Computational Fluid Dynamics for Incompressible Flows
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Module 3: Finite Difference Method
Lecture 1
Finite Difference by Taylor Series Expansion

Hello everyone, so today we will start module 3, finite difference method and in today's lecture we will learn, finite difference by Taylor Series Expansion. In last lecture, we have learned the classifications of partial differential equations and you have seen that in heat transfer and fluid mechanics problems there appears first derivative and second derivative of dependent variables.

The differential of dependent variables appearing in the partial differential equation must be expressed as approximate expression, so that a digital computer can be employed to obtain a solution.

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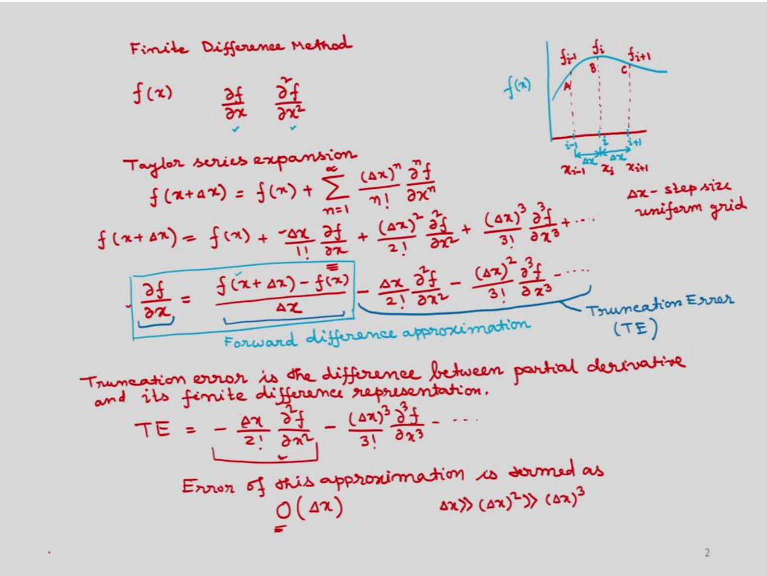

So, for that purpose let us take one function f x. So, we will use finite difference method. So, there are many ways to discretize this partial derivatives, but in today's lecture will use Taylor series expansion. So, let us considered any function f x, it is a continuous function. Now, we need to find its derivative del f by del x and del2 f by del x square using this finite difference approximation. So, before doing so, we need to have discrete points or we have to divide the domains into discrete points.

So, those points generally we represent by index i in x direction, and j in y direction. So, if we consider 1 Dimensional domain, so, if we discretize into discrete points, so these are known as grids. So, this is one point, this is one point, this is one point. So, these are the discrete points and separated by a distance, uniform distance delta x.

And, these index of these discrete points we are telling it is i, this is your i plus 1 and this is i minus 1. As you are considering only 1 Dimensional, we are using index i, but if it is 2 Dimensional then we will use index j in y direction, and if it is 3 Dimensional then index k in third direction.

So, these derivatives whatever we have written the f x is a continuous function, then if we need to find these derivatives, at in the values of these discrete points, then let us say that these f x varies like this. So, let us say that this point, let us say this is you're A, B and C and at this points you have the values known fi minus 1, fi and fi plus 1. So, we will use now Taylor series expansion just to represent this derivative in some approximate values. So, what is Taylor series expansion. Let us write first, let us say that f x plus delta x, we need to expand using Taylor series about the point x. So, if this is your xi, so about the xi we want to expand.

So, you will get f x plus delta x, you can write f x plus summation of n is equal to 1 to infinity delta x to the power n by factorial n deln f by del xn. So, this is the Taylor series expansion of f x plus delta x about x. So, we can expand it as f x plus delta x as f x plus delta x by factorial 1 del f by del x plus delta x square by factorial 2 del2 f by del x square plus del x cube by factorial 3 by del cube f by del x cube and there will be other terms.

Now, if you want to represent this derivative del f by del x so, you can write del f by del x as so this if you take on the left hand side then you will get f x plus delta x minus f x. So, there delta x is there you divided, divided by delta x and other terms will be there. So, this side it will go so, it will be minus delta x by factorial 2 del2 f by del x square minus.

So, we are dividing by delta x so, it will be delta x square by factorial 3 del cube f by del x cube and so on. So, you can see that we have represented this first derivative of f with respect to x, about the point x as f x plus delta x minus f x divided by delta x delta x is your step size and it is uniform.

So, it is uniform grid. So, these discrete points are known as grid, that we have approximated as the value at point f x plus delta x minus f x divided by delta x and some other terms are there. So, if you truncate it up to this point, so this is the finite difference approximation, of this first derivative del f by del x. And as you are using one forward point, f x plus delta x, it is known as forward difference approximation.

But we are neglecting the other terms. So, these terms are known as truncation error. So, truncation error, if this is your truncation error because you have truncated this after that you are neglecting this term. So, this is your truncation error. So, this is the truncation error, what is truncation error, so you can see that truncation error is the difference between the partial derivative and the finite difference approximation. Because finite difference approximation is this one, and this is your partial derivative. So, the truncation error is the difference between partial derivative and the finite difference approximation.

