Aircraft Propulsion

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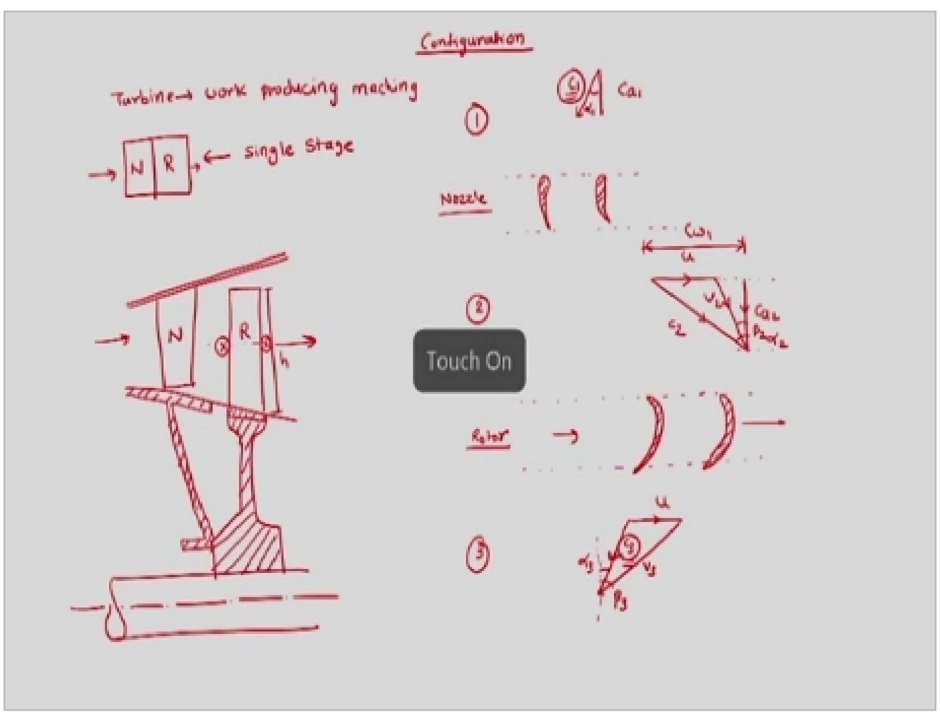
Lecture-34
Axial Turbines

Welcome to the class. Now, we are going towards the discussion for axial turbines. We have seen that axial turbine is an integral part of aircraft engines if we have chosen it to be, otherwise, there can be other option of the turbine which is part of other lectures. So, axial turbines, as the name suggest, the fluid is in the axial direction of the turbine. So, we will see how is its configuration, axial turbine. (refer time: 01:06)



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As it was already discussed, we are talking about configuration. As it was already discussed, a turbine is work producing machine and hence if I say a turbine, it will have two components, or rather one stage or single stage. And those two components are first is nozzle and the second is rotor. So, this is stator or nozzle, second is rotor. So, flow will first enter into the nozzle, gain the kinetic energy and it will enter into the rotor and lose the kinetic energy and it does the work.

This is how the configuration of a turbine would be, and hence the axial turbine, as the name suggests, it will also have two parts. So, the nozzle would be the stationary part, this is the nozzle suppose, this is stationary part of the turbine, and then we have rotor in the schematic which is the moving part of the turbine. And then this rotor would be attached to a shaft. So, in all this is the schematic of a axial turbine where flow is parallel to the axis.

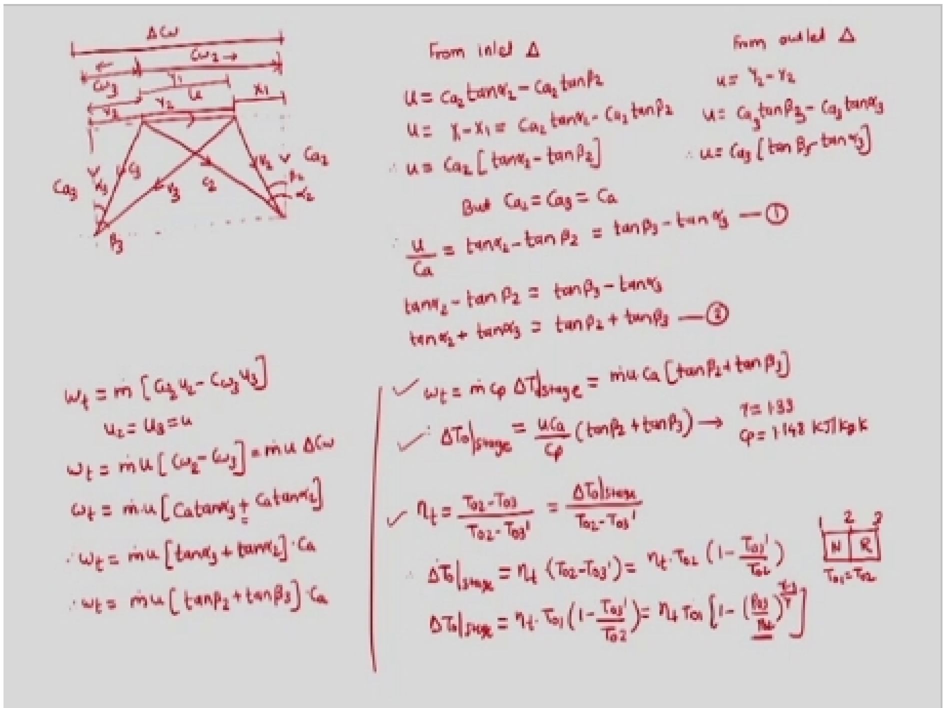
Now, it is understood that first the flow has to pass through the nozzle and then it has to go to the rotor. Let us see how the flow pattern would be. So, this will be the direction of the flow. So, we will first have a flow which is entering with velocity C_1 into the nozzle. This is nozzle ring or stator ring where fluid is supposed to enter, and then this is velocity C_{a1} and angle α_1. Then, secondly, it will enter into the rotor blades.

