# Aircraft Propulsion

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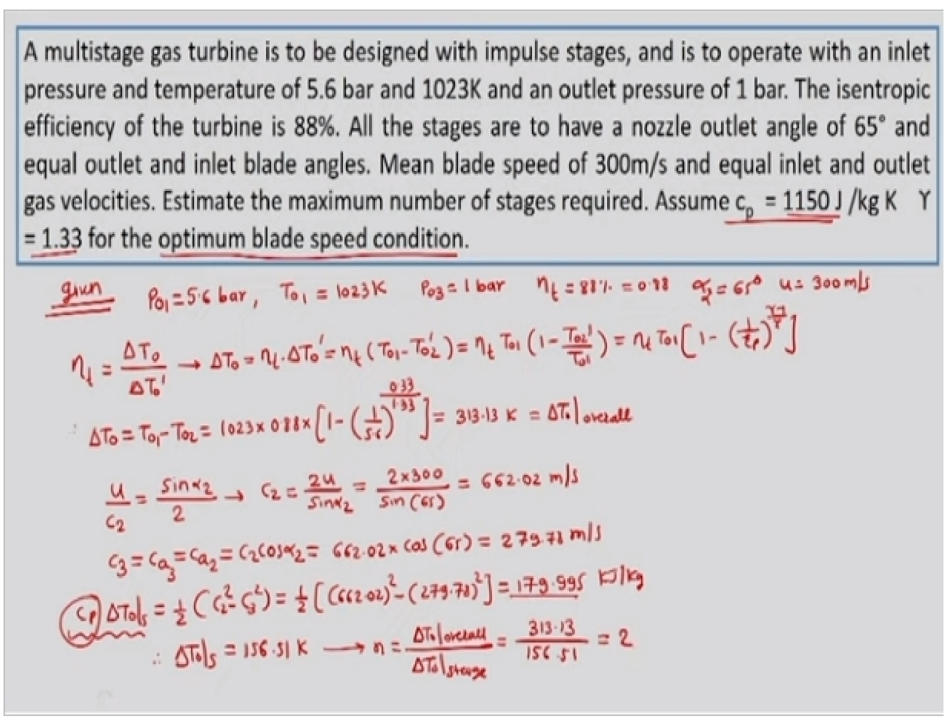
**Lecture-37**
**Practice Examples of Axial Turbine and Centrifugal Compressor**

Welcome to the class. As what it is going on, we will continue with some more practice examples. Now, we are going towards turbo machinery examples. We will solve the first example.

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(refer time : 00:46). It is states that a multistage gas turbine is to be designed with impulse stages and it is to operate with an inlet pressure and temperature of 5.6 bar and 1023 K and the outlet pressure of 1 bar. Isentropic efficiency of the turbine is 88%. All the stages are to have a nozzle outlet angle of 65° and equal outlet and inlet blade angles. Mean blade speed of 300 m/s equal inlet and outlet gas velocities. Estimate maximum number of of stages required.

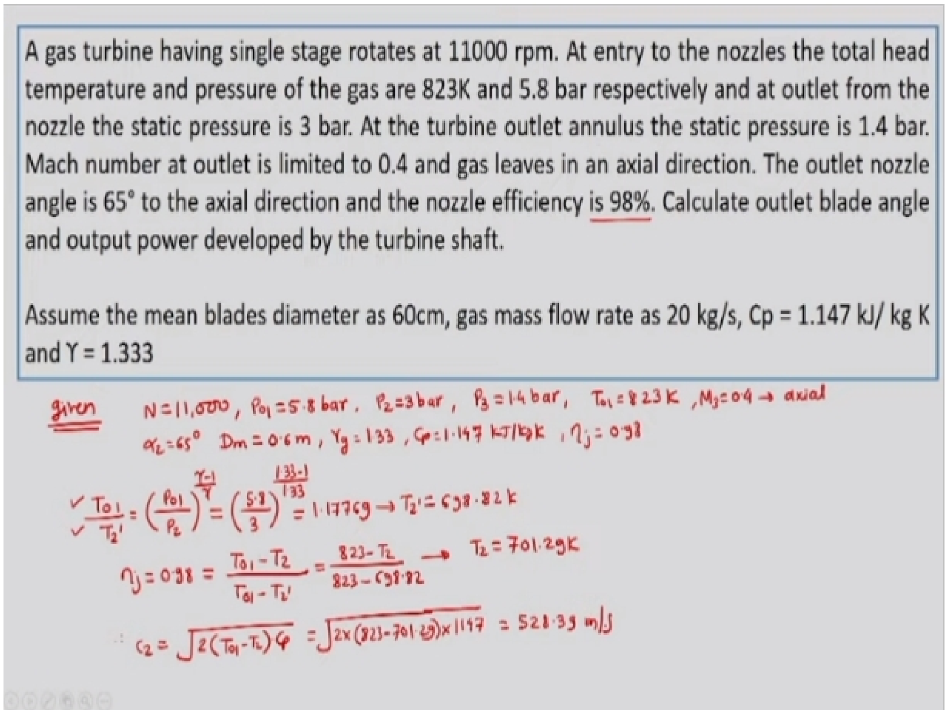
Assume $C_p$ as 1150 J/kg.K, $\gamma$ is equal to 1.33 for optimum blade speed ratio. So, we will see what is given given thing in this example, is $P_{01}$ as 5.6 bar, $T_{01}$ as 1023 K. We are told that outlet pressure means $P_{03}$ is $P_{03}$ is 1 bar. Then turbine efficiency is told as 88%, so it is 0.88, $\alpha_1$ is $\alpha$ basically 2, $\alpha_2$ is 65° and u is 300 m/s. Further $C_p$ and $\gamma$ are given. Let us work out with this example.