So, we can write it here, the truncation error is the difference between partial derivative and its finite difference representation. So now, you can see that so truncation error you can write in this case as, minus delta x by factorial 2 del2 f by del x square minus del x cube by factorial 3 del cube f by del x cube and so on. So, in this case, you can see in the truncation error the leading term, first term is the leading term in this leading term the delta x order is one.

So, it is actually the error of this approximation is termed as, what type of approximation is O delta x. So, this is the notation, O. So, this is order of delta x. So, in the leading term, whatever you have this delta x or delta x square so, whatever is the highest order is there in the first term, so, that is known as order of delta x.

So, you can see, that if delta x is very small, then delta x will be much, much greater than delta x square and delta x square will be much, much greater than delta x cube. So obviously, the first term or the leading term in the truncation error will contribute maximum error. So, that is why, the error of this approximation is termed as the order of whatever delta x is there in the leading term.

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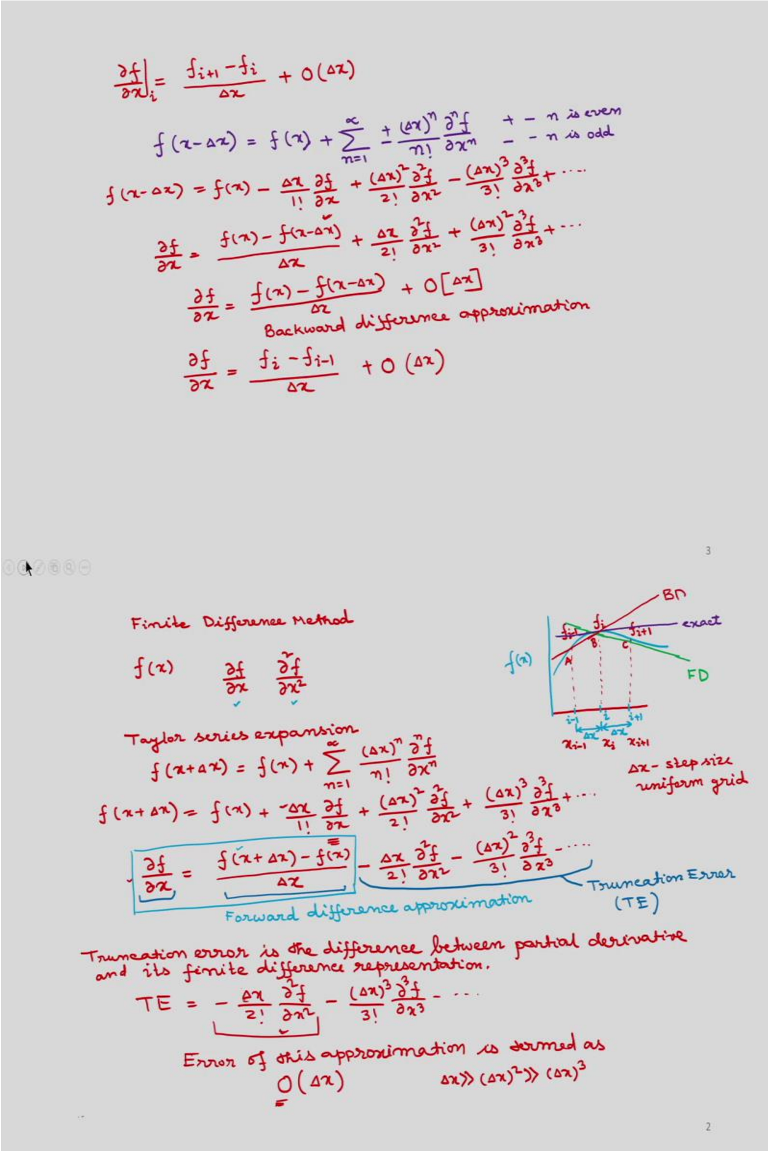

So, now, if you write in terms of the index i, then you can write del f by del x is equal to, at about point i as fi plus 1 minus fi divided by delta x and what is the order of approximation, it is delta x. So, this is the finite difference approximation and it is forward difference approximation of the first derivative del f by del x.

Now, if you want to graphically represent it what does is it mean? So, if you see in the previous graph so, about this point B actually you are writing this approximation. So, you are using the point C and B. So, the slope what it represent graphically, it is the slope if you join the points B and C. So, this is your forward difference and at point B if you want to exactly find the slope of this tangent what will have.

So, if you want to plot it so, it will be just tangent at this point B. So, this is your exact representation of the slope of this tangent. But when you are using forward difference and using a points B and C so, this is your approximation, forward difference approximation. Now, similarly, if we use f x minus delta x if you expand it using Taylor series about the point x, then you can write f x plus summation of n is equal to 1 to infinity plus minus delta x to the power n by factorial n deln f by del xn, where plus is if n is even and minus sign will come if n is odd.

So, if you expand it, you will get f x minus delta x is equal to f x so, minus delta x by factorial 1 del f by del x plus delta x square by factorial 2 del2 f by del x square then minus delta x cube by factorial 3 del cube f by del x cube and so on.

