Computational Fluid Dynamics for Incompressible Flows
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Module 11: Finite Volume Method- 2
Lecture 1
Finite Volume Discretization of Steady Convection- Diffusion

Hello everyone, so in last module we discussed about the Finite Volume Discretization of diffusion equation. So now, we will add the convection term and we will discretize the steady convection diffusion equation using finite volume method in this module, in this lecture so, first let us write the governing equation.

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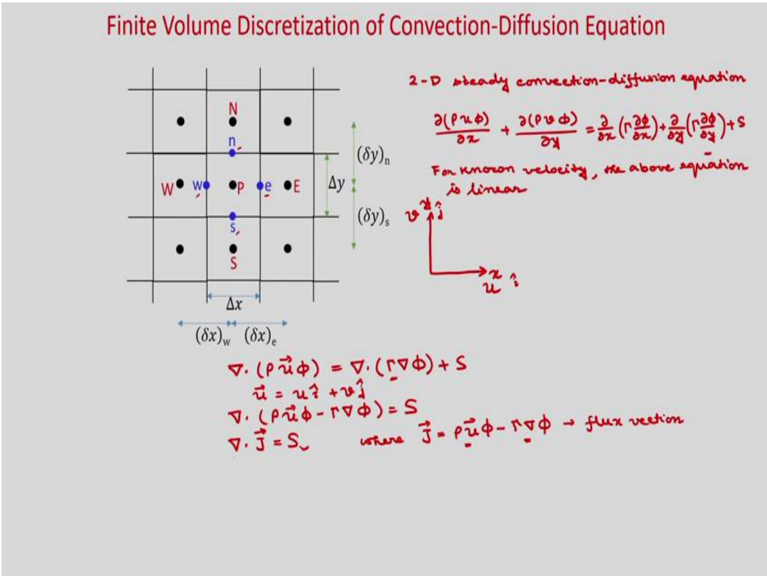

So, we are considering 2D steady, so we have convection terms as well as diffusion term. So, convection diffusion equation so, the governing equation is rho u phi by del x, where rho is the fluid density and u is the velocity in x direction, then rho v phi by del y, v is the velocity in y direction is equal to del of del x gamma del phi del x, gamma is the diffusion coefficient plus del of del y gamma del phi by del y plus s.

So, we have written this governing equation for general variable phi and u, v are the velocities in x and y direction. So, if velocities are known, obviously this equation is linear. So, for known velocity, the above equation is linear.

So, we discussed that if this is your x direction, this is your y direction then, u is the velocity in x direction, v is the velocity in y direction and i is the unit vector in x direction and j is the unit vector in y direction. Now, let us consider this finite volume of cell P, so, you can see this is the main control volume, this P and the face centers are small e, small n, small w and small s.

So, we will integrate this governing equation over this control volume main cell P. So, let us write this governing equation in vector from first and write in terms of the flux vector. So, the governing equation you can write in vector for as divergence of rho u phi is equal to divergence of gamma grand phi plus the source term. So, you can see that u is obviously your velocity vector and now we can write as divergence of rho u phi and this diffusion term you bring in left hand side, so what you will get? Divergence of minus gamma grand phi is equal to S.

So, now we will introduce the flux vector j is equal to S where j is the flux vector rho u phi minus gamma grand phi so, j is the flux vector. So, in last module you have seen this flux vector contain only the diffusion term, but in this lecture you can see the flux vector contains convection terms as well as the diffusion term.

So, the first term is the convection term and second term is the diffusion term and this equation divergence of j is equal to S, you have already discretize in last class and you know all the assumptions involved while integrating this equation in the main cell P. So, we will follow the same procedure, we will have the same assumptions valid here and we will write the discretize equation for this divergence of j is equal to S.

