Aircraft Propulsion

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Lecture 07
Examples for Ideal Brayton Cycle

Welcome to the class till time we have seen that how ideal Brayton cycle is, how to estimate the state variables and what is the particular expression for calculating different interactions like work and heat interaction in different processes. Now, let us execute it through following some examples on ideal Brayton cycle.

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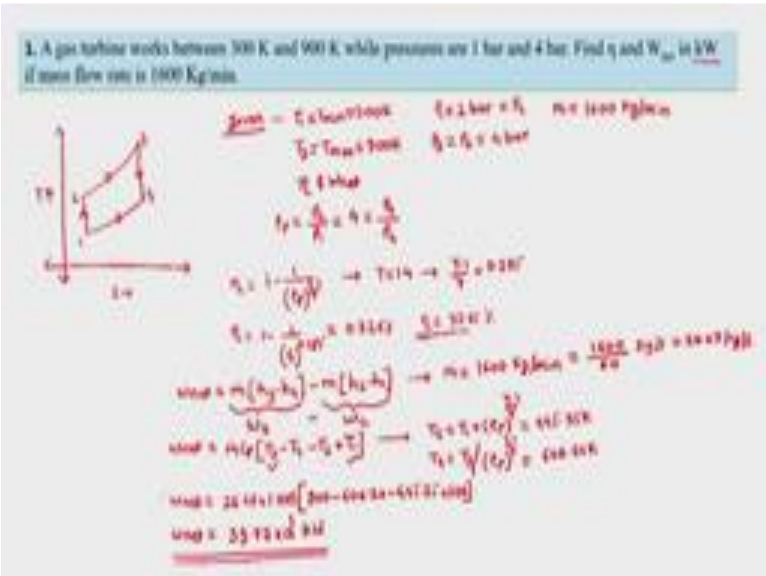

So, here is the first example, it states that a gas turbine works between 300 Kelvin and 900 Kelvin temperature while pressures are 1 bar and 4 bar find out efficiency net work in kW if mass flow rate is 1600 kg per minute. Let us plot our TS diagram for the Brayton cycle. So, this is T and this is as we know that processes 1 to 2, 2 to 3, 3 to 4 and 4 to 1. So, given thing are

$$T_1 = T_{min} = 300\ K; P_1 = 1\ bar = P_4\ ; \dot{m} = 1600\ kg/min$$

$$T_3 = T_{max} = 900\ K\ ; P_3 = P_2 = 4\ bar; \eta = ?\ ;\ W_{net} = ?$$

$$r_P = \frac{P_2}{P_1} = 4 = \frac{P_3}{P_4}$$

$$\eta = 1 - \frac{1}{(\epsilon_P)^{\frac{\gamma-1}{\gamma}}} \rightarrow \gamma = 1.4 \rightarrow \frac{\gamma - 1}{\gamma} = 0.285$$

$$\eta = 1 - \frac{1}{(4)^{0.285}} = 0.3263\ ; \eta = 32.65\%$$

$$W_{net} = \dot{m}[h_3 - h_4] - \dot{m}[h_2 - h_1] \rightarrow \dot{m} = 1600\frac{kg}{min} = \frac{1600}{60}\frac{kg}{s} = 26.67\frac{kg}{s}$$

$$W_{net} = \dot{m}C_p[T_3 - T_4 - T_2 + T_1] \rightarrow T_2 = T_1 * (\epsilon_P)^{\frac{\gamma-1}{\gamma}} = 445.35\ K$$

$$W_{net} = 26.67 * 1.005\ [900 - 606.26 - 445.35 + 300]$$

$$W_{net} = 39.77 * 10^3\ kW$$

This is other answer of the example. This is how we will execute the known expressions to solve an example.