So here, now if you want to represent the first derivative del f by del x so, you can write it as a f x minus f x minus delta x. So, I have taken del f by del x in the left hand side and f x minus delta x in right hand side divide by delta x and this will be this side only so, it will be plus delta x by factorial 2 del2 f by delta x square plus delta x square by factorial 3 del cube f by del x cube.

So, if you truncate it up to this, then your finite difference approximation is del f by del x is equal to f x minus f x minus delta x divided by delta x and what is the order of this truncation error, it is the delta x. So, you can see that we are using 2 points f x and 1 backward points, f x minus delta x. So, as you are using a backward point it is known as backward difference approximation.

Now, if you want to represent in terms of the discrete points i, i plus 1 and i minus 1 you can write del f by del x. So, you can write fi minus the backward point fi minus 1 divided by delta x and order up delta x. So, if you want to graphically represent it, so you can actually join the points A and B.

So, you are using point A xi minus 1, and point B xi. So, it will be the graphical representation of this approximation. So, this is the slope using backward difference.

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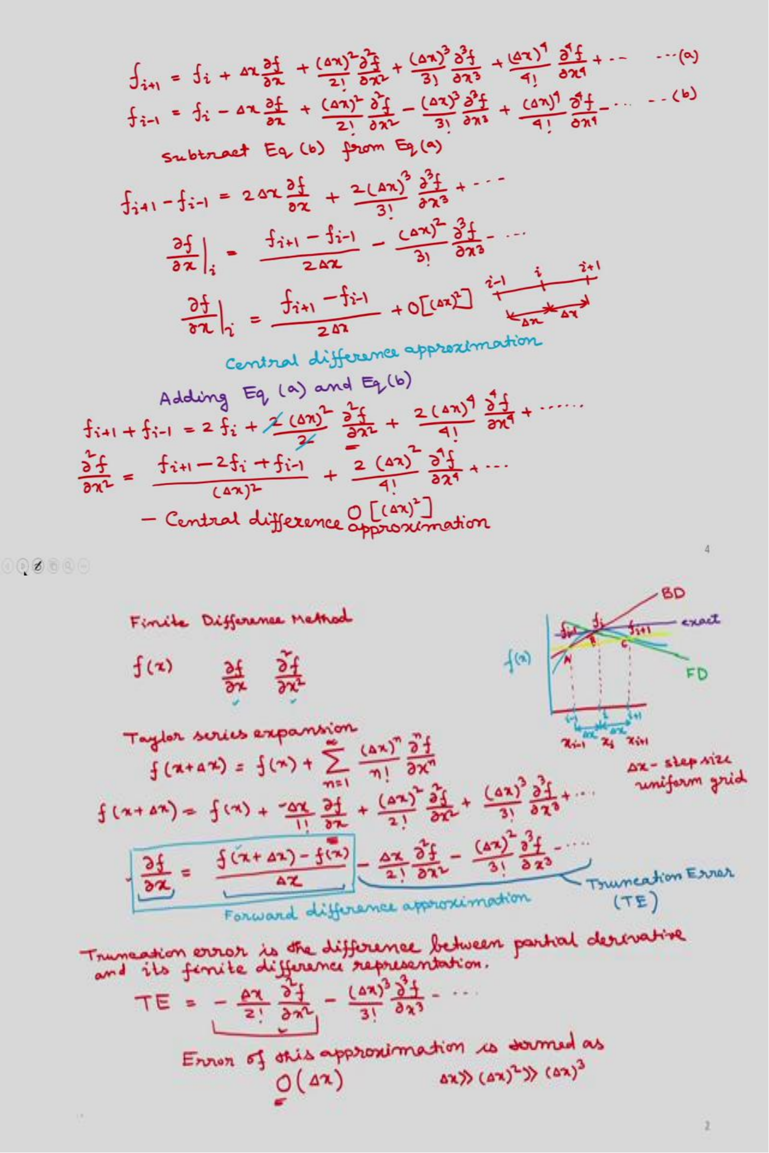

Now, if you want to write fi plus 1 and fi minus 1, you expand this two using Taylor series about point xi, then you will get fi plus delta x del f by del x plus delta x square by factorial 2 del2 f by del x square plus del x cube by factorial 3 del cube f by del x cube plus so on. So, there will be other terms. And similarly, you can write fi minus delta x del f by del x plus delta x square by factorial 2 del2 f by del x square minus del x cube by factorial 3 del cube f by del x cube you just tried the other, another term del x 4 by factorial 4 del 4 f by del x to the power 4 plus other terms and here del x 4 by factorial 4 del 4 f by del x to the power 4 minus so on.

So now, if you subtract the second one from the first one, so if we equation, let us say this is your a and this is equation b, then subtract equation b from equation a, so subtract equation b from equation a. So, what you will get, so if you subtract it then it will be fi plus 1 minus fi minus 1 in left hand side.

