Computational Fluid Dynamics for Incompressible Flows
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Lecture 37
Boundary Conditions

Hello everyone, so in last class, we discussed about the simple algorithm and also we have derived the pressure correction equation from the continuity equation. Simple algorithm has been widely used in the literature, but there are number of attempts to accelerate the convergence. So, today we will discuss two such algorithms, one is simpler and another is simplec. So, there is a drawback a simpler algorithm.

So, dropping the summation of anb U prime nb slows down the convergence. So today we will discuss how to accelerate this simple algorithm. So, first we will discuss about the simpler algorithm. So, what is simpler? It is simple devised. So, semi-implicit method for pressure limit equation revised.

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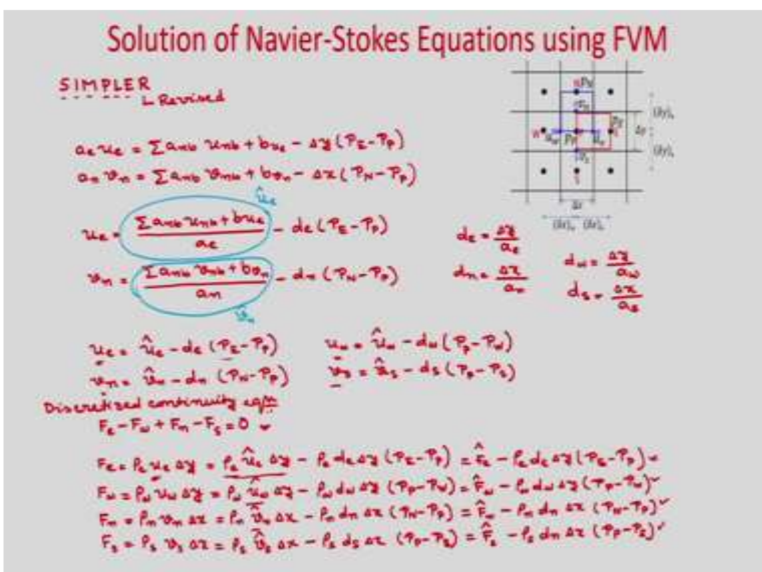

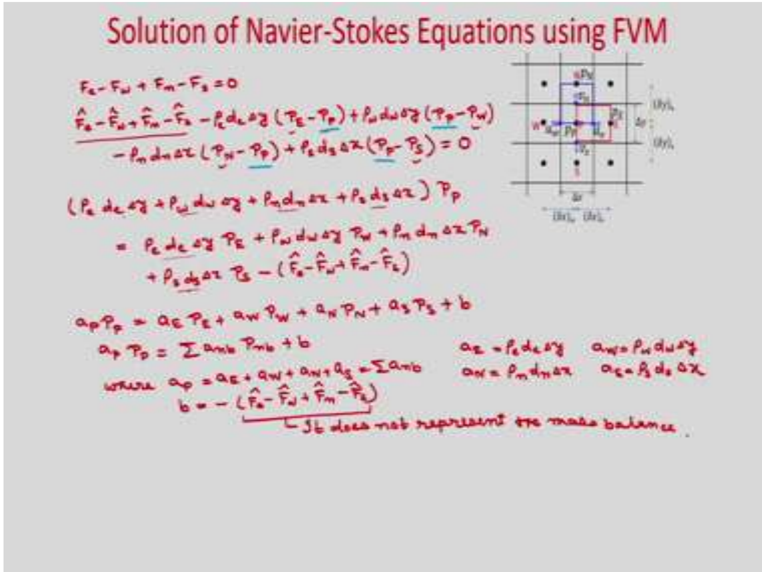

SIMPLE R, okay. So, this is your simple semi-implicit method for pressure linked equation and R for revised. So, this is your simpler algorithm. So, you have seen that in simple algorithm, first we need to guess the pressure field and if you have a good pressure field obviously the simple algorithm will converge faster.