So, the configuration of rotor blades would be this and this rotor rotates in this direction. This is direction of rotation of the rotor which will be accounted while considering the u velocity or the tangential velocity of the rotor. So, this is u velocity or tangential velocity of the rotor. After entering into the nozzle it would get accelerated, and then the flow would attain a velocity which is C_2. Hence it will have relative velocity V_2.

So, practically, flow will have C_{a2} as the axial velocity, the angles, C_2 angle is α_2, V_2 angle is β_2. Hence this component of absolute velocity will be C_{w1} or whirling velocity. Now, the flow would come out of the rotor with tangential velocity which is V_3 and there will be absolute velocity which the flow would have. This is V_3. Then, we will have u velocity, and then this is C_3.

So, the angle of C_3 is α_3, and angle of V_3 is β_3, and then this is the velocity representation. Again, here, we are considering everything at the mid plane height of the blade. So, if this h is the height of the blade, we are working in the mid plane. So, we are plotting inlet velocity triangle at this location, outlet velocity triangle at the same location. So, u velocity is same at the inlet and at the outlet. So, this is one section, this is section two, and this is section three for the one stage of the turbine. (refer time: 08:24)

Having said this, we can proceed and plot the coupled velocity triangle, inlet and outlet velocity triangles together. Since u velocity is common we can plot both together. So, this is u velocity and then we have this as C_2, this as V_2, and we are all set, we have this as C_{a2} which is axial velocity, and then this is α_2 and this is β_2. Having said this, we will plot the outlet



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velocity triangle and in the outlet velocity triangle this is V_3, this is C_3.

And hence angles are α_3 and β_3. Parallelly, we will have the velocity C_{a3}. Considering the components, we have this complete component is whirl velocity 2 and the small component is whirl velocity 3. So, this complete width is ΔC_w which is change in whirl velocity. So, from inlet triangle we can say u which is the blade velocity is equal to C_{a2} tan α_2.

So, this C_{a2} tan α_2 is this complete which is C_{w2} practically, - C_{a2}tan β_2 which is this distance. And then this is y_1, this is x_1. So, we are seeing u = y_1 - x_1 which is C_{a2}tan α_2 - C_{a2}tan β_2. So, u = C_{a2}tan α_2 - C_{a2}tan β_2. From outlet velocity triangle we have u is equal to, again the same thing. Outlet velocity triangle we can represent it as x_2.

So, we have this complete would be y_2, y_2 - x_2. So, y_2 in this case is C_{a3}tan β_3 - C_{a3} tan α_3. So, u = C_{a3}tan β_3 - C_{a3} tan α_3, but C_{a2} = C_{a3} = C_a, u/C_a = tan α_2 - tan β_2 = tan β_3 - tan α_3. So, practically, we have tan α_2 - tan β_2 = tan β_3 - tan α_3.

So, we have tan α_2 - tan β_2 = tan β_3 - tan α_3. We name this as equation number 1 and this is equation number 2. We know that turbine work W_t is equal to $\dot{m}$ × u ×, basically, it is $\dot{m}(C_{w2}u_2 - C_{w3}u_3)$, but $u_2 = u_3$, so turbine work is equal to $\dot{m}u(C_{w2} - C_{w3})$. So, we can write down C_{w2} and C_{w3} in terms of C_a.