So, first term will get cancelled then you will get 2delta x delta f by delta x, this third time will get cancelled then fourth time you will get, plus 2 delta x cube by factorial 3 del cube f by del x cube and so on. So now, if you want to represent this derivative del f by del x about point i then you can write fi plus 1 minus fi minus 1 divided by 2 delta x and the truncation error will be, minus so it will be delta x square because we have divided by 2 delta x divided by factorial 3 del cube f by del x cube and so on.

So, you can see if you truncated here, then you are actually using 2 points fi plus 1 and fi minus 1 while finding the derivatives at point i, and the leading order term is having delta x square, so the truncation error order is delta x square. So, you can write del f by delta x i is fi plus 1 minus fi minus 1 divided by 2 delta x because these are separated or by distance 2 delta x. So, this is your i, this is your i plus 1, i minus 1, so fi plus 1 minus fi minus 1 divided by 2 delta x and order of accuracy is delta x square.

So, graphical if you want to represent it, so you are using point xi minus 1 and xi plus 1. So, that means A and C, so if you join these 2 points A and C, so it will give a slope using central difference because that approximation is known as central difference. So, you can see that central difference approximation is close to the exact solution of this delta f by delta x. So, this approximation is known as central difference approximation.

So, now if you want to find the second derivative approximation del2 f by delta x square, so you can see from these two equations. So, if you add this equation a and b, then what you will get, so adding equation a and equation b, what you will get you just see, so it will be fi plus 1 plus fi minus 1 left hand side, right hand side first term to fi. The second term will get cancelled, then you will get 2 delta x square divided by factorial 2 means 2 del2 f by del x square. And in this case, this 2, 2 will get cancelled out and then fourth term will get canceled, then the next term will be 2delta x to the power 4 by factorial 4 del4 f by del x to the power 4 and other terms.

So, here you can see now if you represent this, second derivative of f with respect to x, so you can write del2 f by del x square. So, you can write fi plus 1 minus twice fi plus fi minus 1 now you divide by delta x square and the first term will be 2 as you are dividing by delta x square, so it will be 2 delta x square by factorial 4 del4 f by del x to the power 4 and other terms.

So, the leading order term in the truncation error is order of delta x square. So, you can see that this is the order of delta x square, so it is a second order accurate. So, del2 f by delta x square you are representing using 3 discrete point values, one is fi, fi plus 1 and fi minus 1 and this approximation also is known as central difference. So, central difference approximation of this second derivative.

In other way also you can also derive, the second order approximation so that let us see in the next slide.

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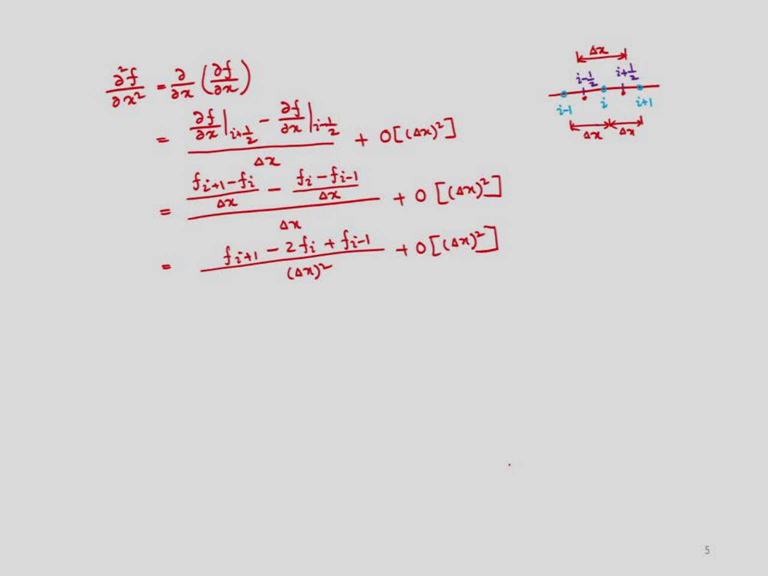

Say, you have this discrete points, this is your uniform grid. So, it is i this is your i plus 1 this year i minus 1 and you have in between these 2, let us say this is your i plus half and this your i minus half. So, if you want to represent this second derivatives. So, you can write del2 f by del x square. So, you can write del of del x of del f by del x. So, you can write del f by del x you can use a central difference so, i plus half minus del f by del x i minus half So, it is your central difference about the point i divided by.

So, what is that distance, so if this is your delta x, this is the step size. This is your delta x and the distance between these 2 points are also, the distance between these 2 points is also delta x. So, this is the first derivative we are writing and, so you can write delta x and what is the order of accuracy. So, we have seen that it is second order accurate, so it will be order of delta x square.

So, now, you can see that del f by del x now, you can again use central difference of this. So, you can write del f by del x at i plus half. So, this is your i plus half. So, this point is i plus half. So, this if you use the central difference then you will get fi plus 1 minus fi divided by delta x. So, it is the central difference approximation of the first derivative. So, fi plus 1 minus fi divided by delta x about the point i plus half.

So, what will be the order of accuracy, it is delta x square and minus del f by del x about point i minus half. So, about this point you take the central difference approximation. So, you will get fi minus fi minus 1 divided by delta x and these 2 approximations also delta x square, earlier it was also delta x square. So, order of accuracy of this approximation will be delta x square.