In simpler algorithm, we calculate the pressure from the known velocity field. Now, let us write the discretized momentum equation for ue and vn, okay. So for ue we can see at this main control volume for velocity ue we are writing the discretized equation ae ue is equal to summation of anb, unb, okay, for all the neighbors plus the source term for u at point ue and the pressure gradient term.

What is the pressure gradient term? So, you can see so, in the, for u velocity. So, the pressure gradient is Pe minus Pp into delta y. So, minus delta y into Pe minus PP, okay, and an vn is equal to so for the V momentum equation you can see here we are solving for vn. So, discretized equation you can write for this control column as summation of anb vnb plus b source term for b at point n minus, so what is the area delta x and the gradient is Pn minus Pp, okay.

So, now, let us write the velocities dividing the diagonal term, so, you can write ue as summation of in anb unb plus bue divided by ae, okay, and minus we will write this as a de Pe minus Pp, okay where de is your delta y by ae, delta y by ae. Similarly for vn you can write vn is equal to summation of anb vnb plus bvn divided by an minus dn Pn minus Pp, where dn is your delta x divided by the diagonal coefficient an, okay.

So, this term, okay, will represent as ue hat, okay and these term will represent as vn hat. So, if you represent like this, then you will then we can write ue is equal to ue hat minus de Pe minus Pp and vn is equal to be vn hat minus dn Pn minus Pp, okay. So, similarly you can write for the buw and bs. So, uw you can write uw hat minus dw into P.

So, for the west space you can see the pressure gradient is Pp minus Pw, so, it is Pp minus Pw and for vs you can write and the pressure gradient for this south space bs you can write Pp minus Ps and dw is your delta y by a aw and ds, is your delta x divided by as, okay. So, now let us write the discretized continuity equation. So, integrating the continuity equation about this main control volume P, okay.

In last class we have written the equation as Fe minus Fw plus Fn minus Fs is equal to 0. So, this is your discretized continuity equation, okay. So, now if you put this mass flow rate Fe is equal to rho e ue ae then what you can write, so Fe you can write as, so, Fe will be, okay, rho e ue and what is ae? ae is delta y, okay, ae is delta y.

So, you can write this as, so now, ue you just substitute ue as ue hat minus de Pe minus Pp. So, if you write that then you can write rho e ue hat delta y, okay, minus rho e. So, now, this is the term so, it is de you have delta y into Pe minus Pp, okay. So, now you can see this rho e ue hat delta y we can write it as Fe hat minus rho e de delta y Pe minus Pp. Now, similarly, you just write Fw, Fn and Fs.

So, Fw will be v rho w uw delta y and uw is this one. So, it will be rho w uw hat delta y minus rho w dw delta y Pp minus Pw and this if you represent with if Fe hat w then it will be rho w dw delta y Pp minus Pw. Similarly, we write a Fn is equal to rho n vn delta x, ok. So, vn so, vn is this one. So, you can write, rho n vn hat delta x minus rho n dn delta x into Pn minus Pp and this will represent as Fn hat.

So, it will be rho n dn delta x Pn minus Pp and Fs is equal to rho s vs delta x is equal to rho s, so, this one sorry this is your ve, so, it will be vs hat delta x minus rho s ds delta x Pp minus Ps is equal to Fs hat minus rho s ds delta x into Pp minus Ps. So now we will substitute in the discretized continuity equation, okay. So, if you substitute what you will get?

So, we have Fe minus Fw plus Fn minus Fs is equal to 0, okay, and now you substitute all these four and write Fe plus Fw plus Fn plus Fs. So, it will be. So, now, first you write Fe minus Fw plus a Fn minus Fs, okay. So, if you write it, so we will get Fe hat minus Fw hat plus Fn hat minus Fs hat, okay, and rest we have minus rho de delta y Pe minus Pp, okay. For Fw, so it is minus Fw so minus-minus plus so rho w dw delta y Pp minus Pw, okay.

