Computational Fluid Dynamics for Incompressible Flows
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Lecture 02
Solution of Navier-Stokes Equations using FVM-2

Hello everyone, so in last class, we integrated the pressure term or pressure gradient term appears in the Navier-Stokes equation using finite volume method and also we have integrated the continuity equation using finite volume method and we have shown why there is a need to go for staggered grid. So, today we will integrate the full Navier-Stokes equation using finite volume method in staggered grid.

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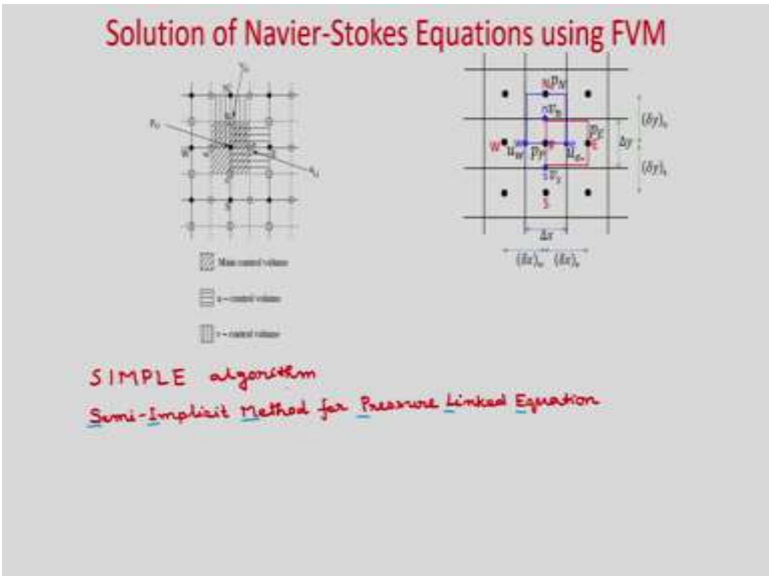

So, you can see that in staggered grid, the pressure is solved in the main control volume. So, you can see this is the main control volume and at cell centre capital P, this Pp is solved. Now, at the face centre of this control volume, small e, now, we have another this red colour cell. So, in that cell we solve for the u velocity for the face centre small e Ue.

Similarly, the V velocity we solve in this blue colour control volume and at the face centre small n for the velocity Vn and similarly we solve Vs at this face centre, small s and UW, this face centre w. So, we can see that, for this u velocity when you are solving in this control volume, the pressure difference Pe minus Pp is the natural driving force.

Similarly, for velocity Vn, pressure difference between Pn and Pp is the driving force for this velocity Vn. And for the main control volume p, you can see that we can satisfy the continuity equation, Uv minus UW plus Vn minus Vs in that way. So, that is the advantage of using staggered grid. So, we will learn today simple algorithm due to Patankar and the main idea behind this simple algorithm is to create a discrete equation for pressure or related quantity called as pressure correction from the discrete continuity equation.

So, what is the full form of SIMPLE? So, we will learn today, SIMPLE algorithm. The full form is Semi Implicit Method for Pressure Link Equation. So, you can see this is the S I M P L E so this is called as simple algorithm and it was proposed by Professor SB Patankar. Now let us write the discretized momentum equation for U velocity in the control volume Ue.

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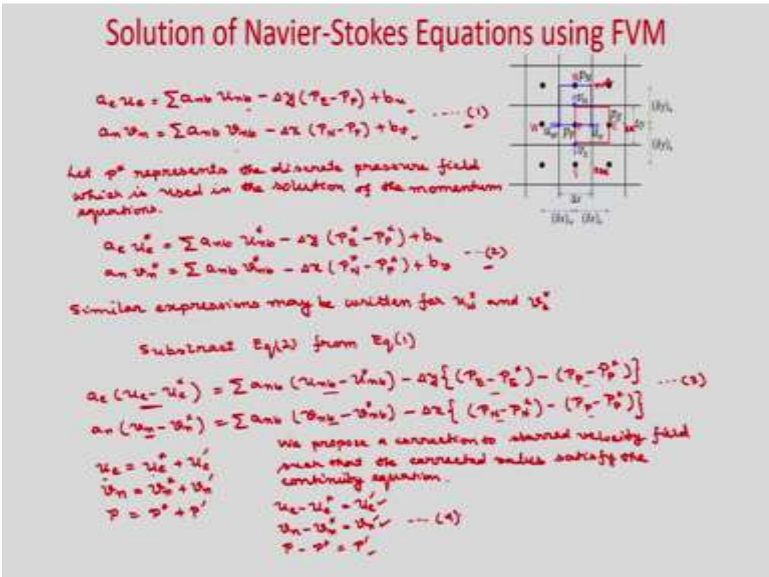

