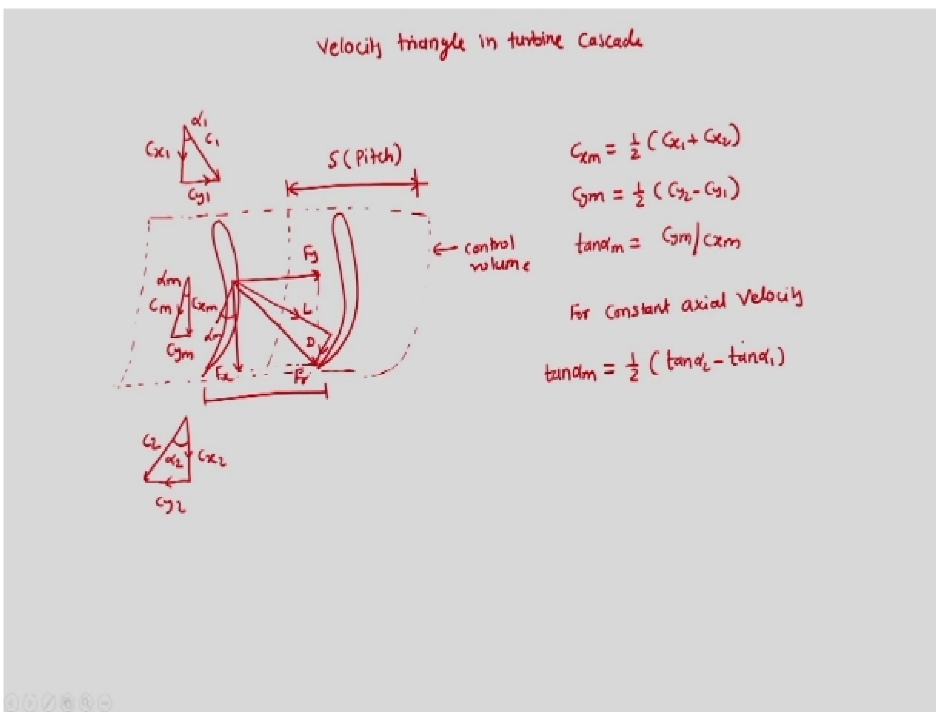
Aircraft Propulsion

Prof. Vinayak N. Kulkarni

Department of Mechanical Engineering
Indian Institute of Technology-Guwahati

Lecture-40
Velocity diagrams of Turbine Cascade, Compressor cascade.

Welcome to the class, we will continue our discussion about blade design and cascade theory till time what we had seen was the cascade tunnel parts of cascade tunnel and then we had seen about the turbine cascade, different angles of the turbine cascade.



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(refer time: 00:47). Now, we will go ahead and talk about velocity triangle. Triangle in turbine cascade we will take a control volume in a turbine cascade which includes Aerofoil sections for 2 nearby blades and then this we will use for our further analysis. So, this is our control volume now which includes 1 pitch at least so this is turbine cascade and now this is control volume now this is spacing between the blades and then this what we call as the pitch S.

And here, we have velocity absolute approaching like this, which is C_1. So, this is C_{x1} and then this is C_{y1} approaching at an angle α_1, we have seen these are the α_1 and α_2 are the air angles at inlet and outlet. So, these are the angles made by the absolute velocity with axial direction. So, similarly this is absolute velocity C_2 leaving the blade, then this is α_2, since this is C_{x2} and this is C_{y2} here from these 2 velocity triangles or mean velocity triangle is constructed.

So, this is C_{xm} and then this is Cm and this is C_{ym} and the angle included is alpha m. In such case the forces is acting are this is F_y, this is F_x and hence the resultant is F_r but this resultant can further be decomposed into 2 more components, which are lift and drag. So, for that these components will be made, which are normal and perpendicular to the velocity mean velocity. So, this is lift and this is drag.

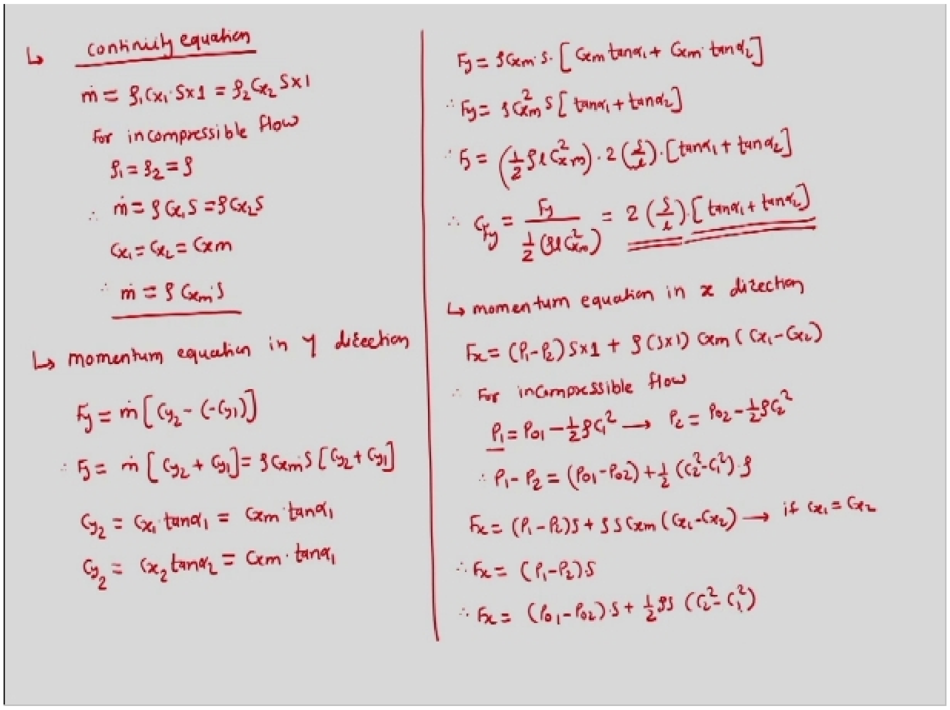
So, this total force F_r actually gets decomposed into two ways one is F_x, F_y which has the coordinate system associated with laboratory coordinate system and then it gets further decomposed into 2 more components which are lift and drag, which are normal to the velocity which are based upon the velocity direction. So, this angle so, this included angle is alpha m. Now, having said this we will write down the expressions.

