

Aircraft Propulsion
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Lecture 09

Examples: Non-Ideal Brayton Cycle

Welcome to the class. Today we will see some examples on non ideal Brayton cycle here. We have seen that in the class we mean by non ideal as the Brayton cycle where compressor and turbines are not 100% efficient and we have derived the optimum case and also for the work and heat interactions. Now, let us solve some examples.

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1. A gas turbine power plant has air entering the turbine at 800 kPa and 1100K. Air leaves the turbine at 100 kPa and 670K. Heat is rejected at the rate of 6700 kW. Air flow rate is 18 kg/s and compressor efficiency is 80%. Find out power output and compressor efficiency if net work is zero.

given

$P_3 = 800 \text{ kPa}$ $T_3 = 1100 \text{ K}$ $P_4 = P_1 = 100 \text{ kPa}$ $T_4 = 670 \text{ K}$
 $Q_{out} = 6700 \text{ kW}$ $\dot{m} = 18 \text{ kg/s}$ $\eta_c = 0.8$
 $r_p = \frac{P_3}{P_1} = \frac{P_2}{P_1} = 8 \rightarrow (r_p)^{\frac{\gamma-1}{\gamma}} = (8)^{0.2877} = 1.808$
 $Q_{out} = 6700 \times 10^3 = \dot{m} (h_4 - h_1) = 18 \times 1001 \times 10^3 (670 - T_1)$
 $T_1 = 299.62 \text{ K} \rightarrow \eta_c = \frac{h_2' - h_1}{h_2 - h_1} = \frac{T_2' - T_1}{T_2 - T_1}$ $\frac{T_2'}{T_1} = (r_p)^{\frac{\gamma}{\gamma-1}}$
 $T_2' = T_1 \times 1.808 = 541.71 \text{ K} \rightarrow 0.8 = \frac{541.71 - 299.62}{T_2 - 299.62} \rightarrow T_2 = 602.23 \text{ K}$
 $w_{net} = w_t - w_c = (h_3 - h_4) \dot{m} - \dot{m} (h_2 - h_1)$
 $w_{net} = \dot{m} c_p [(T_3 - T_4) - (T_2 - T_1)]$
 $w_{net} = 2304.48 \text{ kW}$
 $w_{net} = 0 \Rightarrow w_t = w_c \Rightarrow \dot{m} c_p (T_3 - T_4) = \dot{m} c_p (T_2 - T_1)$
 $T_2 = (T_3 - T_4) + T_1 = 729.62$
 $\eta_c = \frac{T_2' - T_1}{T_2 - T_1} = \frac{541.71 - 299.62}{729.62 - 299.62} = 0.563$
 $\eta_c = 56.3\%$

First example says that a gas turbine power plant has air entering the turbine at 800 kPa and 1100 kelvin air leaves the turbine at 100 kPa and 670 Kelvin heat is rejected at the rate of 6700 kilowatt air flow rate is 18 kg per second and compressor efficiency is 80%. And our power output and compressor efficiency if net work is 0, so let us what is given thing for us. There are two parts of the example in first part we have to find out power output and in the second part we have to find out compressor efficiency.

If net work is 0 we will first plot the TS diagram, this is 1, this is 2, 3, 4. But as per our convention this is 2 dash. This is 2 and this is 4 and this is 4 dash. So this is known to us. Then we are given that

$$P_3 = 800 \text{ kPa}; T_3 = 110 \text{ K}; P_4 = P_1 = 100 \text{ kPa}; T_4 = 670 \text{ K}$$

$$Q_{\text{out}} = 6700 \text{ kW}; \dot{m} = 18 \frac{\text{kg}}{\text{s}}; \eta_c = 0.8$$

$$r_p = \frac{P_3}{P_1} = \frac{P_2}{P_1} = 8 \rightarrow (r_p)^{\frac{\gamma-1}{\gamma}} = (8)^{0.285} = 1.808$$

$$Q_{\text{out}} = 6700 * 10^3 = \dot{m}(h_4 - h_1) = 18 * 1.005 * 10^3 (670 - T_1)$$

$$T_1 = 299.62 \text{ K} \rightarrow \eta_c = \frac{h'_2 - h_1}{h_2 - h_1} = \frac{T'_2 - T_1}{T_2 - T_1}; \frac{T'_2}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}}$$

$$T'_2 = T_1 * 1.808 = 541.71 \text{ K} \rightarrow 0.8 = \frac{541.71 - 299.62}{T_2 - 299.62} \rightarrow T_2 = 602.23 \text{ K}$$

$$W_{\text{net}} = W_t - W_c = (h_3 - h_4)\dot{m} - \dot{m}(h_2 - h_1)$$

$$W_{\text{net}} = \dot{m}C_p [T_3 - T_4 - T_2 + T_1]$$

$$W_{\text{net}} = 2304.48 \text{ kW}$$

$$W - \text{net} = 0 \rightarrow W_t = W_c \rightarrow \dot{m}C_p(T_3 - T_4) = \dot{m}C_p(T_2 - T_1)$$