We know $\eta_t$ is $\frac{\Delta T_0}{\Delta T_0'}$. So, we can find out $\Delta T_0 = \eta_t\ \Delta T_0'$, which is equal to $\Delta T_0.(T_{01} - T_{02}')$. So here, we can take $T_{01}$ common. So, it will be $\eta_t\ T_{01}\left[1-\frac{T_{02}'}{T_{01}}\right]$. So, it is $\eta_t\ T_{01}\left[1-\left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}}\right]$. So, we can get $\Delta T_0$, which is $T_{01} - T_{02}$ is equal to everything is known to us over here, 1023 $\times$ 0.88 $\times$ (1 - $\left(\frac{1}{5.6}\right)^{\frac{0.33}{1.33}}$.

So, we will get total temperature change across the turbine as 313.13 K. It is basically $\Delta T_0|_{overall}$. Now, we are told that we have to work with optimum condition. For optimum condition, in one of the earlier examples, in similar way, we have designed, we have derived the condition as $\frac{u}{C_2}$ is equal to $\frac{sin\alpha_2}{2}$. So, $C_2 = \frac{2u}{sin\alpha_2}$. So, u is given, it is 300 divided by sin 65 and then we get $C_2$ as 662.02 m/s.

We can find out now $C_3$, but for optimum condition, $C_3$ is equal to axial velocity and axial velocity at the inlet is equal to the axial velocity at the outlet, but at the inlet it is $C_2$ cos $\alpha_2$. So, it is 662.02 $\times$ cos 65. So, it is 279.78 m/s. So, we know basically that change of $\Delta T_0|_s \times C_p = \frac{1}{2}(C_2^2 - C_3^2)$. Change in kinetic energy is going to drop, the enthalpy in a stage. So half $C_2^2$ is known. Now, to us as $(662.02)^2$, $C_3$ is known $(279.78)^2$.
So this kinetic energy change is 179.999, 179.995 kJ/kg. So this gives us $\Delta T_0|_s$ as 156.51 K, we know $C_p$, it is 1150. So, if we divide this by 1150, we get 156.51. So, we can find out the number of stages n = $\frac{\Delta T_0|_{overall}}{\Delta T_0|_{stage}}$ and this is equal to 313.13/156.51 so this gives you number of stages as 2. So, the answer as number of stages is equal to 2.

(refer time : 07:41). So, next example says that a gas turbine having single stage rotates at 11000 rpm. At entry to the nozzles, the total head temperature and pressure of the gas are 823 K and 5.8 bar respectively and outlet from the nozzle, the static pressure is 3 bar. At the turbine outlet annulus, the static pressure is 1.4 bar. Mach number at the outlet is limited to 0.4 and gas leaves in an axial direction. The outlet nozzle angle is 65° to the axial direction and nozzle efficiency is 98%.
Calculate outlet blade angle and outlet output power developed by the turbine shaft. Assume

