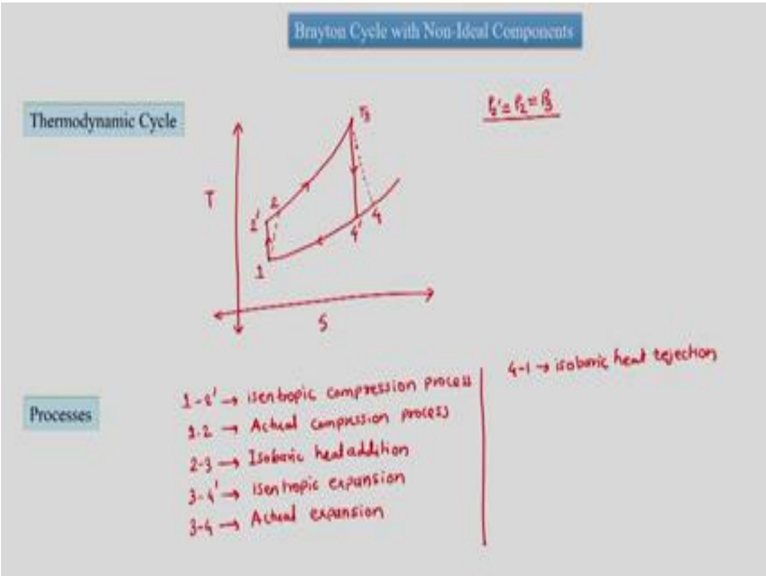
Aircraft Propulsion
Vinayak N. Kulkarni
Department of Mechanical Engineering
Indian Institute of Technology- Guwahati

Lecture 08
Brayton Cycle With Non-Ideal Brayton Cycle

Welcome to the class. We have seen till time about Brayton cycle and also have seen about the ideal Brayton cycle, which is a standard cycle. These cycles are also called as shaft power cycles since the net work whatever we calculate is available at the shaft. Now after considering the ideal cycle will move on towards the Brayton cycle with non-ideal components.

(Refer Slide Time: 00:54)



Specifically we mean here as non-ideal compressor and non-ideal turbine. So the real compressor and real turbines will be considered. So accordingly the processes associated with the compressor and turbine will get changed. So let us consider it first the thermodynamics cycle for the non-ideal components here as we know we have this as ideal cycle. Where we have first compression isobaric heat addition isentropic expansion and isobaric heat rejection so this was minimum temperature and pressure this was maximum temperature and pressure.

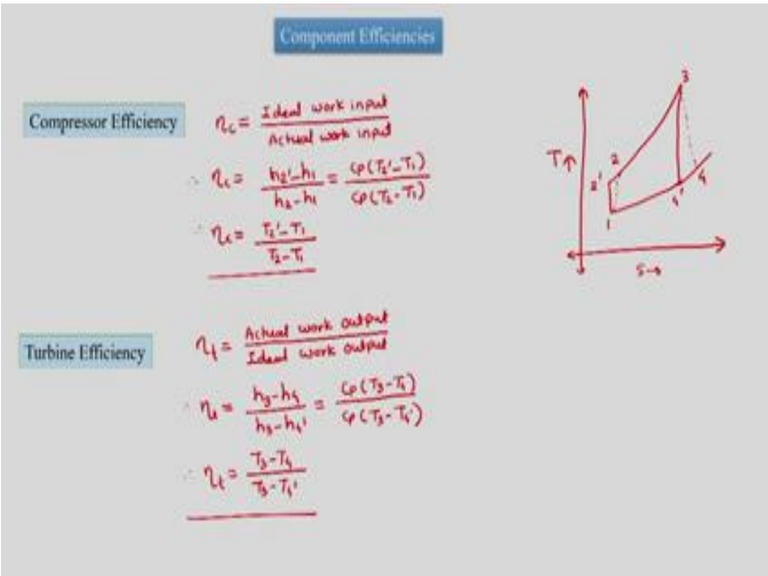
Now, we are going to consider the non-ideal compressor if there would have been ideal compressor, then this would have been the point at the end of the compressor. Let us consider it as 2 dash which corresponds to the ideal point at the end of isentropic compression but in reality the process is not isentropic. So the process will follow such that entropy will increase and we will basically come along the same pressure, which is basically $P_2' = P_2 = P_3$ so the pressures are same.

