Aircraft Propulsion

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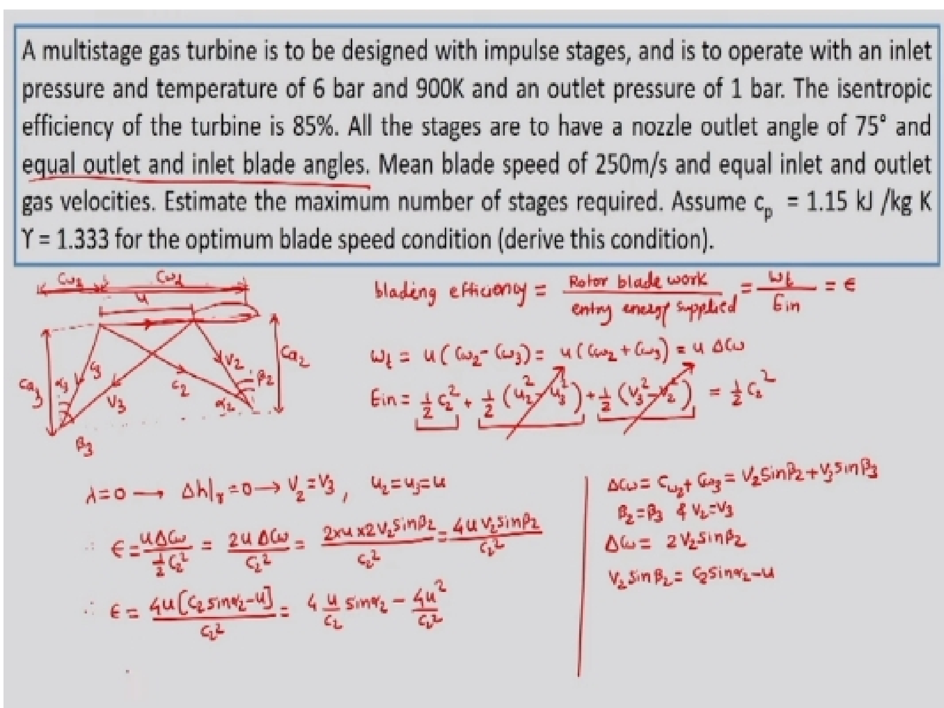
Lecture-36
Axial Turbine: Examples

Welcome to the class. Now, we are going to work about the examples related to axial turbine. Let us see the first example.



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(refer time : 00:38). This example states that a multistage turbine is to be designed with impulse stages. So, it is said in this example that we are going to design an impulse turbine.

And it is to operate with an inlet pressure and temperature of 6 bar and 900 Kelvin with an outlet pressure of 1 bar. Isentropic efficiency of turbine is 85%. All the stages are to have nozzle outlet angle of 75° and equal outlet and inlet blade angles. Mean blade speed of 250 m/s and equal inlet and outlet gas velocities are also said.

Estimate maximum number of stages required. Assume C_p is 1.15 kJ/kg and gamma as 1.33. So, this is given thing for us and with this given thing, we are also said that this has to be solved for optimum blade speed condition and we have to derive this condition. So, let us derive first what is optimum blade speed condition. We know that the velocity triangles for the turbines are this way, this is inlet velocity triangle, this is outlet velocity triangle. And we are working with frictionless and same C_a.

So, this is Ca, this is C_2, this is C_3. Sorry this is V_3, this is C_3, this is V_2. So what we have is, this angle is α_2, this angle is β_2, this angle is α_3, this angle is β_3. Then we have this as u velocity and this we have as C_{w1} and then this as C_{w2}. This is C_{a3}. Basically this is C_{w2} and this is C_{w3} and then this is C_{a2}. So, with this, we have to define for optimum blade speed condition. For that we are going to define a new term, which is blade, blading efficiency.

Blading efficiency of a turbine is defined as rotor blade work divided by entry energy supplied to the rotor. So, we know that W_t is rotor work and E_{in} is entry energy given. So, for us, we know in general, from Euler turbine turbo machinery question, we have specific work done is u(C_{w2} - C_{w3}), but in this specific case, we have C_{w3} in opposite direction to C_{w2}. So, we have C_{w2} + C_{w3}, which is what we are calling it as ΔC_w. Now, we have to define what is E_{in}.