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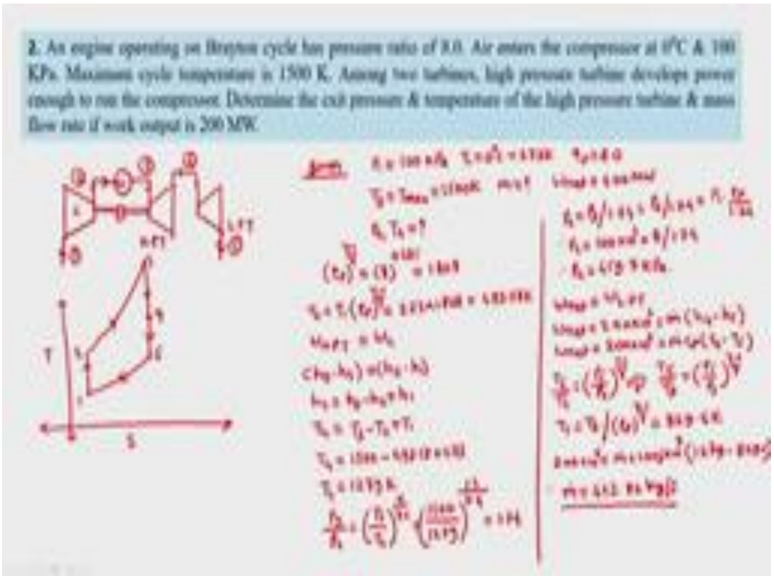

So, let us go to the next example, it states that an engine operating on Brayton cycle pressure ratio of 8, air enters the compressor at 0 degrees Celsius and 100 kPa pressure, maximum cycle temperature is 1500 Kelvin among two turbines, high pressure turbine develops power enough to run the compressor. Determine the exit pressure and temperature of high pressure turbine and mass flow rate if the work output is 200 mega watt.

Now, this example says that there is a compressor which gets air at station 1 and then air goes to the combustion chamber from the combustion chamber it goes to the turbine and then these two are connected to each other. So, this is high pressure turbine and from that air goes to the low pressure turbine and then it goes as an exhaust, low pressure turbine. So, 1 to 2 is compression process 2 to 3 is heat addition 3 to 4 is high pressure turbine and 4 to 5 is the low pressure turbine.

So, we have TS diagram for this example, as similar 1 to 2, 2 to 3, but we have intermediate point 4 and then 5. So, let us evaluate for the known things here given things are

$$P_1 = 100\ kPa\ ; T_1 = 0^0C = 273\ K; r_P = 8.0$$

$$T_3 = T_{max} = 1500\ K\ ; \dot{m} = ?\ ; W_{net} = 200\ MW$$

$$P_4, T_4 = ?$$

$$(r_P)^{\frac{\gamma-1}{\gamma}} = (8)^{0.285} = 1.808$$

$$T_2 = T_1(\epsilon_P)^{\frac{\gamma-1}{\gamma}} = 273 * 1.808 = 493.58\ K$$

$$W_{H.P.T} = W_c$$

$$(h_3 - h_4) = (h_2 - h_1)$$

$$h_4 = h_3 - h_2 + h_1$$

$$T_4 = T_3 - T_2 + T_1 = 1500 - 493.58 + 273 = 1279\ K$$

$$\frac{P_3}{P_4} = \left(\frac{T_3}{T_4}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1500}{1279}\right)^{\frac{1.4}{0.4}} = 1.74$$

$$P_4 = \frac{P_3}{1.74} = P_1\frac{\epsilon_P}{1.74}$$

$$P_4 = 100 * 10^3 * \frac{8}{1.74}$$

$$P_4 = 459.7\ kPa$$

$$W_{net} = W_{L.P.T}$$

$$W_{net} = 200 * 10^6 = \dot{m}(h_4 - h_5)$$

$$W_{net} = 200 * 10^6 = \dot{m}C_p(T_4 - T_5)$$

$$\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow \frac{T_5}{T_3} = \left(\frac{P_5}{P_3}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_5 = \frac{T_3}{(r_P)^{\frac{\gamma-1}{\gamma}}} = 829.6\ K$$

$$200 * 10^6 = \dot{m} * 1.005 * 10^3 * (1279 - 829.6)$$

$$\dot{m} = 442.82\ kg/s$$

So, this is other answer of example, we evaluated pressure temperature at the exit of the turbine and we also found out what is the mass flow rate in the circuit. So, this is how we could solve the example.