So, we can write it down as $C_a\tan\,\alpha_3$. So, the directions are different in this case. So, we practically have plus sign here, plus $C_a\tan\,\alpha_2$. So, we have $W_t = \dot{m}u(\tan\,\alpha_3 + \tan\,\alpha_2)C_a$. Here, if we are considering negative sign, we are practically writing $\dot{m}u\Delta C_w$, but if we see here, C_{w2} is in this direction and C_{w3} is in this direction.

So, ΔC_w will get addition of $C_{w2} + C_{w3}$, so there is plus sign which is getting the plus by minus. We know $\alpha_2 + \alpha_3$ is $\beta_2 + \beta_3$. So, $W_t = \dot{m}u(\tan\,\beta_2 + \tan\,\beta_3)$. We have C_a also along with this. So, this is the turbine work. Now, we know that this turbine work is equal to the one more thing which is $\dot{m}C_p\,\Delta T_0|_s$.

So, it is equal to $\dot{m}u(\tan\,\beta_2 + \tan\,\beta_3)$. We would have written it for $\alpha_2 + \alpha_3$ as well. So, $\Delta T_0|_s$ which is stage temperature drop is equal to $\frac{uC_a}{C_p}(\tan\,\beta_2 + \tan\,\beta_3)$. We should remember from the concepts of Brayton cycle and its components. Again from the aircraft engines we have seen that γ over here in the case of turbine we generally take as 1.33, C_p we generally take for gas as 1.148 kJ/kg.K.

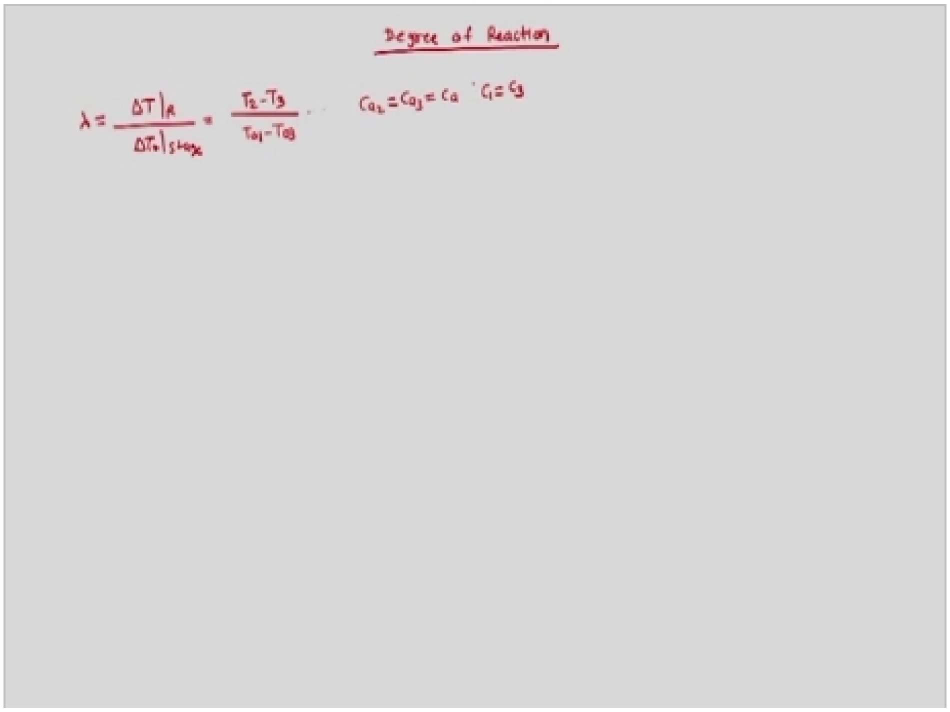
So, this is what is the input for C_p and γ. Then, we can define the ΔT_0 by one more fact, $\Delta T_0|_s$ using thermal efficiency of turbine. It is $\frac{T_{02}-T_{03}}{T_{02}-T_{03}^{'}}$. So, practically, turbine efficiency is actual work divided by ideal work. So, this gives us efficiency is equal to $\Delta T_0|_s/(T_{02} - T_{03}^{'})$. So, $\Delta T_0|_s = \eta_t(T_{02} - T_{03}^{'})$.

We can take T_{02} common. So, it is $T_{02}\left(1 - \frac{T_{03}^{'}}{T_{02}}\right)$. We should remember that we have turbine means we have nozzle and rotor. So, 1 is here, 2 is here, and 3 is here. These are the three states. But 1 to 2 is no work interaction, so we have $T_{01} = T_{02}$. So, we have $\Delta T_0|_s = \eta_t.T_{01}\left(1 - \frac{T_{03}^{'}}{T_{02}}\right)$ bracket raised to, $T_{02} = T_{01}$.