So, however, we will come higher temperature at higher entropy point which is two. Similarly in case of expansion we will start with 3 but we will not end with 4 dash. So but will end at higher entropy point since the process 3 to 4 dash is isentropic and we are considering real process. So 4 is a real point at the end of isentropic compression? So this is thermodynamic cycle corresponding to the non-ideal components.

So let us consider the processes here, so as we have told process 1 to 2 dash is isentropic compression process but the process 1 to 2 is actual or real compression process then we have 2 root 3 is isobaric heat addition this process is still ideal for us. Since we are considering static properties we are going to discuss about it in the next chapter. Then the process 3 to 4 dash is isentropic expansion. And 3 to 4 is actual expansion then we have 4 to 1 is again isobaric heat rejection and this also is an ideal process for us.

This is how we have the thermodynamic cycle for the processes of compression and expansion if they are non-ideal.

(Refer Slide Time: 04:45)



Now let us move to the next point which is component efficiencies. Since we have considered we have considered non ideal components which are compressor and turbine we have to define their efficiencies. So let us plot for the reference TS diagram as what we plotted so T is an y-axis entropy is in x axis. First we will draw the real process 1 2 dash 3 4 dash and then this is 2 and this is 4.

So, compressor efficiency is denoted by η_c which is again also called as isentropic efficiency of compressor and it is ideal work input to the actual work input. Actually, in case of compression we are supplying the work to the system. So if efficiency is not the ideal which is 100% then we mean that actually we need to supply more work than the ideal work. So the efficiency for compressor has ideal work in the numerator and actual work, which is higher than ideal in the denominator.

$$\eta_c = \frac{Ideal\ work\ input}{Actual\ work\ input} = \frac{h_2' - h_1}{h_2 - h_1} = \frac{C_p(T_2' - T_1)}{C_p(T_2 - T_1)} = \frac{(T_2' - T_1)}{(T_2 - T_1)}$$

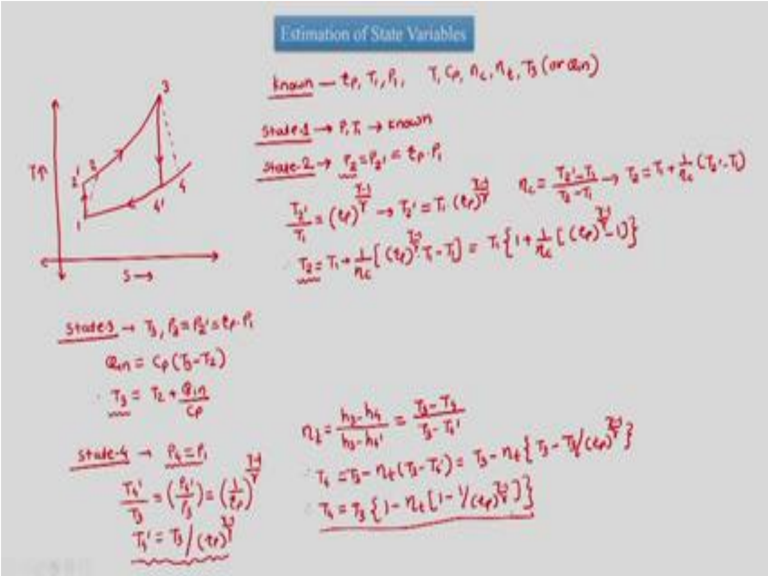
Having said this we will move towards the next component which is turbine and again, we are considering it to be a real turbine and its efficiency is eta t isentropic efficiency of turbine and turbine is a work producing machine.

So ideally it will produce lower more work than the actual case. Actually it is going to produce lower work. So we have

$$\eta_t = \frac{Actual\ work\ output}{Ideal\ work\ output} = \frac{h_3 - h_4}{h_3 - h_4'} = \frac{C_p(T_3 - T_4)}{C_p(T_3 - T_4')} = \frac{T_3 - T_4}{T_3 - T_4'}$$

These are the two formulas, which we should be using for rest of the part of this non ideal component chapter. This in principle while solving the example or in reality we would be knowing the compressor efficiency or turbine efficiency and using that we have to estimate the performance of the cycle or the engine.