So, C_{xm} which is axial mean velocity is $\frac{1}{2}(C_{x1} + C_{x2})$ C_{ym}. Since Y_ms are in opposite direction, we will have C_{y2} - C_{y1} and tan $\alpha_m = \frac{C_{ym}}{C_{xm}}$ and then we have for constant axial velocity we have tan $\alpha_m = \frac{1}{2}$ (tan α_2 - tan α_1). So, this is for the case of constant axial velocity. So, now the having these control volume with us, we will go ahead and find out what are the forces acting on the cascade.

(refer time: 05:49). So, let us apply mass conservation or continuity equation and in the continuity equation we have $\dot{m} = \rho_1$ C_{x1} $\times$ S is the pitch $\times$ 1 is the blade height and we are consulting it to be unit. So, similarly we have ρ_2 C_{x2} $\times$ S $\times$ 1 for incompressible flow we have $\rho_1 = \rho_2 = \rho$. So, we have $\dot{m} = \rho C_{x1} S = \rho C_{x2} S$. So, we have $C_{x1} = C_{x2} = C_{xm}$. So, $\dot{m} = \rho C_{xm} S$.

Now, let us apply Y moment Y momentum equation or let us apply momentum equation in Y direction. Here by this we can write the force F_y will have its magnitude as $\dot{m}(C_{y2} - (-C_{y1}))$. So, this is F_y so, what we will have is $F_y = \dot{m}(C_{y2} + C_{y1})$. We can put the expression for $\dot{m}$, $\rho C_{xm} S[C_{y2} + C_{y1}]$ from the triangle we can write down $C_{y2} = C_{x1}$tan α_1.

So, it is equal to C_{xm}tan α_1 similarly, C_{y1} sorry, similarly, $C_{y2} = C_{x2}$tan $\alpha_2 = C_{xm}$tan α_2. So, putting this in the expression for momentum, we can get $F_y = \rho C_{xm}$S [C_{xm}tan α_1 + C_{xm}tan



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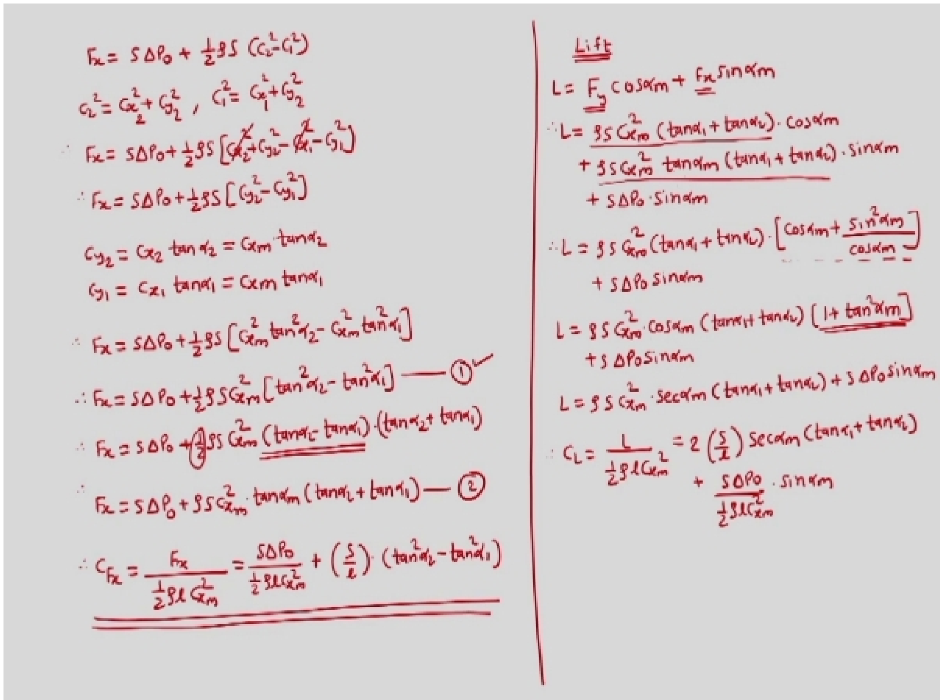
α_2]. So, F$_y$ = ρC$^2_{xm}$S [tan α_1 + tan α_2], we can rearrange the term and write down as ($\frac{1}{2}$ ρ l C$^2_{xm}$) × 2 × $\left(\frac{S}{l}\right)$ × [tan α_1 + tan α_2].

We are purposely doing since we can write down these coefficient of force C$_{F_y}$ = $\frac{F_y}{\frac{1}{2}(\rho l C^2_{xm})}$ and then this gives us answer as 2 × $\left(\frac{S}{l}\right)$ × [tan α_1 + tan α_2]. so, if we know α_1 and α_2, for given geometry and arrangement, we can find out what is the coefficient of the normal force, which is acting under blades. Then we can write down the momentum equation in x direction.