Then for Fn you will get minus rho n dn delta x Pn minus Pp and plus rho s ds delta y Pp minus Ps, okay, is equal to 0. So, this is your delta x equal to 0, okay. So, now, you see we are deriving the pressure equation from the known velocity field, so, now you can write in terms of Fe Phi P is equal to summation of nb Pp nb. So, you can write a Pp, Pp is equal to summation of anb Pnb plus b in this form. So, all the pressure Pp you take in the left hand side and all other terms you take in the right hand side.

So, if you do that you can see, we have these Pp, Pp, and Pp. So, these coefficients you take in the left hand side. So, if you write it, it is minus-minus plus it will be rho e de delta y then this plus rho w dw delta y minus-minus plus. So, it will be rho n dn delta x and this is your plus rho s ds delta x, okay. So, this is the coefficient of Pp and now you take these terms in the right hand side, okay.

So, what you will write, so it will be if you take in the right hand side it will be positive rho e de delta y Pe, okay, plus rho w dw delta y Pw plus rho n dn delta y, sorry, delta x Pn and we have rho s ds delta x Ps and these terms you take in the right hand side. So, you can write minus Fe hat minus Fw hat plus Fn hat minus Fs hat, okay. So, now, these you can write as ap Pp is equal to ae Pe plus aw Pw plus an Pn plus as Ps plus b, okay.

Or ap Pp is equal to summation of anb Pnb plus b where ap is equal to ae plus aw plus an plus as you can see the coefficient ae, this is your ae aw, aw, this is you are an, this is your an and this is your as and this is your as. So, ap is equal to summation of all the neighbor and b is equal to minus if Fe hat minus Fw hat plus a Fn hat minus Fs hat, okay, and we have ae as rho e de delta y aw as rho w dw delta y an is equal to rho n dn delta x and as is equal to rho s ds delta x, okay.

So, in the simpler algorithm from the known velocity field first we derive this pressure equation. So, first you solve this pressure equation in simpler algorithm, which we are actually first guessing the pressure field, instead of that, you solve this equation and get a better pressure field so that it converges faster. So, you can see whatever this, whatever we have written, it actually does not represent the mass balance, okay.

Because ap hat is your rho e ve hat, so, it will not actually satisfy the continuity equation. So, it does not represent the mass balance and you have seen that up to these we have not made any approximation while deriving the pressure equation. So, this pressure with the appropriate boundary condition, this pressure equation with appropriate boundary condition you need to solve to get a pressure field and using this pressure field now you follow the simple algorithm, okay.

So, then all the rest algorithm will be similar to that simple algorithm except you do not correct the pressure, okay. When you solve the pressure correction equation in the simple algorithm. So, here when you are using simpler algorithm, you do not correct the pressure, because already you are solving the pressure equation, ok. But you need to correct the velocity field. So, now we will write the simpler algorithm.

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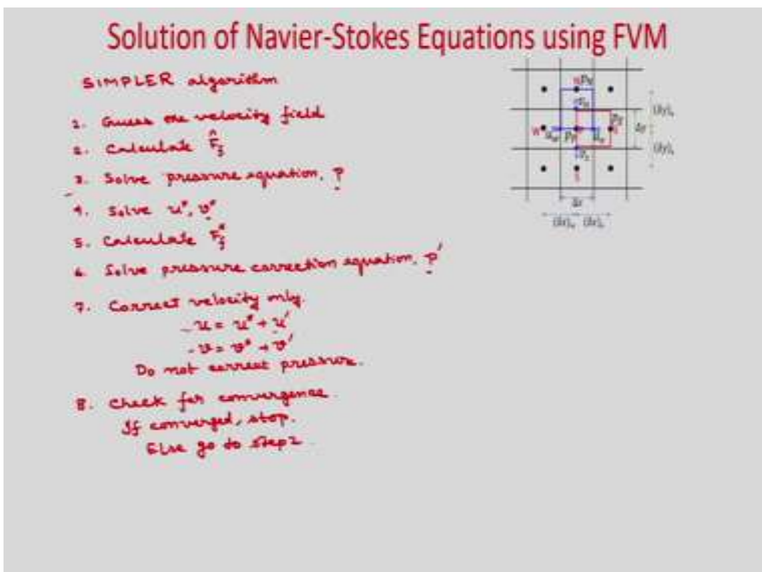