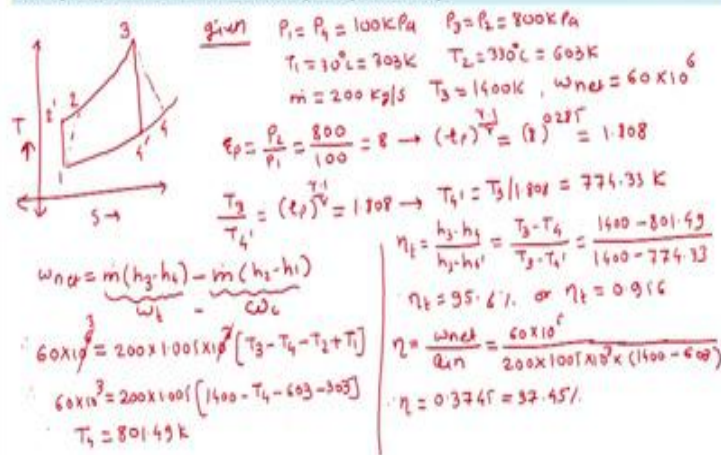
$$T_2 = (T_3 - T_4) + T_1 = 729.62 \text{ K}$$

$$\eta_c = \frac{T'_2 - T_1}{T_2 - T_1} = \frac{541.71 - 299.62}{729.62 - 299.62} = 0.563 = 56.3\%$$

So this is what the way we can solve this example. And in this example we have seen that how to use the compressor and turbine efficiencies. Basically in this example compressor efficiency was given knowing the compressor efficiency we can find out the actual temperature at the end of the compressor or knowing the turbine efficiency we can as well find out the turbine outlet temperature.

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2. A Brayton cycle has limits as 100 and 800 kPa. Air enters the compressor at 30°C and leaves at 330°C with a mass flow rate of 200 kg/s. Maximum cycle temperature is 1400K and power output is 60 MW which is actually measured. Find out turbine efficiency and cycle efficiency.



Let us move on to the next example next example says that the Brayton cycle has limits 100 and 800 kPa and air enters the compressor at 30 degree Celsius and leaves at 330 degree Celsius and then mass flow rate is 200 kg per second. It is given in the example maximum cycle temperature is 1400 Kelvin; power output is 60 megawatt which is actually measured. So this is net power output which is actual find out turbine efficiency and cycle efficiency, so we can proceed with the example.

Using given things before that we plotted the TS diagram and noted 1, 2, 3, 4 points which are corners so given as

$$P_1 = P_4 = 100 \text{ kPa} ; P_3 = P_2 = 800 \text{ kPa}$$

$$T_1 = 30^\circ\text{C} = 303 \text{ K} ; T_2 = 330^\circ\text{C} = 603 \text{ K}$$

$$\dot{m} = 200 \frac{\text{kg}}{\text{s}} ; T_3 = 1400 \text{ K} ; W_{net} = 60 \times 10^6$$

$$r_p = \frac{P_2}{P_1} = 8 \rightarrow (r_p)^{\frac{\gamma-1}{\gamma}} = 1.808 \rightarrow T_4' = \frac{T_3}{1.808} = 774.33 \text{ K}$$

$$W_{net} = \dot{m}(h_3 - h_4) - \dot{m}(h_2 - h_1)$$

$$60 \times 10^6 = 200 \times 1.005 \times 10^3 [T_3 - T_4 - T_2 + T_1]$$

$$60 \times 10^6 = 200 \times 1.005 \times [1400 - T_4 - 603 - 303]$$

$$T_4 = 801.49 \text{ K}$$

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_4'} = \frac{T_3 - T_4}{T_3 - T_4'} = \frac{1400 - 801.49}{1400 - 774.33}$$

$$\eta_t = 95.6\% ; \eta_t = 0.956$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{60 * 10^6}{200 * 1.005 * 10^3 * (1400 - 600)} = 37.45\%$$

This is how we can solve this example which includes the limits and also the evaluation of turbine efficiency.

