Aircraft Propulsion

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Lecture 06
Ideal Brayton Cycle

Welcome to the class today. We are going to focus on the Ideal Brayton Cycle. Till time we have seen that there is gas turbine the different attachments to the gas turbine and it is possible that this gas turbine can be considered as a possible engine for the aircraft propulsion. Now let us understand the gas turbine cycle from thermodynamics viewpoint.

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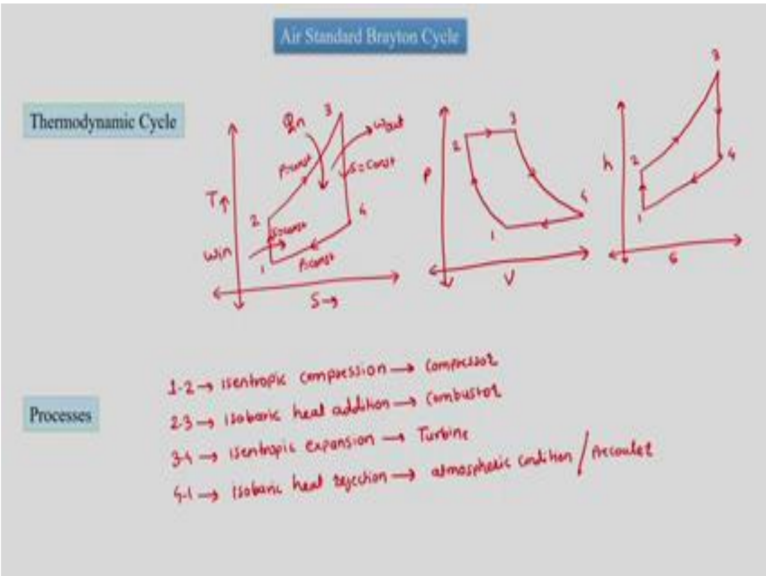

So, standard Ideal Brayton cycle here we are assuming that air is the working medium and it is not changing its properties in the process of the engine or in the thermodynamic cyclic process and further we are also considering that the thermodynamic properties of air which are specific heat and specific heat ratio they are not going to change with respect to temperature we are considering air as calorically perfect gas.

So, we are considering this let us plot thermodynamics cycle we have already considered the thermodynamics cycle while considering the different cycles in the overview of thermodynamics. So, in this process, we will re-plot thermodynamic cycle, which is Brayton cycle. Let us first draw this TS diagram on y-axis temperature and x axis we have entropy so first process is 1 to 2 it is an isentropic process.

And process 2 to 3 in isobaric process then process 3 to 4 is an isentropic process and process 4 to 1 is an isobaric process, but this is entropy constant process 3 and 4 also entropy constant process. 2 to 3 is pressure constant process and 4 to 1 is also pressure constant process. This is the air standard cycle. Let us plot PV diagram for the brayton cycle. As per the PV diagram process 1 to 2 is isentropic compression process 2 to 3 is isobaric heat addition.

Process 3 to 4 isentropic expansion and process 4 to 1 is isentropic heat rejection. This is PV diagram for Brayton cycle. Now let us plot HS diagram for Brayton cycle. HS diagram is similar to the TS diagram except slope is different. So, geometrically it look like similar 1 to 2 is isentropic compression 2 to 3 is constant pressure heat addition 3 to 4 isentropic expansion and 4 to 1 is constant pressure heat rejection.

Now, what are the processes? Process 1 to 2 it is isentropic compression process and we had also said in our earlier classes and it is unlike SI engine, there are in SI engine there are different processes, but all the processes are conducted in one unit and in gas turbine each process will be conducted in different unit. Isentropic compression will be conducted in compressor. And process 2 to 3 is isobaric heat addition.

So, we add heat here, this is Qin this is Win and this process is conducted in combustor or combustion chamber versus 3 to 4 is isentropic expansion. This is a process in which work is done. This is done in turbine when it this is w out. Then process 4 to 1 is isobaric heat rejection. This process is conducted either in atmospheric condition or in case of open cycle or in a pre-cooler. So, we will see this in detail. This is air standard cycle which is a brayton cycle.

Now let us plot the schematic diagram which is possible engine for the brayton cycle we had already considered it multiple times.