Now we can express in terms of pressure ratio saying that $\eta_t.T_{01}\left[1 - \left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}}\right]$. So, we can find out the stage efficiency also like this where the term in the bracket represents the pressure ratio of the turbine. So, now, we will go ahead having said about the temperature rise in a stage, temperature drop in a stage, turbine work input, turbine efficiency, and go ahead and discuss about degree of reaction.(refer time: 19:59)

In case of degree of reaction, we know λ is equal to a degree of reaction that is $\Delta T_0|_R/\Delta T_0|_s$. So it is, T_2 - T_2 and it is equal to T_{01} - T_{03}, but let us assume that we are working for a turbine which has $C_{a2} = C_{a3} = C_a$ and also $C_1 = C_2$.

If you go back and see in the velocity triangle, what we are trying to say is for the first stage we have C_1 which is velocity for the gas entering into the nozzle and C_3 is the velocity of the



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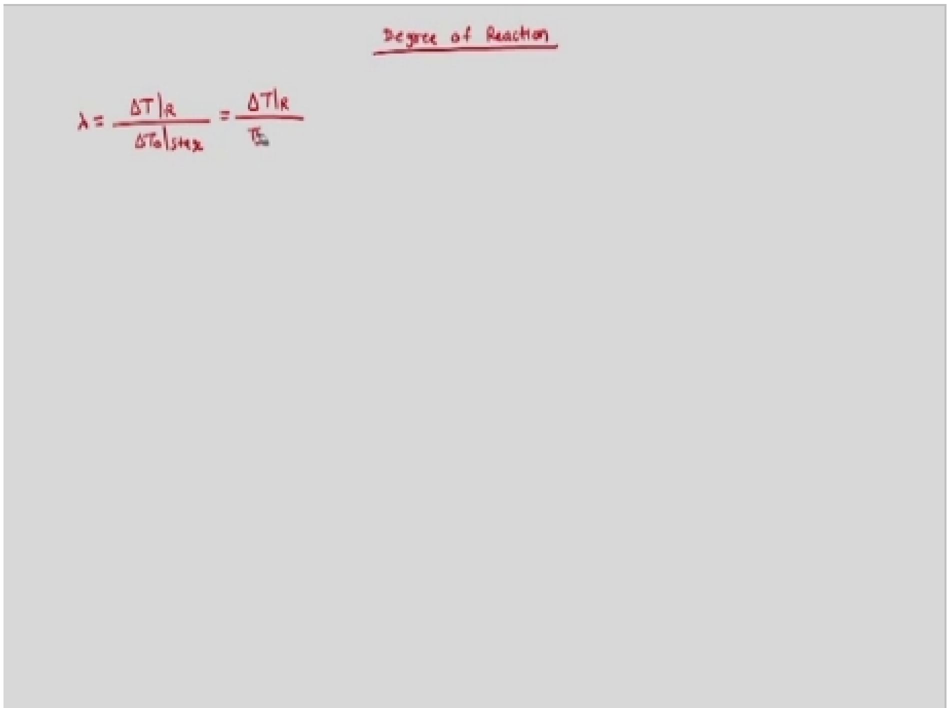
gas which is leaving the rotor. If it has to go to the next stage, we would obviously have $C_1 = C_3$.(refer time: 21:14)

That is same kinetic energy for fluid at the entry of each stage. So, this is $\Delta T_0|_R / \Delta T_0|_s$ which is equal to $\Delta T_0|_R$ divided by, we can mention. (refer time: 21:36)
We will talk about degree of reaction. We know that degree of reaction is defined as $\Delta T_0|_R / \Delta T_0|_s$. We will consider a special case where we have $C_{a2} = C_{a3} = C_a$ and we have $C_3 = C_1$. So, if you go back and see, we are practically seeing that in the velocity triangle the flow which is coming with velocity C_1 into the nozzle of first stage is leaving the rotor of first stage with velocity C_3.

Now, this $C_3 = C_1$, means if there is a next stage, it is receiving the same kinetic energy as what first stage has received. So, if we say so, we can mention it as $\Delta T|_R / \Delta T|_s$. We can bring it in terms of static temperature. Now, $\Delta T|_s$ is practically the work done, $W_t = C_p \, \Delta T_0$ which is equal to $C_p(T_1 - T_3)$ since $C_2 = C_3$, and that is equal to $uC_a(\tan \beta_3 + \tan \beta_2)$.

Having said this, we can consider this for the denominator, but for the numerator we know enthalpy drop which is $C_p(T_2 - T_3)$ = change in relative kinetic energy, so $V_3^2 - V_2^2$. So, practically enthalpy drop in the rotor would lead to change in relative velocity and hence the kinetic energy. So, we have $C_p(T_2 - T_3)$ is equal to half.



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So, let us see what is V_3. If we try to write V_3, then V_3 is this. And we are talking about in terms of C_a, we can write in terms of C_a since C_a is constant between the inlet and the outlet. So, C_a and V_3 has an angle β_3. So, V_3 is equal to C_a sec β_3. So, we have it equal to $C_a^2 \sec^2 \beta_3$ -, same way, V_2 is equal to $C_a^2 \sec^2 \beta_2$. We know that is equal to $C_p(T_2 - T_3)$.