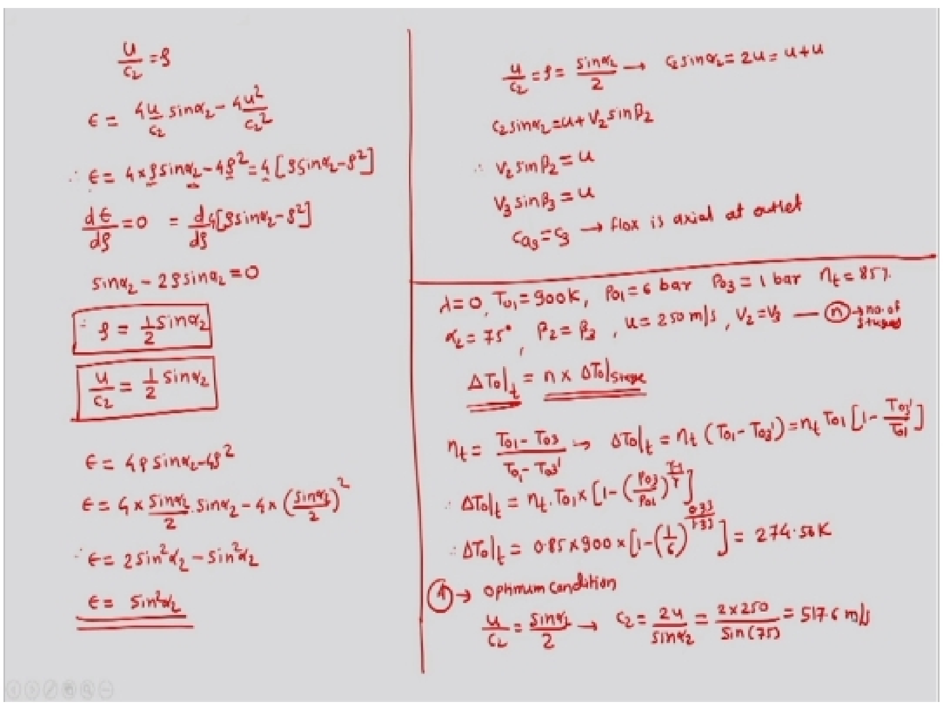
What is the energy available to the blade. Then energy available to the blade is first kinetic energy at the exit of the nozzle, which is $\frac{1}{2}C_2^2$ plus there are two forms of energies; one is change in blade velocity, centrifugal energy, which is based upon the diameter $\frac{\pi DN}{60}$ plus there is enthalpy drop which is going to take place on the blade, which is going to alter the kinetic energy from relative sense. So, this is change in kinetic energy, which is going to happen on the blades.

This is due to the centrifugal energy and this is available kinetic energy in absolute sense at the inlet. At this moment, as what we said, we are working on axial flow turbine and we are having, for this example, we have degree of reaction which is $\lambda = 0$. What is the repercussion of it, this is that delta h rotor is equal to zero, says $\Delta h|_R = 0$ and we are considering frictionless flow, there is no change in enthalpy in the inlet and outlet of the rotor, which is not going to alter.

Therefore, the relative velocity at the inlet and at the outlet further. So, this term is no more present. Further, we are working for the axial flow turbine with mean height or at the mean blade plane and for that we have $u_2 = u_3 = u$, so this term also vanishes. So ultimately, we have $\frac{1}{2}C_2^2$ as the energy. So, blade blading efficiency we are defining it as ε is $\frac{u\Delta C_w}{\frac{1}{2}C_2^2}$, so it is $\frac{2u\Delta C_w}{C_2^2}$.

So, let us find out what do we mean by Δ C_w. For us, it is C_{w2} + C_{w3} and we can know C_{w2} is this, C_{w2} is this complete and C_{w3} is this complete, but this complete length can be also said to be equal to V_2 sin β_2, which is this small length plus V_3 sin β_3, which is this length. So, we are defining it and we will have, suppose we are given in the example that equal blade outlet angle. So, we are given that $\beta_2 = \beta_3$ and also we have $V_2 = V_3$, so Δ C_w = 2 V_2 sin β_2.

So, we can write it as $\frac{2u\times 2V_2 sin\beta_2}{C_2^2}$. So, we have $\frac{4uV_2 sin\beta_2}{C_2^2}$. So, let us find what is V_2 sin β_2. V_2 sin β_2 is this distance, which we can write as V_2 sin β_2 = C_2 sin α_2, which is this big distance minus u. So, we have $\varepsilon = \frac{4u(C_2 sin\alpha_2 - u)}{C_2^2}$. So, which gives this as $4\frac{u}{C_2}sin\alpha_2$ - $4\frac{u}{C_2^2}$.