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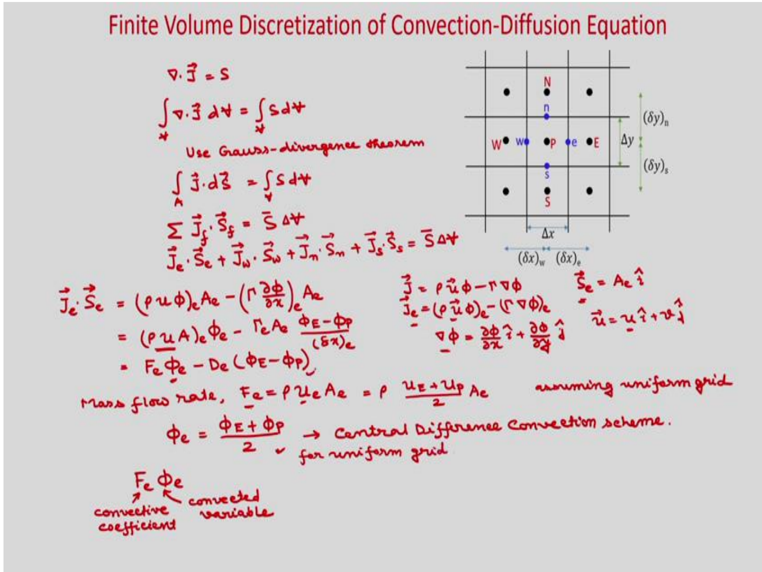

So, first integrate over this control volume so, divergence of j is equal to S where integrating over the main cell P divergence of j dv plus, sorry equal to volumetrical s dv. Now, use Gauss divergence theorem so, you can write integral so, now we are converting the volume integral to surface integral.

So, j dot ds is equal volume integral sdv. We followed the same procedure, till now we have not assumed anything, now we will assume that j varies linearly along the faces and the average value lies at the face center so, face center small e, small n, small w and small s and for the volume integral of the source term also we will have the average value lies at the cell center P.

So, with that now, we can write this integral over the faces that we can write summation of jf dot sf so, sf is the surface vector at the faces equal to S bar is the average value lies at the cell center P and delta v so, this now we can write je so, we have four faces for this two dimensional cases so, East, West, North and South so, we will write Je dot Se plus Jw dot Sw plus Jn dot Sn plus Js dot Ss is equal to S bar delta v.

So now, each flux vector at the faces we have two terms, one is convection and diffusion term. The diffusion term already you know how to discretize so, we will have the assumption that the flux, while calculating the flux vector we will assume that the value of phi varies linearly between the cell centroids. So, obviously the gradient of phi at the face center will calculate using the neighbor cell centered value and its main control volume cell center value.

So, first let us see how that how we will calculate Je dot Se, then similarly we can write for other terms. So, Je dot Se, what is J? So, J is rho u phi minus gamma grand phi so, now Je will be rho u phi at face E minus gamma grand phi at face E, what is the surface vector? Se, Se is Ae i so, now we can see this is your Je, this is your Se now, if you make the dot product, and what is grand phi? Grand phi is del pi by del x i plus del phi by del y j.

So, now you make the dot product and write the terms so, convection term you see, this is the convection term rho u phi e so, u is again ui plus vj. So, if you make the dot product with u and Se then you will get only ue Ae, ue Ae because i dot i will be 1 and i dot j will be 0 so, only this term will remain ue Ae. So, it will be rho u phi at East face into Ae minus gamma del phi by del x because it is grand phi, del phi by del x i and you are making the dot product with Ac so, del phi by del x in to Ae so, so del phi by del x at East face Ae.

So now, we can rearrange it so, we can write as rho uA at East face and phi e you are taking outside of this and minus, now you can see gamma e Ae and del ph by del x at face center E so, that we will use phi E minus phi p divided by the distance between this two cell centers. So, now this will represent as mass flow rate Fe into phi e and gamma Ae by del xe will represent with De into pi E minus P. The last term you have already discretize in the earlier lectures but the first term now, we are introducing with the mass flow rate Fe so, we are introducing mass flow rate Fe is equal to rho Ue Ae.

So, now you can see that Ue, the velocity at the face center E, this is not known because you might have the values available at the cell center P so, you can use a linear average so, you can use as rho if you have a uniform grid then you can write Ue plus Up divided by 2 in to Ae where we are using, assuming uniform grid.