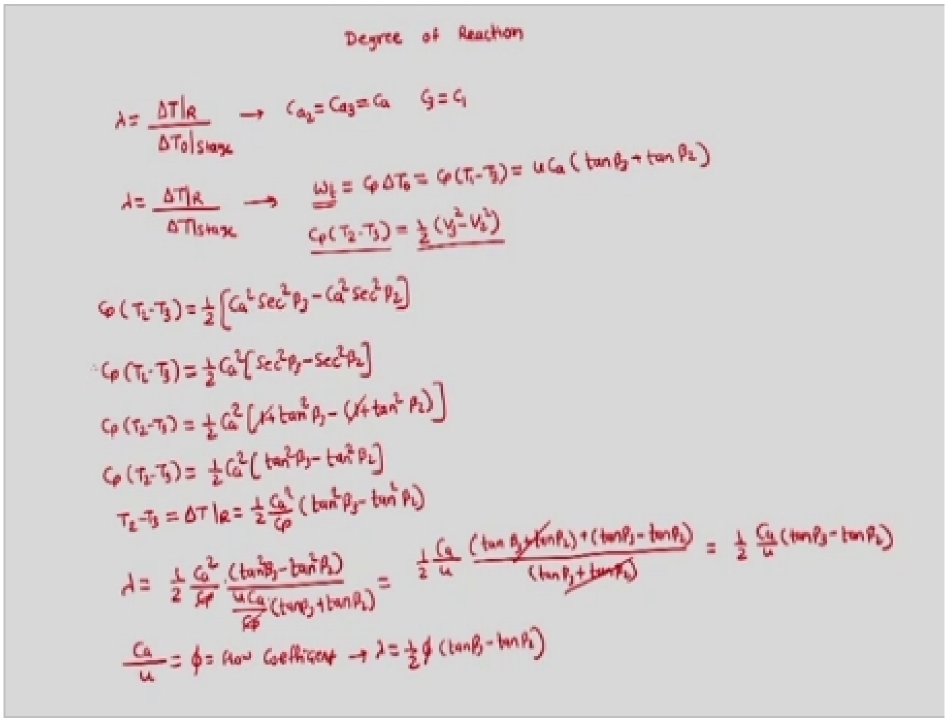
We can take C_a^2 common and the terms in the bracket would remain as it is. So, sec θ we know the relation, what it bears with tan θ, so we have that relation which is 1 + $\tan^2 \beta_3$ - 1 + $\tan^2 \beta_2$. one one would get cancelled. So, we have $C_p(T_2 - T_3) = \frac{1}{2} C_a^2 (\tan^2 \beta_3 - \tan^2 \beta_2)$.

Now we can tell that $T_2 - T_3 = \Delta T|_R = \frac{1}{2} \frac{C_a^2}{C_p}(\tan^2 \beta_3 - \tan^2 \beta_2)$. Having said this, we can represent λ and hence λ becomes $\frac{1}{2} \frac{C_a^2}{C_p} \frac{tan^2\beta_3 - tan^2\beta_2}{\frac{uC_a}{C_p}(tan\beta_3 + tan\beta_2)}$.

So, C_p and C_p will get cancelled, one C_a would get cancelled. So, we have $\frac{1}{2} \frac{C_a}{u} \frac{tan^2\beta_3 - tan^2\beta_2}{(tan\beta_3 + tan\beta_2)}$. $\tan^2 \beta_3 - \tan^2 \beta_2$ can be split and said as $(\tan \beta_3 + \tan \beta_2) \times (\tan \beta_3 - \tan \beta_2)$ divided by $(\tan \beta_3 + \tan \beta_2)$ and this bracket would get cancelled. So, $\frac{1}{2} \frac{C_a}{u} (\tan \beta_3 - \tan \beta_2)$.

So, let us say that C_a which is axial velocity divided by blade speed is equal to ϕ, which is flow coefficient. So, this gives us $\lambda = \frac{1}{2} \phi (\tan \beta_3 - \tan \beta_2)$. (refer time: 28:59)

If we go back and see the derivation for the ψ which is the blade speed ratio, we can define the



