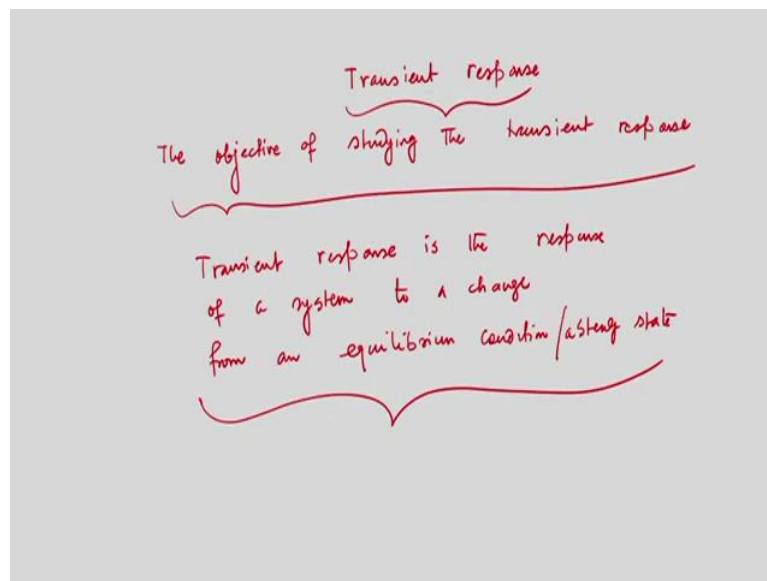
So, what I was telling? That the step change, now, we need to know that transient response. So, that is what we should design. So, that means, if the objective is now thermometer that means, that is what I have purposely drawn the object by a thermometer in the schematic. Whenever we are putting you know, rather we are inserting thermometer in the liquid bath, we are measuring the temperature.

Now, if the temperature of the fluid is constant, then that is what the example now we have solved that it will takes, you know that means, now if we heat up the liquid by a step maybe from 10 degree to 12 degree, 12 degree to 14 degree, then the thermometer, whether the thermometer can measure the temperature, I mean how fast the thermometer can measure the temperature to a change of the system, rather that means system is now getting changed from its steady condition, steady state.

So if which initially it was initially temperature of the bath is 10 degree. Now, if I change the temperature of the thermometer by 2 degree, how fast the thermometer can record, can measure the temperature that is what our objective should be to design the thermometer. So, that is what I am now, would like to tell that for a step change in bath temperature, it will take 3 to 5 time constant for the object to approximately equal to the bath temperature.

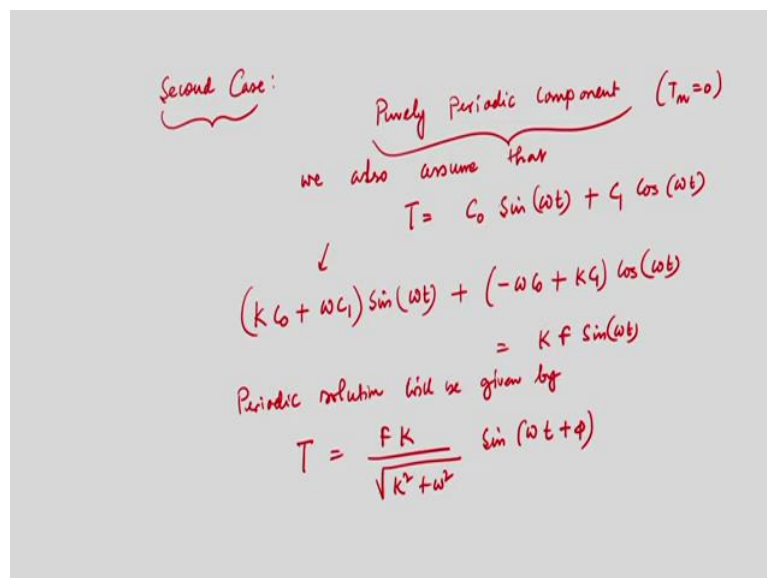
That means, the temperature of the object will definitely be equal to the bath temperature, but for that it requires finite amount of time the time and the required time will depends upon so many factors and that is what our objective is. That means, we have to design that system thermometer in such a way that we can instantly measure the change of a system.

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And that is why in the very beginning I wrote that what you know what is the transient response. The response of a system to a change from an equilibrium steady state. So, this is what for the steady case.