Okay so, if you write that then already we have learned for the convection diffusion equation, similarly, you can write this equation as Ae Ue, now, we are not writing Ap Up because we are solving this u velocity at small e. This is the face centre of the main control volume. So, we are writing Ue is equal to summation of A nb U nb, then the pressure gradient term, we have derived delta y Pe minus Pp and some source term bu.

And similarly, for the velocity Vn you can write an bn is equal to summation of A nb B nb minus delta X Pn minus Pp plus source term for the B momentum equation. So, you can see, so this when you are solving Ae Ue, so, this is the diagonal coefficient Ae and Ue is the velocity e in the face centred small e. So, now the summation of A nb U nb. So, this U nb now it will contain the neighbours of Ue. So, Ue will be at this point, these are the neighbours and similarly at this point, then you have this point and also you have this point.

So, you can see that these are if you represent, so, it will be small e double E, then this will be nne. Then this will be w, it is anyway w and this point, it will be sse. So, in this neighbours you have to get the summation of A nb U nb. Similarly for summation of A nb V nb and when you are solving this equation, so initially pressure is unknown.

So now, let us assume some pressure for the initialization. So that will start with the assumed pressure field P star. So let P star represents the discrete pressure field which is used in the solution of the momentum equations. So, P star is the pressure field then it will satisfy these discrete equations. So if you put it there, then we will get some velocity, which will be the provisional velocity. So, Ue star and Vn star.

So, we can write Ae Ue star is equal to summation of A nb U nb star minus delta y Pe star minus Pp star plus bu. As we are assuming the pressure field, p star, so obviously, after solving the momentum equation, we will get some provisional velocity. That is your u star. So, we have written this equation as Ae V star is equal to summation of A nb U nb star minus delta y p star minus Pp star plus Bu where P star Pp star are the assumed pressure field.

Similarly, for the y momentum equation you can write an Vn star is equal to summation of A nb, all the neighbours B nb star minus delta x Pn star minus Pp star plus V p. So, similar expression you can write for the West face Uw and the South face Vs. So similar expressions maybe written for Uw star and Vs star.

So, now this equation if you tell that equation number 1 and this is your 2. Now subtract 2, equation 2, subtract equation 2 from equation 1. So, what you will get? So, you can see, so if you subtract these equations from the equations 1, so you will get Ae Ue minus Ue star is equal to summation of A nb U nb minus U star nb minus delta y Pe minus Pe star minus Pp minus Pp star and this will get cancelled the source term.

Similarly, you can write An, Vn minus Vn star is equal to summation of A nb V nb minus v star in V minus delta x, then Pn minus Pn star minus Pp minus Pp star. So now, let us introduce the velocity correction. So, what is velocity correction? So, this Ue minus Ue star will tell us velocity correction. So, you can see, this u velocity will satisfy the continuity equation, but this Ue we have got the solution from the assumed pressure field which may not satisfy the continuity equation.

The difference between these two velocities we are calling as velocity correction. So, we propose a correction to star velocity field such that the corrected values satisfy the continuity equation. So, we will propose Ue star so, this way I will write. So, we are proposing a correction in the star velocity, so, Ue is equal to Ue star plus some correction we are making so that it will satisfy the continuity equation.

Similarly, you can write Vn is equal to Vn star plus Vn prime and similarly the pressure, so P will be p star plus pp prime, okay. So, now you can see the difference Uv minus Uv star, you can write in terms of the velocity correction.

So, Uv star you can write the velocity correction and Vn minus Vn star you can write as Vn prime and P minus P star will be your P prime. So, these are the pressure correction and velocity corrections, okay so now these we can represent, these terms we can represent with the correction. So now, if this is equation 3 and this is 4.