So, we will have F$_x$ is equal to, here we just would not have force momentum change equal to force we will have pressure change as well. So, P$_1$ - P$_2$ with the change in pressure in the direction of flow which is S × 1 is the area + ρ × (S × 1) for unit blade height C$_{xm}$ × (C$_{x1}$ - C$_{x2}$). So, we have for incompressible flow we can write down P$_1$ = P$_{01}$ - $\frac{1}{2}$ ρ C$^2_1$.

Basically, this expression says that P$_{01}$ = P$_1$ + $\frac{1}{2}$ ρ V^2. This is strictly for incompressible flows, where total pressure is equal to static plus dynamic pressures. So, we are finding our static pressure from the total pressure and the dynamic pressure after subtracting them. Similarly we can write down P$_2$ = P$_{02}$ - $\frac{1}{2}$ ρ C$^2_2$. So, we can write P$_1$ - P$_2$ is equal to P$_{01}$ - P$_{02}$ + $\frac{1}{2}$(C$^2_2$ - C$^2_1$)ρ.

So, we can write down expression for F$_x$ again (P$_1$ - P$_2$)S + ρSC$_{xm}$(C$_{x1}$ - C$_{x2}$). Take a particular case where we have if $C_{x1} = C_{x2}$ we can write down F_x only as $(P_1 - P_2)S$. So, $F_x = (P_{01} - PP_{02})S + \frac{1}{2}\rho S(C_2^2 - C_1^2)$.



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(refer time: 13:24). We can write down the same expression $F_x = S\Delta P_0 + \frac{1}{2}\rho S(C_2^2 - C_1^2)$ here, we know that $C_2^2 = C_{x_2}^2 + C_{y_2}^2$ and $C_1^2 = C_{x_1}^2 + C_{y_1}^2$. So, $F_x = S\Delta P_0 + \frac{1}{2}\rho S(C_{x_2}^2 + C_{y_2}^2 - C_{x_1}^2 - C_{y_1}^2)$ and both being equal, it would get cancelled. So, $F_x = S\Delta P_0 + \frac{1}{2}\rho S(C_{y_2}^2 - C_{y_1}^2)$, but from velocity triangle. We can write down $C_{y_2} = C_{x_2}\tan\alpha_2 = C_{x_m}\tan\alpha_2$.

Similarly, $C_{y_1} = C_{x_1}\tan\alpha_1 = C_{x_m}\tan\alpha_1$. So, $F_x = S\Delta P_0 + \frac{1}{2}\rho S(C_{x_m}^2\tan^2\alpha_2 - C_{x_m}^2\tan^2\alpha_1)$. So, we will have $F_x = S\Delta P_0 + \frac{1}{2}\rho SC_{x_m}^2(\tan^2\alpha_2 - \tan^2\alpha_1)$. So, $F_x = S\Delta P_0 + \frac{1}{2}\rho SC_{x_m}^2(\tan\alpha_2 - \tan\alpha_1)(\tan\alpha_2 + \tan\alpha_1)$. These are the 2 expressions for us further, we know $\frac{1}{2}$ into this term can lead to $\tan\alpha_m$.

So, we can write down it as $S\Delta P_0 + \frac{1}{2}\rho SC_{x_m}^2.\tan\alpha_m.(\tan\alpha_2 + \tan\alpha_1)$. We will remember this expression and we will name it as number 1 and we will name it as number 2. So, we can write down $C_{F_x} = \frac{F_x}{\frac{1}{2}(\rho l C_{xm}^2)}$ and this gives us if we use equation number 1. We can get as $\frac{S_0}{\frac{1}{2}(\rho l C_{xm}^2)}$ will cancel and we will have $\frac{S}{l}(\tan^2\alpha_2 - \tan^2\alpha_1)$.

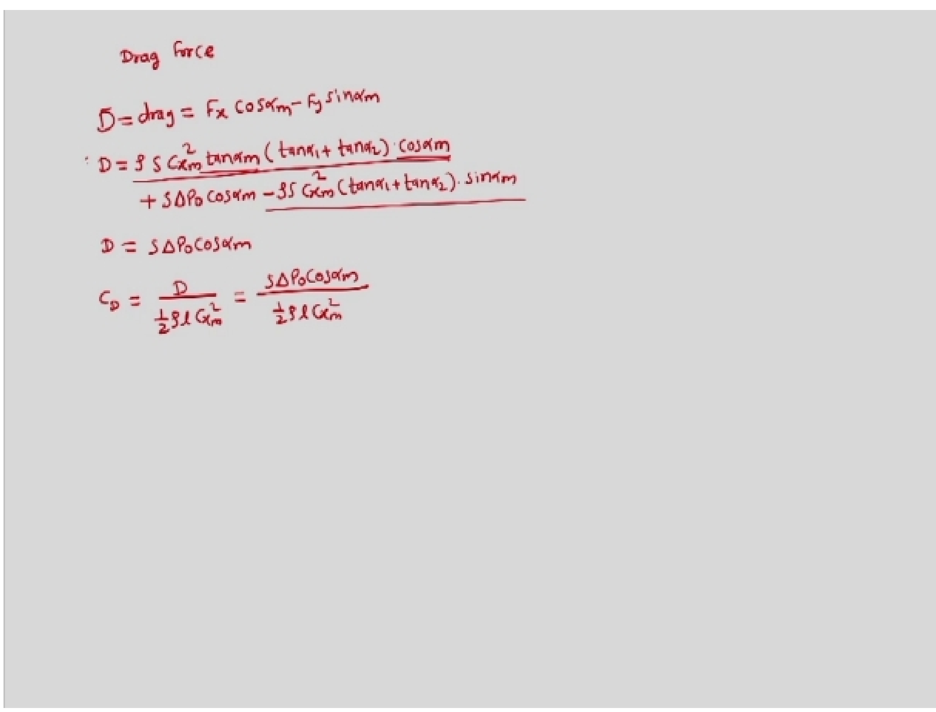
So, this is our non-dimensional form of axial force coefficient having said this, we can move ahead and find out what is lift and drag from the coefficients whatever we have got or from the forces whatever we have arrived at so, L lift from the force resultant force and its decomposition we can clearly mentioned a lift is equal to $F_y \cos \alpha_m + F_x \sin \alpha_m$. So, L is equal to F_y we have found out F_x.