(Refer Slide Time: 08:49)



So, let us start with state estimations we know we did it earlier also in case of ideal. So, now we have to repeat it in case of real gas cycle here as well. We say that few things will be known to us and we have to estimate rest all the things from the known thing and this is what we mean that estimation of state variables. So 1 to 2 dash 2 to 3, 3 to 4 dash 4 to 1, so the known things as what we had considered as

$$known \rightarrow r_P, T_1, P_1, T_3, \gamma, C_P, \eta_c, \eta_t, T_3\ (or\ Q_{in})$$

So, knowing these things we have to find out rest of the quantities at all the corners of the cycle. So, let us feel that we have to know things at State 1. And state 1

$$P_1, T_1 \rightarrow known$$

so we do not have to calculate anything. So, at state 2 since we are considering real case, so we are not considering 2 dash so is is a .2 you have to find out properties at 2, but we know

$$P_2 = P_2' = r_P . P_1$$

Now

$$P_2 = P_2' = r_P . P_1$$

$$\frac{T_2'}{T_1} = (r_P)^{\frac{\gamma-1}{\gamma}} \rightarrow T_2' = T_1 (r_P)^{\frac{\gamma-1}{\gamma}}$$

$$\eta_c = \frac{T_2' - T_1}{T_2 - T_1} \rightarrow T_2 = T_1 + \frac{1}{\eta_c}(T_2' - T_1)$$

$$T_2 = T_1 + \frac{1}{\eta_c}\left[(r_P)^{\frac{\gamma-1}{\gamma}} T_1 - T_1\right] = T_1\left\{1 + \frac{1}{\eta_c}\left[(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right]\right\}$$

So this is how we can estimate temperature at end of the compression and also pressure at the end of the compression. So now we have to estimate variables at state 3 there would be again 2 possibilities that we would be knowing

$$T_3, P_3 = P_2' = r_P . P_1$$

So in this case we do not have to estimate but if instead of that, we are given with Q_{in}

$$Q_{in} = C_p(T_3 - T_2)$$

$$T_3 = T_2 + \frac{Q_{in}}{C_p}$$

So this how we would have known the temperature at the exit of the combustor. Now let us derive the expression for properties at State 4.

$$P_4 = P_1$$

$$T_4' = \left(\frac{P_4'}{P_3}\right) = \left(\frac{1}{r_P}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_4' = \frac{T_3}{(r_P)^{\frac{\gamma-1}{\gamma}}}$$

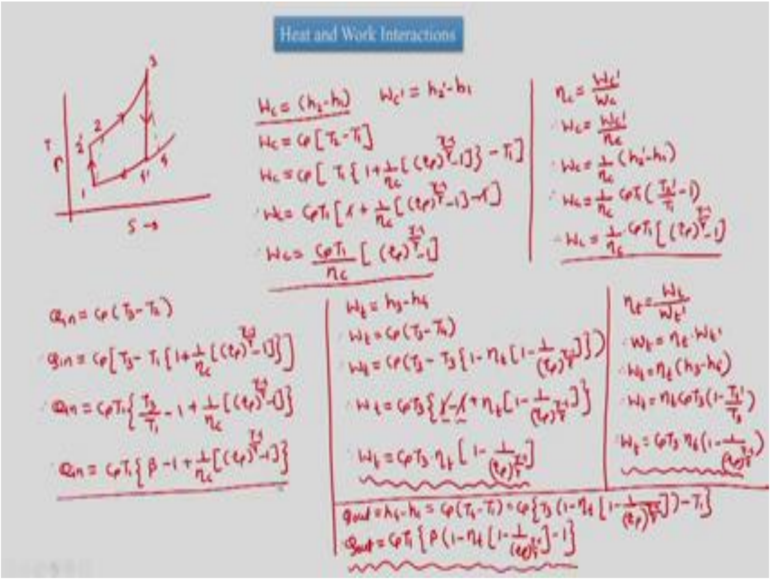
$$\eta_t = \frac{h_3 - h_4}{h_3 - h_4'} = \frac{T_3 - T_4}{T_3 - T_4'}$$

$$\mathrm{T}_4 = T_3 - \eta_t(T_3 - T_4') = T_3 - \eta_t\left\{T_3 - \frac{T_3}{(r_P)^{\frac{\gamma-1}{\gamma}}}\right\}$$

$$T_4 = T_3\left\{1 - \eta_t\left[1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}\right]\right\}$$

So from the known variable unknown values of the quantities we can find out the unknown values of different quantity.