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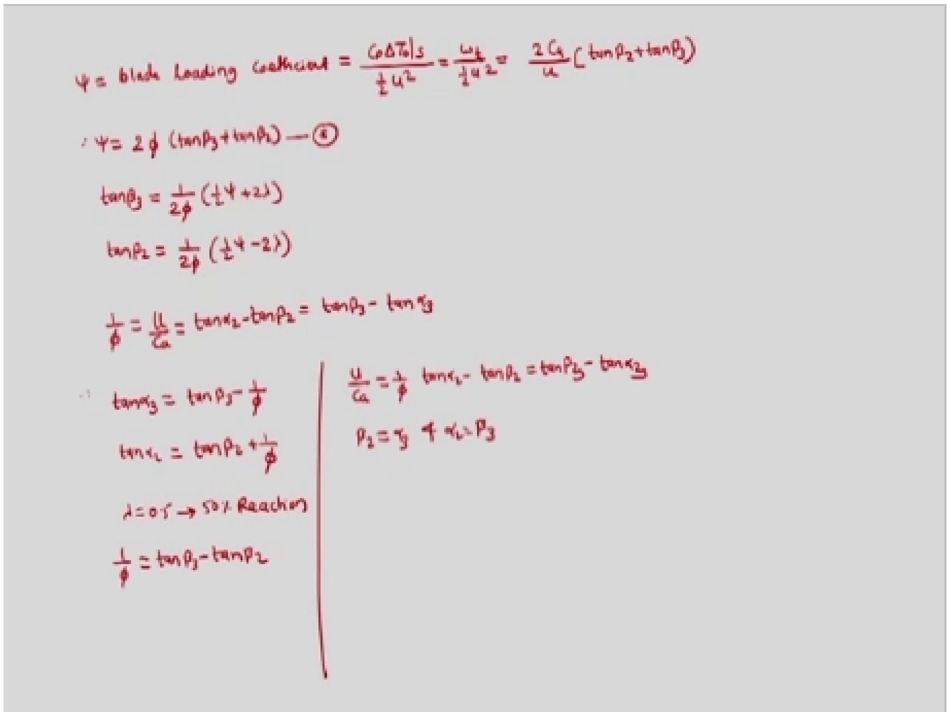
term which is ψ which is called as blade loading coefficient that is defined as $C_p\ \Delta T_0|_s/\frac{1}{2}u^2$ or this is also equal to $W_t/\frac{1}{2}u^2$. So, this is equal to $\frac{2C_a}{u}(\tan\beta_2 + \tan\beta_3)$. So, this ψ is also equal to $2\phi(\tan\beta_3 + \tan\beta_2)$.

Considering the equation this and the previous expression this or lambda, we can write down $\tan\beta_3$ is equal to $\frac{1}{2\phi}(\frac{1}{2}\psi + 2\lambda)$ and $\tan\beta_2$ is equal to $\frac{1}{2\phi}(\frac{1}{2}\psi - 2\lambda)$. Similarly, we can have $\frac{1}{\phi} = \frac{u}{C_a} = \tan\alpha_2 - \tan\beta_2 = \tan\beta_3 - \tan\alpha_3$.

So, we have $\tan\alpha_3 = \tan\beta_3 - \frac{1}{\phi}$ and $\tan\alpha_2 = \tan\beta_2 + \frac{1}{\phi}$. So, if we take a particular case of λ is equal to 0.5 or 50% reaction turbine, then we have $\frac{1}{\phi} = \tan\beta_3 - \tan\beta_2$. Then, $\frac{u}{C_a}$ is equal to $\frac{1}{\phi}\ \tan\alpha_2 - \tan\beta_2 = \tan\beta_3 - \tan\alpha_3$, and hence we have $\beta_2 = \alpha_3$ and $\alpha_2 = \beta_3$. We will move ahead and discuss about the T-s diagram.(refer time: 32:37)

So, if we try to plot T-s diagram, then first we have P_{01}, the gas expands till P_2, this is an isentropic expansion, but from 2 it goes to 3. Initially, gas is at state 1. If that state is isentropically compressed to P_{01}, then this temperature is T_{01}. This gas expands till state 2 and then it responds further till P_3, but in P_2 we can isentropically again compress, then we will have this as P_{02} and the same temperature we will come in that is T_{02} is equal to T_{01}. This is P_{03}.