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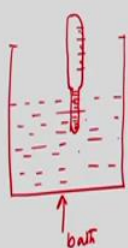
Now, case 2, if we consider, rather second case, then that is purely periodic component. That means T_m equal to 0 then and we also assume that T is equal to T I mean C naught sine ωt plus $C_1 \cos \omega t$. That means, we can write the equation that is K into C naught ω into $C_1 \sin \omega t$ plus minus ω into C naught plus K into $C_1 \cos \omega t$ is equal to $K F \sin \omega t$.

And so, the periodic solution, that means, periodic solution is will be given by T is equal to F times k by root of k square plus ω square sine ωt plus ϕ . So, I did not write the intermediate steps. If it is purely periodic component, T_m is equal to 0 and F sine ωt and we are assume that T is equal to C naught sine ωt plus C_1 sine ωt .

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fluid bath

A small object (a thermometer) is inserted in a fluid bath undergoing transient heat transfer



$$hA(T_b - T) = mC \frac{dT}{dt}$$

$t = \text{time}$

$T_b = \text{temp. of the bath}$
 $T = \text{temp. of the object}$
 $h = \text{heat transfer coeff}$
 $C = \text{sp. heat capacity}$
 $A = \text{surface area}$
 $m = \text{mass}$

Second Case:

Purely Periodic Component ($T_m = 0$)

we also assume that

$$T = C_0 \sin(\omega t) + C_1 \cos(\omega t)$$

↓

$$(kC_0 + \omega C_1) \sin(\omega t) + (-\omega C_0 + kC_1) \cos(\omega t)$$

$$= kF \sin(\omega t)$$

Periodic solution will be given by

$$T = \frac{Fk}{\sqrt{k^2 + \omega^2}} \sin(\omega t + \phi)$$

$\phi = \text{Phase shift}$
 $= \tan^{-1}(\omega/k)$

In the limiting response as $\omega \rightarrow 0$

$$T = F \sin(\omega t)$$

Then, if we try to cross the equation that is what we have written in the previous slide that this equation, then we will get the solution which is given by this. T is equal to F into k by you know k square plus ω square into sine ωt plus ϕ . So, here the ϕ , it is equal to phase shift, that is \tan inverse ω by k and in the limiting case, so we can write that in the limiting case, a limiting response as ω tends to 0, T will be equal to F sine ωt .

That means, in the limiting response when omega is equal to 0 then t is equal to F sine omega t that is what we have considered in the beginning.

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$$hA(T_b - T) = mc \frac{dT}{dt}$$
 If we consider

$$T_a = T_m + F \sin(\omega t)$$
 bath temperature
 is composed of the mean and periodic components
 Then equation which will govern the object temperature

$$\frac{dT}{dt} + \frac{hA}{mc} T = \frac{hA}{mc} \{ T_m + F \sin(\omega t) \}$$

Second Case:
 Purely Periodic Component ($T_m = 0$)
 we also assume that

$$T = C_0 \sin(\omega t) + C_1 \cos(\omega t)$$

$$(K C_0 + \omega C_1) \sin(\omega t) + (-\omega C_0 + K C_1) \cos(\omega t) = K F \sin(\omega t)$$
 Periodic solution will be given by

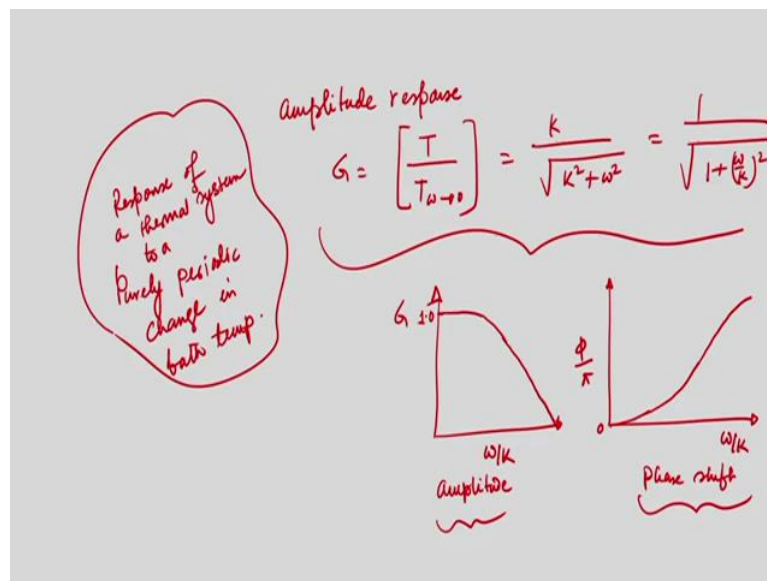
$$T = \frac{F K}{\sqrt{K^2 + \omega^2}} \sin(\omega t + \phi)$$

$$\phi = \text{Phase shift} = \tan^{-1}(\omega/K)$$
 In the limiting response as $\omega \rightarrow 0$

$$T = F \sin(\omega t)$$

That means, if we go to the previous slide where we have written that if the temperature of the bath can be considered, I mean can be considered, you know, I mean by two different components, that means, the temperature of the bath is composed of the mean temperature as well as the periodic component. Now, for the limiting response we can see from the solution that is F sine omega t.

