Computational Fluid Dynamics of Incompressible Flow
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Lecture 2
Boundary conditions for flow problems

Hello everyone. So, in last lecture, we discretize stream function equation and vorticity transport equation using finite difference method. So, both the equations to discretize both the equations we used central difference method. So, the order of accuracy was delta x square, delta y square. Today we will consider two problems, first we will consider lid driven cavity problem. Then we will consider flow between parallel plates, plane Poiseuille flow and we will discuss about the Boundary Conditions.

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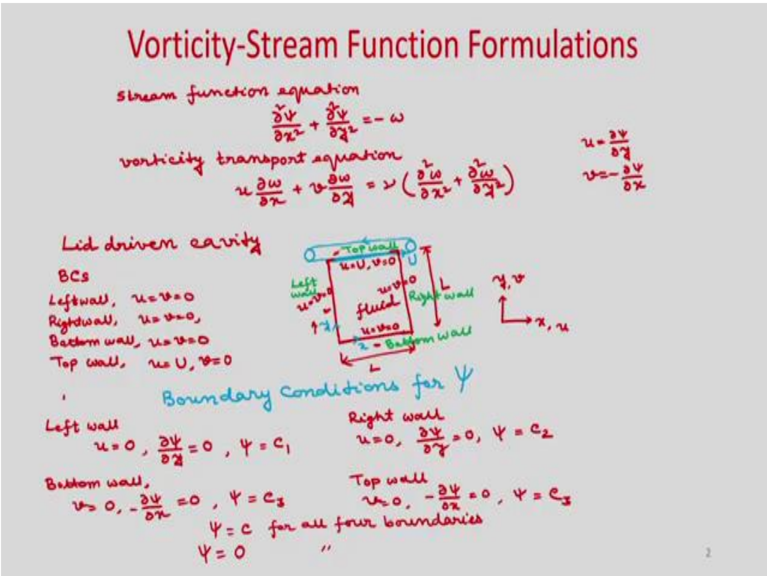

So, our governing equation stream function equation, stream function equation del 2 psi by del x square, plus del 2 psi by delta y square is equal to minus omega, where omega is your vorticity and vorticity transport equation is, vorticity transport equation is u del omega by del x plus v del omega by del y is equal to nu del 2 omega by del x square plus del 2 omega by del y square.

So, now we will derive the boundary conditions for lid driven cavity problem. So, first let us discuss what is lid driven cavity problem? Lid driven cavity problem. So, we have enclosure, square enclosure. So, it is a square enclosure filled with some liquid, some fluid.

So, inside some fluid is there, some fluid is there and this is your square enclosure, say let us say this is your L and this is also L, square enclosure and this upper surface it is connected with some belt let us say and this lid, this is upper plate. So, this is moving with some velocity. So, this belt is moving like this, so we can imagine that these boundary is moving in x direction. So, if it is x and this is your y, then along x direction this lid is moving with some velocity u.

So, you have enclosure, square enclosure left bottom and right surfaces are having no slip boundary conditions, those are stationary walls and the top wall is moving with velocity u. So, in terms of velocity boundary conditions, what are the boundary conditions. So, in terms of velocity boundary conditions.

So, here u is equal to v is equal to 0. The left boundary also u is equal to v is equal to 0 and right also u is equal to v is equal to 0. So, all are no slip condition. Here also it is no slip condition on the top wall. So, it is u is equal to capital U. Because it is moving in the x direction with a velocity U and v is 0.

So, we have applied no slip boundary conditions in all four walls and this is your left wall. So, this is your left wall, this is your left wall, so this is your left wall, this is your right wall. This is your bottom wall and this is your top wall. So, if you write the boundary conditions mathematically for velocity only, then boundary conditions for the problem lid driven cavity.

So, you have left wall, you have u is equal to v is equal to 0, right wall u is equal to v is equal to 0. Bottom wall is u is equal to v is equal to 0 and top wall, u is equal to capital U and v is equal to 0. So, these are the boundary conditions in terms of velocities. But we are solving the governing equations for psi which is your stream function and vorticity omega.

So, we need to know the boundary conditions in terms of psi and omega to solve the discretized equation. So, first let us find the boundary conditions for psi. Stream function boundary conditions. Boundary conditions for psi, psi is stream function. So, we need to find what are the boundary conditions for psi? So, to find the boundary conditions for psi what we will do, first we will impose the normal velocity at each boundary and you will find the boundary conditions for psi.

So, if you will consider left wall, if you consider left wall, what is your normal velocity? Normal velocity will be u, v is your tangential velocity because if this is your x, x direction u have velocity u and in y direction you have velocity v. So, if you consider the left wall then left wall the normal velocity is u.

So, you put normal velocity as 0 that means u is equal to 0. So, now you write u in terms of stream function. So that you know, so the gradient of stream function with respect to y will be 0. So, that means del psi by del y will be 0. So, you can see so this is your y.

In this direction it is y, so along this your, the gradient of psi will be 0, that what does it mean, it means psi is equal to constant, psi is equal to constant. Let us say, some constant C1. So, from these del psi by del y is equal to 0, if you integrate it, then you will get psi is equal to C1, C1 is some constant.