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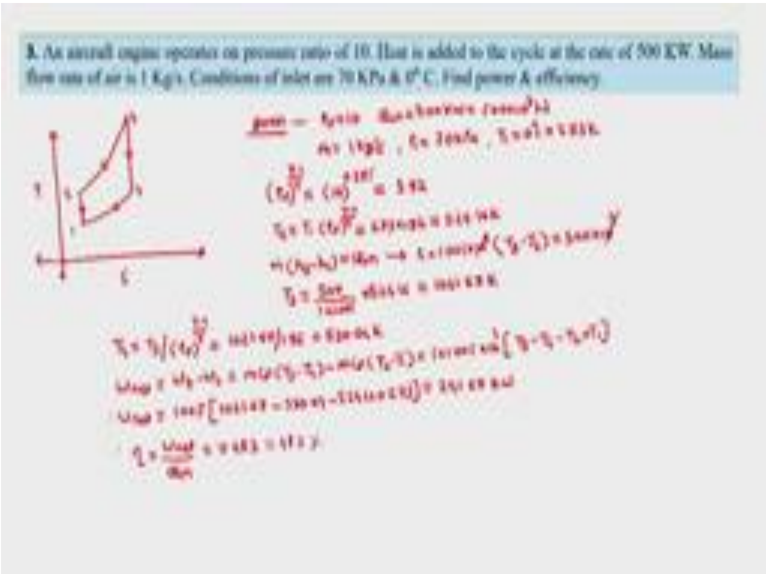

Let us move to the next example. Next example states that an aircraft engine operates on pressure ratio 10 heat is added to the cycle at the rate of 500 kilowatt mass flow rate of air is 1 kg per second conditions at the inlet are 70 kPa and 0 degrees Celsius and we are supposed to find out power and efficiency of the cycle. So, let us plot TS diagram such that we can remember the state for which we are giving the numbers.

So, given things to us are

$$r_P = 10\,;\, Q_{in} = 500\ kW = 500 * 10^3\ W$$

$$\dot{m} = 1\frac{kg}{s}; P_1 = 70\ kPa\ ; T_1 = 0^0C = 273\ K$$

$$(r_P)^{\frac{\gamma-1}{\gamma}} = 10^{0.285} = 1.92$$

$$T_2 = T_1.(r_P)^{\frac{\gamma-1}{\gamma}} = 273 * 1.92 = 524.16\ K$$

$$\dot{m}(h_3 - h_4) = Q_{in}$$

$$1 * 1.005 * 10^3 * (T_3 - T_2) = 500 * 10^3$$

$$T_3 = \frac{500}{1 * 1.005} + 524.16 = 1021.67\ K$$

$$T_4 = \frac{T_3}{(r_P)^{\frac{\gamma-1}{\gamma}}} = \frac{1021.67}{1.92} = 530.04\ K$$

$$W_{net} = W_t - W_c = \dot{m}C_p(T_3 - T_4) - \dot{m}C_p(T_2 - T_1) = 1 * 1.005 * 10^3[T_3 - T_4 - T_2 + T_1]$$