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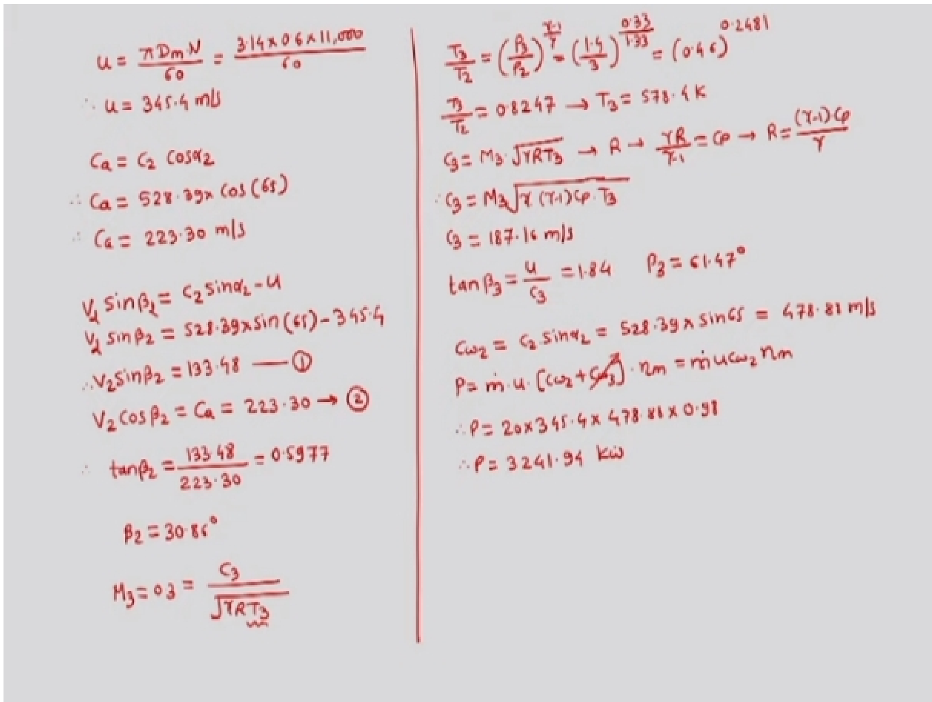
the mean diameter mean blades diameter as 60 cm, gas flow rate as 20 kg/s, $C_p$ as 1.147 kJ/kg.K and $\gamma$ as 1.33. Having said this, we can proceed with the given things and for this axial turbine case, given things are rpm as 11000, $P_{01}$ as 5.8 bar, then two static pressures are given, one is $P_2$ which is 3 bar and other is $P_3$, which is 1.4 bar.

$T_{01}$ is also given as 823 K. We are told that outlet Mach number is maximum 0.4, so let us take it and then direction is told to be axial. $\alpha_2$ is said as 65°, mean diameter is said as 0.6 m, $\gamma_g$ is 1.33 and $C_p$ is 1.147 kJ/kg.K. Let us start first, we will use the nozzle data to find out the nozzle exit absolute velocity. For that, we know that $\frac{T_{01}}{T_2'}$ this is the total temperature at the inlet of the nozzle, this is static temperature ideal at the outlet of the nozzle.

Nozzle being isentropic, this ratio can be $\left(\frac{T_{01}}{T_2'}\right)^{\frac{\gamma-1}{\gamma}}$. $P_{01}$ is 5.8, $P_2$ is 3 bracket raise to $\frac{1.33-1}{1.33}$. So, we can get this ratio as 1.17769 and then this gives us $T_2'$ ideal temperature at the nozzle exit as 698.82 K. Then, we can find out the actual temperature since nozzle efficiency is given as 0.98. So we can use the formula, which is actual conversion to kinetic energy divided by ideal conversion to kinetic energy.

So, $\frac{823-T_2}{823-698.82}$, so this gives us $T_2$ = 701.29 K. Hence we can find out $C_2 = \sqrt{2(T_{01}-T_2)C_p}$, which is basically Δh. So is equal to $\sqrt{2(823-701.29)1147}$ and this gives us absolute velocity as 528.39 m/s.