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(refer time : 09:52).Then let us define $\frac{u}{C_2}$ as ρ, which is blade speed ratio. So, we have ε is equal to, we already had $4\frac{u}{C_2}sin\alpha_2$ - $4\frac{u}{C_2^2}$. This is what we had got the expression for epsilon and hence, we have $\varepsilon = 4\rho sin\alpha_2$ - $4\rho^2$. Now, we can see for given alpha or nozzle angle, blading efficiency is just function of rho. So, $\frac{d\varepsilon}{d\rho}$ if we equate to zero, we can get the condition for this we can also write $\rho sin\alpha_2$ - ρ^2.

This would help to cancel the 4. So, we have this as a $\frac{d\varepsilon}{d\rho}4[\rho sin\alpha_2$ - $\rho^2]$. So, 0 will cancel. We can differentiate with rho, we have sin α_2 - 2ρ sin α_2 = 0. So, we have ρ = $\frac{1}{2}$ sin α_2. So, we have $\frac{u}{C_2}$ = $\frac{1}{2}$ sin α_2. So, this is the condition for blade loading, blading efficiency to be maximum. We can find out maximum blading efficiency, which is ε = 4$\rho sin\alpha_2$ - 4ρ^2. So we have 4 $\times$ $\frac{sin\alpha_2}{2} sin\alpha_2$ - $4\left(\frac{sin\alpha_2}{2}\right)^2$.

So, we have 2sin^2 α_2 - sin^2 α_2. So, ε is equal to we have sin^2 α_2. So, this is the maximum blading efficiency, which we are to obtain for ρ = $\frac{1}{2}$ sin α_2. Now, what is going to happen with this. We have $\frac{u}{C_2}$ = ρ, sorry, $\frac{u}{C_2}$ = ρ = $\frac{1}{2}$ sin α_2. So this gives us C_2sin α_2 = 2u. If we see the velocity triangle, we can see C_2sin α_2 is this length from here till here, within that one u is exit already there.

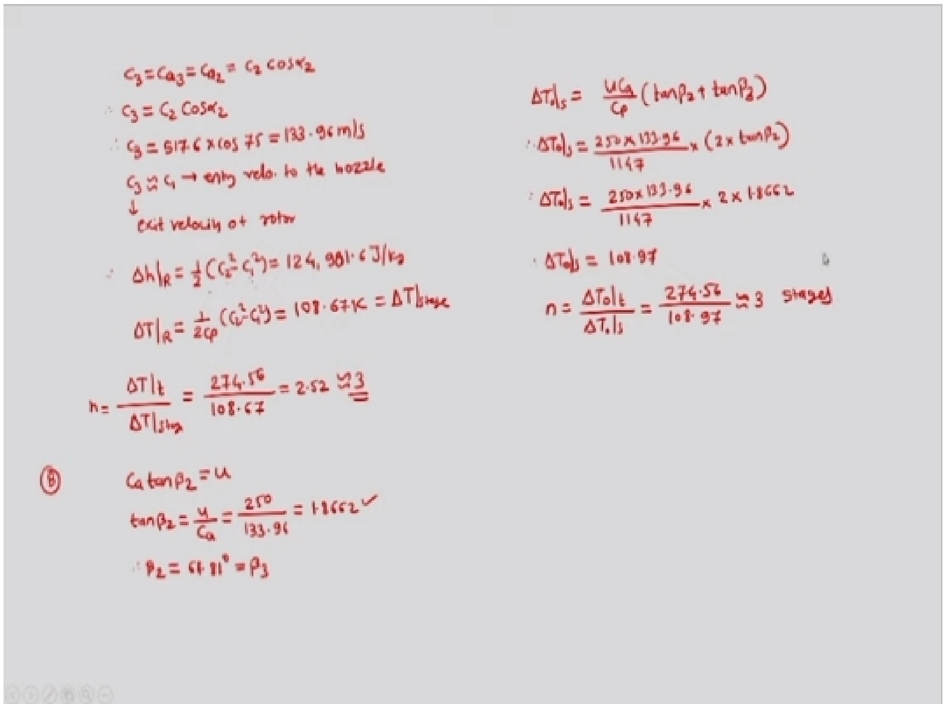
So, we can write down this as u + u. So, second u over here is basically C_2sin α_2 = u + V_2sin β_2. This is known to us. So, we have V_2sin β_2 = u. But we know V_2 = V_3 and sin β_2 = sin β_3 that is also equal to u. So, if that is equal to u, then we can know that, that will only be equal to u, if we have C_a = C_3. So, we can get from C_{a3} = C_3, so flow is axial at outlet for optimum condition, we get this constraint that flow has to be axial. Having said this, now, we can start solving the example for this condition.