We have found out we can keep those expressions written over here $\rho SC^2_{x_m}.(\tan \alpha_1 + \tan \alpha_2).\cos \alpha_m + \rho SC^2_{x_m}.\tan \alpha_m.(\tan \alpha_1 + \tan \alpha_2).\sin \alpha_m + S\Delta P_0.\sin \alpha_m$. Now here, we can rearrange and then we can take $\rho SC^2_{x_m}.(\tan \alpha_1 + \tan \alpha_2)$ common from first 2 terms of this.

And then we can have $\rho SC^2_{x_m}.(\tan \alpha_1 + \tan \alpha_2).[\cos \alpha_m + \frac{sin^2 \alpha_m}{cos \alpha_m} + S\Delta P_0.\sin \alpha_m$. Now, we can take $\cos \alpha_m$ also common from this bracket and then we have $L = \rho SC^2_{x_m}.\cos \alpha_m(\tan \alpha_1 + \tan \alpha_2).\left[1 + tan^2 \alpha_m\right] + S\Delta P_0.\sin \alpha_m$.

So, l but we know $1 + \tan^2 \alpha$ is equal to $1 + \sec^2 \alpha$. So, here we will have $\sec^2 \alpha_m$, but we have one more $\cos \alpha_m$ here. So, that is a reciprocal so, we can write it as $\rho SC^2_{x_m}.\sec \alpha_m(\tan \alpha_1 + \tan \alpha_2) + S\Delta P_0.\sin \alpha_m$. So, this is our expression for the lift, OK.

So, we can write down C_L, which is lift coefficient as $\frac{L}{\frac{1}{2}\rho l C^2_{xm}}$ and then we can get it as $2\left(\frac{S}{l}\right)\sec \alpha_m$ $(\tan \alpha_1 + \tan \alpha_2) + \frac{S_0}{\frac{1}{2}\rho l C^2_{x_m}}.\sin \alpha_m$. So, knowing this, we can find out what is the lift coefficient which is getting acted on the cascade which is a turbine cascade.



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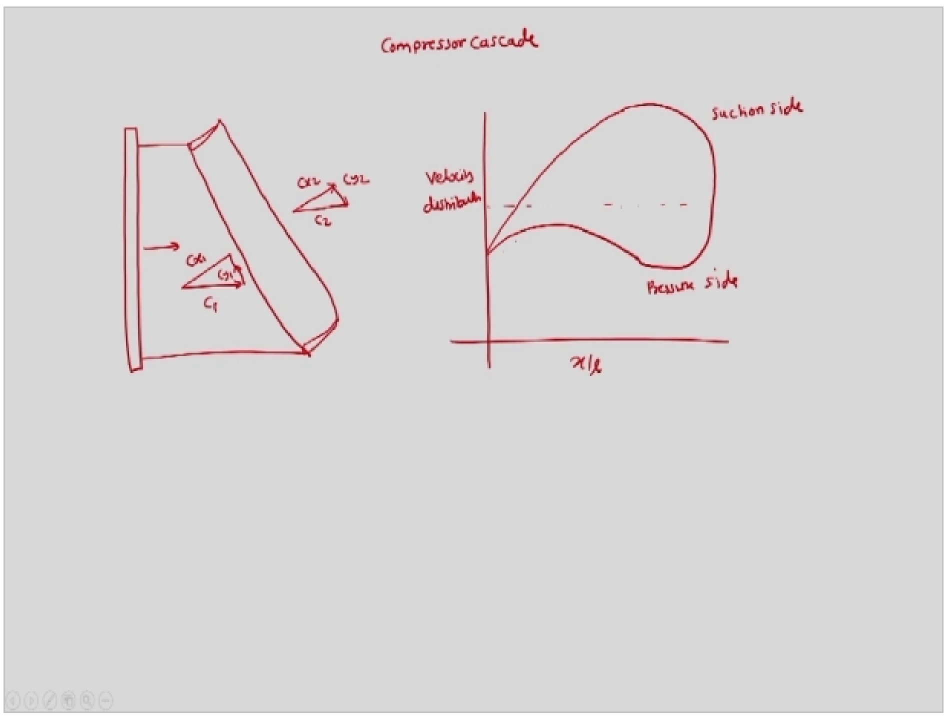
(refer time: 22:40). Now, we will move ahead and then we can find out drag force for the turbine cascade. So, far drag force again from the force resultant force and its decomposition, we can write down D which is a drag which is equal to F_x cos α_m - F_y sin α_m. So, we have D we can put the expression for F_x as $\frac{\rho S C_{x_m}^2 tan\alpha_m (tan\alpha_1 + tan\alpha_2) cos\alpha_m}{S_0 cos\alpha_m - \rho S C_{x_m}^2 (tan\alpha_1 + tan\alpha_2) sin\alpha_m}$.

Here, it is good point to note that we have tan α_m and cosα_m so, this would ultimately be sin α_m. So, we can see that the first term and third term they are same. So, D = $S_0 cos\alpha_m$. So, we can get drag coefficient, C_D as $\frac{D}{\frac{1}{2}\rho l C_{x_m}^2} = \frac{S_0 cos\alpha_m}{\frac{1}{2}\rho l C_{x_m}^2}$ and this is how we would have found out different coefficients and the forces which are acting on the cascade.

And we can see or hear that everything depends upon the setting of the angle and a blade angles and the angles whatever we would have used for the blade or the air and also for the setting.