$$W_{net} = 1005 * [1021.67 - 530.04 - 524.16 + 273] = 241.67\ kW$$

$$\eta = \frac{W_{net}}{Q_{in}} = 0.483 = 48.3\ \%$$

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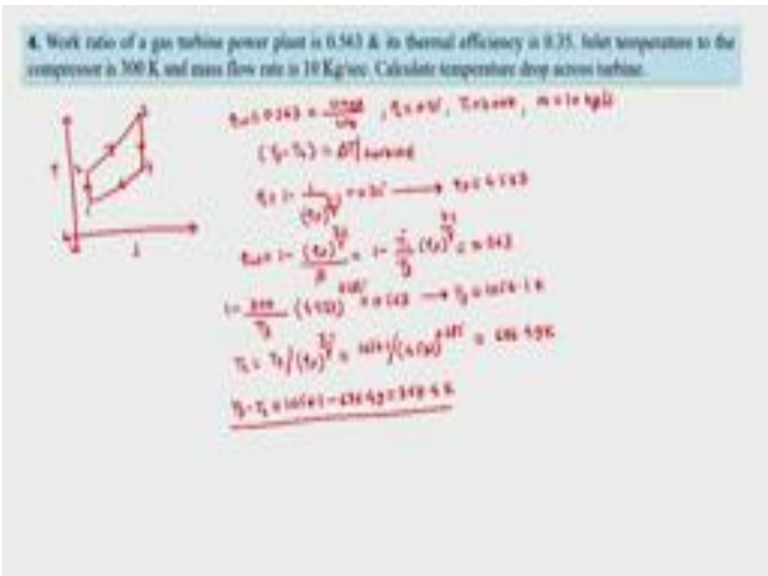

Let us move on to the next example, we have been given as work ratio of a gas turbine plant is .563 its thermal efficiency is .35 inlet temperature the compressor is 300 Kelvin and mass flow rate is 10 kg per second. We are supposed to find out temperature drop across the turbine. So, again let us plot the TS diagram and let us see what are the given things for this cycle it is given to us that

$$r_w = 0.563 = \frac{W_{net}}{W_t}\ ; \eta = 0.35; T_1 = 300\ K; \dot{m} = 10\ kg/s$$

$$(T_3 - T_4) = \Delta T|_{turbine}$$

$$\eta = 1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}} 0.35 \rightarrow r_P = 4.533$$

$$r_w = 1 - \frac{(r_P)^{\frac{\gamma-1}{\gamma}}}{\beta} = 1 - \frac{T_1}{T_3}\ (r_P)^{\frac{\gamma-1}{\gamma}} = 0.563$$

$$1 - \frac{300}{T_3}\ (4.533)^{0.285} = 0.563 \rightarrow T_3 = 1056.1\ K$$

$$T_4 = \frac{T_3}{(r_P)^{\frac{\gamma-1}{\gamma}}} = \frac{1056.1}{4.533^{0.285}} = 686.49\ K$$

$$T_3 - T_4 = 1056.1 - 686.49 = 369.6\ K$$

This is how we can find out the asked quantity which is temperature drop across the turbine.

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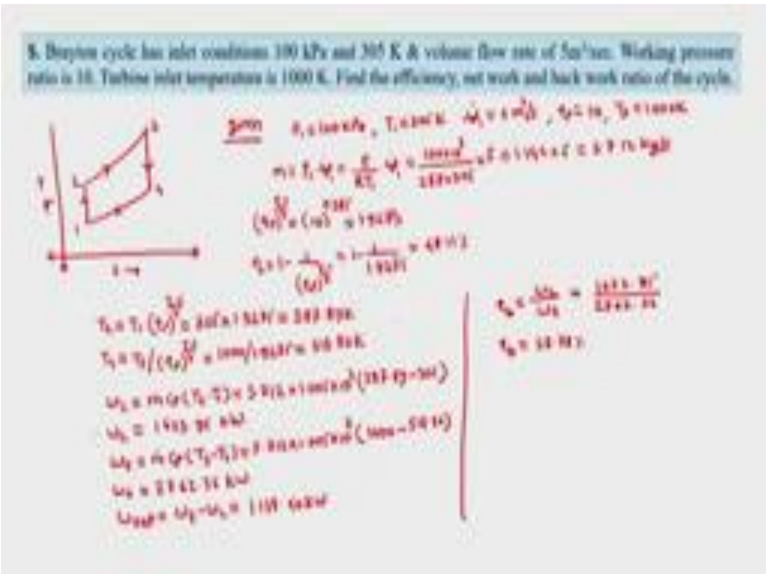

Let us move on to the next example, which states that Brayton cycle as inlet conditions 100 kPa and 305 Kelvin and volume flow rate of 5 meter cube per second working pressure ratio is

10, turbine inlet temperature is 1000 Kelvin find out efficiency net work and back work ratio of the cycle. So, let us see what are the things given to us in the example. Here new thing which is asked to us is back work ratio.