So, in the example, we are told that λ = 0, T_{01} = 900 K, P_{01} = 6 bar, P_{03} = 1 bar. Then we have turbine efficiency is 85%. We have given nozzle angle is 75°. We are told thatβ_2 = β_3 symmetric blades for impulse turbine. We are told that u = 250 m/s. Also, we are told that there is no friction, which is V_2 = V_3. So, we are to find out what is n, which is number of stages. How to find out number of stages? We can find out number of stages by knowing that there is one ΔT_0 maybe complete turbine.

And that Δ T_0 complete turbine is equal to n number of stages into Δ T_0 in one stage. One turbine has many stages, so total temperature drop as a summation of across all stages is equal to Δ T_0 across the turbine. We know that turbine efficiency = T_{02} - T_{03}, sorry $\frac{T_{01}-T_{03}}{T_{01}-T_{03}'}$. So, we have from here as $\Delta T_0|_t$, which is the numerator is equal to η_t $(T_{01} - T_{03}')$. So here, it is $\eta_t T_{01}(1$ - $\frac{T_{03}'}{T_{01}})$.

So, we have $\Delta T_0|_t$ = η_t T_{01} $\left[1-\left(\frac{P_{03}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right]$. Knowing this, we can find out total temperature change across the turbine. So, this is 0.85 $\times$ 900 $\times$ $\left[1-\left(\frac{1}{6}\right)^{\frac{0.33}{1.33}}\right]$ and this gives us to 274.56 K. Now, our objective is to find out stage temperature drop. So, stage temperature drop we can find out by two ways. Let us first consider a way which is A here, we will directly use the optimum condition.

Using optimum condition, we are told that $\frac{u}{C_2} = \frac{sin\alpha_2}{2}$. So, we can say that $C_2 = \frac{2u}{sin\alpha_2}$. So, we know now u, which is $\frac{250}{sin75}$ and this gives us C_2 = 517.6 m/s.



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(refer time : 19:24). Knowing this C_2, we can proceed with the optimum condition that $C_3 = C_{a3} = C_{a2}$, but we know from velocity triangle C_{a2} is $C_2 \cos \alpha_2$. So, we can say $C_3 = C_2 \cos \alpha_2$, so C_3, we know C_2 and C_2 is $517.6 \times \cos 75$ and this gives us C_3 as 133.96 m/s. So, we can find out enthalpy drop in the nozzle. Since we know that C_3 is around is equal to C_1 where C_3 is exit velocity of rotor and C_1 is entry velocity to the nozzle. So, both velocities are around same.

So, we can know the enthalpy drop Δh, which is rotor for the impulse case, specifically it is mainly due to change in kinetic energy. So, it is C_2^2 - C_1^2 and then that is what the enthalpy drop in the nozzle and then there is no enthalpy drop in the rotor. So, we have C_2, we know now C_1. So, we can get enthalpy drop as 124,981.6 J/kg. So, we can write delta at $\Delta T_0|_R$ as $\frac{1}{2C_p}(C_2^2 - C_1^2)$. And this gives us enthalpy drop. Total sorry temperature drop as 108.67 Kelvin.

So we have stage temperature change, we have turbine total temperature, turbine temperature change n is equal to stage and this is equal to stage. And this gives us 274.56/108.67. This gives us 2.52, so it is actually saying that, we need three stages. Now, same thing, we can find out by a method, which is B suppose and in this method, we can now use the concept for the velocity triangle, where we know tan β_2, which is basically $C_a$tan β_2 = u.

We know this from the inlet velocity triangle. So, we have tan $\beta_2 = \frac{u}{C_a}$. So, we u which is 250, we know C_a and that C_a is 133.96. So, this gives us tan β_2 as 1.8662, so gives us β_2 = 61.81 which is also equal to β_3. Here, we can use the formula total temperature change across the stage = $\frac{uC_a}{C_p}$ (tan β_2 + tan β_3). So, $\Delta T_0|_s$, u is known to us, which is 250, C_a is known to us, which is 133.96/C_p and C_p is 1147 (tan β_2 + tan β_3).

So it is 2 tan β_2. $\Delta T_0|_s = \frac{250x133.96}{1147} \times 2$ tan β_2 is 1.8662 and this gives us delta T0 stage is around same, which is 108.97. So, we can find out number we can find out stages and number of stages n = $\Delta T_0|_t$ divided by $\Delta T_0|_s$, so we have 274.56/108.97, which is around three stages. So, this is how we should remember we can solve the example for the turbine, which is told to be impulse turbine and here, we have learned how to derive the expression for the optimum blade speed condition.