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4. A Brayton cycle operates on pressure ratio of 12. Compressor inlet condition is 300 K and turbine inlet condition is 1000 K. Determine required mass flow rate for power output of 70 MW if turbine and compressors are ideal or their efficiencies are same as 85%.

Given - $r_p = 12$, $T_1 = 300\text{K}$, $T_3 = 1000\text{K}$, $W_{net} = 70\text{MW}$
 a) $\eta_t = \eta_c = 100\%$, b) $\eta_t = \eta_c = 0.85$

$$W_{net} = \dot{m}(h_3 - h_4) - \dot{m}(h_2' - h_1)$$

$$W_{net} = \dot{m} \cdot C_p [T_3 - T_4' - T_2' - T_1]$$

$$W_{net} = \dot{m} \cdot C_p [T_3 - T_3 \left(\frac{T_1}{T_3}\right)^{\frac{\gamma}{\gamma-1}} - T_1 \left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}} - T_1]$$

$$\left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = (12)^{\frac{1.4}{1.4-1}} \rightarrow T_2' = 609.8\text{K}, T_4 = 492.41\text{K}$$

$$70 \times 10^6 = \dot{m} \times 1.005 \times 10^3 [1000 - 492.41 - 609.8 + 300] \rightarrow \dot{m} = 352.64\text{ kg/s}$$

b) $W_c' = C_p(T_2' - T_1) = 311.44\text{ kJ/kg} \rightarrow \eta_c = 0.85 = \frac{W_c}{W_c'} \rightarrow W_c = \frac{W_c'}{0.85} = 366.41\text{ kJ/kg}$
 $W_t' = C_p(T_3 - T_4') = 509.92\text{ kJ/kg} \rightarrow \eta_t = 0.85 = \frac{W_t}{W_t'} \rightarrow W_t = 0.85 \times 509.92 = 433.43\text{ kJ/kg}$
 $W_{net} = (W_t - W_c) \times \dot{m} = 70 \times 10^6$
 $\dot{m} = \frac{70 \times 10^6}{67.01\text{ kJ/kg}} = 1044.44\text{ kg/s}$

We will solve the example it reads as Brayton cycle operates on a pressure ratio of 12 compressor inlet condition is 300 Kelvin and turbine inlet condition is 1000 Kelvin determine the mass flow rate for power output of 70 Megawatt if turbine and compressors are ideal or their efficiencies are 85%. So we are told that there are two examples in it one is ideal and another is real. So, we can plot our usual diagram 1, 2 Dash, 2, 3, 4 dash and 4 so this is our ideal and real cycle. We are given that

$$r_p = 12 ; T_1 = 300\text{K} ; T_3 = 1000\text{K} ; W_{net} = 70\text{MW}$$

$$a) \eta_t = \eta_c = 100\%, \quad b) \eta_t = \eta_c = 0.85$$

$$W_{net} = \dot{m}(h_3 - h_4') - \dot{m}(h_2' - h_1)$$

$$W_{net} = \dot{m}C_p [T_3 - T_4' - T_2' - T_1]$$

$$W_{net} = \dot{m}C_p \left[T_3 - \frac{T_3}{(r_p)^{\frac{\gamma}{\gamma-1}}} - T_1 * (r_p)^{\frac{\gamma-1}{\gamma}} - T_1 \right]$$

$$(r_p)^{\frac{\gamma-1}{\gamma}} = (12)^{0.285} \rightarrow T'_2 = 609.9 \text{ K}; T_4 = 492.61 \text{ K}$$

$$70 * 10^6 = \dot{m} * 1.005 * 10^3 * [1000 - 492.61 - 609.9 + 300] \rightarrow \dot{m} = 352.68 \text{ kg/s}$$