So, simpler algorithm like these, okay. So, first is guess the velocity field, okay. Once you guess the velocity field you can calculate the Fs hat. So, if Fe hat, Fw hat, Fn hat, Fs hat you can calculate. Then you solve for the pressure equation, okay. So, if the velocity field is exact the correct pressure field will be recovered, okay. So, towards the convergence when you have the velocity field, correct velocity field anyway the, you will get a correct pressure field.

Then, now, you follow the simple algorithm, okay. So, now we have solve u star, v star, okay. So, this is now guess pressure field, okay. So, what do you have solved the pressure equation that you will use in this equation to solve u star, v star, okay. Then calculate if Ff star and then solve for the pressure correction equation, pressure correction equation, okay. So, whatever we derived in the simple algorithm that you solve, okay. Then correct velocity only.

So, now you know u star, and you have solved the P prime, so from P prime you can calculate the u prime, okay. So, there is a relation and v is equal to v star plus v prime. Do not correct pressure, okay. Because in this step, at third step you have already solved for the correct pressure field with the known velocity field, okay. So, here you do not correct pressure so this is the difference with the simple algorithm.

Then check for convergence, okay so, if converged, stop, okay, else go to step 2. So, you go to step 2 with the new velocity whatever you have got it, okay with this you calculate this Ff star and solve for the pressure equation. So whatever velocity now you are getting so that will be used to solve this pressure equation. So, you can see that in simpler algorithm, although there is you need to solve one additional equation for pressure, but overall it will take less time to converge as you are giving a correct result field, okay.

In a simpler algorithm if you use a correct guessed pressure field then obviously we will get faster result but it is very difficult to get a good guessed pressure field. But here you are solving for the pressure with the known velocity field. So obviously, it will converge faster than the simple algorithm, but you need to solve one additional equation for the pressure. Next, we will discuss another algorithm that is simplec, okay. Simple, corrected.

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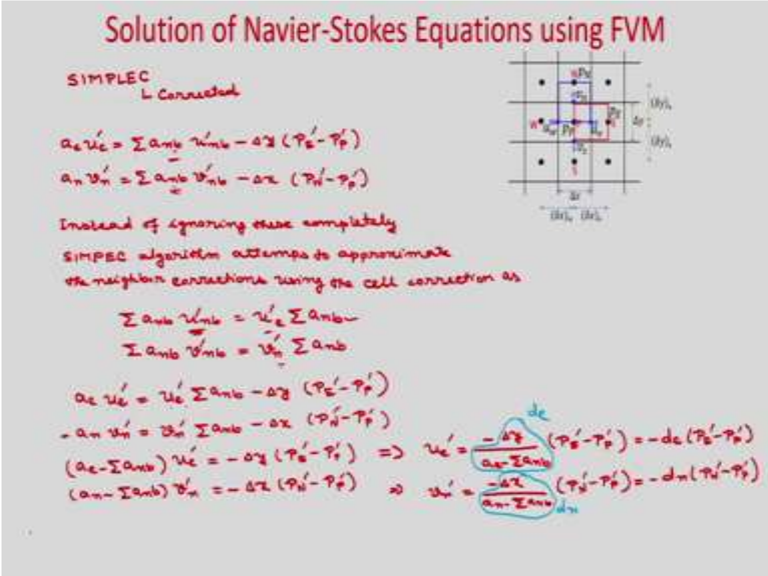

Simple C, so this is your corrected. So, in this algorithm, it is similar to simple algorithm except without dropping the term, we will just use summation of anb u prime nb is equal to u prime, b u prime P summation of nb. So, with the main cell value of u prime, we will use, the main cell value of u prime at cell center small e we will use instead of dropping the full terms.