With certain constraints that equal blade angles without friction and outcome is we will have axial flow velocity and we get $\frac{u}{C_2} = \frac{sin\alpha_2}{2}$.

The following particulars of a single stage turbine of free vertex type is given below. Determine the gas temperatures, velocities and discharge angle at the blade root and tip radii. Assume cp = 1.147 kJ /kg K, ϒ = 1.333

Total head inlet pressure	:	4.6 bar
Static head pressure at mean radius	:	1.6 bar
Mean Blade diameter to height ratio	:	10
Nozzle loss coefficient	:	0.10
Total head inlet temperature	:	973K
Nozzle outlet angle	:	60°
Mass flow rate	:	20 kg/s

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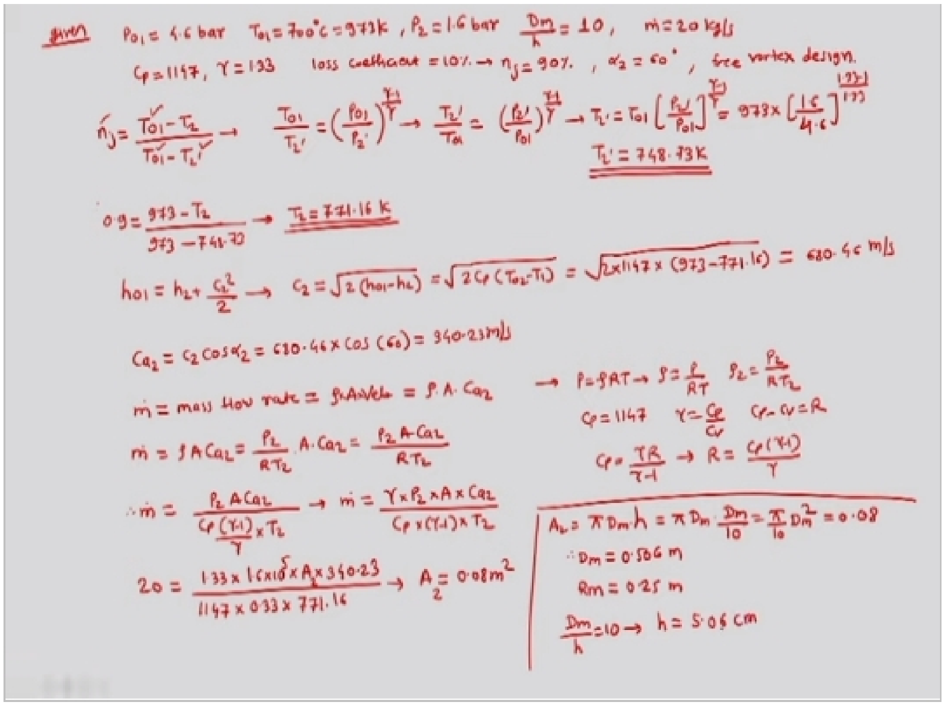
(refer time : 26:00). Knowing this, we can move ahead and solve the next example, which says that following particulars of a single stage turbine of free vortex type is given below.

Determine gas temperatures, velocities and discharge angle at the blade root and tip radii. Assume 1.147 C_p and γ 1.4. Total pressure inlet head is 4.6 bar, static pressure at mean radius is 1.6 bar, mean blade diameter to height ratio is 10, nozzle loss coefficient is 0.1, total head inlet temperature is 973, outlet nozzle outlet angle is 60°, mass low rate is 20 kg/s.



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(refer time : 26:56). So, having said this, we can say that the given things for this example are P_{01} = 4.6 bar, T_{01} = 700° C, which is 973 K. Then P_2 = 1.6 bar, D_m mean diameter to the height of the blade, that ratio is told to be 10, $\dot{m}$ is given as 20 kg/s, C_p is given as 1147, γ is given at 1.33. We are told that loss coefficient is 10%. So, we are practically told that nozzle efficiency is 90%. We are also told that nozzle angle is 60°.

We are supposed to find out temperature and velocity and angles at different heights, where we are told that there is free vortex design. Having said this, we can go ahead and solve the example. We know nozzle efficiency as $\frac{T_{01}-T_2}{T_{01}-T_2'}$, but we know $\frac{T_{01}}{T_2'} = \left(\frac{P_{01}}{P_2'}\right)^{\frac{\gamma-1}{\gamma}}$. Here we are assuming isentropic process in the nozzle where initial the flow had total condition, after expansion it entered with static condition P_2 and T_2, which is for isentropic is P_2' and T_2'.