Similarly, for right wall what is the normal velocity u, u is equal to 0. Similarly, del psi by del y is equal to 0 and you can write psi is equal to C1 or psi is equal to some another constant. Let us say psi is equal to C2 and if you write the, so on the right wall psi is equal to C2 another constant let us say.

Now bottom wall, so for the bottom wall you see here. So, normal velocity will be in y direction that means v is equal to 0. So, you can write v is equal to 0 and v is in terms of psi it is minus del psi by del x is equal to 0 and you can see so along x it is 0. So, psi is also on the bottom wall will be constant and let us say the constant is C3.

Similarly, on the top wall, if you consider top wall, what is the normal velocity v is equal to 0 and you can write minus del psi by del x is equal to 0, why? Because u is del psi by del y and v is equal to minus del psi by del x you know right.

So, minus del psi by del x is equal to 0, so psi is equal to C3. So, from the normal velocity boundary conditions we have found that in all boundaries you have some constant stream lines. So, constant stream functions, constant stream functions. So, left wall is C1, right wall is C2, bottom wall is C3 and top wall is C4. Now you see, so these walls are connected.

So, now the constant stream function denotes to the stream line, we know that constant stream function is a stream line and two stream lines cannot cross each other that means you have. So, if you see right wall, left wall, so in the left wall, so this is continuous and to the bottom walls and bottom wall to right wall and right wall to top wall and top wall to left wall.

So, these are all connected, what does it mean? That means it is a single stream line and the stream function should be same. So, along all these four boundaries psi will be constant. So, you can write psi is equal to C for all four boundaries. So, that means psi is equal to constant and it is a same constant. Because two stream lines cannot cross each other. So, obviously it should be continuous.

So, that means psi is constant single value in all four boundaries and for simplicity let us take that psi is equal to 0 this constant value is 0 on these four walls. So, we are considering psi is equal to 0 for all four boundaries. Because we are considering that constant as 0. Because it does not matter, because you can take any constant. So, it will not effect your solution. So, we are considering the stream function value on the walls as 0 and accordingly your stream function value will be there at the interior points.

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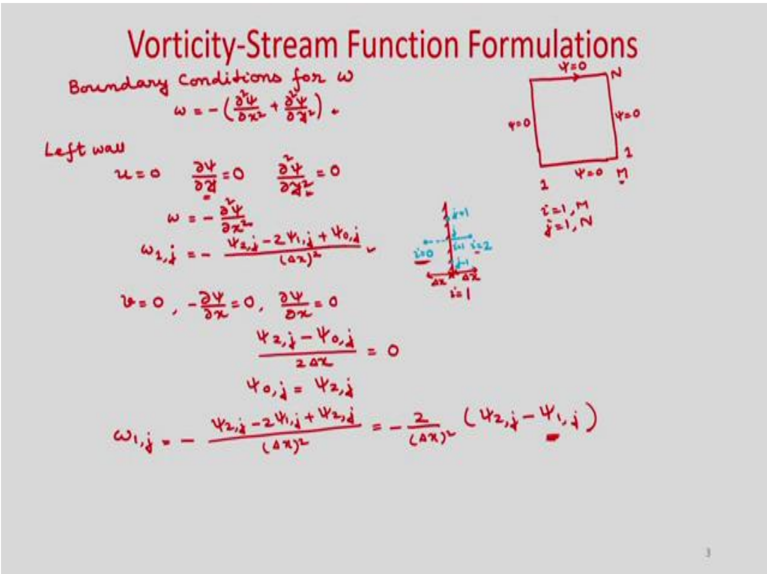

So, now let us find the boundary conditions for vorticity. Boundary conditions for omega. So, we know that omega in terms of stream function, so minus del 2 psi by del x square plus del 2 psi by del y square and you have this square enclosure. Top wall is moving, top wall is moving and psi is equal to 0, we have considered on four walls and let us say that you have the values, i is equal to 1 to M and 1 to N.

So, i is equal to 1 to M it is varying and j is equal to 1 to N. So, i is equal to 1 that means it is j is varying. So, it is a lid boundary and i is equal to M and j is varying, so it is your right boundary. So, now let us consider left wall. So, in the left wall, if you consider, so what is normal velocity? Normal velocity is u is equal to 0. That means del psi by del y will be 0.

So, on the left wall del psi by del y is 0 and you can see that psi is constant along this line. Along this line, so its gradient is 0 and its second derivative also will be 0. So, if you take the gradient of del psi by del y that will be also 0 along this line. So, del 2 psi by del y square will be 0.

Because it is constant there is no variation and its gradient is 0 everywhere. So, obviously if you would take the derivative with respect to y of this gradient del psi by del y that will be also 0 along this left wall. So, del 2 psi by del y is 0. So, now you can see this equation, vorticity equation. So, if you put del 2 psi by del y square is equal to 0 for the left wall.