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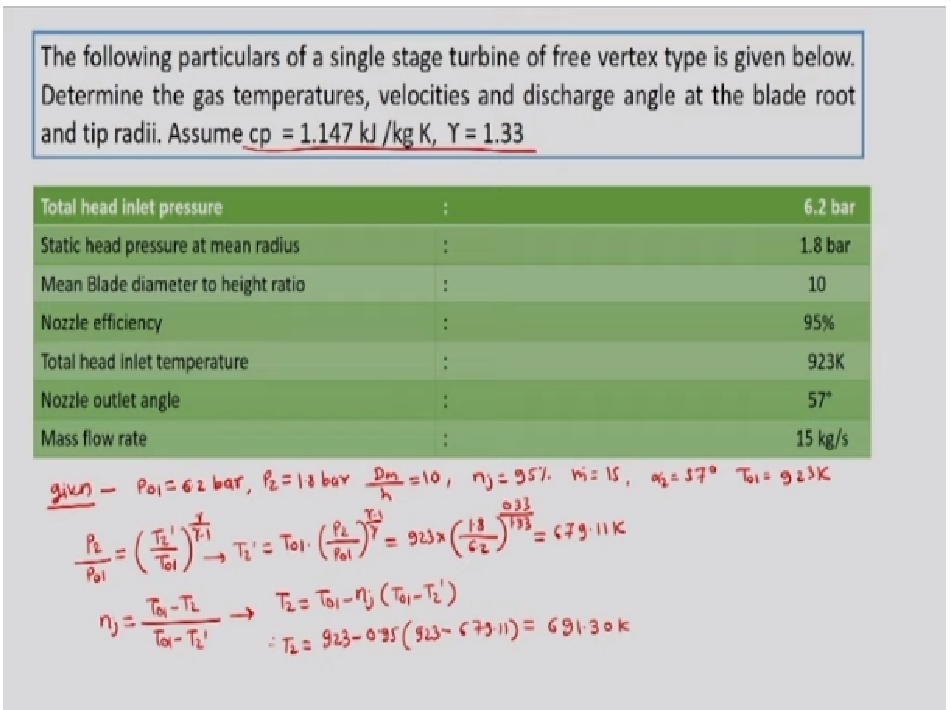
(refer time : 12:42). Now, we can also find out, from the known data, the tangential velocity of the blade, which is $\frac{\pi D_m N}{60}$, which is 3.14 mean diameter is given as 0.6 into N which is told as 11000 divided by 60. So we get u as 345.4 m/s. Now, we know that axial velocity is $C_2$cos $\alpha_2$, so we get axial velocity from 528.39 × cos 65. So, we will have axial velocity as 223.30 m/s. Now, we need to find out angles. So for that, we can make use of the axial velocity and the inlet velocity triangle.

Which says that $C_a$ is equal to here, we can make use of the fact that $V_1$ sin $\beta_1$ = $C_2$ sin $\alpha_2$ - u. And here, we know $V_1$, basically $V_2$, $V_2$ sin $\beta_2$ = 528.39 × sin $\alpha_2$ is 65 - u. And u is 345.4. So this gives us, $V_2$ sin $\beta_2$ as 133.48. This is equation number 1. Similarly, $V_2$ cos $\beta_2$ = $C_a$ and $C_a$ is 223.30. This is equation 2. We can divide both the equation and we can get tan $\beta_2$ = 133.48/223.30 and we can get it as 0.5977.

Thus we get $\beta_2$ as 30.86°. Having said this, we can make use of Mach number, where we are told that exit Mach number is 0.3. So, it is $\frac{C_3}{\sqrt{\gamma R T_3}}$. So, we can make use of this. Now, for this, we should know T3, but we are not aware about T3, but we can make use of isentropic relation, which says that $\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$. So, it is $\left(\frac{1.4}{3}\right)^{\frac{0.33}{1.33}}$. So, it is $(0.46)^{0.2481}$.

And we get $\frac{T_3}{T_2}$ = 0.8247, so we get $T_3$ = 578.4 K. So, having said this, we can now find out $C_3$ = $M_3$ $\sqrt{\gamma R T_3}$, but further R is unknown and for R, we can use $\frac{\gamma R}{\gamma - 1}$ = $C_p$, so R = $\frac{(\gamma-1)C_p}{\gamma}$. So, $C_3$ = $M_3$ $\sqrt{(\gamma-1)C_p T_3}$. And we can get $C_3$ as 187.16 m/s. So, we can find out for the from the exit velocity triangle which has outlet velocity as axial tan $\beta_3$ = $\frac{u_3}{C_3}$ = 1.84, $u_3$ is known, $u_3$ is basically u and $C_3$ is just now evaluated.

So, $\beta_3$ = 61.47°, now we are supposed to find out power output, so for that we need tangential velocity. It can be calculated as $C_2$ sin $\alpha_2$. So $C_2$ is 528.39 × sin 65, so this gives us $C_2$ as 478.88 m/s. So, power output will be $\dot{m}$u($C_{w2}$ + $C_{w3}$) but $C_{w3}$ is 0 × $\eta_m$. So it is$\dot{m}$u($C_{w2}$) × $\eta_m$. So, power output is 20 × 345.4 × 478.88 × 0.98 and we get power output as 3241.94 kW.