But $P_2 = P_2'$. In reality flow will attain same pressure but not the same temperature. So, we

have $\frac{T_2'}{T_{01}} = \left(\frac{P_2'}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$ So, we have T_2' = T_{01} $\left(\frac{P_2'}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$ and it is going to give us 973 × 4.6 divided by, sorry P_2' is $(1.6/4.6)^{\frac{1.33-1}{1.33}}$. So, this gives us T_2' and this T_2' = 748.73 K. It was required over here since T_{01} and n_j are already given.
So, this is 0.9 and T_{01} is $\frac{973-T_2}{973-748.73}$. So, this gives us T_2 and T_2 comes to be 771.16 K. Having said this, we can write down the expression for total enthalpy at state one is equal to h_2 + $C_2^2/2$, this is from the energy equation. So, we can find out C_2 which is equal to $\sqrt{2(h_{01} h_2)}$. So, this is equal to $\sqrt{2(T_{02} - T_{01})}$, sorry T_{02} - T_1. So, having said this we can calculate C_2, so it is 2 × C_p, we are told that it is 1147.

Sorry, this 2 is inside the square root, 2 × 1147 × T02 and T02 is 973 T2 is told or calculated 771.16 and this gives us C_2 680.46 m/s. From velocity triangle, we know C_{a2} = C_2 cos α_2. So, we know C_2 now, which is 680.46. We are told that α_2 is 60. So, this gives us C_2 as 340.23 m/s. Now, we have to find out different dimensions, which will be used for the free vortex design. So, for that, we can make use of $\dot{m}$, which is mass flow rate.

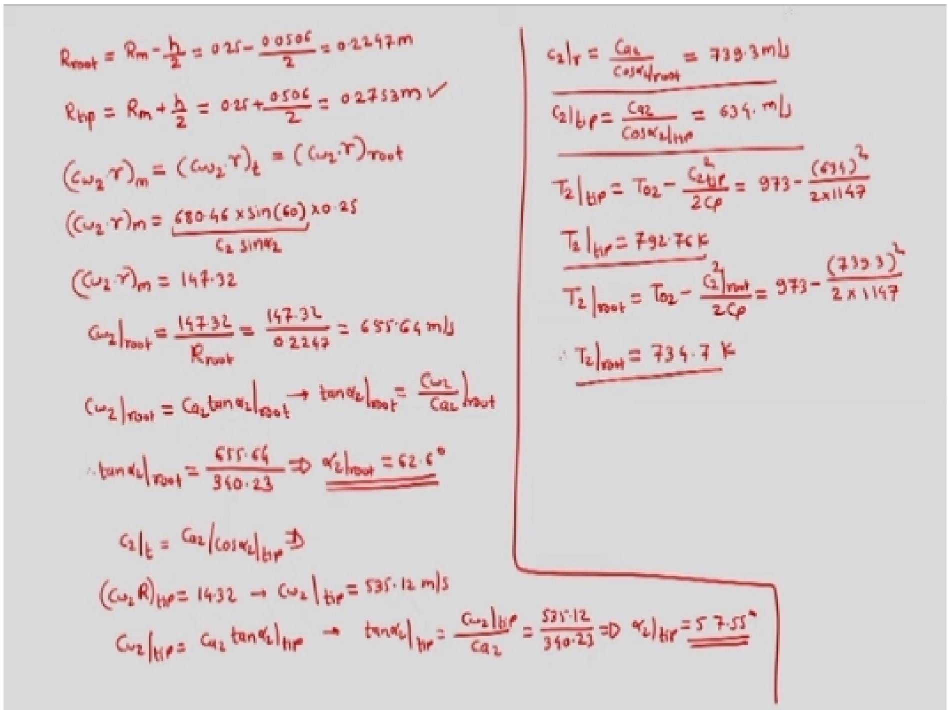
Which is equal to ρAV, this is velocity into area into density. But here, we have to take velocity as C_2, here we have to take ρAC_{a2}. But we have to find out ρ. So, for that, we know that P = ρRT. So, this gives us ρ = P/RT. So, we have ρ_2 = P_2/RT_2. But we do not know R. What we are given with C_p, which is 1147. We know that γ = C_p/C_v and C_p - C_v = R. So, having said this, we can find out R by replacing C_v with C_p and then we can get $C_p = \frac{\gamma R}{\gamma-1}$.