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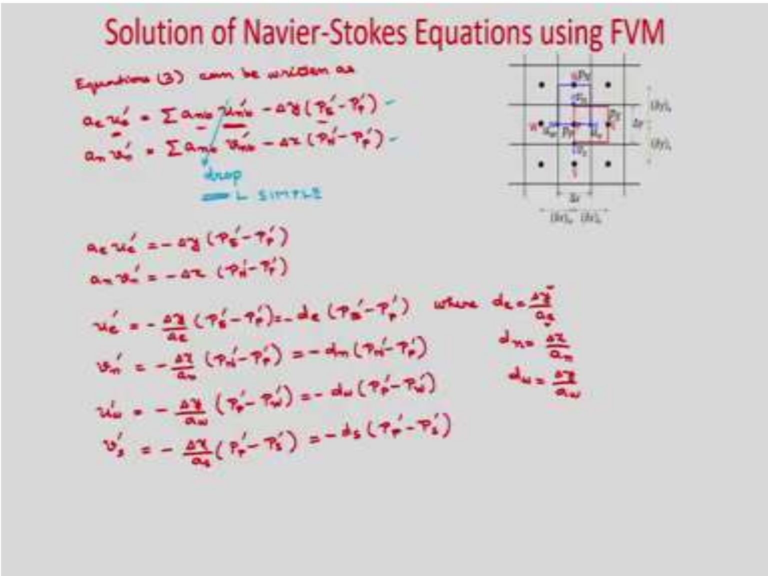

Now equation 3 can be written as. So, now we are introducing the pressure correction and velocity correction. So, it will be Ae U e prime is equal to summation up A nb U nb prime minus delta y Pe prime minus Pp prime. And An Vn prime is equal to summation of anb V nb prime minus delta x Pn prime minus Pp prime. And similar expression may be written for Uw prime and Vs prime.

So, here now we will make one important assumption. So here now we will make the important simplification, you can see this term summation of A nb U nb prime. So, if you are solving for the (pressure), if you are solving for the velocity correction, Ue prime at this small face centre, at the face centre small e, then you need to know the velocity correction from the neighbour points as well, so this U nb prime.

So obviously, these neighbour points again it will involve the other neighbour points so it will be very cumbersome to calculate this Ue prime. So now we will make this important simplification that will drop these terms. So these terms will make as 0, so now these two terms will drop. So, this is the important simplification and this is known as simple algorithm.

So, why it is semi-implicit, now let us understand, you see this is the implicit equation, it is fully implicit equation because Ue prime it depends on the neighbor velocity U nb prime. But now we are dropping these terms so now some semi-implicitness is coming because it is not full-implicit. Okay as we are dropping this term, that is why this is known as semi implicit method for pressure linked equation.

So now, if you drop this term, so you can write the velocities in terms of only pressure correction. So, if you drop these terms, then you can write the velocities only in terms of the pressure correction. So, you can write Ae Ue prime is equal to minus delta y Pe prime minus Pp prime and An Vn prime you can write as minus delta x Pn prime minus Pp prime. So, now we can write Ue prime as minus delta y by Ae Pe prime minus Pp prime and this we will write as de into Pe prime minus Pp prime where de is equal to delta y by a. So, this is your face area and this is your diagonal coefficient Ae.

Similarly for Vn prime what you can write? It will be minus delta x by n Vn prime minus Pp prime is equal to dn sorry here minus sign will be there equal to minus d n into Pn prime minus Pp prime and dn is equal to delta x by An. Now this velocity correction let us write for the west face and south face as well.

So, you can write U prime W so, at this W face we are writing for the velocity correction U W prime. So, this will be in terms of the pressure correction so, it will be just Pp minus Pw prime, Pp prime minus Pp w prime. So, this will be delta y by Aw so, W Pp prime minus Pw prime and we will write as minus dw Pp prime minus Pw prime where dw is delta y by Aw.

And similarly V prime s so, at this face centre we will solve for Vs and this will be Vs prime is equal to minus delta x by As. And what is the pressure correction difference? It will be Pp prime minus Ps prime, so it will be p prime minus Ps prime is equal to minus ds Pp prime minus Ps prime.