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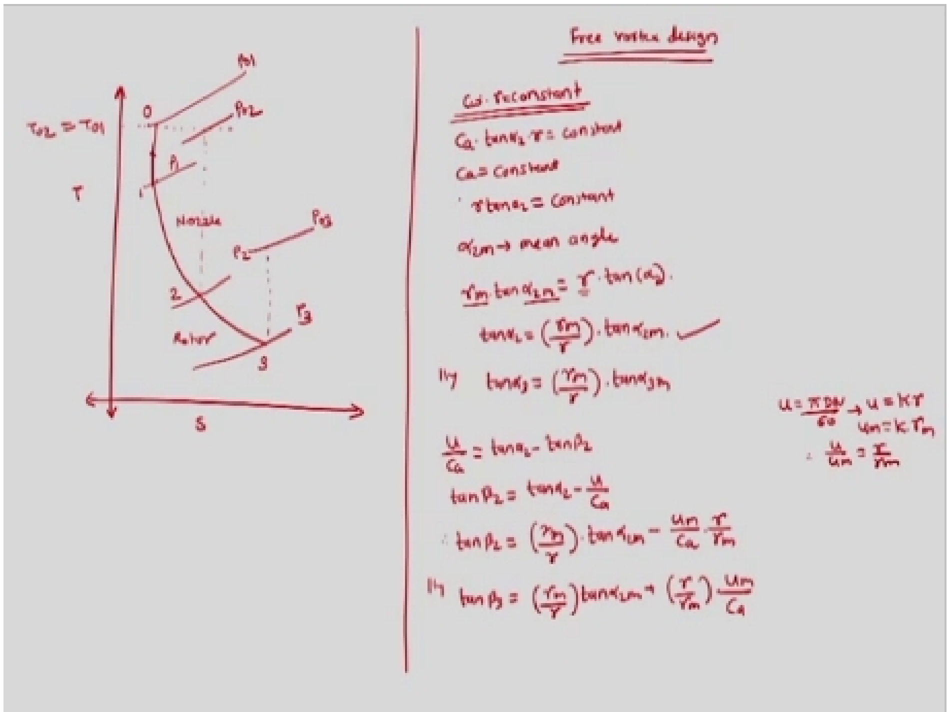
This is 0. This is 1. This is 2. This is 3. Here, we are having nozzle. Here we are having rotor. Now, we will talk about the nozzle which we have discussed, that is free vortex design. In case of free vortex design we have seen that C_wr = constant. So, we have $C_a \tan \alpha_2$r = constant, but C_a is again itself a constant, so our number of constants increase. r tan α_2 = constant.

Let α_{2m} is the mean angle at the inlet of the absolute velocity. So, we have r_m tan α_{2m} = r tan α. Here, this is alpha. This r is any radius r at which α_2 is measured. This is the mean radius and this is the mean angle. So, we have tan $\alpha_2 = \frac{r_m}{r}$(tan α_{2m}). Similarly, tan $\alpha_3 = \frac{r_m}{r}$(tan α_{3m}).

Further, we know that $\frac{u}{C_a}$ = tan α_2 - tan β_2. So, we have tan β_2 = tan α_2 - $\frac{u}{C_a}$. So, we have tan β_2, we know this is tan α_2, it is $\frac{r_m}{r}$(tan α_{2m}) - $\frac{u}{C_a}$. This u is going to vary at different r. So, we know that u =$\frac{\pi DN}{60}$. So, we know u is equal to some constant into r. So, u_m is equal to that constant × r_m.
So, we have $\frac{u}{u_m}$ is equal to $\frac{r}{r_m}$. So, tan β_2 = $\frac{r_m}{r}$(tan α_{2m}) - $\frac{u}{C_a}$. So, we can now say it as $\frac{u_m}{C_a}$ $\frac{r}{r_m}$, that is $\frac{u}{C_a}$. Similarly, tan β_3 = $\frac{r_m}{r}$ (tan α_{2m}) + $\frac{r}{r_m}$ $\frac{u_m}{C_a}$. So, this is the thing what we know from the free vortex design which we have seen, that for free vortex design where we had considered radial equilibrium of the flow element and then we assumed that the work done at all the heights is same.



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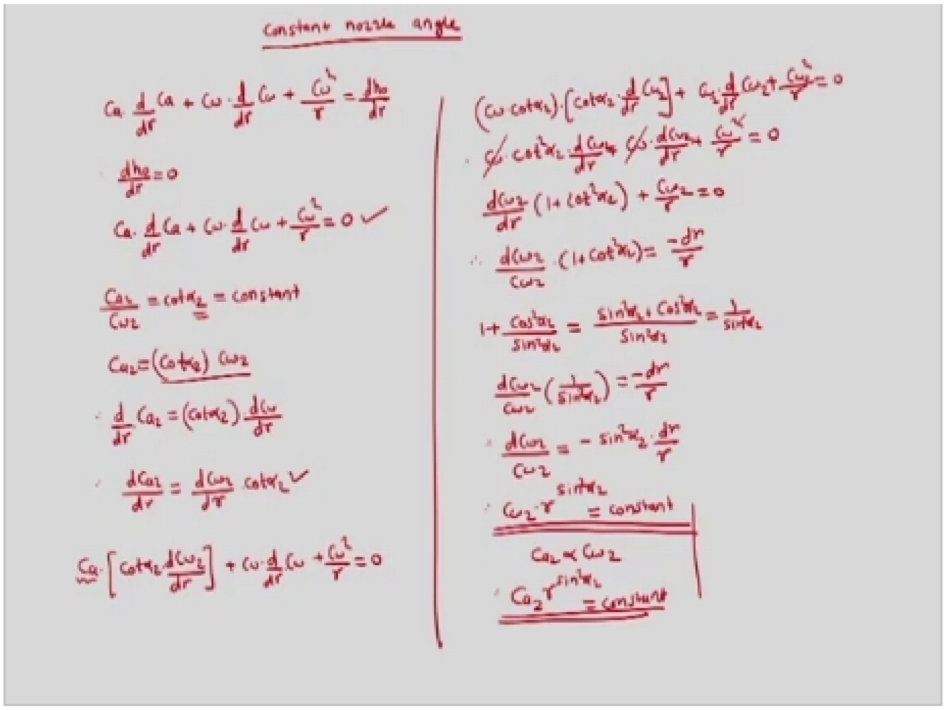
So, we have got C_wr as constant, but now this is not the sole philosophy in which the turbines are designed. (refer time: 39:10)