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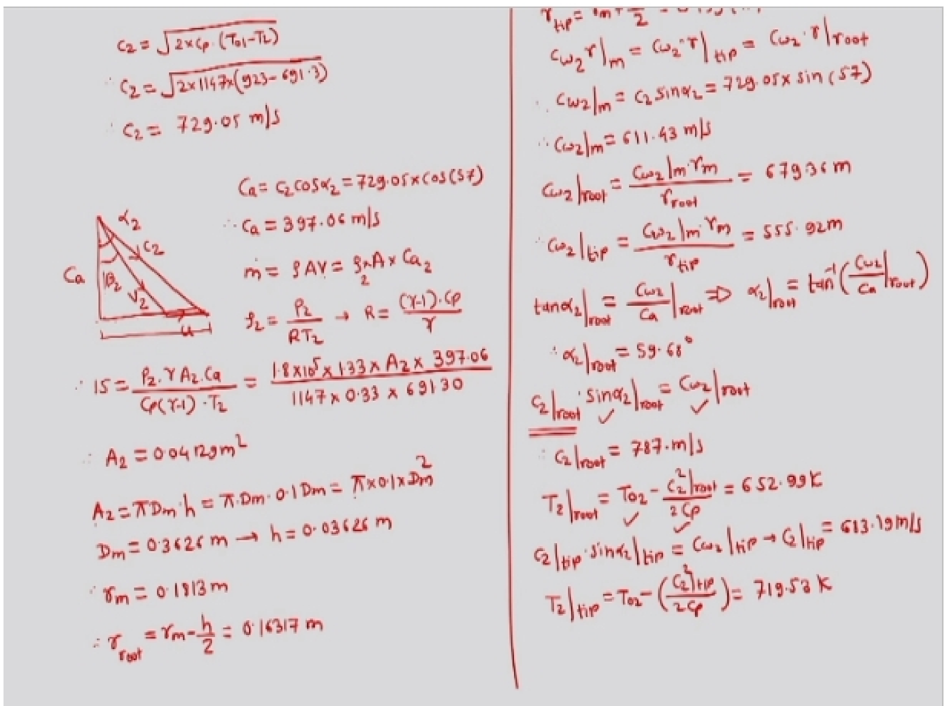
(refer time : 20:19). We will move ahead with the next example. It says that the following particulars of a single stage turbine of free vortex type is given below. Determine the gas temperatures, velocities and discharge angle at the blade root and the tip radii. Assume $C_p$ as 1.147 kJ/kg.K and $\gamma$ is 1.33. Given that, total pressure head at the inlet is 6.2 bar, static pressure at mean radius is 1.8 bar, mean blade diameter to the height ratio is 10, nozzle efficiency is 95%.

Total head inlet temperature is 923 K, nozzle outlet angle is 57° and mass flow rate is 15 kg/s. So, let us mention what are the given things. We are told that $P_{01}$ is 6.2 bar, $P_2$ is 1.8 bar, then

$\frac{D_m}{h}$ that height is given as 10, nozzle efficiency is told as 95%, mass flow rate is 15, then $\alpha_2$ is 57°, $T_{01}$ is 923 K. Further $C_p$ and $\gamma$ are given. Here first, again we can make use of nozzle quantities to find out absolute velocity at the nozzle outlet.

So, for that $\frac{P_2}{P_{01}}$, $\frac{P_2}{P_{01}} = \left(\frac{T_2'}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$. This gives us $T_2' = T_{01}\left(\frac{P_2}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$, so $T_{01}$ is 923, $P_2$ is 1.8 divided by 6.2 bracket raise to $\frac{0.33}{1.33}$. So $T_2'$ turns out to be 679.11 K. Then nozzle efficiency is $\frac{T_{01}-T_2}{T_{01}-T_2'}$ and this gives us $T_2 = T_{01} - \eta_j(T_{01} - T_2')$. So, $T_2 = 923 - 0.95(923 - 679.11)$. So this gives us $T_2$ as 691.30 K.

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(refer time : 23:54). Hence, we can find out absolute velocity at the nozzle angle, at the nozzle as $C_2$ bracket $\sqrt{2C_p(T_{01} - T_2)}$. So, we will have $C_2 = \sqrt{2 \times 1147(923 - 691.3)}$. So, $C_2$ = 729.05 m/s. We can try with velocity triangle, so this is velocity inlet velocity triangle, where this is $C_1$, this is $V_1$, this is u. So this height will be $C_a$, this angle is $\alpha$, basically this is everything is 2. This is $C_2$, this is $V_2$, this angle is $\alpha_2$, this angle is $\beta_2$. So basically $C_a$ is equal to $C_a \cos\alpha_2$, which is $729.05 \times \cos 57$, so we get $C_a$ as 397.06 m/s.