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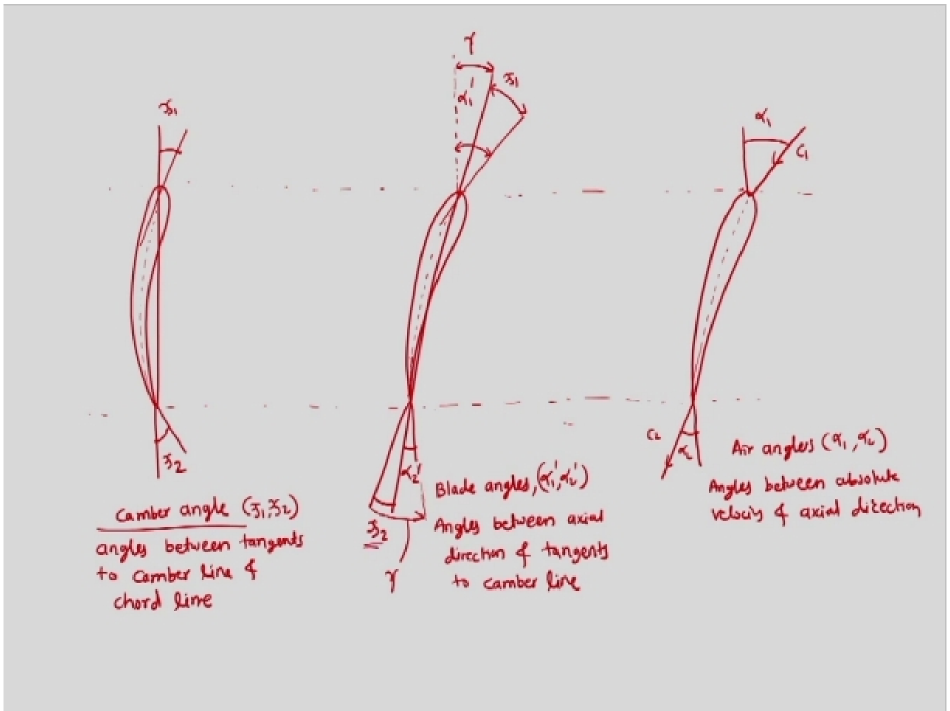
(refer time: 25:08). So, we will move ahead for the compressor cascade and we can note that for the compressor cascade first it will be the same wind tunnel which is the cascade tunnel getting applied for the application of the compressor. So, we will change our turbine cascade to the compressor cascade in case of compressors. So, this is the C_1 so, this will be C_{x1} this will be C_{y1} this will be C_{x2} this will be C_{y2} and then this will be C_2 on a particular Aerofoil.

If we plot the velocity distribution then this velocity distribution for $\frac{x}{l}$ can be seen like this. This is a typical velocity distribution for suction side on top and for pressure side on bottom, pressure side now again.



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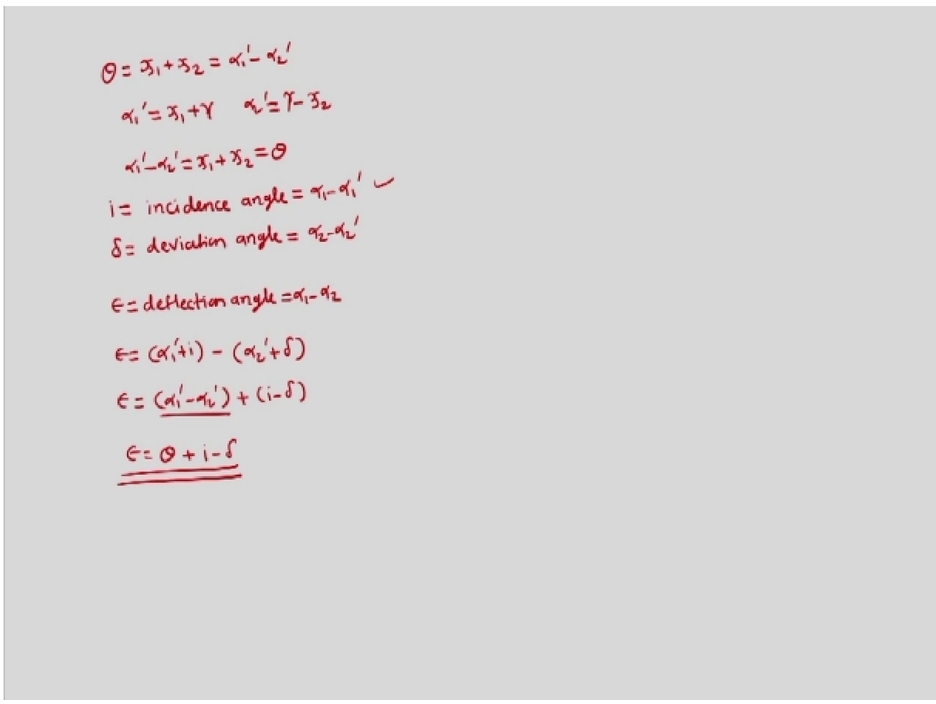
(refer time: 27:11).We will draw the angles for the cascade which is a compressor cascade as what we did earlier. In case of turbine we do 3 types of angles first angle was camber angle. So, for camber angle so, this is camber line, so, this is the vertical line. So, this is the tangent to the camber at the inlet, so, this is χ_1 and this is χ_2. Basically this vertical line is the chord line in this specific case. So, this is chi 2.

So, this is camber angle as what we had seen earlier this is χ_1 and χ_2, they are the angles between tangents to camber line and chord line then, we will show the blade angles. So, for the blade angle we will make the chord little bit tilted. So, now this is chord and now this is camber line and now this is tangent to the camber. So, as what we know this angle is χ_1 and this is we will put it here χ_2.