(Refer Slide Time: 16:49)



In knowing this we can then find out what is the work and heat interaction in the process? Or in different processes of Brayton cycle knowing the different state variables as what we had just now found out. So this is the cycle which is known to us and from this cycle. We can calculate W_c which is the compressor work for compressor work at this moment is

$$W_c = (h_2 - h_1)\,; W_c' = h_2' - h_1$$

$$W_c = C_p(T_2 - T_1)$$

$$W_c = C_p\left[T_1\left\{1 + \frac{1}{\eta_c}\left[(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right]\right\} - T_1\right]$$

$$W_c = C_pT_1\left[1 + \frac{1}{\eta_c}\left[(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right] - 1\right]$$

$$W_c = \frac{C_PT_1}{\eta_c}\left[(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\eta_c = \frac{W_c'}{W_c}$$

$$W_c = \frac{W_c'}{\eta_c} = \frac{1}{\eta_c}(h_2' - h_1) = \frac{1}{\eta_c}\;C_pT_1\left(\frac{T_2'}{T_1} - 1\right) = \frac{1}{\eta_c}\;C_pT_1\left[(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$\mathrm{Q_{in}} = C_p(T_3 - T_2)$$

$$\mathrm{Q_{in}} = C_P\left[T_3 - T_1\left\{1 + \frac{1}{\eta_c}\left[(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right]\right\}\right]$$

$$Q_{in} = C_P T_1\left\{\frac{T_3}{T_1} - 1 + \frac{1}{\eta_c}\left[(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right]\right\}$$

$$Q_{in} = C_P T_1\left\{\beta - 1 + \frac{1}{\eta_c}\left[(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right]\right\}$$

$$W_t = h_3 - h_4$$

$$W_t = C_p(T_3 - T_4)$$

$$W_t = C_P\left(T_3 - T_3\left\{1 - \eta_t\left[1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}\right]\right\}\right)$$

$$W_t = C_P T_3\left\{n_t\left[1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}\right]\right\} = C_P T_3 n_t\left[1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}\right]$$

$$\eta_t = \frac{W_t}{W_t'}$$

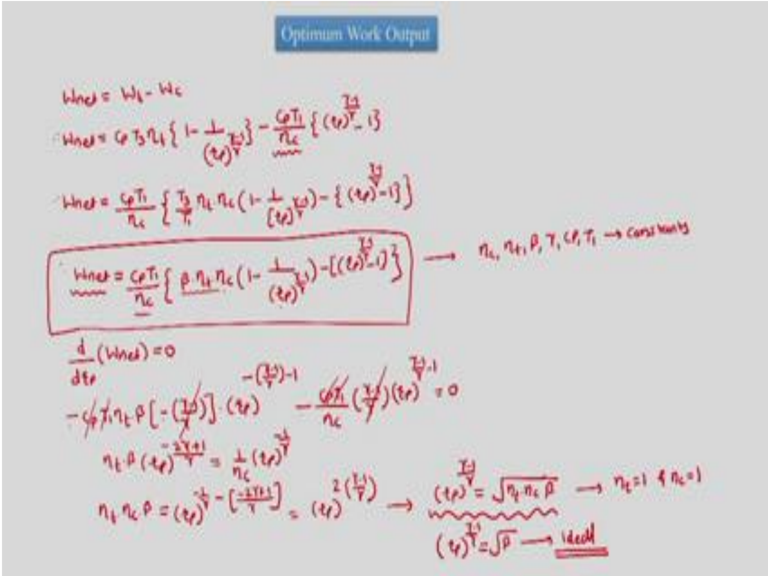
$$W_t = \eta_t . W_t' = \eta_t(h_3 - h_4') = \eta_t C_P T_3\left(1 - \frac{T_4'}{T_3}\right) = \eta_t C_P T_3\left(1 - \frac{T_4'}{T_3}\right) = \eta_t C_P T_3\left(1 - \frac{T_4'}{(r_P)^{\frac{\gamma-1}{\gamma}}}\right)$$

Further we can also evaluate the compressor the heat output which is

$$Q_{out} = h_4 - h_1 = C_p(T_4 - T_1) = C_p\left\{T_3\left(1 - \eta_t\left[1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}\right] - T_1\right)\right\}$$

$$Q_{out} = C_P T_1\left\{\beta\left(1 - \eta_t\left[1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}\right]\right) - 1\right\}$$