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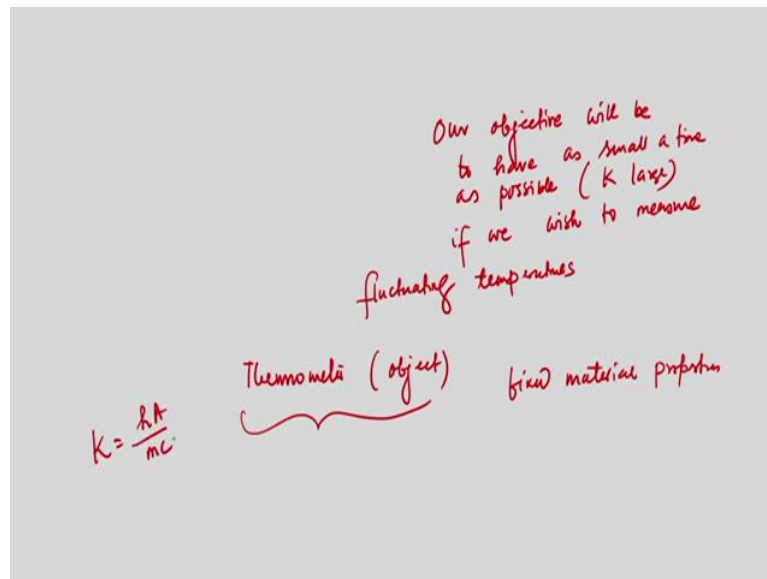


So, now question is what will be the amplitude response and that is G that will be equal to T by t when ω tends to 0 and that is k by root k square plus ω square, that is 1 upon root of 1 plus ω by k the whole square. So, this is the amplitude response. So, this is important because that is what I am not going to discuss, we have learned this in our mathematics course.

So, the amplitude and phase shift characteristics if we try to you know so, graphically you know, so that means, if we try to plot g versus ω by k and this is ϕ by π , this is also ω by k , so we will get you know, profile like this and if it is 1, say 1.0 and so this is 0 and I am not going to put the numerical values, but the train will be like this. So, these are the phase shift and amplitude.

So, this is amplitude, this is phase shift, these are the you know graphical representation of the amplitude and phase shift of a thermal system to a purely periodic changing bath temperature. So, these are the purely bath temperature. So, thermal response I can write purely change in bath temperature. So, that is the response of a thermal system to a purely periodic changing bath temperature. So, these are the you know graphical representation.

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What we can tell that, I am writing that our objective will be to have as small as you know a time as possible, as small a time as possible, that is k is large if we wish to measure fluctuating temperature. So, that means our objective when we are designing a thermometer that is the object, our objective should be to design in such a way that we can correctly capture the fluctuating temperatures.

If that is the object, from this simple analysis, from this graphical representation of the phase shift and amplitude of a thermal system or rather response of a thermal system where only the change is periodic, a change in temperature of the bath is having you know periodic change. That means, then the case would be as case would be large essentially to have as small as a time as possible stop that means we need to measure the fluctuating temperatures correctly, the response would be very fast.

Otherwise the fluctuations will die out. That means we are inserting a thermometer in the liquid bath and our objective should be to record the fluctuating temperature instantaneously. For that we need to we need to design thermometer accordingly and for that what you can see from this know expression, from the graphical representation, ks would be very large and if that is the large, so that we can you know measure the fluctuating temperature and we will have as small as a time as possible.

Now, if we talk about thermometer, so if we talk about thermometer that is the object and that is what I was telling that in fact if we look at the schematic where we have shown the

thermometer and that is inserted, so if thermometer is the object then it has fixed material properties. So, if we now look at k that is nothing but hA by mc.