Now, I will plot the axial direction here and then we will have the tangent and the tangent and the axial direction we will have angle $\alpha_1^{'}$. So, similarly here also it is $\alpha_2^{'}$ so, this is blade angle which are $\alpha_1^{'}$ and $\alpha_2^{'}$ angles between axial direction and tangents to camber line and then we can have third which is for air angle. So, same way we will replicate.

This is camber line and this is axial direction and this is supposed the direction of velocity, which is C_1 so, this angle is α_1. Similarly, if velocity is leaving, absolute velocity in this direction and this is α_2 this is the axial direction. So, we have air angles which are α_1, α_2. So they are angles between absolute velocity and axial direction. So, having said this, we can continue here showing the other angles among that first was the stagger angle.

And, stagger angle can be shown over here this γ will be the stagger angle which is between the chord line and the axial direction.



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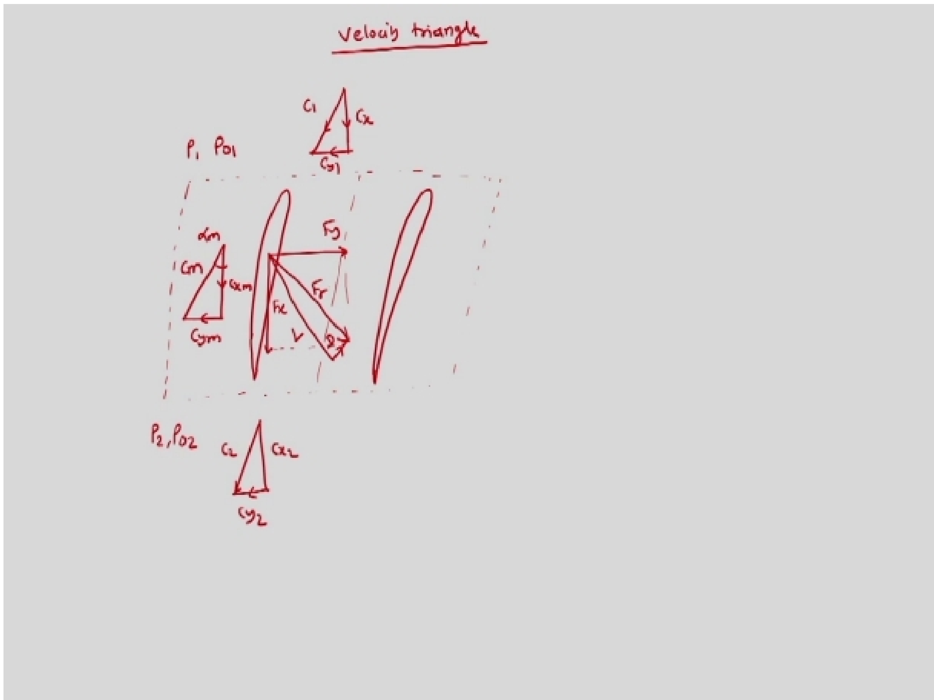
(refer time: 32:58). So, other settings we can easily make out as what we did earlier camber angle $\theta = \chi_1 + \chi_2$, which is $\alpha_1^{'} - \alpha_2^{'}$ further $\alpha_1^{'} = \chi_1 + \gamma$ and $\alpha_2^{'} = \gamma - \chi_2$, it should be noted here that, this angle which is between the tangent which is between the so, this big angle is γ in the outside in the outlet. So, this γ is equal to sorry this $\gamma - \chi_2$ is $\alpha_2^{'}$.

So, we can write down $\alpha_1^{'} - \alpha_2^{'} = \chi_1 + \chi_2$, which is θ. So, i we have defined as incidence angle so, it is $\alpha_1 - \alpha_1^{'}$, we have defined δ which is deviation angle and then that is $\alpha_2 - \alpha_2^{'}$. So, we have ε, which is deflection angle and then that is $\alpha_1 - \alpha_2$. So, $\varepsilon = \alpha_1^{'} + \mathrm{i}$, from the expression of this $(\alpha_1^{'} + \mathrm{i}) - (\alpha_2^{'} + \delta)$.

So, deflection is $(\alpha_1^{'} - \alpha_2^{'}) + (i - \delta)$, but $\alpha_1^{'} - \alpha_2^{'}$ is equal to θ. So, it is $\theta + i - \delta$. This is the deflection angle so, now for the compressor cascade.



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We can again plot the velocity triangle. So, velocity triangle for compressor cascade here as well, we will draw a control volume so, far that let us first plot the Aerofoil and let us complete the control volume. So, this is the control volume for the compressor cascade. So, here we will say this is P_{01} and P_1 and this is P_2 and P_{02} say this is inlet, so, for the inlet we know that this is the direction of absolute velocity.

This is direction of axial velocities, this is C_{y1}. Similarly, if we take outlet, then for outlet, this is C_2, this is C_{x2} and this is C_{y2}. So, we can draw the mean velocity triangle as C_{x_m} this is C_m and this is C_{y_m} and this angle mean is α_m. So, this is the typical velocity triangle for compressor cascade. Now, we can show the forces same way as what we did for turbine, this is F_y this is F_x.

So, this becomes the resultant F_r now here we are again needs to decompose them into lift and drag. So, this is lift and this is drag this resultant is F_r.