So, omega will be minus del 2 psi by del x square. So, now you use central difference method to discretize this second derivative. So, if you use second derivative, so it will be, so now you can write omega for which wall, left wall, where i is equal to 1 and j will vary. So, omega 1 j will vary from 1 to N. So, for this it will be now minus second derivative, so you discretize using central difference method that means if you have, what we will do. So, if this is the left boundary, so it is your, i is equal to 1.

We will take a fictitious point outside this boundary. So, this is your one grid, and this is your interior grid. So, this is your fictitious point, so here you have i is equal to 1, here i is equal to 2 and this is i is equal to 0. So, and this way it is, if it is j, so it is j plus 1 and this is your j minus 1.

But it is del 2 psi by del x square, so it is be for point j only. So, you can write with this fictitious point. So, i is equal to 0 is fictitious point outside the boundary and with the same distant delta x. So, this is also delta x and this is also delta x.

So, if you considered this then you can write discretize form of the second derivative as psi. So, 2 j minus 2 psi 1 j plus psi 0 j divided by delta x square. But we do not know the value of psi 0 j because it is a fictitious point. So, to find the value of psi 0 j we will use another velocity boundary condition which is v is equal to 0.

So, you know that on the left wall v is equal to 0, what does it mean? Minus del psi by del x is 0. That means del psi by del x is 0. So, if del psi by del x is 0, so you can write the central difference. So, psi 2 j minus psi 0 j divided by distance between these two points 2 delta x. Because here i is equal to 2 and i is equal 0. So, 2 delta x is equal to 0.

So, we have used central difference. So, now psi 0 j you can find as psi 0 j is equal to psi 2 j. So, now psi 0 j value you know, so you put it in this expression. So, you will get psi omega 1 j is equal to minus psi 2 j minus 2 psi 1 j and this is also psi 2 j divided by delta x square.

So, that means you can write as minus 2 by delta x square psi 2 j minus psi 1 j. So, psi 1 j, where it is psi 1 j on the boundary. So, if you assume psi is 0, then you can put it 0. But if you can take, if you consider some other values constant values on the wall of psi then psi 1 j will be having some constant value.

So, we are not going to type 0 here. Because how you are considering the constant depending on that psi 1 j value will depend. So, that means if you considered stream function on the walls, then it will be 0 and if you consider some constant value then you can put it that constant value here. So, we are not going to put it 0 here. So, let us keep in terms of psi 1 j. So, omega 1 j we have found. Similarly, now you do for the right wall.

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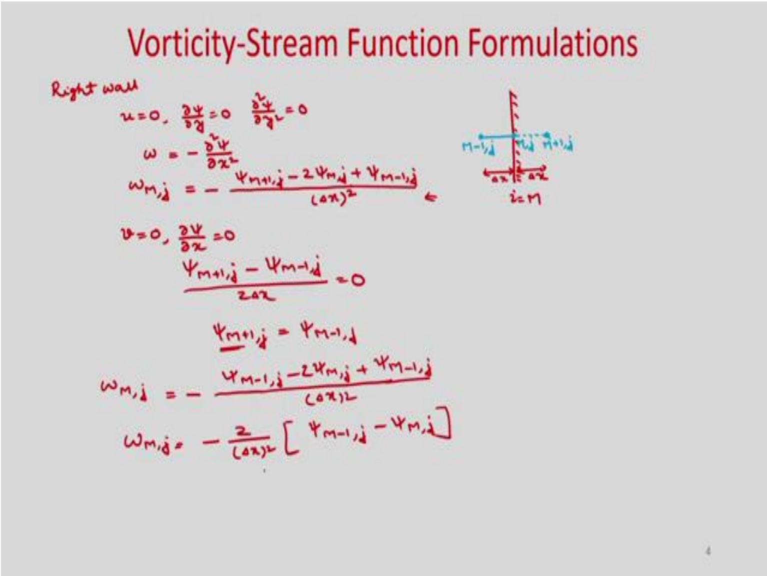

So, right wall also, so for the right wall, so if it is right wall, i is equal to M in this case and you have, this is your original interior grid and this is your fictitious grid outside the boundary. So, you can see this is your i j this is your i plus 1 j and this is your i minus 1 j. So, i minus 1 j means M minus 1 j and this is your, i plus 1 j means it is your M plus 1 j and this is your M j, this is your M j.

So, similarly we will use the normal velocity. So, normal velocity is u is equal to 0. So, del psi by del y is 0 and del 2 psi by del y square is equal to 0 and from the vorticity equation you can write omega is equal to minus del 2 psi by del x square. Now if we discretize it, so you will get omega M j on the right wall is equal to minus now central difference we are using. So, you can write psi M plus 1 j minus twice psi M j plus psi M minus 1 j divided by delta x square.

Now psi M plus 1 j is unknown, because it is a fictitious point value. So, we will use again the tangential velocity v is equal to 0 del psi by del x is 0 and if you discretize it. So, you will get psi M plus 1 j minus psi M minus 1 j divided by 2 delta x. Because we have considered at the same distance this fictitious point. So, 2 delta x is equal to 0 and psi M plus 1 j which is outside the boundary is equal to psi M minus 1j and this value if you put it in this omega expression., then you will get omega M j is equal to minus.