And rest of the things are known to us and we also know what is back work ratio. So

$$P_1 = 100\ kPa\ ; T_1 = 305\ K\ ; \dot{V} = 5\frac{m^3}{s}; r_p = 10\ ; T_3 = 1000\ K$$

$$\dot{m} = \rho_1 \dot{V} = \frac{P_1}{RT_1}\dot{V} = \frac{100 * 10^3}{287 * 305} * 5 = 1.142 * 5 = 5.712\ kg/s$$

$$(r_P)^{\frac{\gamma-1}{\gamma}} = (10)^{0.285} = 1.9275$$

$$\eta = 1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{1.9275} = 48.11\ \%$$

$$\mathrm{T}_2 = T_1 (r_P)^{\frac{\gamma-1}{\gamma}} = 305 * 1.9275 = 587.89\ K$$

$$\mathrm{T}_4 = \frac{T_3}{(r_P)^{\frac{\gamma-1}{\gamma}}} = \frac{1000}{1.9275} = 518.80\ K$$

$$W_c = \dot{m} c_P (T_2 - T_1) = 5.712 * 1.005 * 10^3 (587.89 - 305) = 1623.95\ kW$$

$$W_t = \dot{m} c_P (T_3 - T_4) = 5.712 * 1.005 * 10^3 (1000 - 518.80)$$

$$W_t = 2762.36\ kW$$

$$W_{net} = W_t - W_c = 1138.40\ kW$$

$$r_b = \frac{W_c}{W_t} = \frac{1623.95}{2762.96} = 58.78\ \%$$

So, this is how we would have solved the example on Brayton cycle. New thing in this example was we were given as volume flow rate in meter cube per second. So, we were needing it to find out in kg per second.

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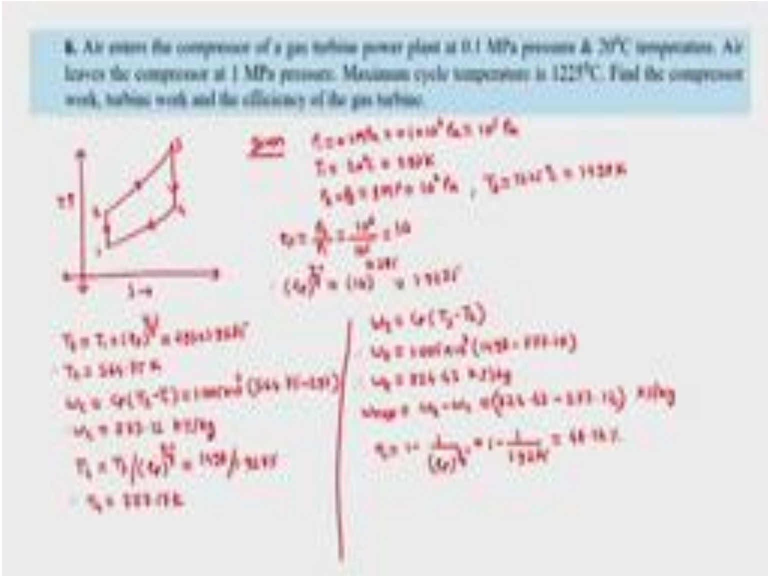

So, next example it states that air enters the compressor of a gas turbine at .1 mega Pascal pressure and 20 degree Celsius and leaves the compressor at 1 mega Pascal pressure maximum cycle temperature is 1225 degrees Celsius and out compressor work turbine work efficiency of the gas turbine. So, we are given implicitly everything and then the thing includes the pressure ratio also.