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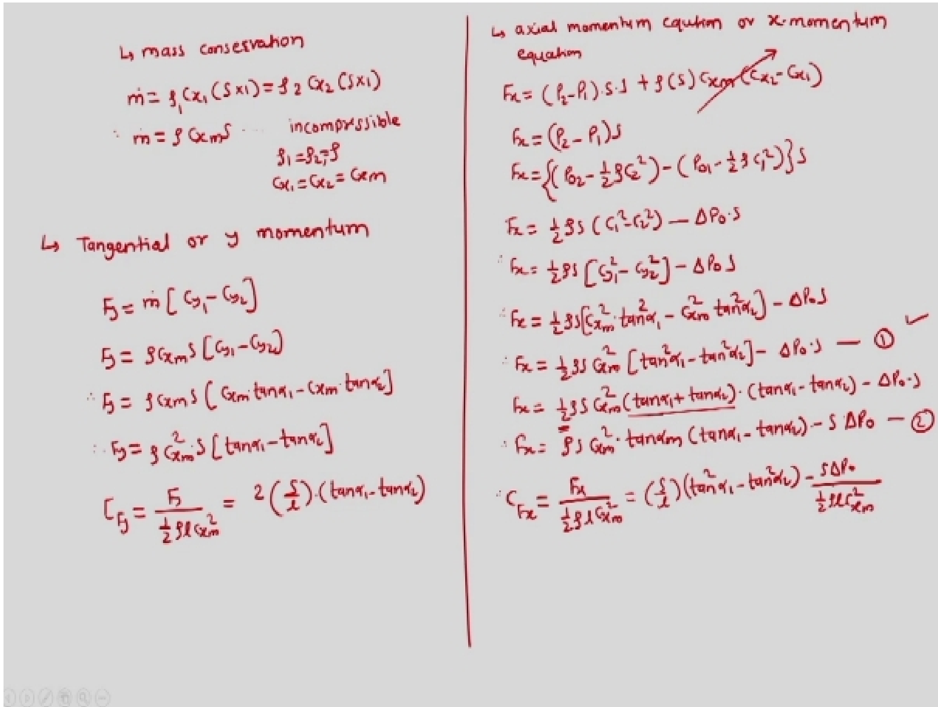

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we know at the outlet ρ_2 C_{x2} S $\times$ 1 for unit blade height but again we will say that C_{x1} = C_{x2} = C_{xm} for the constant axial velocity, we can get it as rρ C_{xm}S where we are saying it as incompressible which has led to $\rho_1 = \rho_2 = \rho$ and then we have said $C_{x1} = C_{x2} = C_{xm}$.

So, this is how we would have the mass conservation equation for compressor cascade which is same as what for the turbine cascade. Now, let us apply tangential or y momentum equation if we apply we can get $F_y = \dot{m}(C_{y1} - C_{y2})$ here it should be remembered that both y components are in the same direction, which was in the case of turbine both were in opposing, so, we had $C_{y1} + C_{y2}$.

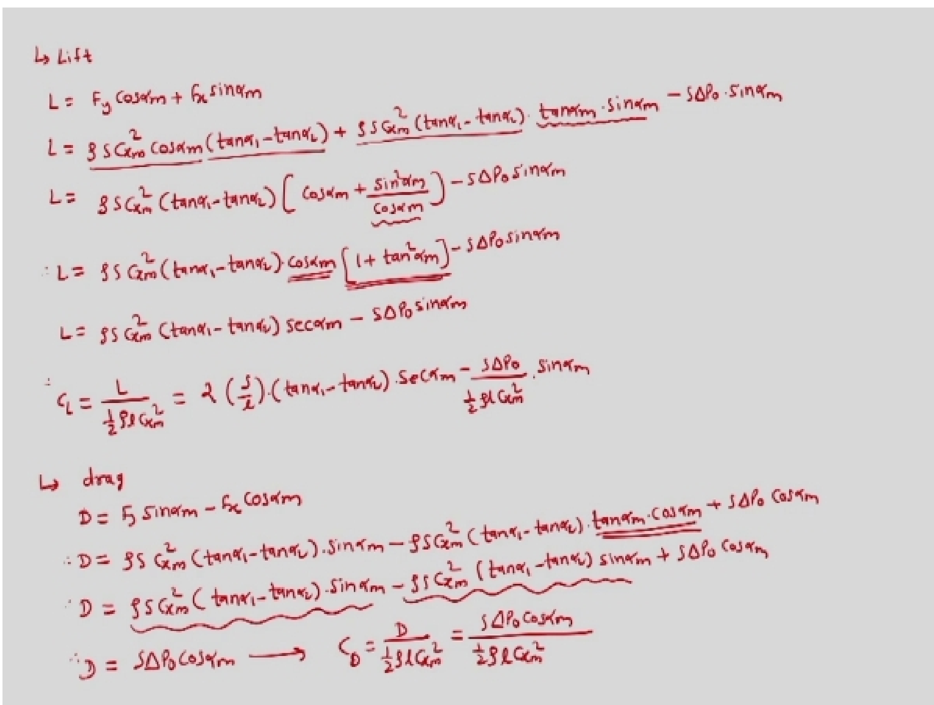
So, we can write down put m dot expression for $\dot{m}$, $\rho C_{xm}S(C_{y1} - C_{y2})$. So, we will have $F_y = \rho C_{xm}S$, we can put C_y as $[C_{xm}\tan\alpha_1 + C_{xm}\tan\alpha_2]$. So, $F_y = \rho C_{xm}^2 S\ [\tan\alpha_1 - \tan\alpha_2]$. So, we can write down coefficient of force F_y, $C_{F_y} = \frac{F_y}{\frac{1}{2}(\rho l C_{xm}^2)} = 2 \times \left(\frac{S}{l}\right) \times [\tan\alpha_1 - \tan\alpha_2]$. Similarly, we can apply the axial momentum equation or x momentum equation.