So, it will be psi M plus 1 j minus twice psi M j and plus psi, now, this M minus 1 j you just write because now we are putting the value of psi M plus 1 j as psi M minus

1 j here M minus 1 j divided by delta x square. So, you are going to get omega M j is equal to minus 2 by delta x square psi M minus 1 j minus psi M j.

So, in this case, again psi M j actually the values of psi on the right wall and if you choose 0. Then it will be 0, otherwise it will have some other value. Now let us consider the bottom boundary. Similarly, we will consider the fictitious point outside this bottom wall and first we will make the normal velocity 0.

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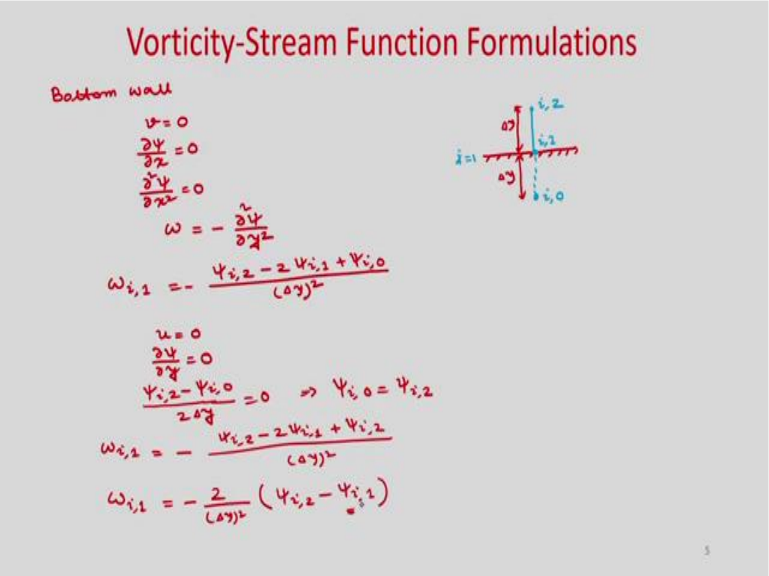

So, for the bottom wall. So, normal velocity is, so this is your bottom wall and you have the fictitious point outside. So, this is your grid, so this is your j is equal to 1. So, it will be i 1, this is your i 0 and this is your i 2. i 2 is the interior point, i 1 is the, on the wall and i 0 is the fictitious point outside the boundary. So, similarly first we will consider the normal velocity as 0 that means v is the normal velocity. So, it is 0 and from here you will get del psi by del x is 0.

So, you can see along this bottom wall psi is constant, psi is constant del psi by del x is 0 and obviously del psi by del x is 0 everywhere. So, if you take the gradient of this del psi by del x with respect to x that will be also 0 that means you will get del 2 psi by del x square is equal to 0.

So, your omega equation will be minus del 2 psi by del y square. So, now if you discretize using central difference method. Then for omega i 1 you can write psi i 2 minus twice psi i 1 and psi i 0 and divided by delta y square. So, this delta y square, delta y is the constant step size.

So, similarly now psi i 0 is the value of psi at the fictitious point. Let us find using the tangential velocity condition. So, tangential velocity is u is equal to 0 that means del psi by del y is equal to 0 and if you discretize del psi by del y using second approximation then you will get psi i 2 minus psi i 0 divided by 2 delta y is equal to 0 and that will give psi i 0 is equal to psi i 2.

So, you can see that fictitious value is in, as the interior points psi i 2. So, now this value you put it in the omega expression. So, you will get omega i 1 is equal to psi i 2 minus twice psi i 1 and psi i 0 is psi i 2 divided by delta y square. So, you will get omega i 1 is equal to minus 2 by delta y square. So, you will get psi i 2 minus psi i 1 and here you can see psi i 1 is the boundary value. So, for our case we have chosen as 0. But you keep it as it is, so that if you have a, some constant value you can use it here so psi i 1.

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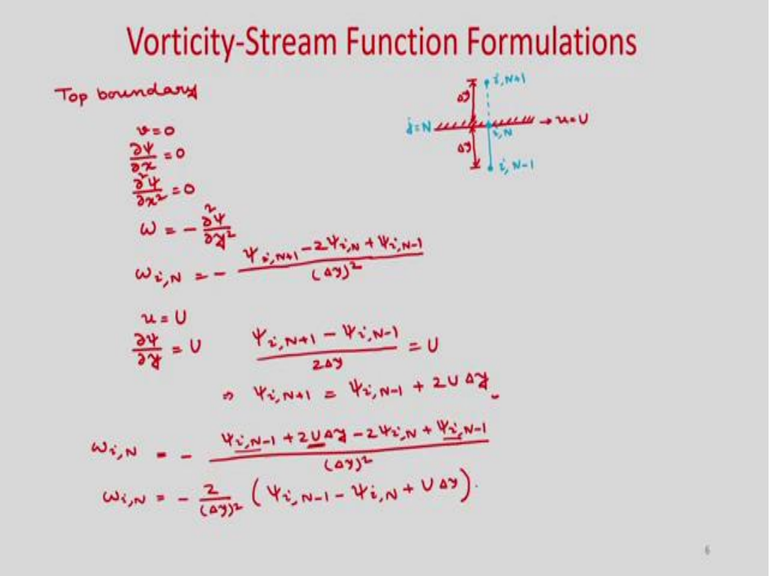

Similarly, now considered the top boundary, so top boundary, so on the top boundary so you can see this is our boundary, and it is moving in the x direction, this is moving in this x direction with u is equal to capital U. So, now if you take the grid, so this is your fictitious point, this is on the wall and you have interior point.