So, that is the difference with simple algorithm. So if you see, so we have the corrected momentum equation, we have derived in the simple algorithm like this, okay. If you see that we have written first the momentum equation, then we have guessed the pressure field then we have written ae u star and subtracting the second one from the first we got this corrected momentum equation.

So, that is ae ue prime is equal to summation of anb u prime nb minus delta y P prime minus Pp prime and an vn prime is equal to summation of anb v prime nb, okay, minus delta x into Pn prime minus Pp prime. So, if you see, in a simple algorithm what we did, we drop these 2 terms, okay. So, we drop these terms, but now we will use some simplification here. So, instead of ignoring this completely simplec algorithm attempts to approximate the neighbor corrections using cell correction as, so we will use summation of anb u prime nb we will write as u prime e summation of anb, okay.

So, now you can see. So, these actually it has to be summation of all these anb u prime nb so neighbor (coefficient) neighbor values but neighbor corrections. So that we are approximating as the cell correction e prime similarly summation of anb, v prime nb we will write it as vn prime summation of anb, okay. So vn prime obviously, you can see, so, this is your correction at the cell of this v control volume.

So, now, from this equation now, if you write it then you can write, ae ue prime. So, this now we are approximating as this. So, it will be ue prime summation of anb minus delta y Pe prime minus Pp prime. Similarly, an vn prime you can write to this we are now approximating as this. So vn prime summation of anb minus delta x Pn prime minus Pp prime, okay. So, now if you take in left hand side, so, what you can write ae minus summation of anb, okay.

So, now you can take common this ue prime, so, ue prime is equal to minus delta y Pe prime minus Pp prime and now, this you can write as you ue prime is equal to minus delta y divided by ae minus summation of anb Pe prime minus Pp prime and this we will denote as de, okay. So it will be minus de, minus de into Pe prime minus Pp prime. Similarly, for this equation you can write an minus summation of anb v prime n is equal to minus delta x Pn prime minus Pp prime.

So, that will give you vn prime is equal to minus delta x divided by an minus summation of anb Pn prime minus Pp prime and this we will denote as minus dw, sorry minus dn into Pn prime minus Pp prime, okay. So, these we have represented as de and these we have represented as dn, okay. So, now, you can see that we have approximated these Navier correction value, okay, which we dropped in the simpler algorithm.

So, that is the difference with the simple algorithm. So, in the simplec algorithm now we follow the same procedure as simple except calculating this term de and dn.

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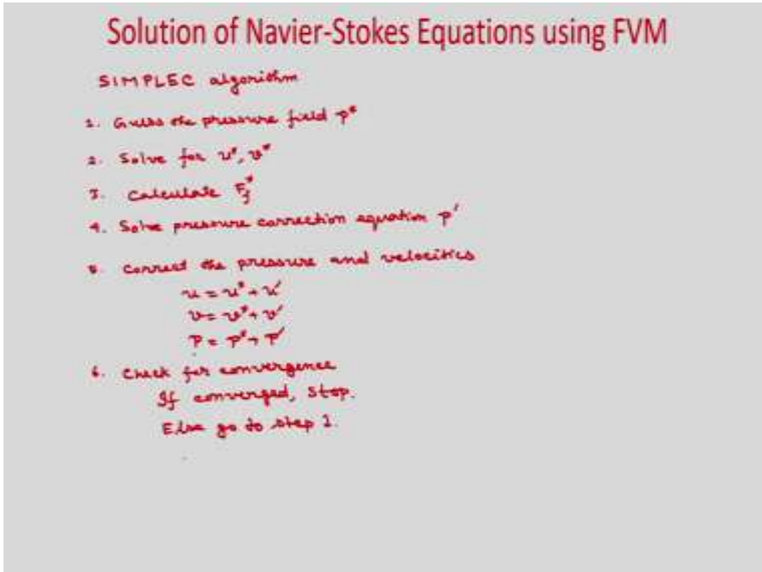