There, we can write down F_x is equal to $(P_2 - P_1)S\ .1 + \rho S C_{xm}(C_{x2} - C_{x1})$ but again, we would have same axial coefficient velocity so, $F_x = P_2 - P_1$ we already know that $P_2 = [P_{02} - \frac{1}{2}\ \rho\ C_2^2 - (P_{01} - \frac{1}{2}\ \rho\ C_1^2)] \times S$. Here, we can mention F_x as $\frac{1}{2}\ \rho S\ (C_1^2 - C_1^2) - \Delta P_0 S$. So, ΔP_0 is $P_{01} - P_{02}$ are here, since we have minus sign here. So, Fx can be written as the $\frac{1}{2}\ \rho S$.

So, C_1^2 can be split into $C_{x1}^2 + C_{y1}^2$. Similarly C_2^2 will also be split into $C_{x2}^2 + C_{y2}^2$ and C_{x1} and C_{x2} both are equal. So, we will have only y_2 remaining so, C_{y1}^2 - C_{y1}^2 - $\Delta P_0 S$, we can put the value of C_{y1} using the velocity triangle as $[C_{x_m}^2 \tan^2 \alpha_1$ - $C_{x_m}^2 \tan^2 \alpha_2]$ - $\Delta P_0 S$. So, $F_x = \frac{1}{2}$ $\rho S C_{xm}^2$ $[\tan^2 \alpha_1$ - $\tan^2 \alpha_2]$ - $\Delta P_0 S$.

Similarly, we can decompose we can keep this equation 1 remembered, we can decompose the bracket and then write $\frac{1}{2}$ $\rho S C_{xm}^2$ $[\tan \alpha_1 + \tan \alpha_2]$ $[\tan \alpha_1$ - $\tan \alpha_2]$ - $\Delta P_0 S$. So, F_x we can make this $\frac{1}{2}$ and this bracket and then we can have $\rho S C_{xm}^2.\tan \alpha_m$ $[\tan \alpha_1$ - $\tan \alpha_2]$ - $\Delta P_0 S$.

So, we can write down force coefficient in x direction as $\frac{F_x}{\frac{1}{2}(\rho l C_{xm}^2)}$, and then we will get it as if we use this expression. This will be remember as 2 and then we can get it as, $\frac{S}{l}[\tan^2 \alpha_1$ - $\tan^2 \alpha_2]$ - $\frac{S_0}{\frac{1}{2}(\rho l C_{xm}^2)}$. Now, having this force coefficient in axial direction known and similarly force coefficient in normal direction known.



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(refer time: 46:27). We can find out lift and drag so, lift L is equal to from the decomposition of forces, $F_y \cos \alpha_m + F_x \sin \alpha_m$. So, lift from the F_y expression $\rho S C_{x_m}^2.\cos \alpha_m.(\tan \alpha_1$ - $\tan \alpha_2) + \rho S C_{x_m}^2.(\tan \alpha_1$ - $\tan \alpha_2).\tan \alpha_m.\sin \alpha_m$ - $S\Delta P_0.\sin \alpha_m$. So, L is equal to we can take this common from both the terms and then we can take.

We can write down $\rho SC^2_{x_m}$.(tan α_1 - tan α_2).[cos α_m + $\frac{sin^2\alpha_m}{cos\alpha_m}$. Here, we have 2 terms, these 2 terms mean this single term which is sin α_m divided by cos α_m. So, SΔP$_0$.sin α_m. so, we have $\rho SC^2_{x_m}$(tan α_1 + tan α_2), we can still take cos α_m common once, we take cos α_m common.

We know it will decompose into tan^2, 1 + tan^2 α_m - SΔP$_0$.sin α_m. So, we can say that $\rho SC^2_{x_m}$(tan α_1 - tan α_2) so, this cos α_m and then this term is sec^2 α_m. So, we will have sec α_m - SΔP$_0$.sin α_m. So, we can write down L as this and hence, we can further say that we can further find out the lift coefficient as $\frac{L}{\frac{1}{2}\rho l C^2_{xm}}$, and then we can have it as 2 $\times$ $\frac{S}{l}$ ρ C$_{xm}$ will get cancelled.

Then, we have (tan α_1 - tan α_2) sec α_m - $\frac{S_0}{\frac{1}{2}\rho l C^2_{xm}}$.sin α_m. So, this is the expression for lift coefficient now, we can find out drag so, drag from the decomposition of force is same F$_y$ sin α_m - F$_x$ cos α_m. So, we can put the expression for F$_y$, $\rho SC^2_{x_m}$.(tan α_1 - tan α_2).sin α_m - $\rho SC^2_{x_m}$.(tan α_1 - tan α_2).tan α_m.cos α_m + SΔP$_0$.cos α_m.

So, we have D = $\rho SC^2_{x_m}$.(tan α_1 - tan α_2).sin α_m - $\rho SC^2_{x_m}$.(tan α_1 - tan α_2).sin α_m + SΔP$_0$.cos α_m. So, we will have SΔP$_0$.cos α_m, we can see that the first 2 terms are same so, we have drag is equal to SΔP$_0$.cos α_m. So, we can write down drag coefficient C$_D$ = $\frac{D}{\frac{1}{2}\rho l C^2_{xm}}$. So, it is $\frac{S\Delta P_0 cos\alpha_m}{\frac{1}{2}\rho l C^2_{xm}}$.

So, this is the method by which we can find out the different forces acting on the cascade. So, here what we have seen that what are the different angles associated with the cascade testing or blade design or blade testing the angles like camber angle, air angle, blade angle. So, these are the angles and once we set these angles, we measure the direction of velocities and we can get the drag forces acting on the drag and lift or the F$_x$ and F$_y$ forces acting on the cascade.

So, having said this, we have talked about the complete blade design and cascade tunnel and the gas flow through a cascade. Here, we have completed the portion which is related with the blade design and cascade theory. Thank you.