So, this is your j is equal to N. So, it will be i N, i N minus 1 and i N plus 1. So, similarly now first we will use the normal velocity condition. So, what is the velocity condition? Normal velocity is v is equal to 0, so normal boundary condition is v is equal to 0 that means del psi by del x is equal to 0.

So, along this boundary obviously del psi by del x is 0. So, you can write del 2 psi by del x square is also 0. So, your omega condition is minus del 2 psi by del y square and if you use central difference method, then you can write omega i N is equal to minus psi i N plus 1 minus twice psi i N plus psi i N minus 1 divided by delta y square.

So, now we have to find the value of psi i N plus 1 which is the value of psi at the fictitious point. So, for that now we will use the tangential velocity, so what is tangential velocity u is equal to U. Because it is moving with a velocity u in the positive x direction.

So, that means del psi by del y is equal to U. So, that you can now discretize using central difference. So, it will be psi i N plus 1 minus psi i N minus 1 divided 2 delta y. So, this is your delta y and this is also delta y. So, 2 delta y is equal to U. So, you can write psi i N plus 1 is equal to psi i N minus 1 plus U 2 U delta y, So 2 U delta y psi i N plus 1 s equal to psi i N minus 1 plus 2 U delta y

Now this value you substitute in the omega expression and find the omega i N. So, omega i N will be, it will be minus. So, psi i N plus 1 is equal to psi i N minus 1 plus twice U delta y minus twice psi i N plus psi i N minus 1 divided by delta y square. So, now you can see psi i N minus 1 psi i N , psi i N minus 1 so it will be 2. So, you can write omega i N is equal to minus 2 delta y square. So, it will be psi i N minus 1 minus psi i N plus twice U, U into delta y, U into delta Y, this term U into delta y, 2 we have taken outside so it will be 2 into delta y.

So, along this top boundary, so the value of omega will be with this expression. Where U is the lead velocity. Now you have seen that to solve the vorticity transport equation we could derive all the boundary conditions for omega as well as psi we have already found.

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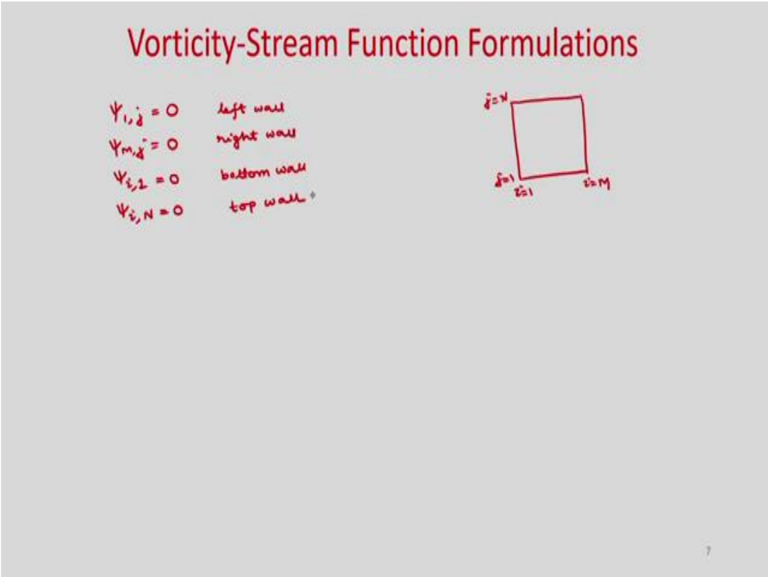

So, psi for all the boundaries this is psi, we have found, so it is i is equal to 1 to i is equal to M and j is equal to 1 to j is equal to N. So, you can see that now for psi, psi is 0 in everywhere. So, it will be psi 1, psi 1 j is equal to 0, psi. So, it is for the left boundary, left wall, psi, so this is your M, so psi M j is equal to 0 for right boundary, right wall, psi i 1 is equal to 0 for bottom wall and psi i N is equal to 0 for top wall, for all i.

So, these are the discretize boundary conditions for psi and also we have shown the discretize boundary conditions for omega. So, you have the discretized equation for the interior points with these boundary conditions, you can solve using some linear solver and you will get the values of psi and omega for this lid driven cavity problem.

But you have to remember that psi and omega are coupled. Because when you are solving the stream function equation you have omega and when you are solving the vorticity transport equation you have psi. So, together this you have to converge and get the solution.