So, R = Cp $\frac{\gamma-1}{\gamma}$. So, this we can use to find out R and then we can make use of it for the calculation of area. So, we can make use of this over here for $\dot{m}$ = ρAC_2, so we have now ρ as $\frac{P_2}{RT_2}AC_{a2}$, so this we can have as $\frac{P_2 AC_{a2}}{RT_2}$. So $\dot{m} = \frac{P_2 AC_{a2}}{C_p \frac{\gamma-1}{\gamma} T_2}$ and so this gives us $\dot{m} = \frac{\gamma P_2 AC_{a2}}{C_p(\gamma-1)T_2}$. So, we know $\dot{m}$ is 20 as given in the example and that is equal to 1.33, P_2 is given as 1.6 x 10^5.

The area is to be evaluated. C_{a2} is 340.23 divided by C_p is 1147 × γ -1, which is 0.33 × T_2 which is 771.16 and this gives us area is equal to basically A_2 and 0.08 m^2. But, we know that the area A_2 = π D_m h. This is the area across which flow is flowing. But we know h given in terms of D as the $D_m/10$. So it is $(\pi/10)\times$ D_m^2. And then this is equal to 0.08 and this gives us D_m = 0.506. So, we can know R_m, which is radius as 0.25 m. Further, we know D_m/h = 10. So, we get h = 5.06 cm. It is very small number.

(refer time : 38:47). Having said this we can proceed for further calculations. Now, we can find out root, R_{root}, what is the radius at root and then that radius at the route is equal to R_m - h/2. So, it comes out to be 0.25 - 0.0506/2 which is 0.2247 m. Similarly, radius at the tip is equal to R_m + h/2 and so we have 0.25 + 0.506/2 = 0.2753 m. Now, we know all radii, we can use the free vortex expression, which says that $(C_{w2}r)_m = (C_{w2}r)_t = (C_{w2}r)_{root}$.

So, we have basically we can mention here we can find out C_{w2}r for mean radius. So $(C_{w2}r)_m$,



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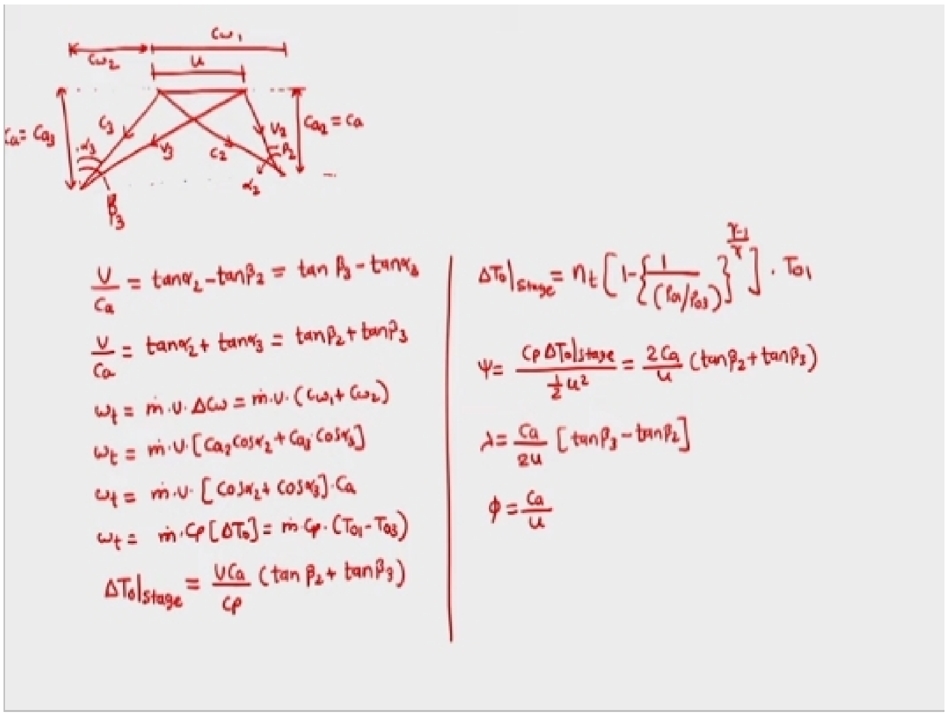
we can write down C_{w2} is 680.46 $\times$ sin 60 $\times$ 0.25. we know that we are writing C_{w2} as C_2sin α_2. So $(C_{w2}r)_m$ is coming to be 147.32 units. So, we know that $C_{w2}|_{root} = \frac{147.32}{R_{root}}$ and R_{root} is 147.32/0.2247. So C_{w2} at root is 655.64 m/s. But we know that $C_{w2}|_{root} = C_{a2}$ tan $\alpha_2|_{root}$.