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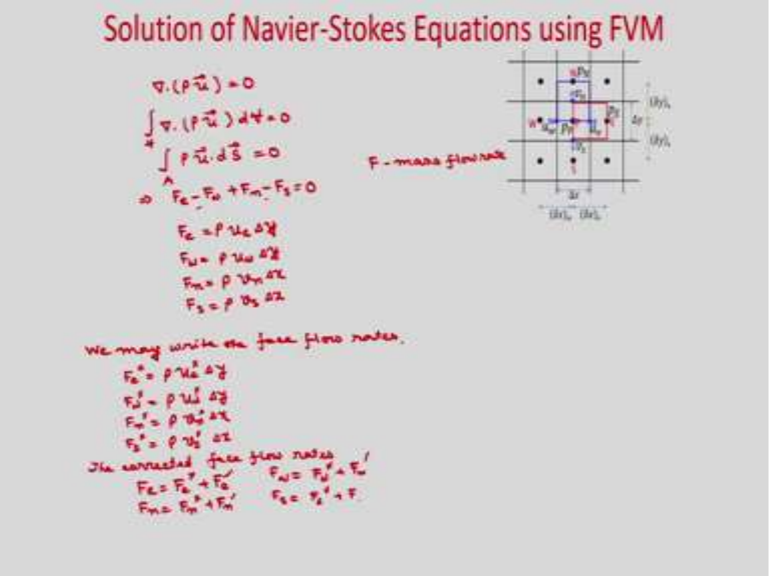

Now, let us write the discrete continuity equation. So, if you see that continuity equation, so it will be divergence of Rho U is equal to 0, so we integrate over the control volume, then you can write divergence of Rho U is equal to 0, dV is equal to 0 and if you convert to the surface integral, then it will be Rho u dot d s is equal to 0.

Now you can write in terms of mass flow rate Fe minus Fw plus Fn minus Fs is equal to 0. So this is the mass flow rate Fe, okay so F is your mass flow rate. So you can see Fe will be your Rho, if it is constant then you can write only Rho Ue delta y, Fw is Rho Uw delta y Fn is your Rho Un Vn delta x, we are writing for the uniform grid, so delta x is same and delta y is same.

So, it will be Rho Vs delta x and this minus sign you know, because U dot ds when you are doing so, obviously, you can see that the face normal is outward direction, so it is a negative x direction that is why it is minus and here also you are getting minus so, this is the continuity equation.

So now, you see that, obviously your velocities Ue, Uw, Vn and Vs will satisfy the continuity equation, but when we assume the pressure field P star and whatever provisional velocity we got, U star and V star those will not satisfy the continuity equation. So, now, this equation let us write in terms of the start quantity.

Now, we may write the face flow rates from the U star quantity, so, it will be Fe star is equal to Rho Ue star delta y. Fw star will be your Rho Uw star delta y and Fn star will be Rho Vn star into delta x and Fs star will be Rho V star into delta x. And now the corrected face flow rates will be now Fe is equal to Fe star plus Fe prime.

So, Fn is equal to Fn star plus Fn prime, Fw is equal to Fw star plus Fw prime and Fs is equal to Fs star plus Fs prime. So, now, you can see from this equation Ue prime, Vn prime, Uw prime and V s prime. So, this we have written in terms of the pressure correction only.

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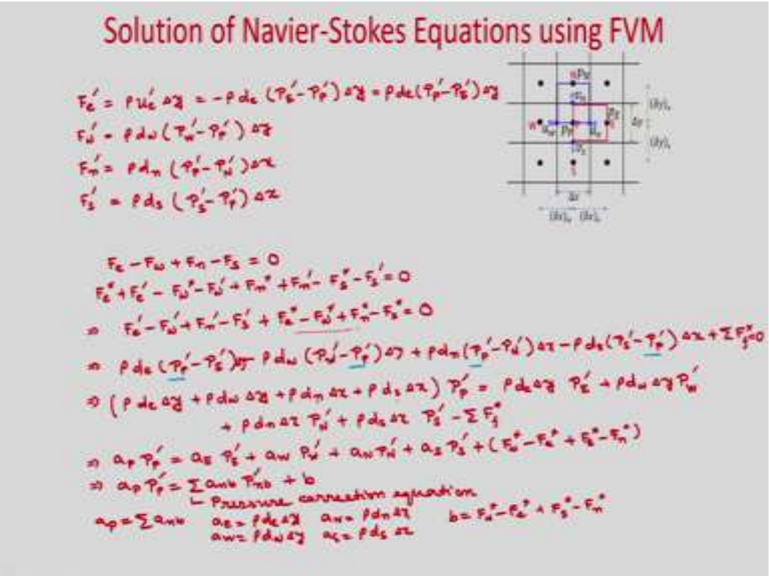

So, now, we will write the corrected mass flow rate Fe prime as Rho e Rho Ue prime delta y. So, now Ue prime you just write in terms of the pressure correction so, it will be minus Rho de Pe prime minus Pp prime into delta y. Or you can write negative sign if you take inside, then it will be Rho into de Pp prime minus Pe prime into delta y.