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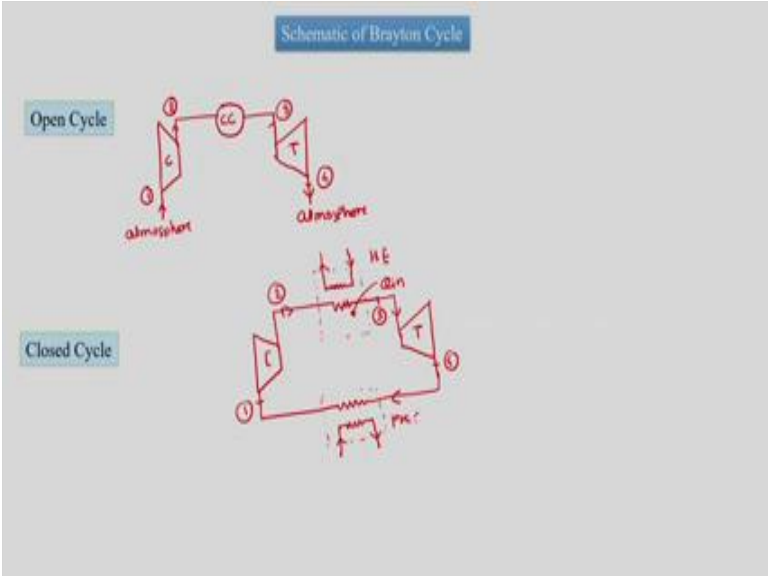

So, first is open cycle, in case of open cycle first process as what we said process 1 to 2 is compression. This is 1 to 2 integrating done in compressor then we have combustion chamber. Combustion chamber gas goes to the turbine then this gas goes back into the atmosphere. This is taken from the atmosphere here to practically feel that this is an open cycle since the same gas which is a coming and exhaust of the turbine is not going into the compressor. We are having this is an open circuit so this is a open cycle for gas turbine power plant.

However, we consider same amount of mass flow rate of the gas is coming into the compressor at initial temperature 1. So, we will say that this is a open and this is cycle. Then there is closed cycle gas turbine power plant and in case of closed cycle gas turbine power plant again there will be compressor first. And from the compressor gas will go air will go into heat exchanger. So, this is a heat exchanger.

And then from the heat exchanger it will go to the turbine and then on the turbine it will go again to heat exchanger, which is called as pre- cooler. But these two heat exchangers will have another field which will pass the heat or which will take the heat. So, in case of heat exchanger which is plotted over here. We are interested in getting heat into our system. That is where the hot gas will be passed in the heat exchangers.

So, that gas give heat to the air at constant pressure process 2 to 3 after 3 to 4 expansion pre cooler will have a cold gas which will be cold fluid which will be taking heat from the air which is undergoing the process 4 to 1 which is a constant pressure heat rejection process this is a frequent there are the diagrams for schematic diagram for brayton cycle. Having seen this we move ahead and see what are the relations for thermodynamics processes and associated work and heat interactions.

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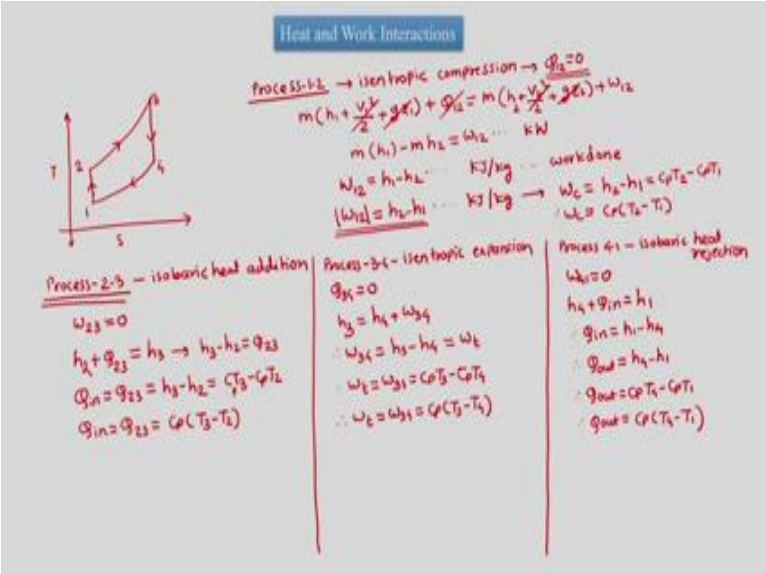

For that let us first Plot for TS diagram in case of Brayton cycle, you would see that we repeatedly plot the TS diagram since it helps us to analyse understand basics of Brayton cycle. So, in each case we will plot the TS diagram so heat and work interactions as what we said that let us first consider the process 1 to 2, so process 1 to 2 in the process 1 to 2 it is isentropic compression.