(Refer Slide Time: 27:39)



Having said this we can move ahead and find out the formula for W_{net}

$$W_{net} = W_t - W_c$$

$$\mathrm{W_{net}} = C_P T_3 n_t \left[1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}\right] - \frac{C_p T_1}{\eta_c}\left\{(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right\}$$

$$W_{net} = \frac{C_p T_1}{\eta_c}\left\{\frac{T_3}{T_1}\eta_t.\eta_c\left(1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}\right) - \left\{(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right\}\right\}$$

$$W_{net} = \frac{C_p T_1}{\eta_c}\left\{\beta.\eta_t.\eta_c\left(1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}\right) - \left\{(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right\}\right\} \rightarrow \eta_c, \eta_t, \beta, \gamma, C_p, T_1 \rightarrow constants$$

$$\frac{d}{dr_p}\,(W_{net}) = 0$$

$$-C_P T_1 \eta_t \beta\left[-\left(\frac{\gamma-1}{\gamma}\right)\right].(r_P)^{\frac{\gamma-1}{\gamma}} - \frac{C_P T_1}{\eta_c}\left(\frac{\gamma-1}{\gamma}\right)(r_P)^{\frac{\gamma-1}{\gamma}} = 0$$

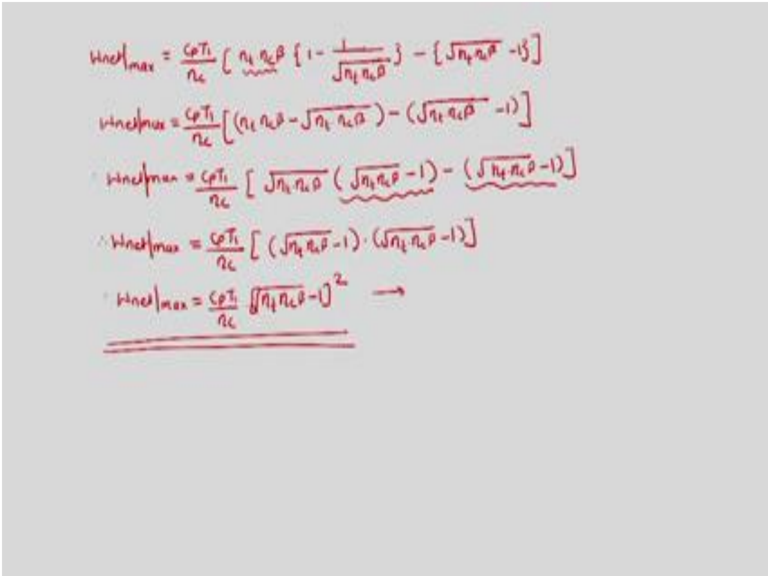
$$\eta_t.\beta(r_P)^{\frac{-2\gamma+1}{\gamma}} = \frac{1}{\eta_c}\,(r_p)^{-\frac{1}{\gamma}}$$

$$\eta_t.\eta_c.\beta = (r_P)^{-\frac{1}{\gamma}-\left[\frac{-2\gamma+1}{\gamma}\right]} = (r_P)^{2\left(\frac{\gamma-1}{\gamma}\right)}$$

$$(r_P)^{\frac{\gamma-1}{\gamma}} = \sqrt{\eta_t.\eta_t.\beta} \rightarrow \eta_t = 1 \; and \; \eta_c = 1$$

$$(r_P)^{\frac{\gamma-1}{\gamma}} = \sqrt{\beta}$$

(Refer Slide Time: 36:01)