So, now Fe is the mass flow rate and phi e is the value of phi at the face center. So, this is also unknown, phi is also unknown because phi you are solving only at the cell centroids but you need to find the value of phi at the face center small e so, for simplicity let us use the central difference and phi e we can use the value of phi E plus phi p divided by 2 so, this is known as central difference convection scheme.

If you see Fe phi, the value of phi e sometime we will determine depending on the value of Fe, whether it is positive or negative so, Fe is known as convective coefficient and phi e is known as convected variable. So, you can see phi e is the convected variable so, this mass flow rate whatever Fe is there it is known as convective coefficient and due to the mass flow rate this phi e is transported, so that is why it is convected variable and to determine the value of phi e from the sin of Fe, we will use some convection schemes and then we will discuss later in later classes.

So, now for these if you use the central difference convection scheme phi is equal to phi e plus phi P divided by 2 for uniform grid, for uniform grid then what you can write now Je dot Se so, let us write down.

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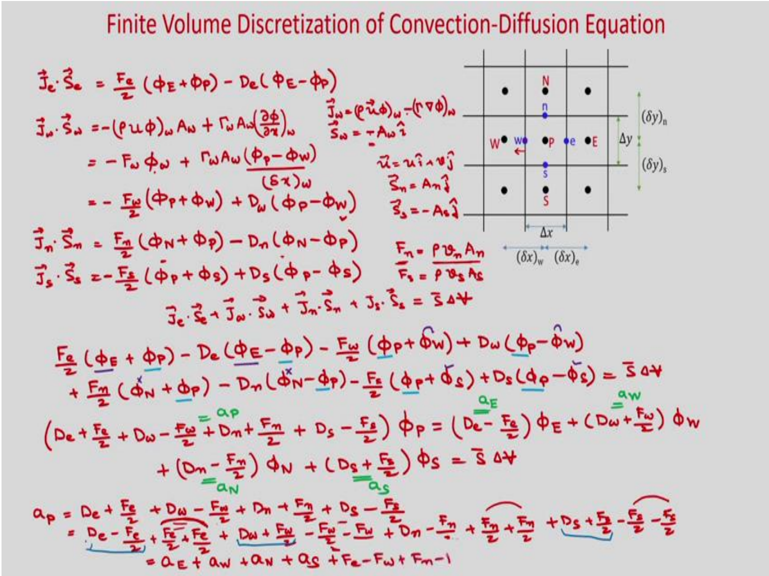

So, Je dot Se now, we can write Fe by 2 then we have a average value phi E plus phi P minus De convection term and phi E minus phi P. So, we have written this flux vector dot product with the surface vector at the East face. Now, let us write for West face. So, Jw dot Sw, similarly now you can see it will be rho u phi at West face. Now, if you make that dot product with Jw with the Sw. So, Jw is rho u phi w minus gamma grand phi at w and Sw is minus Aw i.

So, this you have to remember because this is minus because at this space you face normal in negative x direction so, that is why it is minus Aw i so, now if you make a dot product obviously it will become minus Aw similarly, here also you will get minus 1 convection term is there and another minus will come it will be a dot product and you will get plus and you write gamma w Aw and del phi del x at w. So now, you can write it so, now you use the convection minus Fw phi w and plus gamma w Aw. Now, del phi by del x at w so, it will be phi p minus phi w divided by delta xw.

So, now rearrange it so, you will get Fw will take the convection scheme, central difference convection scheme, so minus Fw by 2 so, it will average phi p plus phi w by 2. So, it is phi p plus phi w and this is your plus De and phi p minus phi w. Now, similarly if you do Jn dot Sn so, you can write as Fn by 2 phi N plus phi P minus Dn phi N minus phi P.

So, this is your convection term, diffusion term where Fn is now you have rho Vn into An because now you have the u velocity and J Sn is your an J. So, now if you make a dot product so it will be v J dot nJ so it will be VAn and that rho Vn An we are writing as the mass flow rate at North face Fn.