Then, we can make use of formula of $\dot{m}$, which is $\rho$AV, where we have to take, V is velocity generalized. So, we have to take rho into area into $C_a$. So, for that we need to find out the $\rho_2$ and $\rho_2$ can found out from the $\frac{P_2}{RT_2}$. So, for that, we need to again find out R and as we have said in earlier example, R is equal to $\frac{(\gamma-1)C_p}{\gamma}$. So, we can put everything in the mass flow rate formula and then we can put the mass flow rate value as $15 = \frac{P_2 \gamma A_2 C_a}{C_p(\gamma-1)T_2}$.

So, Pc is $1.8 \times 10^5$ Pa, $\gamma$ is 1.33, so we have to find out $A_a$. $C_a$ is $C_{a2}$ is 397.06 divided by $C_p$ is 1147, $\gamma$ -1 is 0.33 and $T_2$ is 691.3. Knowing this, we can find out $A_2$ as 0.04129 $m^2$. And then this area can be taken as $\pi$ $D_m$ $\times$ height and this height and ratio with diameter is known to us, so we can put that. So $\pi$ $D_m$ $\times$ 0.1 $D_m$, so it is $\pi \times 0.1D_m^2$. We know $\pi$ is 3.14, $A_2$ is found out.

So from that we can calculate $D_m$ and $D_m$ turns out to be 0.3626 m so it gives us h, height of the blade as 0.03626 m. That is why we can $r_m$ as 0.1813 m. So we can find out $r_{root}$ as $r_m$ - $\frac{h}{2}$ and root radius becomes 0.16317 m. Similarly, we can find out $r_{tip}$ as $r_m$ + $\frac{h}{2}$ and it gives us the value of 0.1994 m. We are told that this turbine is based on the free vortex concept, so $C_{w2}r|_{mean} = C_{w2}r|_{tip} = C_{w2}r|_{root}$.

So, let us find out first $C_{w2}$. Here for us, at mean is equal to $C_2$ sin $\alpha_2$. It is this and then $C_2$ is known 729.05 into sin of $\alpha_2$ is 57. So, $C_{w2}|_m$ is 611.43 m/s. Then, we can find out $C_{w2}|_{root}$ $= \frac{C_{w2}|_m r_m}{r_{root}}$. And this gives us $C_{w2}|_{root}$ as 679.36 m/s. Similarly, we can find out $C_{w2}|_{tip}$ with same $\frac{C_{w2}|_m r_m}{r_{tip}}$ and we get it as 555.92 m/s.

Knowing this, we can find find out we know $C_w$ is this, in the inlet angle $C_a$ is same for all, so we can find out that $\tan \alpha_2|_{root} = \frac{C_{w2}}{C_a}|_{root}$. So $C_{w2}$ at root is known, $C_a$ is known, we can find out the $\alpha$ and this gives us $\alpha_2|_{root} = \tan^{-1}\left(\frac{C_{w2}}{C_a}|_{root}\right)$. We know both the velocities, so $\alpha_2|_{root}$ can be found out and it is 59.68°. Similarly, knowing this angle we can find out $C_2|_{root}.sin\alpha_2|_{root} = C_{w2}|_{root}$.

So, we know $C_{w2}|_{root}$, we know sin $\alpha_2|_{root}$, so we can find out $C_2|_{root}$ and this gives us $C_2|_{root}$ as 787 m/s. Knowing the absolute velocity at the root, we can find out the temperature at the static temperature at the root using total temperature, which is $T_{02} - \frac{C_2^2|_{root}}{2C_p}$. We know total temperature, we know the value of $C_2|_{root}$ and then this gives us the value of static temperature at the root as 652.99 K.

We can do this exercise for the tip as well. So for that we can find out $C_2|_{tip}.sin\alpha_2|_{tip} = C_{w2}|_{tip}$. So, this gives us $C_2|_{tip}$, that is 683.19 m/s. So, same way, we can find out $T_2|_{tip}$ as $T_{02} - \frac{C_2^2|_{tip}}{2C_p}$ and it turns out to be 719.53 K. So, this is how we can make use of the concept of free vortex, where if we know the quantities at one mean radius, we can find out the quantities at any angle.

And here, most of the quantities, whatever we have found out, they are only from the inlet velocity triangle, where we are interested to find out inlet temperatures and inlet blade angles.