So now, you can write it delta x square, so it will be fi plus 1 again minus twice fi, so plus fi minus 1 and order of delta x square. So, this way also you can derive the finite difference approximation of the second derivative of f with respect to x.

Now, let us see that if you use more than 2 points for the first derivative then in forward difference and backward difference how you can write.

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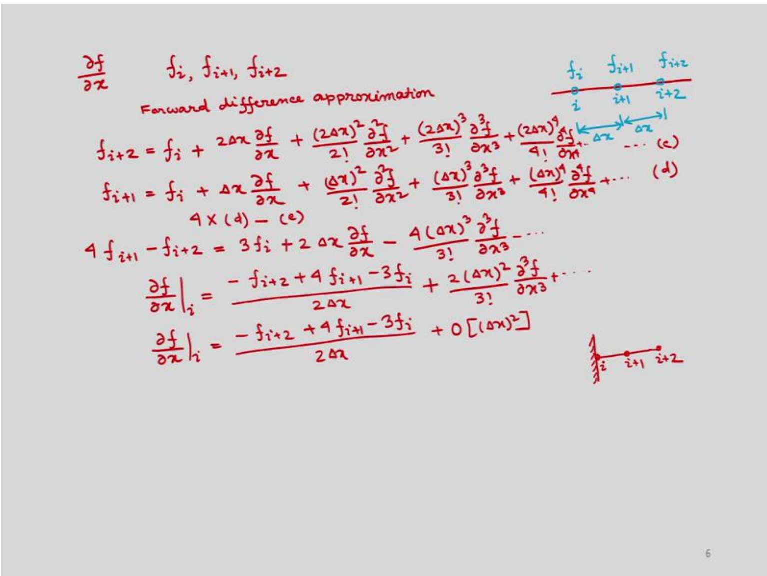

So, for these, let us say that we will use 3 discrete points. So, we will find the value of del f by del x. So, this is the finite difference approximation of del f by del x using 3 discrete points, and we will use fi, fi plus 1 and fi plus 2. So, using fi, fi plus 1, and fi plus 2. So, we are using forward points i plus 1 and i plus 2. So, it is forward difference approximation.

So, we will use uniform grid, so the spacing between these points, i, i plus 1 and i plus 2, let us say. So, this is your i, this is your i plus 1 and this is your i plus 2 and it is uniform steps size so it is delta x and this is also delta x and the values at this point fi plus 1 this is fi plus 2 and this your fi.

So, we need to represent this first derivative del f by del x using the values at discrete points i, i plus 1 and i plus 2. So, first we will use Taylor series expansion of fi plus 2 and fi plus 1. So, if you write fi plus 2, so it is f x plus 2 delta x, so you can write fi plus 2 delta x del f by del x plus 2 delta x square by factorial 2 del2 f by del x square plus 2 delta x cube by factorial 3 del cube f by del x cube plus 2 del x to the power 4 by factorial 4 del 4 f by del x to the power 4 and other higher order terms.

So, now let us expand fi plus 1 about the point xi so, you will use fi plus 1 is fi plus delta x del f by del x plus delta x square by factorial 2 del2 f by del x square plus del x cube by factorial 3 del cube f by del x cube plus del x to the power 4 by factorial 4 del 4 f by del x 4 and other higher order terms.

Now, what we will do we need to find the values of del f by del x. So, what we will do, we will just write 4 fi plus 1 minus fi plus 2. So, we will do this algebra for fi plus 1 minus fi plus 2, so what you will get, so if you multiply, so if equation this as c and this is your d, then you write 4 into equation d and minus equation e.

So, if you do that, then what you will get, so for fi plus 1, so it will be 3 fi then you will get, this is your del f by del x so it will be 4 into d so it will be 4. So, 4 minus 2 so it will be 2 delta x del f by del x. Then if you write these terms it will be multiplied by 4, so it will be 4 and here 4.

So, this term will become 0 because this is your 4, and this is also your multiplied by 4 and if we have subtracted, so this term will get 0. So, next term will be, so it will be plus 0. So, next time you write, so it will be multiplied by 4, so it will be 4 and this is your 8, so you will get minus, so it will be 4 delta x cube by factorial 3 del cube f by del x cube and so on.

So, now you represent del f by del x so del f by del x about point i. So, 3 fi will come this side it will be minus. So, you can write minus fi plus 2 plus 4 fi plus 1 minus 3 fi you divided by 2 delta x it will be 2 delta x and minus. So, this will come in this right hand side, to show it will be plus. So, you divide by 2 delta x it will be 2 delta x square by factorial 3 del cube f by del x cube and other terms.

So, you can see that in this case, the truncation error order is delta x square. So, you have represented this first derivative del f by del x, using forward difference approximation fi plus 2 plus 4 fi plus 1 minus 3 fi by 2 delta x order up delta x square. So, it is a second order accurate. So, you have used 3 points fi, fi plus 1 and fi plus 2. So, you are getting the approximation, finite difference approximation of this derivative del f by del x as second order accurate.