Now, let us write the simplec algorithm, simplec algorithm. So first, now similar to simple algorithm, guess the pressure field, guess the pressure field, p star, okay. Once you guess the pressure field solve for u star and v star, okay. So, once you solve for u star, v star you can calculate Ff star, once you know the Ff star you can solve for the pressure correction equation, pressure correction equation, p prime, okay.

Now you correct the pressure and the velocities but do not under relax it, okay the pressure, correct the pressure and velocities. So, u correct u as u star plus u prime v is equal to v star plus v prime and P is equal to P star plus P prime, okay. Now check for convergence. So, if converged stop, if converged stop, else go to step 1, okay. So with the, now this pressure you can take as the guessed pressure field, okay.

So, the simplec algorithm is actually shown as faster than the simple algorithm because you are not fully dropping this summation of anb u prime nb. But also here you do not need to solve additional equation as you do in the simpler algorithm. So, now, we will discuss about the boundary conditions while solving these discretized equations. Mostly you know the velocity boundary conditions again we will discuss, but we have derived the pressure correction equation. So, we need to know the boundary conditions for this pressure correction p prime.

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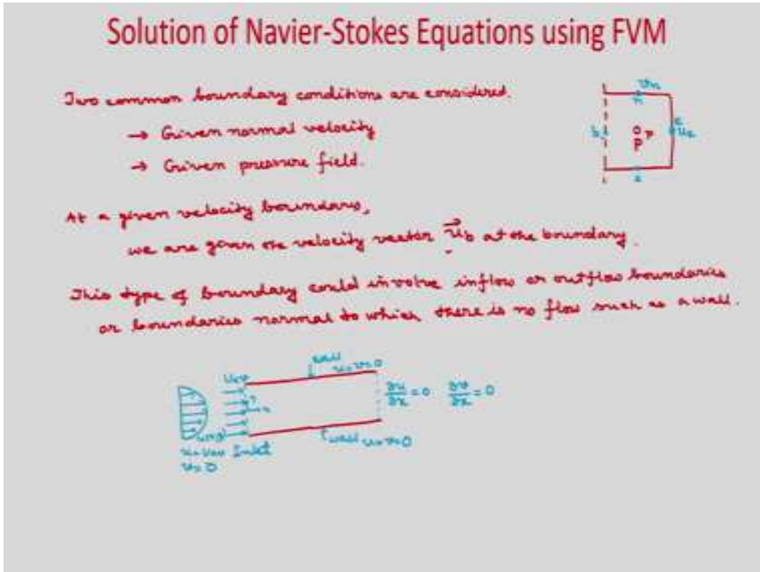

So, if you consider boundary in the left side let us say this is your cell, okay. So, this is your main control volume P where you are solving small P and here you are solving ue here are you solving vn and this is your boundary, okay. So this is your small e, this your small n, this your small s and the west space is now boundary, okay, for this particular case let us write the boundary conditions.

So, generally two boundary conditions we will discuss, one is normal velocity is given or the pressure is given, okay.

So two common boundary conditions are considered, one is your given normal velocity, another is given pressure field, ok. So, you can see that the velocity is known at the boundary then you can represent for the fluid flow problem in flow, okay, where velocity will be specified at the inlet, outflow where velocity may be specified at the outlet and the no slip walls where you have velocities are considered to be 0.

So, at a given velocity boundary we are given the velocity vector u prime b. So, at this boundary, okay, ub is specified okay. So, in that now if it is inflow then velocity will be provided if it is outflow, if you know the outflow boundary condition then also we can use the velocities specified or we will can use some outflow boundary condition and if it is a wall then there will be no penetration through the boundary and velocities will be 0.