Similarly, Fw prime you can write Rho dw. So, it will be Pw prime minus Pp prime into delta y, Fn prime will be Rho d n Pp prime minus Pn prime delta x and Fs prime will be your Rho into d s into Ps prime minus Pp prime into delta x. So now, we have written the corrected mass flux, we have also written the star quantity mass flux F star, now let us put these into the main continuity equation.

So, what is the discrete main continuity equation? That is your Fe minus Fw plus Fn minus Fs is equal to 0. So if you put it, so what you are going to get? So Fe is Fe star plus Fe prime minus Fw star minus if w prime, then Fn star plus Fn prime and minus Fs star minus Fs prime is equal to 0.

So now you can write Fe prime minus Fw prime plus Fn prime minus Fs prime, so this we will write in terms of pressure correction and you have plus Fe star minus Fw star plus Fn star minus Fs star is equal to 0. Now, let us substitute the corrected mass flux in terms of the pressure correction. So, if you write that, so Fe prime what is that? So, you can write Rho into de Pp prime minus Pe prime.

And Fw prime is minus Rho dw Pw prime minus Pp prime here delta y will be there and here delta y, then Fn prime is Rho d n Pp prime minus Pn prime into delta x and minus Rho d s Ps prime minus Pp prime into delta x. Now, you can see this is your, you can write plus summation up Ff star, all the faces it is the star velocities when it satisfied the continuity equation. So, you have written summation of Fs star.

So, initially this will not be 0 summation of Fs star, but when at convergence your velocity correction will be 0 that time your Ue will become Ue star and hence this will become 0. So now, you write the equation for pressure correction. So, now all the Pp term you just take together, so you can see, so this is your Pp term, this is your Pp term, Pp and Pp.

So, this you take in the left hand side and all other terms you in the right hand side, so that you can write in terms of Ap phi p. So, you can see these so, you can see the coefficient is Rho de into delta y for this it is minus-minus plus so, it will be plus Rho dw into delta y for this it is plus Rho d n into delta x and for this it is plus, minus-minus plus Rho ds into delta x. So, this is the coefficient for the Pp prime.

Now you, all other times you take in the right hand side, so for P, so it is minus so it will be plus, so Rho de delta y and it is for Pe prime This is your minus, so this will be plus so it will be Rho dw delta y Pw prime. Then you have north so, it is minus so this side will become plus Rho d n delta x Pn prime. And similarly for south face Rho ds delta x into Ps prime.

So, we have taken all the terms right hand side and these also you have taken the right hand side, so it will be minus summation of Ff star. So, now you just write in terms of coefficient, diagonal coefficient Ap in the left hand side, so it will be Ap Pp prime is equal to Ae Pe prime plus Aw Pw prime plus An Pn prime plus As Ps prime minus or you can write plus Aw star minus Fe star plus Fs star minus Fw star.

At minus Ff star we have written so, it will be Fw star minus Fe star. So, now you can write it as Ap Pp prime is equal to summation of A nb Phi nb plus source term B, sorry not phi, summation of A nb, P nb prime plus source term B.

So, now you can see that we have written this equation for pressure correction. So, this equation is known as pressure correction equation, where Ap is equal to summation of Anb, you can see your Ae is your Rho de delta y Aw is Rho dw delta y, An is Rho d n delta x and As is Rho d s delta x. So, you can see in left hand side you have Ae plus Aw plus An plus As so, Fe is equal to summation of A nb and b is your Fe, so b is equal to Fw star minus Fe star plus Fs. So, this is sorry this will be north, so plus Fs star minus Fn star.

So, now you can see that you can solve this equation. Once you solve this equation, you will get the pressure correction. Now, if pressure corrections are known now you can find the velocity correction and that velocity correction now, you can actually use to correct the velocity with the U star, because we assume the pressure P star, we solved for the velocities U star and V star and now, once we know the correction, then we can add U star plus U prime and V star plus V prime so, that we get the correct velocity field U and V.