So, in today's lecture we will show later the computer code for this problem lid driven cavity and later we will show the results for this lid driven cavity and plane poiseuille flow. So, now let us consider plane poiseuille flow where the flow is taking place between two infinite parallel plates.

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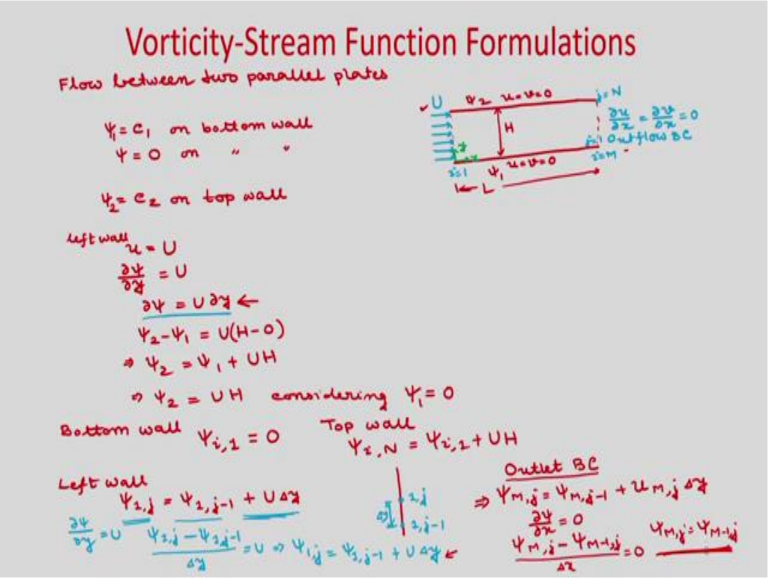

So, let us consider another problem, flow between two parallel plates. So, this is a flow between two parallel plates. So, we are considering two plates, these are two plates, parallel to each other, parallel to each other. Let us say that length of the plate is L and height is H.

This is your domain, this is your competitional domain. Now obviously on the wall velocity will be 0. So, that means in terms of velocity if you give, so on the bottom wall u is equal to v is equal to 0. On the top wall also u is equal to v is equal to 0 and let us say that you have a uniform velocity inlet at the inlet.

So, this is your inlet, so you have a uniform velocity. So, this is your uniform velocity. So, this is your capital U with this velocity it is entering. So, now you can see that boundary layers will be developed over the plates and you will get a fully developed flow after a certain distance.

So, and obviously you will get a parabolic profile at the, when it will become fully developed. So, now numerical will solve this problem. So, we are not considering plane poiseuille flow, where we consider two infinite parallel plates and the assumption is that you have fully developed flow.

But here it is a initially it will be a developing flow, after that it will become fully developed. So, at the outlet now we will consider the gradients of velocity will be 0.

Because the flow, whatever it is going we are assuming that it is a fully developed and whatever way it is coming it will go out.

So, that means at the outlet we will consider del u by del x is equal to del v by del x is 0. So, it is a fully developed condition and this is your at the outlet. So, it is outflow boundary conditions, outflow boundary conditions. So, inlet U and let us say that you have i is equal to 1, i is equal to M and here j is equal to 1, j is equal to N and obviously your, let us say coordinate is here. So, it is your x and this is your y.

So, the out flow condition is fully developed. So, we have written del u by del x is equal to, del u by del x is equal to 0 because x is along the flow direction. So, now you can see that psi value again we can consider some constant value on the bottom wall.

So, psi is C1 on bottom wall. So, that you can find from the normal velocity. So, normal velocity is v, v is equal to 0 that means del psi by del x is 0 and so del psi by del x is 0 that means psi will be constant along x. So, psi is equal to C1 and for simplicity, let us say psi is equal to 0 on bottom wall.

So, you can take any value. Because it does not matter, so whatever value you will consider on the bottom wall accordingly we will get the psi on top wall. So, now we have to find what is the value of psi on the top wall. But anyway from the normal velocity condition you can find that psi will be some another constant C2 on top wall. Because that will be also constant, because from normal velocity v is equal to 0. So, obviously del psi by del x is 0, so along this x direction. So, psi will be constant, so let us say psi is equal to C2 on top wall.

Now how we will find this constant. So, now you know that inlet velocity U from there you can consider that how you can find the C2. So, now you can see that what is this condition, so you have the, on the left wall, you have u is equal to U which is your normal velocity that means del psi by del y is equal to U. So, that means you can write that del psi is equal to U del y.

So, if you will integrate it from 0 to H, then you will get. So, if it is psi 1 and this is your psi 2, if it is psi 1 and this is your psi 2 then you can get that psi 2 minus psi 1 is equal to U into H. So, y is varying 0 to H, so it will be H minus 0 let us say. So, that means psi 2 will be psi 1 plus U H. So, psi 2 value we have found, so psi 2 is equal to psi 1 plus UH.