As the name suggests in this process $Q_{12} = 0$. And then we need to find out work interaction in the process 1 to 2 we had seen that for the steady flow energy equation. If we apply for the process 1 to 2 you can see that

Process 1-2

$$Isentropic\ compression \rightarrow Q_{!2} = 0$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) + Q_{12} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) + W_{12}$$

$$\dot{m}(h_1) - \dot{m}h_2 = W_{12} \ldots kW$$

$$W_{12} = h_1 - h_2 \ldots \frac{kJ}{kg} \ldots work\ done$$

$$|W_{12}| = h_2 - h_1 \ldots \frac{kJ}{kg} \rightarrow W_c = h_2 - h_1 = C_p(T_2 - T_1)$$

Process 2-3: Isobaric heat addition

$$W_{23} = 0$$

$$h_2 + Q_{23} = h_3 \rightarrow h_3 - h_2 = Q_{23}$$

$$Q_{in} = Q_{23} = h_3 - h_2 = C_pT_3 - C_pT_2$$

$$Q_{in} = Q_{23} = C_p(T_3 - T_2)$$

Process 3-4: Isentropic expansion

$$Q_{34} = 0$$

$$h_3 = h_4 + W_{34}$$

$$W_{34} = h_3 - h_4 = W_t$$

$$W_t = W_{34} = C_p(T_3 - T_4)$$

Process 4-1: Isobaric heat rejection

$$W_{41} = 0$$

$$h_4 + Q_{in} = h_1$$

$$Q_{in} = h_1 - h_4$$

$$Q_{out} = h_4 - h_1$$

$$Q_{out} = C_p(T_4 - T_1)$$

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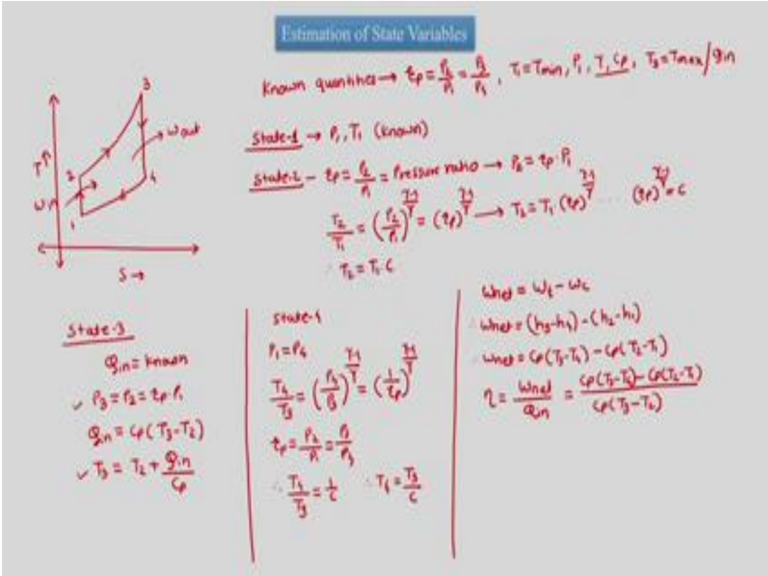

Let us move on to find out state variables for the ideal Brayton cycle again for that let us plot TS diagram for Brayton cycle and we are considering as what we are here have said simple air standard cycle with 4 processes for Brayton cycle and they include 2 isentropic processes and 2 isobaric process. So in case of Brayton cycle whenever we are dealing with examples or in general for design perspective the known quantities are in general pressure ratio which is

Known quantities $\rightarrow r_P = \frac{P_2}{P_1} = \frac{P_3}{P_4}$; $T_1 = T_{min}, P_1, \gamma, C_p, T_3 = \frac{T_{max}}{Q_{in}}$

State 1: $\rightarrow P_1, T_1\ (known)$

State 2:$\rightarrow r_P = \frac{P_2}{P_1} = Pressure\ ratio \rightarrow P_2 = r_P.P_1$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_P)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1.(r_P)^{\frac{\gamma-1}{\gamma}} = T_1.C$$