So 1, 2, 3, 4 so, we are given that

$$P_1 = 0.1\ MPa = 0.1 * 10^6\ Pa = 10^5\ Pa$$

$$T_1 = 20^0 C = 293\ K$$

$$P_2 = P_3 = 1\ Mpa = 10^6\ Pa; T_3 = 1225^0 C = 1498\ K$$

$$r_P = \frac{P_2}{P_1} = 10;\ (r_P)^{\frac{\gamma-1}{\gamma}} = (10)^{0.285} = 1.9275$$

$$T_2 = T_1 * (r_P)^{\frac{\gamma-1}{\gamma}} = 293 * 1.9275 = 564.75\ K$$

$$W_c = C_P(T_2 - T_1) = 1.005 * 10^3 * (564.75 - 293)$$

$$W_c = 273.12\ kJ/kg$$

$$T_4 = \frac{T_3}{(r_P)^{\frac{\gamma-1}{\gamma}}} = \frac{1498}{1.9275} = 777.17\ K$$

$$W_t = C_P(T_3 - T_4) = 1.005 * 10^3(1498 - 777.17) = 724.43\ kJ/kg$$

$$W - net = W_t - W_c = 724.43 - 273.12\ kJ/kg$$

$$\eta = 1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{1.9275} = 48.12\,\%$$

So, we found out all the necessary numbers which were required.

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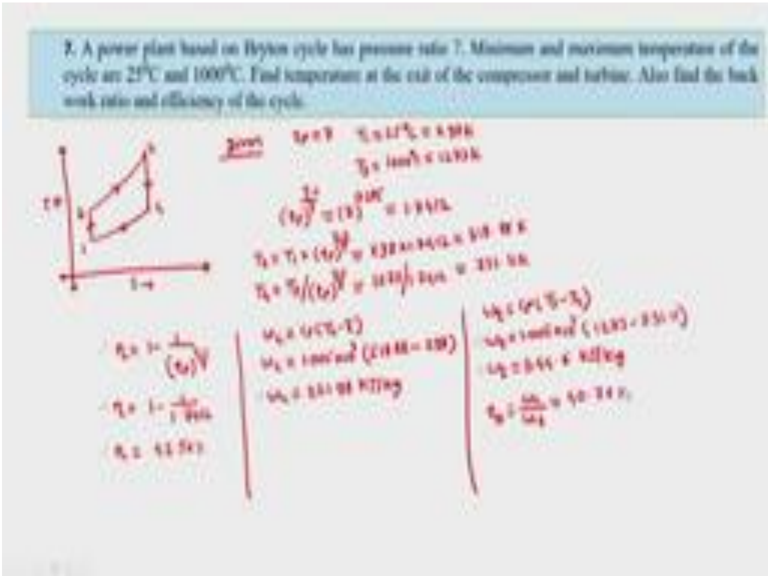

Now, having said this will move to the next example and which states that the power plant based on Brayton cycle as pressure ratio 7 minimum and maximum temperature of the cycle are 25 degrees Celsius and 1000 degrees Celsius find the temperature at the exit of the compressor and turbine. Also find out back work ratio and efficiency of the cycle. Here we are explicitly asked to find out different temperatures, we would have used different formulas directly to find out the performance parameters.

But we cannot use since we are asked intermediate numbers. So, given things are

$$r_P = 25^0 C = 298\, K$$

$$T_3 = 1000^0 C = 1273\, K$$

$$(r_P)^{\frac{\gamma-1}{\gamma}} = 7^{0.285} = 1.7412$$

$$T_2 = T_1 * (r_P)^{\frac{\gamma-1}{\gamma}} = 298 * 1.7412 = 518.88\, K$$

$$\mathrm{T}_4 = \frac{T_3}{(r_P)^{\frac{\gamma-1}{\gamma}}} = \frac{1273}{1.7412} = 731.11\, K$$

$$\eta = 1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{1.7412} = 42.56\,\%$$

