

# Aircraft Propulsion

Prof. Vinayak N. Kulkarni

Department of Mechanical Engineering  
Indian Institute of Technology-Guwahati

## Lecture-33 Examples of Axial Flow Compressor

Let us proceed with these some more examples practice.(refer time: 00:30)

A 8 stage axial flow compressor provides an overall pressure ratio of 6.5:1 with an overall isentropic efficiency of 82%. The temperature of air at inlet is 292K. The work is equally divided between the stages. Compressor has 50% reaction with a blade speed of 225 m/s and a constant axial velocity of 145 m/s. Estimate the blade angles. Assume a work done factor of 1.

$n = 8$   $\eta_c = 0.82$   $u = 225 \text{ m/s}$   $\lambda = 0.5$   $r_p = 6.5$   $T_{01} = 292 \text{ K}$   $C_a = 145 \text{ m/s}$

$\Delta T_{01} |_{\text{stage}} = \frac{\Delta T_{01} |_{\text{overall}}}{n} = \frac{T_{01} \left[ (r_p)^{\frac{1}{n}} - 1 \right]}{n}$

$\Delta T_{01} |_{\text{stage}} = \frac{25.6}{8 \times 0.82} \left[ \left( \frac{6.5}{1.4} \right)^{\frac{1}{8}} - 1 \right]$

$\Delta T_{01} |_{\text{stage}} = 31.47 \text{ K}$

$T_{03} = T_{01} + \Delta T_{01} |_{\text{stage}} = 323.47 \text{ K}$

$\Delta T_{01} |_{\text{stage}} = \frac{2 \cdot u \cdot C_a}{C_p} (\tan \beta_1 - \tan \beta_2)$

$31.47 = \frac{2 \times 225 \times 145}{1005} (\tan \beta_1 - \tan \beta_2)$

$\tan \beta_1 - \tan \beta_2 = 0.9834 \text{ --- (1)}$

$\lambda = \frac{C_a}{2u} (\tan \beta_1 + \tan \beta_2) = \frac{1}{2}$

$\frac{145}{2 \times 225} (\tan \beta_1 + \tan \beta_2) = \frac{1}{2} \text{ --- (2)}$

$2 \times \tan \beta_1 = 2.52 \rightarrow \tan \beta_1 = 1.26 \rightarrow \beta_1 = 51.76^\circ$

$\tan \beta_2 = 1.15 - \tan \beta_1 \rightarrow \tan \beta_2 = 0.299 \beta_2 = 16.17^\circ$

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(refer time: 00:00) Let us proceed with these some more examples practice.(refer time: 00:30) We are seeing some examples here, first example states that 8 stage axial flow compressor provides an overall pressure ratio of 6.5:1 with an overall isentropic efficiency of 82%, a temperature drop of air at inlet temperature of air at inlet is 292 K, the work is equally divided between all the stages compressor has 50% reaction with a blade speed up to 25 m/s and constant axial velocity of 145 m/s.

Estimate the blade angles assume a work done factor to be 1. In this example, given things for us are number of stages 8 compressor's isentropic efficiency is 82% then, we are told

that  $u = 225$  m/s,  $\lambda = 0.5$ , pressure ratio is 6.5 and temperature  $T_{01} = 292$  K, we have  $u$  we have axial velocity 145 m/s ok, having said that we will proceed. So, we know  $\Delta T_{0|overall} = \frac{T_{01}}{\eta_c} \left( (r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right)$ . This example we are solving for air.

So,  $\gamma$  will be taken as air, unless otherwise said. Further we are working for compressor. So, we should take  $\gamma$  as 1.4, the same  $\gamma$  will change to 1.33 in case of turbine, since we will have reacted products as gases flowing through the turbine. This is overall temperature ratio, temperature change, but this overall temperature change for that it is said that work is equally divided between the stages. So, temperature changes equally divided between the stages and this gives us  $\Delta T_{0|s}$ .