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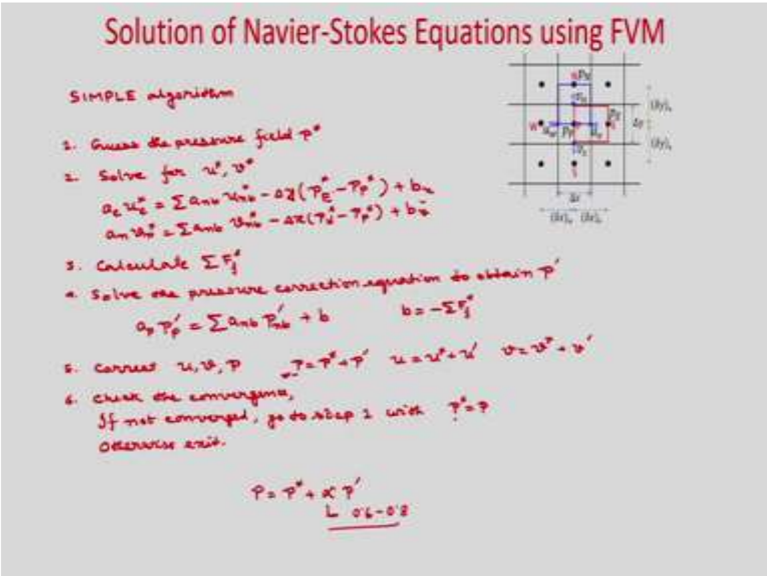

So now, let us write the algorithm whatever we have derived today for the simple algorithm. So first is guess the pressure field P star. Then you solve for U star and V star. So what U star V star so you can write U ae Ue star is equal to summation of A nb Un b star minus delta y Pe minus Pp star with sum source term Bu.

Similarly, A n Vn star is equal to summation of A nb V nb star is equal minus delta x Pn star minus Pp star plus bV. So, once you solve the equation for U star V star obviously, you can calculate the summation of Ff star which appears in the pressure correction equation, right.

So, once these are calculated, you can calculate summation of Ff star. So now, solve for pressure correction equation to obtain P prime.

So what is that equation we have written? Ap Pp prime is equal to summation of A nb P nb prime plus source term summation of Ff star so that is your b, where b is your minus of summation of Ff star. So now, once you know the pressure correction now you correct the velocities, so correct U, V and P so, you can write P is equal to P star plus P prime. So, now this is the guess pressure for the next iteration and U is equal to U star plus U prime and V is equal to V star plus V prime.

Now, you see whether, now you check whether your continuity equation is satisfied or not whether summation Ff star is equal to 0 or not. If not, then you take this pressure field as the guess pressure field for the next iteration and repeat the algorithm. So 6, check the convergence, if not converged go to step 1 with, now you make P star is equal to P.

So, whatever p you have calculated with P star plus P prime so these will be used as a P star for the next iteration otherwise exit. So, today we have learned the simple algorithm due to Patankar. So, main idea of this simple algorithm is to create one equation for pressure or pressure like equation like pressure correction from the continuity equation.

So, today we started with the staggered grid, we solved the velocities Ue, Uw, Vn and Vs with the guess pressure field P star. So, you got some provisional velocity U star and V star, obviously, this will not satisfy that continuity equation. So, we subtracted U e minus U w and wrote the equation for the velocity correction.

So, but in the velocity correction when we wrote in the right hand side, you have the summation of A nb, U nb prime and summation of A nb V nb prime in the V momentum equation. So, as we have dropped these terms so, now it has become semi implicit, it is no longer full implicit. Now, we have retained the velocity correction in terms of the pressure correction only.

We substituted these velocity correction in the terms of corrected mass flow rate to satisfy the continuity equation and we have derived the pressure correction equation. And that pressure correction equation once you solve, you will get the P prime and once you know that P prime you can calculate the velocity correction U prime and V prime.

And with those velocity corrections under pressure correction, you can update the velocity U and V and P. So, if you check for the convergence, if it is not converged, then this P you take a P star for the next iteration. So, as we have dropped that term summation of A nb U prime A nb, so pressure may diverge during the solution of the pressure correction equation.

So, pressure correction equation may diverge for that you can use some under relaxation factor while correcting the pressure. So, where you have written this P is equal to P star plus P prime, you can use actually P is equal to P star. And you would use some under relaxation factor Alpha into p prime and these under relaxation you can take in the range of 0.6 to 0.8 as you have dropped the summation of A nb U nb prime, for that reason, it tends to divert. So if you use some under relaxation then it will not diverge. Thank you.