State-3:

$$Q_{in} = known$$

$$P_3 = P_2 = \epsilon_P.P_1$$

$$Q_{in} = C_p(T_3 - T_2)$$

$$T_3 = T_2 + \frac{Q_{in}}{C_p}$$

State-4:

$$P_1 = P_4$$

$$\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} \left(\frac{1}{r_P}\right)^{\frac{\gamma-1}{\gamma}}$$

$$r_P = \frac{P_2}{P_1} = \frac{P_3}{P_4}$$

$$\frac{T_4}{T_3} = \frac{1}{C} \rightarrow T_4 = \frac{T_3}{C}$$

$$\mathrm{W_{net}} = W_t - W_c$$

$$\mathrm{W_{net}} = (h_3 - h_4) - (h_2 - h_1)$$

$$W_{net} = C_P(T_3 - T_4) - C_P(T_2 - T_1)$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{C_P(T_3 - T_4) - C_P(T_2 - T_1)}{C_p(T_3 - T_2)}$$

So this is how once we know some design variables we can estimate other variables at different states. And then using those variables we can find out the net work Brayton cycle and the efficiency of Brayton cycle.

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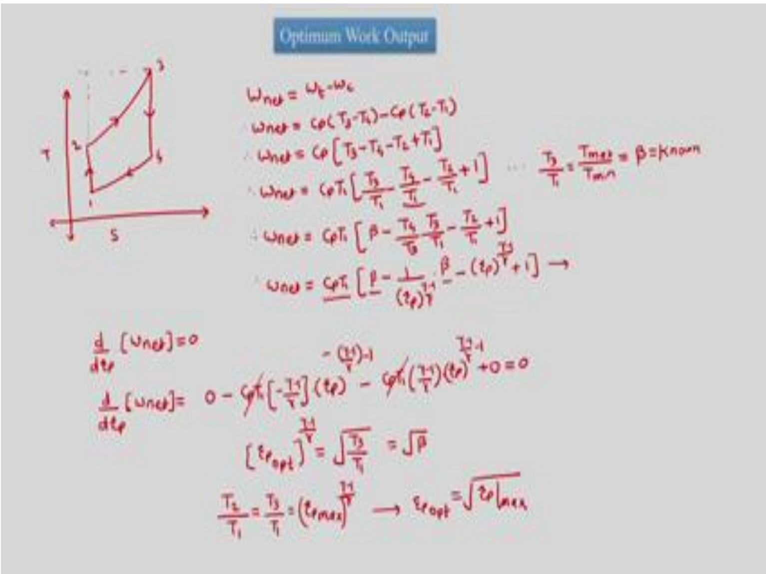

As we said this we can move ahead and find out what is this optimum work done in case of the Brayton cycle. So again let us plot Brayton cycle which is known to us from the TS diagram process 1 to 2, 2 to 3, 3 to 4 so here we said that

$$\mathrm{W_{net}} = W_t - W_c$$

$$W_{net} = C_P(T_3 - T_4) - C_P(T_2 - T_1)$$

$$W_{net} = C_P[T_3 - T_4 - T_2 + T_1]$$

$$W_{net} = C_P T_1 \left[\frac{T_3}{T_1} - \frac{T_4}{T_1} - \frac{T_2}{T_1} + 1\right]; \ \frac{T_3}{T_1} = \frac{T_{max}}{T_{min}} = \beta = known$$

$$W_{net} = C_P T_1 \left[\beta - \frac{T_4}{T_3}\frac{T_3}{T_1} - \frac{T_2}{T_1} + 1\right]$$

$$W_{net} = C_P T_1 \left[\beta - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}.\beta - (r_P)^{\frac{\gamma-1}{\gamma}} + 1\right]$$

$$\frac{d}{dr_P}[W_{net}] = 0$$

$$\frac{d}{dr_P}[W_{net}] = 0 - C_P T_1 \left[\frac{-(\gamma-1)}{\gamma}\right].(r_P)^{-\left(\frac{\gamma-1}{\gamma}\right)-1} - C_p T_1 \left(\frac{\gamma-1}{\gamma}\right)(r_P)^{\frac{\gamma-1}{\gamma}} + 0 = 0$$

$$\left[r_{P_{opt}}\right]^{\frac{\gamma-1}{\gamma}} = \sqrt{\frac{T_3}{T_1}} = \sqrt{\beta}$$

So, this is condition for optimum r_P, if we know T3 if we know T1 we can take square root of that ratio, that would be our optimum r_P and that will lead to maximum net work. Further we know that

$$\frac{T_2}{T_1} = \frac{T_3}{T_1} = \left[r_{P_{max}}\right]^{\frac{\gamma-1}{\gamma}} \rightarrow r_{P_{opt}} = \sqrt{r_P|_{max}}$$

So my this we can find out what are the basic things about $r_{P_{opt}}$.