So, similarly you can write Js dot Ss so, it will by Fs by 2 so, you remember we are using central difference convection scheme, it will be phi P plus phi S and here minus sign will be there because Ss will be minus AsJ so, due to that minus sign will come and you have plus Ds phi P minus phi S. So, we have written all the terms.

Now, let us put in the summation of J dot SF is equal to S equation. So, we have Je dot Se so, your Fs will be rho Vs As so, Se plus Jw dot Sw plus Jn dot Sn plus Js dot Ss equal to S bar delta V. So, now let us put all this values so, you can see here it will be Fe by 2 phi E plus phi P minus De phi E minus phi P. So, this is your Je dot Se plus Jw dot Sw, this one you write so, you can write minus Fw by 2 phi P plus phi W plus, sorry, here it will be Dw so, it will be Dw phi P minus phi W. So, this for the West face.

Now, for the North face the flux is, plus Fn by 2 phi N plus phi P minus Dn phi N minus phi P minus Fs by 2 phi P plus phi S and you have plus Ds phi P minus phi S equal to S bar delta V. So now, we will write this equation in the form of Ap phi p so, you collect all the terms of phi p in the left hand side and the rest all other terms you take in the right hand side.

So, which are the p terms? So, this is so, these are the phi p terms so, all this coefficient will write in the left hand side. So, if you write that so, you see it will be Fe by 2 and minus minus plus so, this will be plus De so, we will write De plus Fe by 2, similarly here you see this is your Dw plus Dw so, plus Dw and here p coefficient minus Fw2.

So, minus Fw by 2, for north face you can see this is your minus minus plus so it will be Dn plus Fn by 2 plus for South, so it will be phi p so, it will be Ds and it will be minus Fs by 2. These are the coefficient of phi P, so this is your diagonal coefficient equal to now, other terms you take in the right hand side so, if you take phi e term then you take phi w, then you take phi n and then you take phi s in the right hand side.

So, you can see, if you write the coefficient of phi e then you can see phi is here minus De phi, so, right hand side if you take it will be positive D and here if you see Fe by 2 phi so, it will be minus Fe by 2 so, it is coefficient of phi similarly now you see phi w so, here minus Dw phi w so, it will be, right hand side if you bring it, it will be Dw and here you have minus Fw by 2 so, it will become plus Fw by 2.

Similarly, for North it is Dn, it will be minus because the left hand side you are brining it in to the right hand side so, minus Fn by 2 phi N and phi S, so it will be plus Ds plus Fs by 2 phi S is equal to S bar delta V. So now, these coefficient you just write it as coefficient ae, this is as aw, this is as an and this coefficient as as. So, if you write that then what you can write the left hand side term, this first coefficient you can see so, we will denote it as ap, so this will denote as ap, so ap you can write as De plus Fe by 2 plus Dw minus Fw by 2 plus Dn plus Fn by 2 plus Ds minus Fs by 2.

So, we will rearrange it now, you can see ae is De minus Fe by 2 so, we will write first this terms as De minus Fe by 2. So, if you write De minus Fe by 2 then another Fe by 2 we have to add so, Fe by 2 plus Fe by 2. This will get cancelled so, Fe by 2 will remain. Now, this together you can see it is Fe, this is Fe. So, similarly you can write Dw so, now we have in aw coefficient Dw plus Fw by 2 so we will write Fw by 2 and we will subtract Fw by 2 and another minus Fw by 2 is there, so now this together will give minus Fw.

Similarly, Dn so n counters minus Fn by 2 so, we will write minus Fn by 2 plus Fn by 2 and another Fn by 2 is there so this together will give Fn and plus Ds plus Fs by 2 and we have now subtract minus Fs by 2 and another Fs by 2 is there so, this together will give Fs. So, now this ap we can write as this term, this is an, this is term this is your Aw, this is your An and this is your As, so, we can write as aE plus this is your aW plus aN plus aS so, these are the coefficient and we have now this together, this together will give plus Fe minus Fw plus Fn minus Fs.