So, this sometime it is required at the boundary points. So, if you have any boundary and you need to find the derivative or if you want to apply the boundary conditions in terms of derivative, if you want to have secondary accurate of this approximation, then you can use it. Because you will get these 3 points as this is your i, this is your i plus 1 and this your i plus 2.

So, if you require secondary approximation at the boundary points or first derivative, then you can use this approximation.

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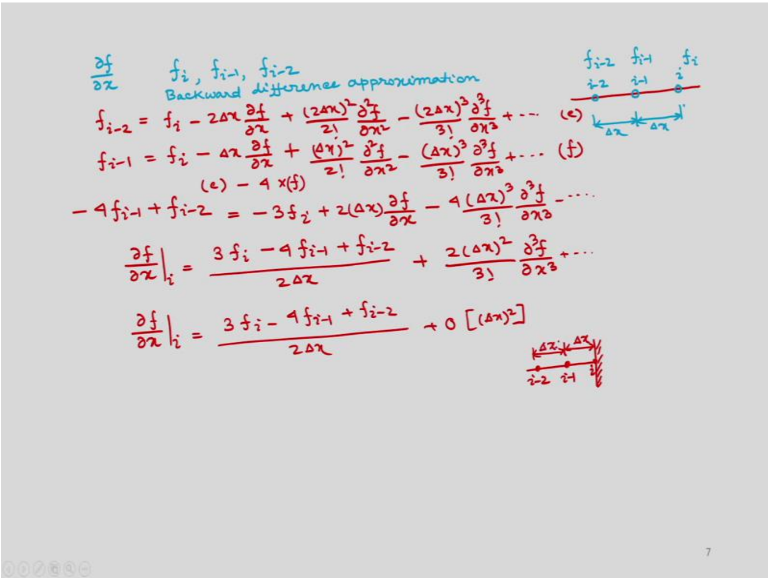

So, similarly if you want to use 3 backward points so, fi, fi minus 1 and fi minus 2, then similarly you can find this derivative del f by del x using 3 values fi, fi minus 1, and fi minus 2. So similarly, if you use it, this Taylor series expansion for fi minus 2 so, you have 3 points.

So, this is your 3 points, discrete points separated by distance, uniform distance delta x. So, this is your i, this is your i minus 1, and this your i minus 2.

So, the values at this point is fi, fi minus 1, and fi minus 2. So similarly, whatever way we have derived the forward difference approximation using 3 points, we can use the backward difference approximation, to find the del f by del x using 3 discrete point values fi, fi minus 1, and fi minus 2.

So, here fi minus 2 you can write, fi minus 2 delta x del f by del x plus 2 delta x square by factorial 2 del2 f by del x square minus 2 delta x cube by factorial 3 del cube f by del x cube and so on. Similarly, fi minus 1 you can expand using Taylor series about point xi. So, you will get fi minus delta x del f by del x plus delta x square by factorial 2 del2 f by del x square minus del x cube by factorial 3 del cube f by del x cube and other higher order terms.

So, if you do this algebra, let us say minus 4 fi minus 1 plus fi minus 2. So, that means if it is your equation e, this is your equation f, then you multiply equation f with 4 and subtract it from c, so it will be c minus 4 into f. So, what you can write, 4 fi minus 1, so 4 fi minus 2 minus 4 fi minus 1, so if you multiply this f with 4, so you will get minus 3 fi. Then you will get here, 4 and 4 so you will get 2, so you will get plus 2 delta x del f by delta x. You can see here, the third term in the right hand side will get cancelled because you are multiplying 4 with f when you are subtracting so, you will get 0.

Then the other term will be here, so it will be 8 minus 8 plus 4 so, it will be minus 4 delta x cube by factorial 3 del cube f by del x cube and other terms. So, now if we want to represent del f by del x so, you can write this 3 fi you can take it in the right hand side. So, it will be 3 fi minus 4 fi minus 1 plus fi minus 2 divided by so, 2 delta x. And this will be in the right hand side, so it will be, you are dividing by 2 delta x it will be 2 delta x square by factorial 3 del cube f by del x cube and other higher order terms. So, you can see the leading term, in the truncation error is having the order of delta x square.

So, this approximation del f by del x, you are using backward difference approximation and you are using 3 discrete values fi minus 1 and fi minus 2, which is having the order of accuracy delta x square. Similar way, so, you can actually use these for the boundary points to having these first derivative with this accuracy delta x square.

So, if you want to use or if you want to discretize any derivative, first derivative del f by del x at the boundary points and you need second order accuracy then you can use this, you can see this is your boundary. So, this is your i, this is your i minus 1 and this is your i minus 2 and the distance is delta x and this is your delta x. So, till now we have found the derivatives of a single variable.