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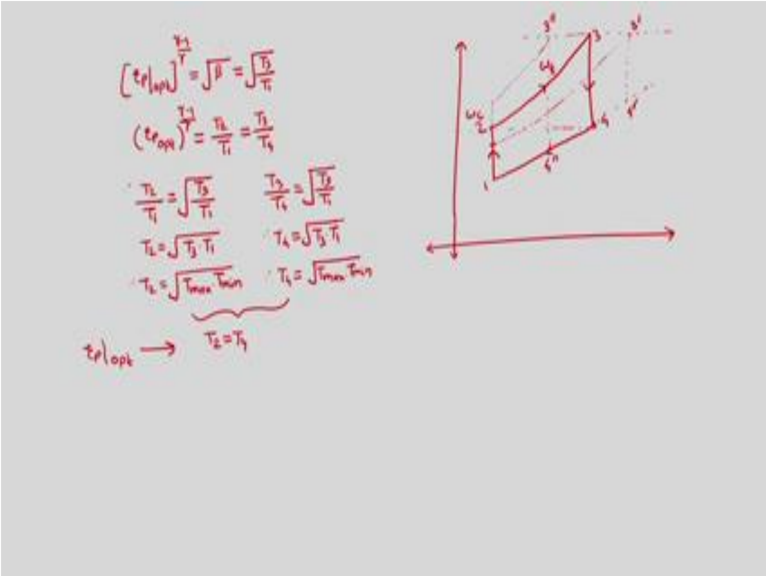

$$\left[r_{P_{opt}}\right]^{\frac{\gamma-1}{\gamma}} = \sqrt{\beta} = \sqrt{\frac{T_3}{T_1}}$$

$$\left[r_{P_{opt}}\right]^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1} = \frac{T_3}{T_4}$$

$$\frac{T_2}{T_1} = \sqrt{\frac{T_3}{T_1}}\,;\frac{T_3}{T_4} = \sqrt{\frac{T_3}{T_1}}$$

$$T_2 = \sqrt{T_3.T_1}\,;T_4 = \sqrt{T_3.T_1}$$

$$T_2 = \sqrt{T_{max}T_{min}}\,;T_4 = \sqrt{T_{max}T_{min}}$$

$$r_{P_{opt}} \rightarrow T_2 = T_4$$

Practically we mean that TS diagram will have two points which are at say height. So, this is compression, heat addition, expansion then T2 = T4 then we have heat rejection if we have the pressure ratio lower than RP optimum in that case we will go like this after compression and let us assume that T max is same we have to go till here then we come back and we have some R dash.

And then this is still 3 dash, which will have same temperature. Now this case will have low efficient net work output since exhaust temperature is more, so more heat is rejected. So that is why we have lower work output. Further, in this case If we have more compression ratio, then optimum for the same T3 we will go like this and expand and this becomes 3 double dash. Here we can see that more compressor work is there in comparison to turbine work.

Turbine work is here and compressor work is here since compressor work is more than turbine work we will have lower net work than the optimum case so only one case has maximum net work output.

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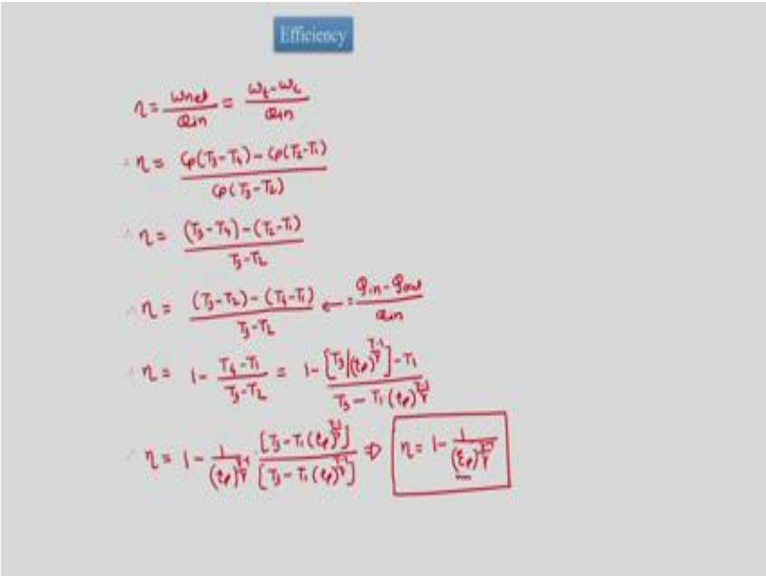