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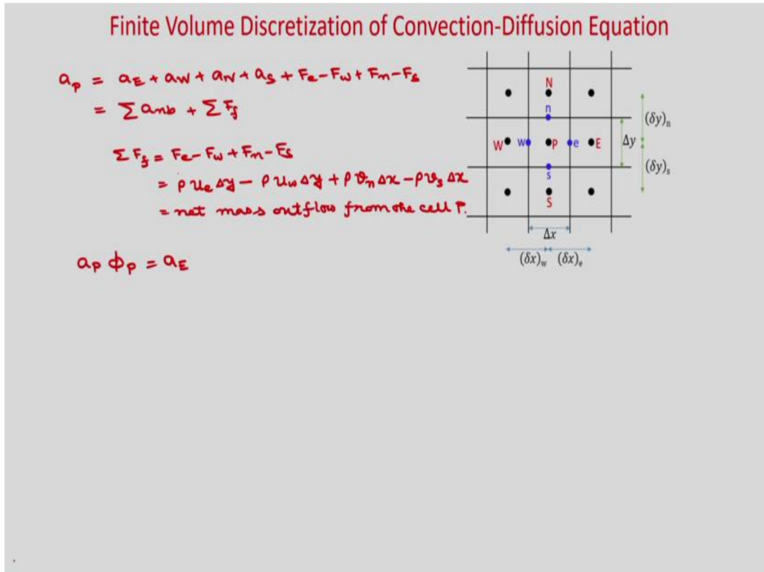