So, if you consider psi 1 as 0 then psi 2 will be UH, for considering psi 1 is equal to 0, considering psi 1 is equal to 0. So, in terms of discretize boundary condition you can write psi, so on the left wall, bottom wall. So, bottom wall is psi 1 is equal to 0 we have assumed. So, psi 1 is nothing but, so i, j is equal to 1. So, psi i it will vary from 1 to M this is equal to 0 and top wall psi will be psi i, j is equal to N. So, it will be U into H, or you can write psi i 1 plus U into H.

So, for a given flow inlet velocity U and the distance between two parallel plates each you can find the value of psi i N. Now we need to find what is the value of psi on the left wall? So, left wall you can find from the integration of this equation. So, that means you can write psi, so i is equal to 1, so psi 1 j is nothing but psi 1 j minus 1 plus U into delta y.

So, delta y is the grid point, so you can see. So, it is your left wall, left boundary, inlet boundary. So, you have, if this is your 1 j and this is your 1 j minus 1. So, you can write, so from this integration you can write from here. You can write psi 1 j is equal to psi 1 j minus 1 into U delta y.

So, we have used first order scheme. So, you can see del psi by del y. So, del psi by del y is you have written psi. So, you can write here also del psi by del y is equal to U that means psi 1 j minus psi 1 j minus 1. So, backward difference we have used , we have used backward difference and distance between these two points is delta y.

So, is equal to U, so you can write psi 1 j is equal to psi 1 j minus 1 plus U into delta Y and on the outlet boundary. So, at outlet what you can write for the psi value. Outlet boundary conditions. So, these you can actually find from this equation only so you can write psi i is equal to M, M j is equal to psi M j minus 1 plus, if you know the value of U, so then you can find u, small u.

So, you have to find at which point, at M j into delta y. So, this way also you can use. But better condition is that it is fully developed, so del psi by del x also will be 0 so that you can write. Del psi by del x is equal to 0, so del psi by del x is equal to 0, so del psi by del x is equal to 0, so you can write psi, so j will vary. So, it will psi j will vary so it will be M j is equal to or minus psi M minus 1 j. So, if you use first order, so it will be delta x is equal to 0 that means psi M j will be psi M minus 1 j.

So, it will be easy to use this condition, so this you can psi M j is equal to psi M minus 1 j. So, we have found all the stream function boundary conditions in all four boundaries. So, on the bottom wall we have considered some constant value of psi that is 0.

So, we can see here and on the top wall you can calculate from the given inlet boundary condition and the distance between two parallel plates. Left wall you can find from this relation and outlet it is easier to use this one. So, you can use psi M j is equal to psi M minus 1 j.

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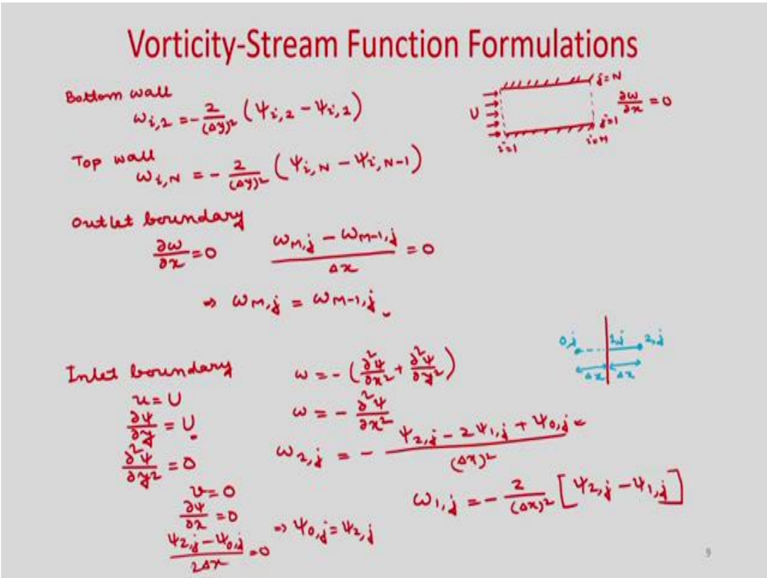