Having said this let us move on for the definition of efficiency

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{W_t - W_c}{Q_{in}}$$

$$\eta = \frac{(T_3 - T_4) - (T_2 - T_1)}{(T_3 - T_2)}$$

$$\eta = \frac{(T_3 - T_2) - (T_4 - T_1)}{(T_3 - T_2)} = \frac{Q_{in} - Q_{out}}{Q_{in}}$$

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{\left[\frac{T_3}{[r_P]^{\frac{\gamma-1}{\gamma}}}\right] - T_1}{T_3 - T_1 [r_P]^{\frac{\gamma-1}{\gamma}}}$$

$$\eta = 1 - \frac{1}{\left[r_{P_{opt}}\right]^{\frac{\gamma-1}{\gamma}}} \frac{\left[T_3 - T_1 [r_P]^{\frac{\gamma-1}{\gamma}}\right]}{\left[T_3 - T_1 [r_P]^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\eta = 1 - \frac{1}{[r_P]^{\frac{\gamma-1}{\gamma}}}$$

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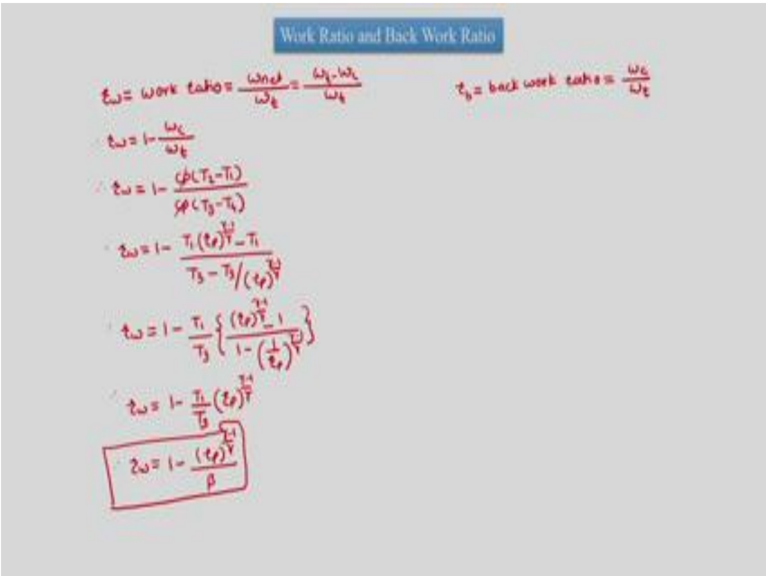

Now let us find out a new term which is called as work ratio or back work ratio. Here we define

$$r_w = work\ ratio = \frac{W_{net}}{W_t} = \frac{W_t - W_c}{W_t}$$

$$r_w = 1 - \frac{W_c}{W_t}$$

$$r_w = 1 - \frac{(T_2 - T_1)}{(T_3 - T_4)}$$

$$r_w = 1 - \frac{T_1 [r_P]^{\frac{\gamma-1}{\gamma}} - T_1}{T_3 - \frac{T_3}{[r_P]^{\frac{\gamma-1}{\gamma}}}}$$

$$r_w = 1 - \frac{T_1}{T_3} \left\{ \frac{[r_P]^{\frac{\gamma-1}{\gamma}} - 1}{1 - [1/r_P]^{\frac{\gamma-1}{\gamma}}} \right\}$$

$$r_w = 1 - \frac{T_1}{T_3}[r_P]^{\frac{\gamma-1}{\gamma}}$$

$$r_w = 1 - \frac{[r_P]^{\frac{\gamma-1}{\gamma}}}{\beta}$$

So this is the formula for work ratio for a Brayton cycle from the known r_P, known beta and known gamma we can find out what is work ratio and there is one more term which is r_b which is back work ratio. Back work ratio is defined as compressor work upon turbine work.

$$r_b = back\ work\ ratio = \frac{W_c}{W_t}$$

Similarly you can derive an expression for back work as well knowing the temperatures and corresponding relations we can find out what is the back work. So these are derivations for work ratio and back work there are certain observations for the Brayton cycle.