So, tan $\alpha_2|_{root} = \frac{C_{w2}}{C_{a2}}|_{root}$. So, tan $\alpha_2|_{root} = C_{w2}$, which is 655.64 divided by C_{a2} and C_a is basically 340.23. We have found out C_a, which is 340.23. And this gives us the angle, which is $\alpha_2|_{root}$ and this angle is 62.6°. So, one of the answers we have found out, what is the angle at root or the nozzle and then we know now we have to find out $C_2|_t$, C_2 at tip is equal to also $\frac{C_{a2}}{cos\alpha_2|_{tip}}$. So we have, but here we have to first find out what is cos α_2 at tip.

So, for that, we have to first equate $(C_{w2}r)_{tip}$ = 147.32 and then we know r at tip and r at tip is 0.2753. This gives $C_{w2}|_{tip}$, which is equal to 535.12 m/s. So, we can find out now, first tan α_2. So, we have C_2 at tip = C_{a2}. We know again that $C_{w2}|_{tip} = C_{a2}$tan $\alpha_2|_{tip}$. So, we have tan $\alpha_2|_{tip} = C_{w2}|_{tip}/C_{a2}$ and so we have 535.12/340.23 and we get from here as $\alpha_2|_{tip}$ and that is 57.55°.

So now, we can work out with the temperatures, so for that, we have to find out absolute velocity. So, $C_2|r = \frac{C_{a2}}{cos\alpha_2|_{root}}$. So this gives us velocity as 739.3 m/s. Similarly, $C_2|_{tip} = \frac{C_{a2}}{cos\alpha_2|_{tip}}$ and this gives us $C_2|_{tip}$ as 634 m/s. Having said this, we can find out temperature $T_2|_{tip} = T_{02}$ - $\frac{C_2^2|_{tip}}{2C_p}$. We know $T_{01} = T_{02}$ which is 973 - $C_2|_{tip}$ as $634^2/(2 \times 1147)$.

So, we get temperature at tip as 792.76 K. Similarly, $T_2|_{root}$ = T_{02} - $\frac{C_2^2|_{root}}{2C_p}$. So, which is 973 - $739.3^2/(2 \times 1147)$. So, $T_2|_{root}$ = 734.7 K. So, this is how we can make use of the basics and the velocity triangle to solve the example, which is for the turbines.



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(refer time : 47:59). In case of turbine, following are the salient features what we learnt. So, this is the velocity triangle for the turbine, where we are assuming that the axial at the inlet and at the outlet is same. So, here we will have this as C_2, this as V_2 and hence this as C_{a2}, further this as C_{a3}, which is equal to C_a and which is equal to again C_a. But then, we will have this as V_3, this as C_3. So, the angles what we can make out over here, this first angle is α_2, the second angle is β_2.

Over here, this angle is β_3 and this internal angle is α_3. So, this is what further we have this as u and then this would be our C_{w1}, then this would be C_{w2}. Then, we have some derived formulas, where we said that $\frac{u}{C_a}$ = tan α_2 - tan β_2 = tan β_3 - tan α_3. So, we could also write $\frac{u}{C_a}$ = tan α_2 + tan α_3, which is equal to tan β_2 + tan β_3. We know that turbine work is $\dot{m}\ \Delta C_w$.

So, which is equal to $\dot{m}$u(C_{w1} + C_{w2}), so we can write it as $\dot{m}$u, C_{w1} is C_a, (C_{a2} cos α_2 + C_{a3} cos α_3). So, W_t = $\dot{m}$u(cos α_2 + cos α_3)C_a. We also know from thermodynamics that work done = $\dot{m}C_p\ \Delta T_0$, which is $\dot{m}C_p$ (T_{01} - T_{03}). Further $\Delta T_0|_s$ = $\frac{uC_a}{C_p}$ (tan β_2 + tan β_3). There is one more formula, which includes pressure ratio.

So, we have $\Delta T_0|_s = \eta_t \left[1 - \left(\frac{1}{\frac{P_{01}}{P_{03}}}\right)^{\frac{\gamma-1}{\gamma}}\right] \times T_{01}$. We know blade loading side is equal to $\frac{C_p \Delta_0|_{stage}}{\frac{1}{2}u^2}$, and so it is $\frac{2C_a}{u}$ (tan β_2 + tan β_3). We know degree of reaction $\lambda = \frac{C_a}{2u}$(tan β_3 - tan β_2). We have also defined ϕ as flow coefficient, which is $\frac{C_a}{u}$.

So, these are the relations and the diagram which are required to solve the examples related to axial turbine.