So, this type of boundary could involve inflow or outflow boundaries or boundaries normal to which there is no flow such as a wall, okay. So, if you see, let us consider a channel. So, let us consider flow between 2 parallel plates. So, third direction is infinite and we are considering flow between 2 parallel plates. So, we can see that if you want to give the velocity boundary condition then on the walls there will be no slip.

So, velocities will be 0, at the inlet you can specify the velocity boundary condition, you can specify as a uniform velocity boundary condition or some time varying or specially varying boundary condition. So, at this boundary okay you can specify their uniform velocity boundary condition. So, you can see let us say u average you are given, or you can specify a parabolic boundary condition also where u is function of y, okay.

So, if it is x and this is your y then u is function of x and y. So, this is your inlet similarly at the outlet you can generally specify the normal direction velocity gradient is 0, del u by del x is equal to 0 and del v by del x is equal to 0, okay. So, it is a kind of fully developed condition. So, no axial change of velocities in this direction and these are the no slip condition. So, wall, so, no slip condition.

So, you can put u is equal to v is equal to 0 and u is equal to v is equal to 0. If you are specifying normal velocity then here u is equal to u average and v is equal to 0. So, this in detail we have discussed already in earlier modules. So, now, let us discuss about the pressure correction boundary condition.

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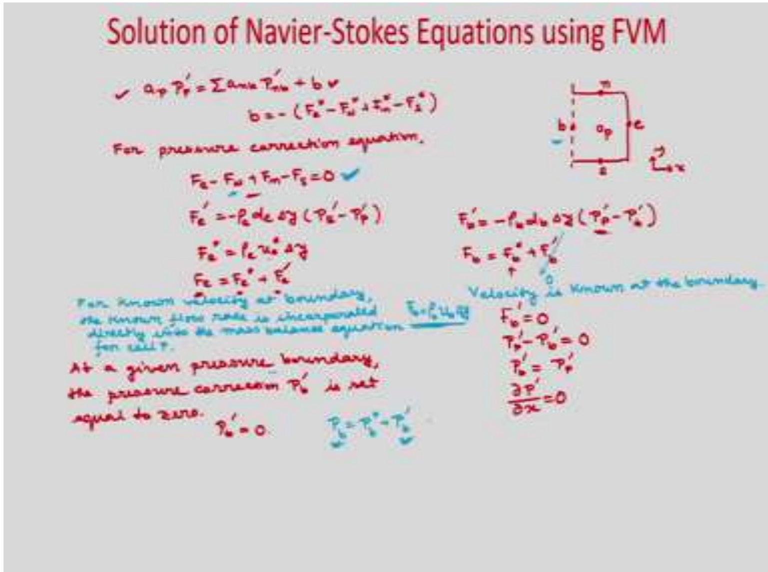

So, if you see the pressure correction equation is ap Pp prime is equal to summation of anb, Pnb prime plus b okay where b is equal to minus, Fe star minus Fw star plus Fn star minus Fs star, okay.

So, for pressure correction equation, if you write the continuity equation in general so it is Fe minus Fw plus Fn minus Fs is equal to 0 and that substituting the star quantities and the prime quantities we have derived these pressure correction equations, okay. So, for this boundary let us say again let us consider these left boundary. So, this is your capital P, this is your small e, small n, small s and this is your west space which is your boundary, okay.

So, for east space if you write the Fe prime then it will be rho e minus rho e de ae is delta y, and Fe star is your rho e ue star into delta y. So, Fe you have written as Fe star plus Fe prime. Now, you see if velocity is specified at the boundary okay then mass flow rate is known. So, you do not need any correction in the mass flow rate, okay. So, you can see this is your Fe is equal to Fe star plus Fe prime and if velocity is known, then this is your correct velocity, okay.