Find impeller diameters and its width at the exit along with power required to drive the centrifugal compressor for following data. Assume equal pressure ratio for the impeller and the diffuser

| | |
|---|---|
| Speed | 10000 rev/min |
| Mass flow rate | 12 kg/s |
| Pressure ratio | 5:1 |
| Isentropic efficiency | 85% |
| Slip factor | 0.88 |
| Flow coefficient at impeller exit | 0.4 |
| Hub diameter at the eye | 17 cm |
| Air velocity at the entry and exit from impeller | 140 m/s |
| Inlet Stagnation temperature | 290K |
| Inlet Stagnation pressure | 1.03 bar |

given $N = 10{,}000$ rpm, $\eta_c = 0.85$, $r_p = 5$, $\sigma = 0.88$, $\phi_2 = 0.4$, $d_h = 0.17$ m, $C_a = 140$ m/s

$T_{01} = 290$ K $\quad P_{01} = 1.03$ bar $\quad \dot{m} = 12$

$$T_{01} = T_1 + \frac{C_a^2}{2c_p} \Rightarrow T_1 = T_{01} - \frac{C_a^2}{2c_p} = 290 - \frac{(140)^2}{2 \times 1005} = 280.29\text{ K}$$

$$\frac{P_{01}}{P_1} = \left(\frac{T_{01}}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{290}{280.29}\right)^{\frac{1.4}{0.4}} = 1.1272 \rightarrow P_1 = P_{01}/1.1272 = 0.9137\text{ bar}$$

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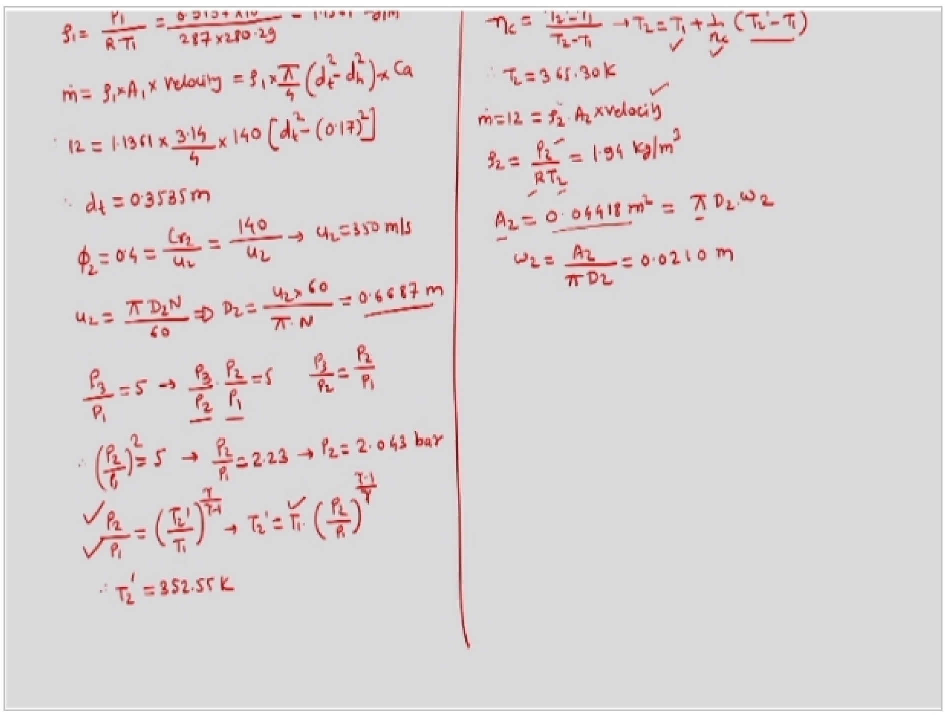
(refer time : 35:04). We will move to the next example, this is about centrifugal compressor, it states that, find the impeller diameters and its width at the exit along with power required to derive to drive the centrifugal compressor as following for following data. Assume equal pressure ratio for impellers and the diffuser. Speed is given as 10000 revolutions per minute, mass flow rate is 12 kg per second, pressure ratio 5:1, isentropic efficiency is 85%, slip factor as 0.88, flow coefficient at impeller exit 0.4.

Hub diameter at the eye is 17 cm, air velocity at the entry and at the exit of the impeller is 140 m/s, inlet stagnation temperature is 290 K, inlet stagnation pressure is 1.03 bar. So, let us see what is given in the example for centrifugal compressor and given thing is, N is equal to 10000 rpm, isentropic efficiency is 0.85, then pressure ratio is 5, then slip factor is 0.88, flow coefficient at the exit is 0.4, diameter at the hub is 0.17 m.

Axial velocity is 140 m/s, $T_{01}$ is 290 K, $P_{01}$ is 1.03 bar. Further, $\dot{m}$ is equal to 12. So, for that all sake, we are supposed to find out impeller diameters. So, we know only the diameter at the hub, we can find out the diameter at the tip from the mass flow rate. So, tip of the eye. So, we can for that we should know density, since we know mass flow rate is equal density into area into velocity, where area we can take at the eye from the hub to the tip of the eye.