$$W_C = C_P(T_2 - T_1) = 1.005 * 10^3(518.88 - 298) = 221.98\,kJ/kg$$

$$W_t = C_P(T_2 - T_1) = 1.005 * 10^3 * (1273 - 731.11) = 544.6\,kJ/kg$$

$$r_b = \frac{W_c}{W_t} = 40.76\,\%$$

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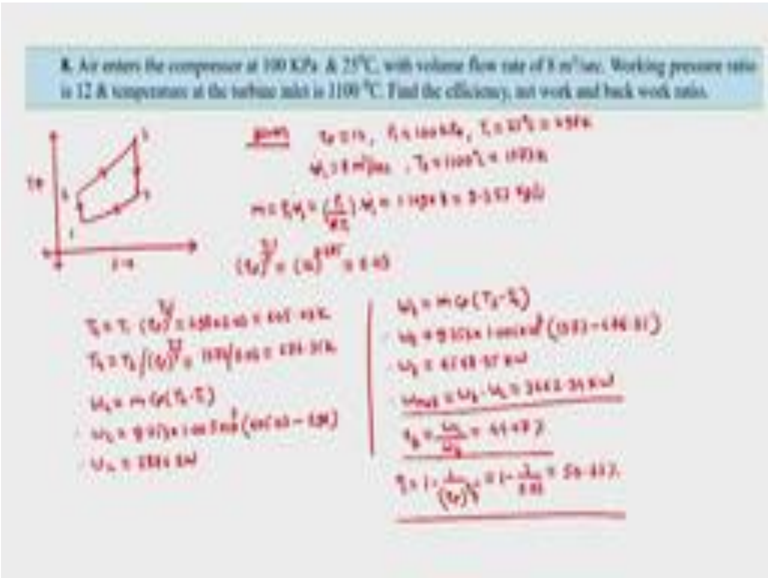

So, we will move on to the last example, here we have said that there is air which enters the compressor at 100 kPa and 25 degrees Celsius with volume flow rate as 8 meter cube per second working pressure ratio is 12 temperature at the inlet of the turbine is 1100 degrees Celsius find out efficiency net work and back work ratio. Again similar thing we are given that there is volume flow rate.

So, using the volume flow rate, we have to find out mass flow rate, once the mass flow rate is known, we can find out net work in kilowatt. So, given things are

$$r_P = 12; P_1 = 100\,kPa\,; T_1 = 25^0C = 298\,K$$

$$\dot{V} = 8\frac{m^3}{s}; T_3 = 1100^0C = 1373\,K$$

$$\dot{m} = \rho_1 \dot{V}_1 = \left(\frac{P_1}{RT_1}\right) \dot{V}_1 = 1.169 * 8 = 9.353\ kg/s$$

$$(r_P)^{\frac{\gamma-1}{\gamma}} = (12)^{0.285} = 2.03$$

$$T_2 = T_1.(r_P)^{\frac{\gamma-1}{\gamma}} = 298 * 2.03 = 605.03\ K$$

$$T_4 = \frac{T_3}{(r_P)^{\frac{\gamma-1}{\gamma}}} = \frac{1373}{2.03} = 676.35\ K$$

$$W_c = \dot{m} C_P\ (T_2 - T_1) = 9.353 * 1.005 * 10^3 * (605.03 - 298) = 2886\ kW$$

$$W_t = \dot{m} C_P\ (T_3 - T_4) = 9.353 * 1.005 * 10^3 * (1373 - 676.35) = 6548.35\ kW$$

$$W_{net} = W_t - W_c = 3662.34\ kW$$

$$r_b = \frac{W_c}{W_t} = 44.07\ \%$$

$$\eta = 1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}} = 1 - \frac{1}{2.03} = 50.73\ \%$$

So, this is the way we solve the examples which are related with the Brayton cycle which is ideal where everywhere we have considered the all the components like turbine and compressor with 100% efficiency we have considered CP CV or gamma is constant. So these things tell us that we worked with air standard Brayton cycle which is ideal, thank you.