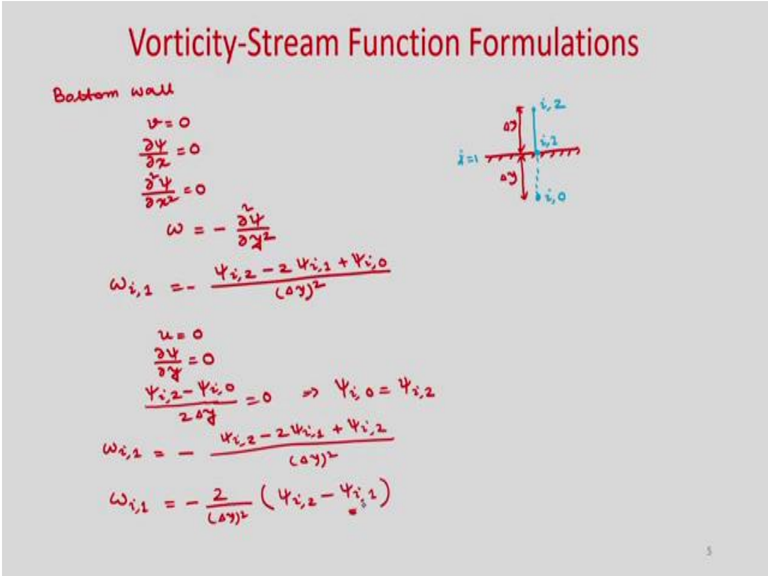

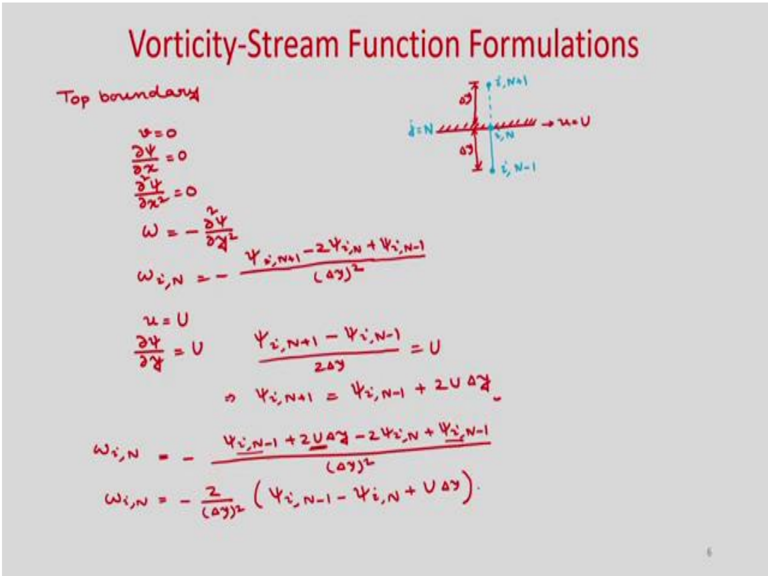

Now similarly, you can find the boundary condition for omega. For omega so if you use, say bottom wall and top wall it will be almost similar whatever we have derived for the lid driven cavity. So, this is your parallel plates and you have inlet velocity U and this is your, i is equal to 1. So, this is your, i is equal to M and j is equal to 1 and j is equal to N. So, for the bottom wall you can write the similar way, so you can see which way you have written for the bottom.

So, bottom wall obviously the no slip condition so v will be 0. So, this equation you can write, omega i 1, so you can write omega i 1, it is bottom, so it will be, so bottom one will be omega, so j is equal to 1. So, i 1 it will be minus 2 by delta y square, so it will be psi i 2 minus psi i 1. This is your omega boundary condition same as the earlier and for the top boundary. So, top boundary also you can write similar way only.

So, top wall will be omega. So, j is equal to N, so i N is equal to minus 2 by delta y square and you will get psi i N minus psi i N minus 1.

So, omega i N you can find like this. Now outlet boundary, so outlet boundary we will consider, at outlet boundary we will consider fully developed flow, so if it is a fully developed flow you can write del omega by del x is 0. So, del omega by del x is 0, so it is a fully developed condition. So, del omega by del x will be 0 and you can use first order accurate scheme. So, it will omega M j minus omega M minus 1 j divided by delta x will be 0.

So, that means psi, omega M j will be just omega M minus 1 j. You can use also second order, one sided difference also. So, but we have used for simplicity the first order equate method. So, you can write omega M j is equal to omega M minus 1 j. What about the inlet boundary?

So, similarly for inlet boundary, so your u is U normal velocity. So, you can write del psi by del y is equal to U. Now you can take the derivative, so it is U is equal to constant. So, you can write del 2 psi by del y square will be 0. So, as you have a constant velocity inlet. So, del 2 psi by del y square will be 0.

So, from the vorticity equation omega is equal to minus del 2 psi by del x square plus del 2 psi by del y square. So, you can put del 2 psi by del y square 0. So, omega is equal to minus del 2 psi by del x square. So, now you can discretize omega, so it will be i is equal to 1.

So, 1 j is equal to minus, so it will be psi. So, if you take a fictitious point, so this is your fictitious and this is your interior grid separated with a uniform step size, delta x, this is also delta x. So, this is your 1 j, this is your 2 j and this is your 0 j. So, you can write psi 2 j minus twice psi 1 j plus psi 0 j.

So, psi 0 j we do not know. So, you can find from tangential velocity. So, that means v is equal to 0. So, that means del psi by del x will be 0, so it means del psi by del x will be 0. Then you can write psi 2 j minus psi 1 j divided by 2 delta x is 0 and that implies, psi 2j by psi 0 j, psi 0 j. So, distance between these points 2 delta x, so it is 0, so psi 0 j is nothing, but psi 2 j.

So, that means omega 1 j will be minus 2 by delta x square, 2 by delta x square. So, it will be psi 0 j you put and psi 2 j here. So, you will get psi 2 j minus psi 1 j. So, now we have found the boundary conditions for omega in all boundaries and in discretize form we have written. Thank you.