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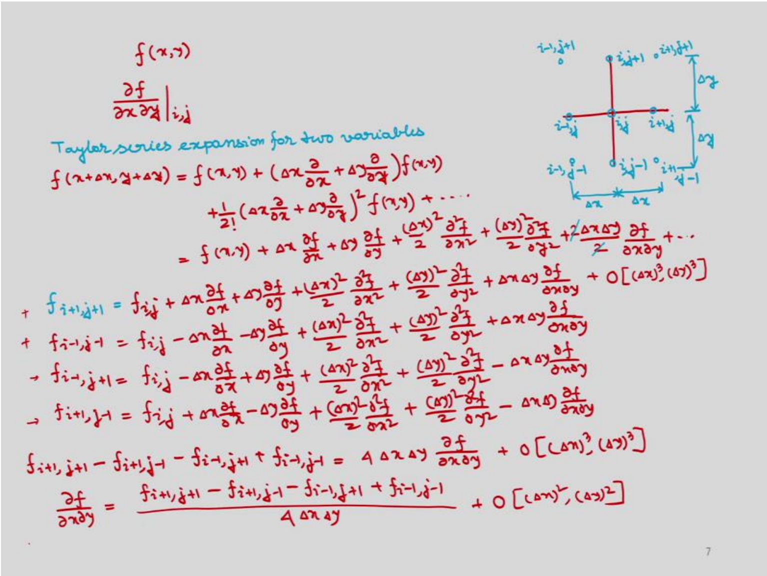

So, now if we want to find the mixed derivatives, so how we will find it? So, mixed derivative mean, say if f is function of x and y. So now, you want to find the mixed derivatives of it. So, you want to find del f by del x del y, about any point let us say i and j. So, i is the index notation in x direction, and j is the index in y direction. So, in this case now the grid points or discrete points, let us draw. So, it will be on uniform grid we are drawing.

So, this is your ij, this, so in right hand side so, it is i plus 1 j so, this is your i minus 1 j, this is your ij minus 1 and this is your ij plus 1. So, these are the discrete points and we want to find this mix derivative about this point ij with the neighboring values. And, the distance these are uniform step size. So, this is your delta x, this is your also delta x and this is your delta y and this is your delta y. So, delta y is also uniform and delta x is also uniform, but delta x may not be equal to delta y.

So, in this case, we will use Taylor series expansion of two variables. So, Taylor series expansion for two variables. So, if you write for two variables, you can write this way, f x plus delta x y plus delta y. So, this you want to expand about the point x y. So, it will be f x y plus delta x del of del x plus del y del of del y f x y plus del x del of del y, so it will be 1 by factorial 2, 1 by factorial 2 plus del y del of del y square f x y and so on.

So, if you expand it, so it will get f x y plus del x del f x y or you just write del f by del x plus del y del f by del y. So, this is your factorial 2, so 1 by factorial 2 is 2. So, 1 by 2 delta x square del2 f by del x square plus delta y square by factorial 2 del2 f by del y square and you have del x del y, y factorial 2 so it will be 2 here, here 2 del f del x del y and so on.

So, you will have now, these 2, 2 will get cancel out. So, if you now expand all this fi plus 1 j fi minus 1 j fi, j plus 1 and fi, j minus 1, then you can write all these points. So, another points you write here, i plus 1 j plus 1 here another point i plus 1 j minus 1 this is your i minus 1 j minus 1 and this is your i minus 1 j plus 1.

So, if you write fi plus 1 j plus 1, so you can expand it using this fi, j plus del x del f by del x plus del y del f by del y plus del x square by 2 del2 f by del x square plus del y square by factorial 2 del2 f by del y square plus del x del y del f del x del y and whatever you have neglected so, that will be order of accuracy as delta x cube delta y cube.

So, now if you write for fi minus 1 j minus 1. Similarly, you can write fi, j minus delta x del f by del x minus delta y del f by del y. So, it will be plus delta x square by 2 del2 f by del x square then it will be plus del y square by 2 del2 f by del y square and it will be minus, minus plus it will be delta x delta y del f del x del y.

Similarly, you can write fi minus 1 j plus 1. So, it will be fi, j, so it will be minus delta x del f by del x plus delta y del f by del y and it will be delta x square by 2 del2 f by del x square. Then it will be, plus delta y square by 2 del2 f by del y square and del x del y del f by del x del y. And fi plus 1 j minus 1 if you write then it will get fi, j, so it will be plus del x del f del y, this is x and this will be minus delta y del f del y plus delta x square by 2 del2 f by del x square plus del y square by 2 del2 f by del y square plus x del y del f by del x del y.

So, here it will be minus and this is will be also minus. So now, if you do this algebra, fi plus 1 j plus 1 minus fi plus 1 j minus 1 minus fi minus 1 j plus 1 plus fi minus 1 j minus 1. So, what you will get, so you can see so, fi plus 1 j plus 1 minus fi plus 1 so, this is your minus then this is minus and this is plus. So, this is plus, this is plus and this is minus, this is minus.

So, if you do it you will see that these, these f will cancel out. So, if these are 2 plus and these are 2 minus of these will also cancel out, these will cancel out, these will cancel out, these will cancel out so, you will get only 4, so it is minus, minus plus, so 4 delta x delta y del f by del x del y. So, if you write it so, it will be order of del x cube del y cube. So now, if you want to find the mix derivative, so, you can write del f by del x del y is nothing but fi plus 1 j plus 1 minus fi plus 1 j minus 1 minus fi minus 1 j plus 1 plus fi minus 1 j minus 1.