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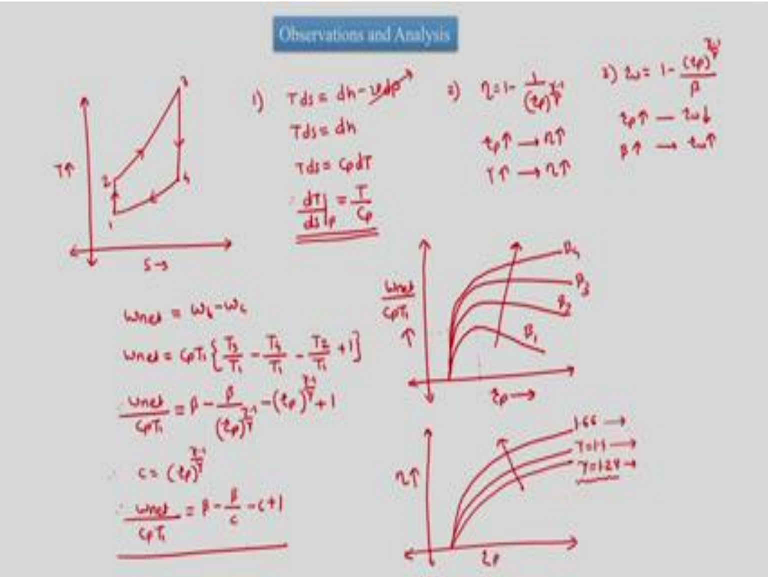

As per those observations we can try to analyse was do we get an extreme site of the Brayton cycle. So, here watching the Brayton cycle we have to take an extreme care such that the process. 1 to 2 to 3 and process 4 to 1 they are isobaric process. So, let us write down the combined first and second law formula which is

$$Tds = dh - vdP$$

$$Tds = dh$$

$$Tds = C_p dT$$

$$\left(\frac{dT}{ds}\right)_P = \frac{T}{C_p}$$

So this is the slope of constant pressure line and the TS diagram, so the slope of process 2 to 3 is increasing as the temperature increases. This is a curved line which has increasing slope from 2 to 3. Similarly process 4 to 1 has decreasing slope from 4 to 1. Hence, the constant pressure lines diverge on the TS diagram this is first observation. Second observation, we have seen that

$$\eta = 1 - \frac{1}{(r_P)^{\frac{\gamma-1}{\gamma}}}$$

$$r_P(\uparrow) \rightarrow \eta(\uparrow)$$

$$\gamma(\uparrow) \rightarrow \eta(\uparrow)$$

From the formula for the work ratio

$$r_w = 1 - \frac{(r_P)^{\frac{\gamma-1}{\gamma}}}{\beta}$$

$$r_P(\uparrow) \rightarrow r_w(\downarrow)$$

$$\beta(\uparrow) \rightarrow r_w(\uparrow)$$

$$W_{net} = W_t - W_c$$

$$\mathrm{W_{net}} = C_p T_1 \left\{\frac{T_3}{T_1} - \frac{T_4}{T_1} - \frac{T_2}{T_1} + 1\right\}$$

$$\frac{W_{net}}{C_p T_1} = \beta - \frac{\beta}{(r_P)^{\frac{\gamma-1}{\gamma}}} - (r_P)^{\frac{\gamma-1}{\gamma}} + 1$$

$$c = (r_P)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{W_{net}}{C_p T_1} = \beta - \frac{\beta}{c} - c + 1$$

Here we can take this formula and we can plot for relation of W_{net} for different beta so this β $\frac{W_{net}}{C_p T_1}$ y axis and r_p on x axis we will get something like this. This is for particular value β_1 this is for particular value β_2 and then this value is β_4, betas are increasing in this direction.

If we would have plotted efficiency versus RP then we would have got plot like this, then this, then this is for gamma 1.28. This is for gamma 1.4 this is for gamma 1.6 so in this direction gamma increases as we have said efficiency increases with RP efficiency also increases with gamma. In case of W net we have seen that for particular RP W net is maximum.

Then for this gamma 1.28 represents triatomic molecule, gamma 1.46 represents diatomic molecule and gamma 1.66 this is means monatomic molecule and then thus we have seen that how to deal with ideal Brayton cycle or air standard Brayton cycle. How to evaluate different state variables performance parameters and what are the observations from the derivations. We will see next things next class, thank you.