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First Case: Consider only the purely transient component (i.e. $F=0$)

$$\frac{dT}{dt} + \frac{hA}{mc} (T - T_m) = 0$$

or, $\frac{dT}{dt} + K (T - T_m) = 0$ [where $\frac{hA}{mc} = K$]

$$\frac{(T - T_m)}{(T_0 - T_m)} = e^{-Kt}$$

[where T_0 is the initial temp.]

for a step change in bath temp, it will take 3 to 5 time constant ($1/K$) for the object to approximately equal to the bath temp

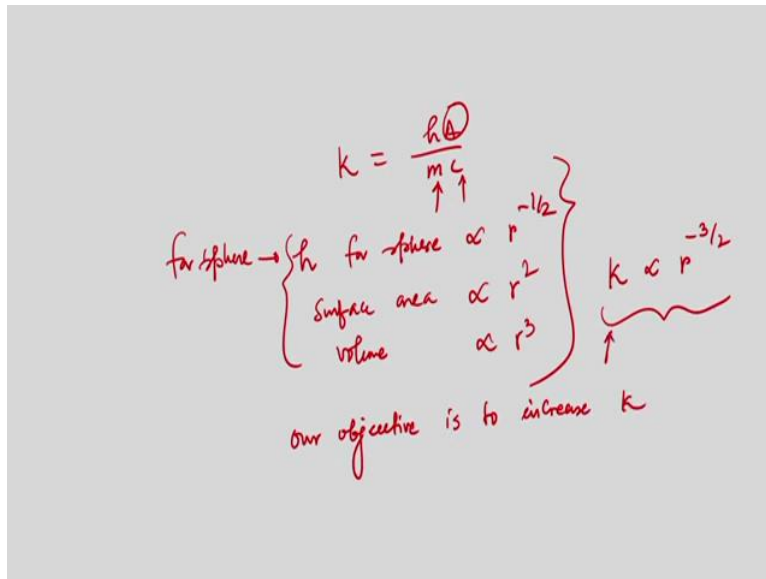
Our objective will be to have as small a time as possible (K large) if we wish to measure fluctuating temperatures

Thermometer (object) fixed material properties density & sp. heat can not be altered only we can change the surface area

$$K = \frac{hA}{mc}$$

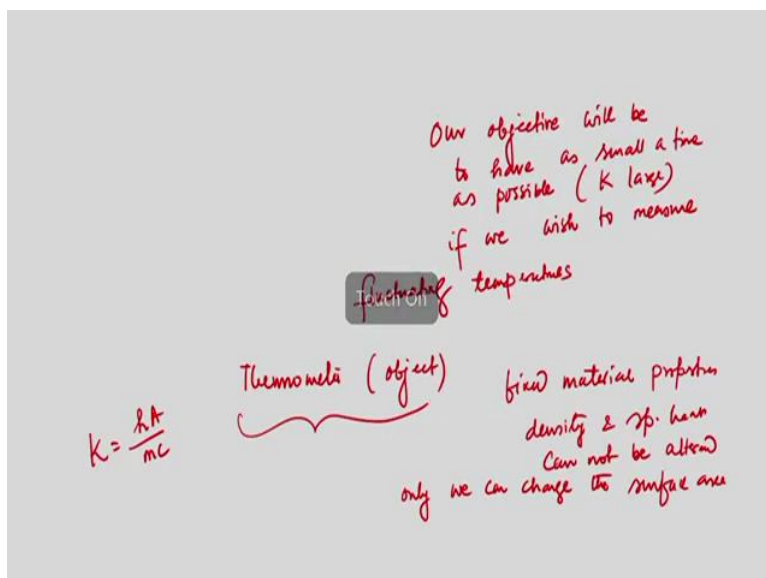
If we go to the previous slide, where k is hA by mc and now, thermometer object, it has fixed material properties, that means density and specific heat cannot be altered. So, what we can change? We can change the surface area. Density, we cannot alter, we cannot alter T_m , specific heat cannot be altered, so only we can change the area. So, that means, we can change the area. Now, this k which is heat transfer coefficient heat transfer coefficient is you know. So, I am writing in the next slide. So, we can alter only the surface change the surface area.