So, you divide by 4 delta x delta y. So, what will be the order of accuracy, because the truncated term whatever we have seen that it is order of delta x cube and delta y cube. So, now we are dividing by delta x and delta y. So, obviously then order of accuracy will be delta x square and delta y square. So, you can write order of accuracy is delta x square and delta y square. So, this mix derivative we are found using the Taylor series expansion.

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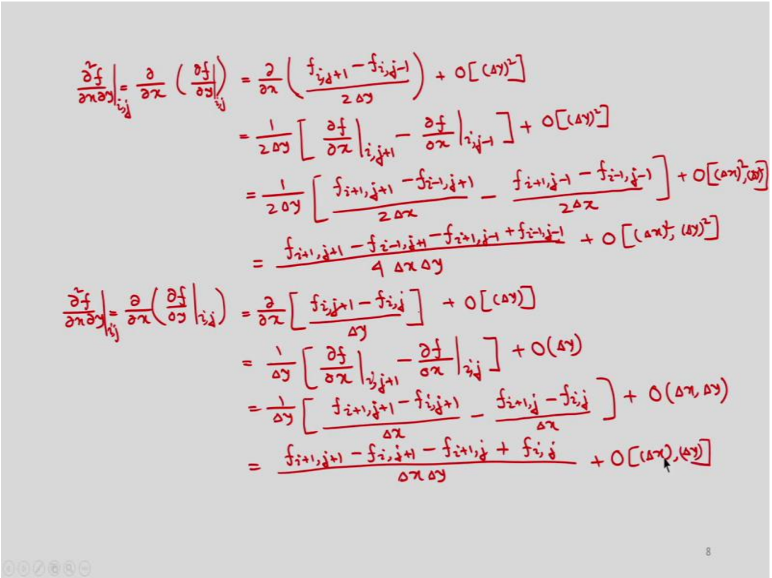

But you can also derive it using this way. So, you can write del2 f by del x del y, you can write it as a del of del x of del of del y. Now, you can use the central difference. So, if you use the central difference, then it will be order of accuracy delta x, y square. So, if del of del x, so now we are using del f by del y, so that we are using f, so it is at i, j. So, this we are writing, so at i, j. So, del f by del y you can write, fi, j plus 1 minus fi, j minus 1 divided by so 2 delta y.

So, what is the order of accuracy because we have a central difference of this first derivative, so you can write order of delta y square. Now, this we can write 1 by to delta y and now you can write del f by del x about i, j plus 1 minus del f by del x i, j minus 1 which is order of delta y square.

So, now again it is first derivative. So, now you can use central difference so, that it will be, order of accuracy will be delta x square so, you can write 1 by 2 delta y. So, for del f by del x you can write at i, j plus 1. So, if you use the central difference, you can write fi plus 1 j plus 1 minus fi minus 1 j plus 1 divided by 2 delta x. This is the central difference and obviously, the order of accuracy is delta x square.

And similarly, this you can write del f by del x at i, j minus 1. So, it will be i plus 1 j minus 1 minus fi minus 1 j minus 1 divided by delta 2 delta x. So, this is also central difference of this first derivative about the point i, j minus 1 and both are having the order of accuracy delta x square, so overall accuracy is delta x square delta y square.

So, if it is so, so now you can write so, it will be 4 delta x delta y it will be fi plus 1 j plus 1 minus fi minus 1 j plus 1 so it will be minus fi plus 1 j minus 1 and this minus, minus plus, so it will be fi minus 1 j minus 1 and order of accuracy second order in both x and y.

Now, let us use forward difference for this mix derivative. So, if you use the forward difference approximation, then similarly you can (())(57:27) del2 f by del x del y as del of del x del f by del y i, j. So, this you can write, if you use the forward difference of this first derivative. So, you can write del of del x del f by del y, so it will be fi, j plus 1 minus fi, j. So, this is your forward difference divided by the distance between this 2 points is delta y. What is the order of accuracy, this is the first order accuracy it will be the order of delta y.

So similarly, you can right now 1 by delta y del f of del x i, j plus 1 minus del f of del x i, j and order of accuracy is delta y. So, now you got the first derivative with respect to x. So now, you use again forward difference, for each derivative.

So, what you will get, 1 by delta y. So, if you use this del f by del x you will get fi plus 1 j plus 1 minus fi, j plus 1 divided by delta x. And what is the order of accuracy, obviously, order of delta x and you will get here fi plus 1 i plus 1 j minus fi, j this is also you have used forward difference, so delta x so order of accuracy is delta x delta y. So, you will get delta x delta y.

So, it will be fi plus 1 j plus 1 minus fi, j plus 1. So, this will be minus fi plus 1 j and minus minus plus it will be fi, j and order of accuracy is delta x delta y. So, today we have used Taylor series expansion to find the finite difference approximation of first derivative and second derivative. So, first we have learned the forward difference approximation, then backward difference approximation and central difference approximation of the first derivative. Then also, we have learned the central difference approximation of second derivative. And also, at the last we have learned the finite difference approximation of mixed derivative. Thank you