For that we need density, so for that we need to find out associated quantities to evaluate density. So, let us first find out temperature, it is said known that $T_{01} = T_1 + \frac{C_a^2}{2C_p}$, so we know it is $T_1 = T_{01} - \frac{C_a^2}{2C_p}$, so it is 290 K - $\frac{140^2}{2\times 1005}$. So, we get $T_1$ as 280.29 K. Having said this, we will find out static pressure also, where we know that $\frac{P_{01}}{P_1} = \left(\frac{P_{01}}{P_1}\right)^{\frac{\gamma}{\gamma-1}}$. $T_{01}$ is 290, $T_1$ is 280.29 bracket raise to 1.4/0.4.

So this ratio turns out to be 1.1272. Then, we know $P_1 = P_{01}/1.1272$ and we get $P_1$ as 0.9137 bar. So, knowing this value of $P_1$ and value of $T_1$, we can find out density.

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(refer time : 39:21). So $\rho_1 = \frac{P_1}{RT_1}$. So $P_1$ is 0.9137/ R 287, it is in Pascal now for the pressure into 280.29. So we get density as 1.1361 kg/m$^3$. Now, we can find out mass flow rate as $\rho_1$ $A_1$ $\times$ velocity where we will have $\rho_1 \times \frac{\pi}{4}(d_t^2 - d_h^2)\times C_a$. Mass flow rate is given, which is 12 is equal to density 1.1361 $\frac{3.14}{4}$, $C_a$ is 140, tip diameter for eye minus hub diameter, which is given $(0.17)^2$. So, we can find out tip of the eye, its diameter as 0.3535 m.

We are told that $\phi_2$ is 0.4, we know $\phi_2$ is $\frac{C_{r2}}{u_2}$ by the definition of flow coefficient, $C_{r2}$, at the exit, we are told that the velocity is same as the inlet axial velocity, so $C_{r2} = C_a$, which is axial velocity. So, $u_2$ can be found out from here and we get $u_2$ is equal to 350 m/s. Knowing

$u_2$, we can find out $D_2$ since we know $u_2 = \frac{\pi DN}{60}$. So, it is $D_2$ is equal to $\frac{u_2 \times 60}{\pi N}$ and we get $D_2$ from here as 0.6687 m. Now, we can find out the other quantities.

For that, we know that pressure ratio is 5, but it is told that the pressures are equally risen in the diffuser and the impeller. So, $\frac{P_3}{P_1}$ can be split into $\frac{P_3}{P_2} \times \frac{P_2}{P_1}$ but $\frac{P_3}{P_2}$ is equal to $\frac{P_2}{P_1}$, so we will have $\left(\frac{P_3}{P_1}\right)^2 = 5$. So, we get $\frac{P_2}{P_1}$ as $\sqrt{5}$, which is 2.23. So, we get $P_2 = 2.043$ bar since we know $P_1$. Now, we can find out $T_2$. Since we know $\frac{P_2}{P_1} = \left(\frac{T_2'}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$. So, we know that $T_2' = T_1 \left(\frac{T_2'}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$.

We know pressure $P_2$, we know pressure $P_1$, we know $T_1$, then $T_2'$ can be found out, and it is 352.55 K. But real temperature is required for us to find out density at the exit and that can be found out from compressor efficiency, which states that $\frac{T_2'-T_1}{T_2-T_1}$. So $T_2 = T_1 + \frac{1}{\eta_c}(T_2' - T_1)$. So, $T_2$ can be found out, we know $T_2$, we know compressor efficiency, and all the temperatures. So $T_2$ turns out to be 365.30 K.

So, we will have the mass flow rate $\dot{m}$ is 12. Now, we are trying to work out with the outlet. So it is equal to $\rho_2$ $A_2 \times$ velocity. But for that again rho2 is required and $\rho_2$ is $\frac{P_2}{RT_2}$, $P_2$ is known, R is known, $T_2$ is known, we get density as 1.94 kg/m$^3$. So, from the above equation, we know velocity, which is $C_a$ or $C_{r2}$, density is known, mass flow rate is known, we can get area from this expression and then this area is basically equal to 0.04418 m$^2$, but it is equal to $\pi$ $D_2$ $w_2$.

And then this w is the width at the outlet. So, w, $D_2$ is found out, $D_2$ is known to us as 0.6687, $\pi$ is 3.14, area is 0.4418. So, we can know $w_2 = \frac{A_2}{\pi D_2}$. We can put all the individual values and find out $D_2$ as 0.0210 m. So, this is how we have evaluated different diameters, like diameter at the eye tip, we have also found out $D_2$ and we have found out the width at the exit of the impeller. So, these are the examples for the practice for this course. Thank you.