Apart from free vortex theory turbine design has one more constraint or method which is constant nozzle angle. Let us try to find out the derivation which is similar to C_wr = constant, but it is for constant nozzle angle design. We know from the vortex equation or vortex flow equation we have $C_a \frac{d}{dr} C_a + C_w \frac{d}{dr} C_w + \frac{C_w^2}{r} = \frac{dh_0}{dr}$.

So, we have $\frac{dh_0}{dr} = 0$ with the fact that work done at all the heights is same. So, we have $C_a \frac{d}{dr} C_a + C_w \frac{d}{dr} C_w + \frac{C_w^2}{r} = 0$. We know that $\frac{C_{a2}}{C_{w2}} = \cot \alpha_2$ from the velocity triangle, okay? Then, let us say that this is constant. Since we are considering constant nozzle angle and this is inlet angle at the nozzle, so we have C_{a2} = constant $\times \tan \alpha_2 \times C_{w2}$.

So, we have $\frac{d}{dr} C_{a2} = \tan \alpha_2$, sorry this is $\cot \alpha_2$, this is also $\cot \alpha_2 \times \frac{dC_{w2}}{dr}$. So, we can write down $\frac{dC_{a2}}{dr} = dC_{a2}$ by, sorry this is dr, $\times \cot \alpha_2$. We can use this for this expression and we can mention it as $C_{a2}(\cot \alpha_2 \frac{dC_{w2}}{dr}) + C_w \frac{d}{dr} C_w + \frac{C_w^2}{r} = 0$.

We know C_{a2} is equal to $C_{w2} \cot \alpha_2$, so we have $C_{w2} \cot \alpha_2(\cot \alpha_2 \frac{dC_{w2}}{dr}) + C_{w2} \frac{d}{dr} C_{w2} + \frac{C_{w2}^2}{r} = 0$. So, we have $C_w \cot^2 \alpha_2 \frac{dC_{w2}}{dr} + C_w \frac{d}{dr} C_{w2} + \frac{C_{w2}^2}{r} = 0$. So, we can divide this expression by C_{w2}, so we have $\frac{dC_{w2}}{dr}(1 + \cot^2 \alpha_2) + \frac{C_{w2}}{r} = 0$.



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So, we have $\frac{dC_{w2}}{C_{w2}}(1 + \cot^2 \alpha_2) = -\frac{dr}{r}$. Having said this, this $\cot^2 \alpha_2$ can be written as $\frac{cos^2\alpha_2}{sin^2\alpha_2}$. So, we have $\frac{sin^2\alpha_2+cos^2\alpha_2}{sin^2\alpha_2}$ which is $\frac{1}{sin^2\alpha_2}$ which is actually going to get replaced here, $\frac{dC_{w2}}{C_{w2}}$ $\frac{1}{sin^2\alpha_2} = -\frac{dr}{r}$.

So, $\frac{dC_{w2}}{C_{w2}} = -\sin^2 \alpha_2 \frac{dr}{r}$. We can integrate both sides and then we can get $C_{w2}r^{sin^2\alpha_2}$ = constant and this is the constraint for constant nozzle angle design of turbine blades. Further, we know that C_{a2} is directly proportional to C_{w2}, so we can write down this as C_{a2} = constant.

So, these two are the expressions of nozzle design with constant angle or constant nozzle design angle for the turbines. So, we will see some examples related to this in one of the lectures. Here, we have seen, if given an axial turbine, how to find out the velocity triangle, from the velocity triangle how to find out the blade loading coefficient, turbine work output, ΔT_0 or temperature drop in a stage, and also degree of reaction.

We have seen how they can be calculated for the axial turbine. Further, we have seen what are the different design criteria for designing an axial turbine. So, there are two ; one is vortex flow theory, another is constant nozzle angle. So, we have seen the expressions for both from fundamental equations. Regarding axial turbine examples we will see in one of the classes. Thank you.