So, that means, you do not need any correction Fe prime. So, Fe prime will be 0, so you can see that your mass flow rate we have written in terms of the star quantities plus the correction quantity. Now, for this boundary if you write then Fb prime is equal to you can write rho b db delta y, Pp prime so this is not being delta y, yes delta y Pp prime minus Pb prime, okay. So, now you can see that if velocity is known then Fe is equal to Fb is equal to Fb star Plus Fb prime.

So, at the boundary velocity is known then the mass flux is known. If mass flow rate is known, so if velocity is known the mass flow rate is known, if mass flow rate is known, you do not need any correction at the boundary. So, that means, in this case your Fb prime will be 0, okay. Because you do not need any correction because you have the velocity field is specified. okay. Velocity is known at the boundary, okay.

So, Fb prime. So, you do not need actually any boundary condition for the pressure correction, because this term you will not add while adding in this equation, okay. So, wherever this Fw which is actually coming as Fb, so, Fb will be just in terms of velocities not in terms of the pressure correction. So, you do not need any boundary condition for pressure correction for the known velocity.

And in other words also you can say that if it is 0, if Fb prime is 0 then this will be 0 that means, it will be Pp prime minus Pb prime will be 0 and hence, you can write these as the Pb prime is equal to Pp prime that means, at this point you are applying the kind of normal boundary condition Pp prime by delta x is equal to 0, okay, so this is x direction this is your y direction so obviously, del p prime by del x0 however these pressure correction values are not required for the boundary.

Because you will not add it in the pressure correction equation while solving it, okay, and if pressure is specified at the boundary what will happen? So, in general when you have a flow inside a channel at the outlet we specify the pressure which is maybe the atmospheric pressure open to the atmosphere, so the, you can set as the atmospheric pressure at the outlet.

So, if it is so, then you do not need any correction at the boundary, right, so, that means P prime will be 0 at the boundary. So, at a given pressure boundary the pressure correction, the pressure correction, Pb prime is set equal to 0 that means Pb prime will be 0, okay. So, you can see that if pressure is specified at the boundary, then you do not need any actually correction in the pressure.

So, Pb prime will be 0 and if velocity specified, then you will know the mass flow rate and if you know the mass flow rate you directly put it in the pressure correction equation so that you do not need any pressure correction boundary condition that means. So, for the known velocity at boundary the known flow rate is incorporated directly into the mass balance equation for cell P, okay.

So, which you have written here. So, there we directly put the value of Fb is equal to rho b ub delta y. So, this we directly put so because Fb prime is 0, okay. So, this we directly put for this Fw which is for this particular case it is Fb and rest we can represent in terms of pressure correction, okay. So, those are interior points so, you and if the pressure is specified then P is P star plus P prime, right.

So, Pb is Pb star plus Pp prime. So, if pressure is specified then Pb is known so you do not need any pressure correction. In todays class we will discuss 2 variants of simple algorithm which are known as simpler, simple revised and simplec, simple corrected. So, these algorithm we have discussed, which are actually to accelerate the convergence rate.

So, in simpler algorithm, we have seen that instead of guessing the pressure field with the known velocity field we solve for the pressure equation and that is obviously a good guess pressure field and it converges faster than the simple algorithm. Rest of the algorithm is similar to simple algorithm except you do not need to correct the pressure because you are solving a pressure equation.

So, that is your correct pressure field from the known velocity field so you do not correct the pressure in the simpler algorithm. In simplec algorithm, it is also faster than the simple algorithm. Here, without dropping the Navier correction term, you write it in terms of the correction term of the cell center other algorithm is similar to simple algorithm, but as you are not dropping these summation of anb u prime nb you get faster convergence than the simple algorithm.

Finally, we discussed the velocity boundary condition and the pressure correction boundary condition. For pressure correction boundary condition if velocity is known, so, you know the mass flux. So, you know the mass flow rate so, you do not need any pressure correction boundary condition, and if pressure is specified at the boundary, then you know do not need any pressure correction. So, that means P prime at the boundary is 0. Thank you.