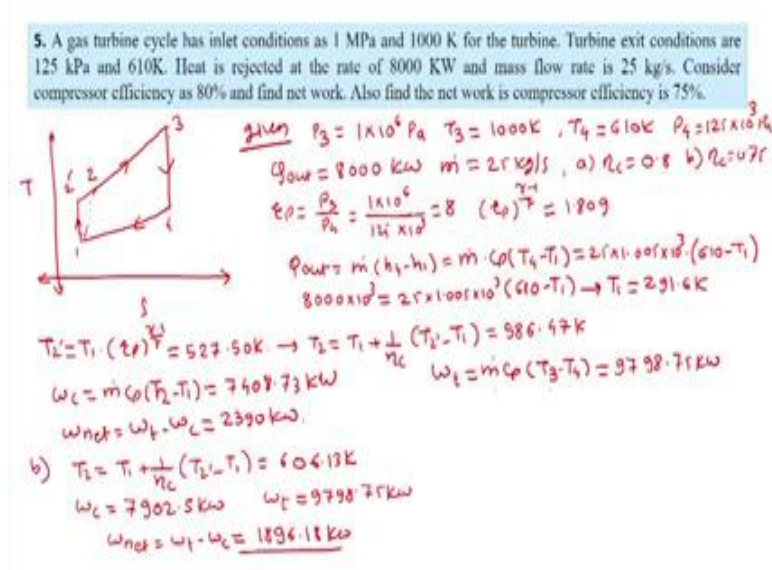
$$b) W'_c = C_p(T'_2 - T_1) = 311.44 \frac{\text{kJ}}{\text{kg}} \rightarrow \eta_c = 0.85 = \frac{W'_c}{W_c} \rightarrow W_c = \frac{W'_c}{0.85} = 366.41 \text{ kJ/kg}$$

$$W'_t = C_p(T_3 - T'_4) = 509.92 \frac{\text{kJ}}{\text{kg}} \rightarrow \eta_t = 0.85 = \frac{W_t}{W'_t} \rightarrow W_t = 0.85 * W'_t = 433.4 \text{ kJ/kg}$$

$$W_{net} = (W_t - W_c) * \dot{m} = 70 * 10^6; \dot{m} = 1044.44 \frac{\text{kg}}{\text{s}}$$

Here we can see that if the components are non-ideal then how the mass flow rate increases for the Brayton cycle. Knowing this we can proceed with the next example.

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Which states that a gas turbine cycle has inlet conditions as 1 mega Pascal and thousand Kelvin for turbine and turbine exit conditions are 125 kPa and 610 Kelvin heat is rejected at the rate of 8000 kilowatt and mass flow rate is 25 kg per second and consider compressor efficiency as 80% and find out net work also find out net work if compressor efficiency is 75%.

So Let us plot TS diagram here although plotting always real cases some time we would not have compressor or turbine which are real in this case specific we are having turbine which is

real and ideal say we have basically 4 dash and 4 points are same. So, we need not plot 4 and 4 dash separately so we can make it as only 4. So, having said this we can go ahead with given

$$P_3 = 10^6 ; T_3 = 1000 K ; T_4 = 610 K ; P_4 = 125 * 10^3 Pa$$

$$Q_{out} = 8000 kW ; \dot{m} = 25 \frac{kg}{s} ; a) \eta_c = 0.8, \quad b) = 0.75$$

$$r_p = \frac{P_3}{P_4} = \frac{10^6}{125 * 10^3} = 8 ; (r_p)^{\frac{\gamma-1}{\gamma}} = 1.809$$

$$Q_{out} = \dot{m}(h_4 - h_1) = \dot{m}C_p(T_4 - T_1) = 25 * 1.005 * 10^3 * (610 - T_1)$$

$$8000 * 10^3 = 25 * 1.005 * 10^3 * (610 - T_1) \rightarrow T_1 = 291.6 K$$

$$T_2' = T_1 (r_p)^{\frac{\gamma-1}{\gamma}} = 527.50 K \rightarrow T_2 = T_1 + \frac{1}{\eta_c} (T_2' - T_1) = 586.47 K$$

$$W_c = \dot{m}C_p(T_2 - T_1) = 7408.73 kW ; W_t = \dot{m}C_p(T_2 - T_1) = 9798.75 kW$$

$$W_{net} = W_t - W_c = 2390 kW$$

$$b) T_2 = T_1 + \frac{1}{\eta_c} (T_2' - T_1) = 606.13 K$$

$$W_c = 7902.5 kW ; W_t = 9798.75 kW$$

$$W_{net} = W_t - W_c = 1896.18 kW$$

This is how turbine efficiency being same if you alter compressor efficiency. We have changing the net work output. Net work output decreases if compressor efficiency is decreased, so we can solve the examples which are for ideal case and also real case using the given efficiency formulas. Thank you.