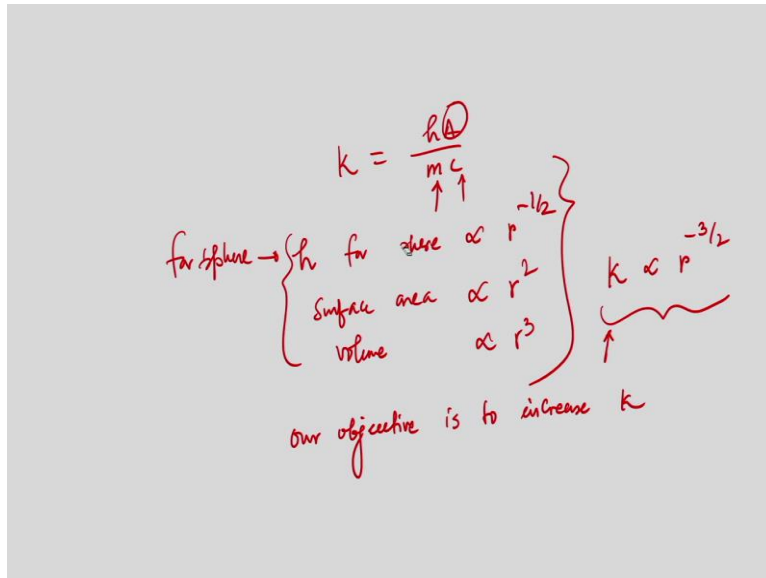
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So, we can write quickly that k is hA by MC . We cannot change this value, we cannot change this value. So, we can change only the H . Now, H for sphere is proportional to r square, sorry r power minus half and you know, surface area for sphere, surface area which is proportional to r square and volume r cube. Surface area r square and volume to r cube, then k will be proportional to r power minus 3 by 2. So, h is equal to you know that is minus 3 by 2. Now, that means, k , our objective is to increase k .

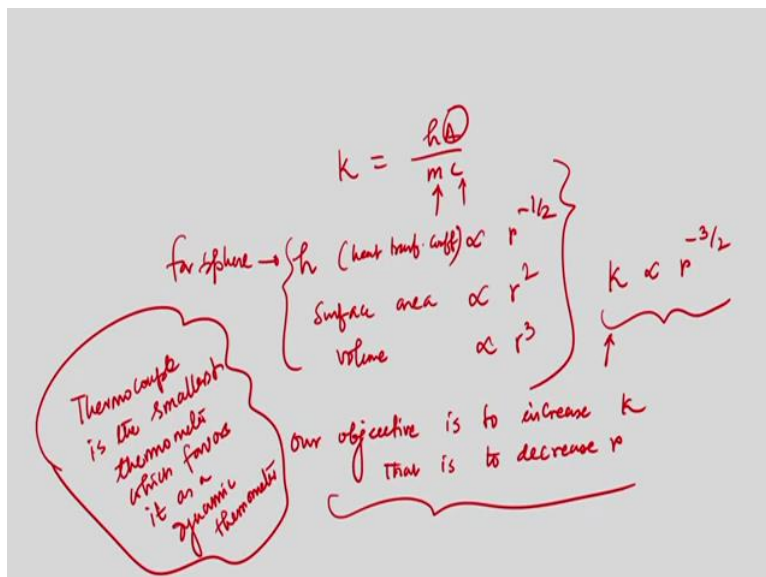
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So, as I said that if we go back to this point, our objectives will be to have as small as a time that is k should be large. That is what we obtain from this graphical representation. Now, k would be large, if we consider object is thermometer, density and specific heat for a given material is constant. So, we can alter the surface area. Sphere, h, you know that is convective heat transfer coefficient which is proportional to r power minus 1/2 and from there we can see k is proportional to r power 3 by 2.

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Now, then objective should be to increase k. That means, if we increase r, that means, that is to increase, sorry to decrease r. That is to decrease r. That means, if we reduce r, we can increase k and if k is increased, our objective will be fulfilled. That means, we can measure

temperature, fluctuating temperatures instantaneously. And that is why thermocouple is the smallest thermometer which favours it as a dynamic thermometer, not static.

So, that is why that means what we can you know see from this analysis, r should be reduced and that is why thermocouples are used and thermocouple is the smallest thermometer which favours it as a dynamic thermometer. That means, not only the static, we also need to know the transient response, how fast we can measure the fluctuating components and for that we need to reduce r .

So, our objective was, for this class was you know either I can say the entire objective for today's class was to see what do we mean by you know transient response characters you know modelling and then taking an example, rather that is what we have considered, a thermal system.

We have tried to obtain the equation which govern the transient response of that particular system. So, the modelling of transient response we have understood from this you know example and also we have seen if we really would like to interested in measuring the temperature, rather fluctuating temperature, then what should be the geometric construction of a thermometer?

That means, when the objective is thermometer, the radius of the sphere of the that bulk, temperature thermometer bulk, should be very small and if this becomes very small, then only we can have larger k and we can measure the fluctuating components instantaneously and that from there, we have tried to conclude that thermocouple is the smallest thermometer which favours it as a dynamic thermometer. So, this I stop my discussion today and we will continue our discussion in the next class. Thank you.