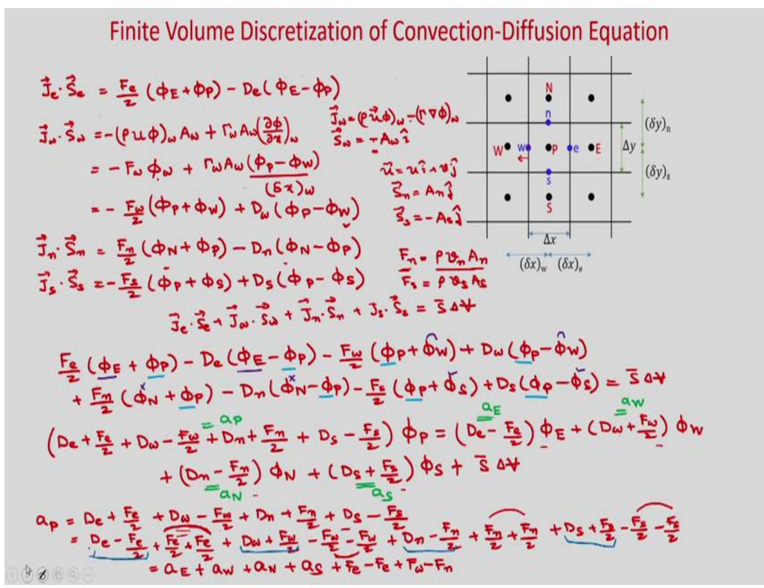

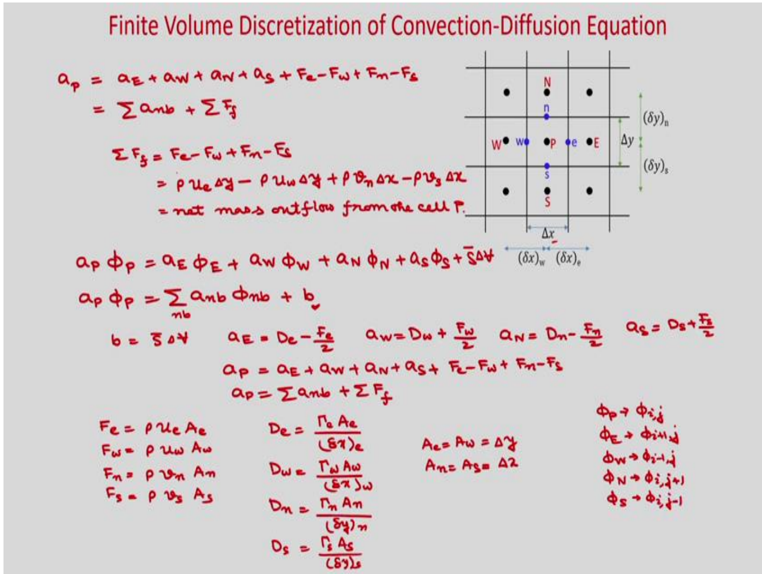

So now, you can write Fe as aE plus aW plus aN plus aS plus Fe minus Fw plus Fn minus Fs and this you can write as summation of an because all the neighbor coefficient summation plus summation of Ff, what is Ff? So, what does it signify? Fe minus Fw minus Fn minus Fs, so this is the net outflow from the control volume P.

So, that means it is the continuity equation, so if you integrate the continuity equation over this volume then you will get this so, that means summation of Ff is Fe minus Fw plus Fn minus Fs, and you know what is Fe? Fe is rho Ue Ae, Ae is your delta y minus Fw is rho Uw delta y plus rho Vn, Vn and the area is delta x and minus rho Vs into delta x and this is the net mass outflow from the cell P. Net mass outflow from the cell P so, you can see that if your flow field satisfy the continuity equation, then summation of Ff will become 0, so, generally at convergence summation of Ff will be tending to 0.

So now, you can write the final form of the discretize equation, so we can write aP phi P is equal to aE, so you look this one so, this is your discretize equation so, you can write aE phi E plus aW phi W plus aN phi N plus aS phi S, sorry this will be plus so, plus S bar delta V, so we can write aE phi E plus aW phi W plus aN phi N plus aS phi S plus S bar delta V.

So, this now you can write as aP phi P is equal to summation of anb phi nb all the neighbor, anb phi nb plus, so S bar delta V will denote as b. So, you have b is equal to S bar delta V aE is your De minus Fe by 2, aw is Dw plus Fw by 2, aN is Dn minus Fn by 2 and aS is Ds plus Fs by 2 and the diagonal coefficient will be just aE plus aW plus aN plus aS minus summation or plus Fe minus Fw plus Fn minus Fs. So, you can write aP as summation of all the neighbor coefficient plus summation of Ff.

And what about your Fe Fw? So, Fe is rho Ue Ae, A is nothing but your delta y, Fw is rho Uw Aw, Fn is rho Vn An, and Fs is rho Vs As, and you have De, so De will be gamma e Ae divided by delta x e, Dw is gamma w Aw divided by delta x w and Dn is gamma n An by delta x n and Ds is gamma s As, sorry here delta y.

So, this is your delta y and gamma s As by delta y s where is Ae is equal to Aw is equal to, this is the surface area so this is your delta y and A north is equal to A south is this distance so, this will be delta x, so it will be delta x. So, you can see so, this is the final algebraic equation for this steady convection diffusion equation and this you need to solve using some linear solver and where P, phi P denotes phi ij, phi E denotes phi i plus 1j, phi West denotes phi i minus 1j, and phi North denotes phi ij plus 1 and phi South denotes phi ij minus 1.

So, in today's class we discretize steady convection diffusion equation, so we considered two dimensional case and we introduce the flux vector which contains diffusion term as well as the convection term. Then we integrated this over the control volume P and we have converted the volume integral to surface integral using Gauss divergence theorem.

Then each flux vector at each faces, the flux vector dot product with the surface vector, we have calculate at each faces East, West, North and South and we defined mass flow rate Fe as rho Ue Ae and similarly for the north face Fn is equal to rho Vn An and finally after rearranging we have written the discretize form in the form of ap phi P.

So, we have written ap phi P is equal to summation anb phi nb plus b, where ap is the diagonal coefficient and this diagonal coefficient ap is equal to summation of all the neighbor coefficient, ap is equal to aE plus aW plus a north plus a south plus the summation of Ff, where summation of Ff denotes the net mass flow outlet from the cell P and it actually is the discretize form of the continuity equation. So, if you flow field satisfy the continuity, then the summation of Ff will become 0. Thank you.