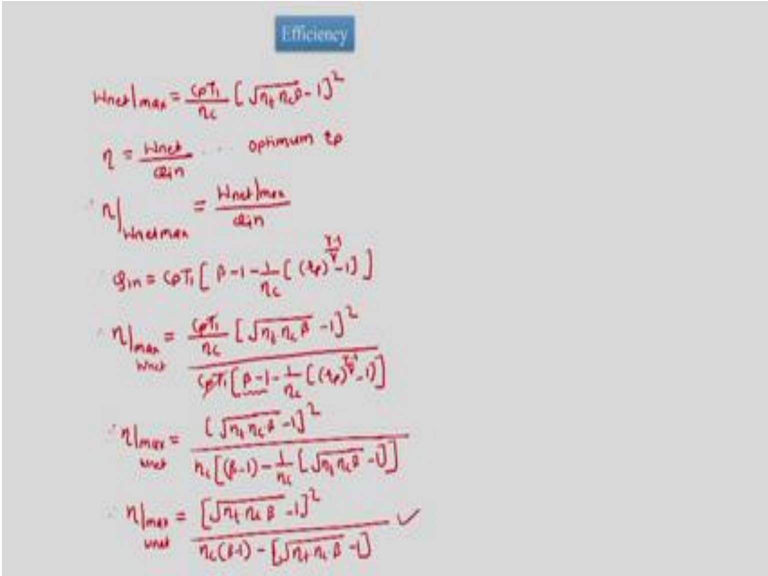
$$W_{net|_{max}} = \frac{C_P T_1}{\eta_c}\left[\eta_t.\eta_c\,\beta\left\{1-\frac{1}{\sqrt{\eta_t.\eta_c\,\beta}}\right\}-\left\{\sqrt{\eta_t.\eta_c\,\beta}-1\right\}\right]$$

$$W_{net|_{max}} = \frac{C_P T_1}{\eta_c}\left[\left(\eta_t.\eta_c\,\beta-\sqrt{\eta_t.\eta_c\,\beta}\right)-\left(\sqrt{\eta_t.\eta_c\,\beta}-1\right)\right]$$

$$W_{net|_{max}} = \frac{C_P T_1}{\eta_c}\left[\sqrt{\eta_t.\eta_c\,\beta}\left(\sqrt{\eta_t.\eta_c\,\beta}-1\right)\left(\sqrt{\eta_t.\eta_c\,\beta}-1\right)\right]$$

$$W_{net|_{max}} = \frac{C_P T_1}{\eta_c}\left[\sqrt{\eta_t.\eta_c\,\beta}-1\right]^2$$

(Refer Slide Time: 39:11)



Having said this need to find out we need to remind this formula since we need to find out what is the corresponding efficiency when we are operating with the maximum compression optimum compression ratio for maximum efficiency. So we had said that

$$W_{net|max} = \frac{C_P T_1}{\eta_c}\left[\sqrt{\eta_t . \eta_c\, \beta} - 1\right]^2$$

Now we have to find out efficiency, efficiency is

$$\eta = \frac{W_{net}}{Q_{in}}\ ; Optimum\ r_P$$

$$\eta|_{W_{net|max}} = \frac{W_{net|max}}{Q_{in}}$$

$$\mathrm{Q_{in}} = C_p T_1\left[\beta - 1 - \frac{1}{\eta_c}\left[(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right]\right]$$

$$\eta|_{max,W_{net}} = \frac{\frac{C_P T_1}{\eta_c}\left[\sqrt{\eta_t . \eta_c\, \beta} - 1\right]^2}{C_P T_1\left[\beta - 1 - \frac{1}{\eta_c}\left[(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right]\right]}$$

$$\eta|_{max,W_{net}} = \frac{\left[\sqrt{\eta_t . \eta_c\, \beta} - 1\right]^2}{\eta_c\left[(\beta - 1) - \frac{1}{\eta_c}\left[(r_P)^{\frac{\gamma-1}{\gamma}} - 1\right]\right]}$$

$$\eta|_{max,W_{net}} = \frac{\left[\sqrt{\eta_t.\eta_c\,\beta} - 1\right]^2}{\eta_c\left[(\beta - 1) - \frac{1}{\eta_c}\left[\sqrt{\eta_t.\eta_c\,\beta} - 1\right]\right]}$$

$$\eta_t = 1; \eta_c = 1$$

$$\eta|_{max,W_{net}} = \frac{\left(\sqrt{\beta} - 1\right)^2}{(\beta - 1) - \left(\sqrt{\beta} - 1\right)} = \frac{\left(\sqrt{\beta} - 1\right)^2}{\sqrt{\beta}\left(\sqrt{\beta} - 1\right)}$$

$$\eta|_{max,W_{net}} = \frac{\left[\sqrt{\eta_t.\eta_c\,\beta} - 1\right]^2}{\eta_c(\beta - 1) - \eta_c\left[\sqrt{\eta_t.\eta_c\,\beta} - 1\right]}$$

This is how we can evaluate different parameters for the case where we are having non ideal compressor and turbine next things which deal with the examples of this part we'll deal in the next class. Thank you.