So,  $\Delta T_{0|s}$ ,  $\Delta T_{0|overall}/n$ . So we can find out  $\Delta T_{0|s}$  and it is equal to 292 divided by 8 stages, compressor efficiency point 0.82, pressure ratio is  $(6.5)^{\frac{0.4}{1.4}} - 1$ . So,  $\Delta T_{0|s} = 31.47$  K. Hence, we can get  $T_{03} = T_{02} + \Delta T_{0|s}$ .  $\Delta T_{01} + \Delta T_{0|s}$  hence it is 323.47 K, we are expected to find out blade angles. So, for that we can make use of new formula which is  $\Delta T_{0|s}$  and that is equal to work done factor  $\frac{u C_a}{C_p} (\tan \beta_1 - \tan \beta_2)$ .

So, we know  $\Delta T_{0|s}$  is 31.47, work done factor is 1,  $u$  is 225 and  $C_a$  is 145,  $C_p$  is  $1005 \times (\tan \beta_1 - \tan \beta_2)$ . So, we get  $(\tan \beta_1 - \tan \beta_2) = 1.55$  we do not know both  $\tan$  of  $\beta_1$  and  $\beta_2$ . So we name it as equation number 1. Further we can make use of the expression for degree of reaction which is  $\lambda = \frac{C_a}{2u} (\tan \beta_1 + \tan \beta_2)$  but we are working with degree of reaction 1/2. So, it turns out to be  $\frac{u}{C_a} = (\tan \beta_1 + \tan \beta_2)$ .

We know  $u$  and  $u$  is 225,  $C_a$  is 145. So,  $u/C_a$  is 1.55 we'll name it as equation number 1. So, we have  $\tan \beta_1 + \tan \beta_2$  we have  $\tan \beta_1$  minus,  $\tan \beta_1 - \tan \beta_2 = 0.9694$ . So, we have  $\tan \beta_1 - \tan \beta_2$ , we have  $\tan \beta_1 + \tan \beta_2$ . So we can solve these equations simultaneously and we can get  $2 \times \tan \beta_1$  is 2.52 which gives us  $\tan \beta_1 = 1.26$ . So, we have  $\beta_1 = 51.56^\circ$ . Similarly, we can get  $\tan \beta_2 = 1.55 - \tan \beta_1$ , so this gives us  $\tan \beta_2 = 0.29$  and hence  $\beta_2 = 16.17^\circ$ . So, we got the required thing from the example.(refer time: 07:49)

We move on to the next example it reads that an axial air compressor of 50% reaction has blades with inlet and outlet angles of  $47^\circ$  and  $12^\circ$  respectively. The compressor is to produce a pressure ratio of 5.5:1 with an overall efficiency isentropic efficiency of 0.87, when inlet temperature is  $30^\circ\text{C}$ . The inlet the blades speed and axial velocity are constant throughout the compressor.

Assume a value of 220 m/s for blade speed, find out number of stages required. If work done factor is unity. So for all this sake whatever we are given with we can make use of we are given that pressure ratio is 5.5 isentropic efficiency is 0.87,  $T_{01}$ , we are told that that is 303 Kelvin, then we are told that  $\beta_1 = 47^\circ$  and  $\beta_2$  is  $12^\circ$ ,  $u$  is 220 m/s,  $\lambda = 0.5$ , work done factor unity.

An axial air compressor of 50% reaction design has blades with inlet and outlet angles of  $47^\circ$  and  $12^\circ$  respectively. The compressor is to produce a pressure ratio of 5.5:1 with an overall isentropic efficiency of 0.87 when inlet temperature is  $30^\circ\text{C}$ . The blade speed and axial velocity are constant throughout the compressor. Assuming a value of 220 m/s for blade speed find the number of stage required if the work done factor is unity for all stages.

$$\begin{aligned} \text{Given: } r_p &= 5.5, \eta_c = 0.87, T_{01} = 303\text{ K}, \beta_1 = 47^\circ, \beta_2 = 12^\circ, u = 220\text{ m/s}, \lambda = 0.5, \gamma = 1.4 \\ \Delta T_{0|_{\text{overall}}} &= \frac{T_{01}}{\eta_c} \left[ (r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right] = \frac{303}{0.87} \left[ (5.5)^{\frac{1.4-1}{1.4}} - 1 \right] \\ \Delta T_{0|_{\text{overall}}} &= 218.55\text{ K} \\ \Delta T_{0|_{\text{stage}}} &= \frac{\lambda u C_a (\tan \beta_1 - \tan \beta_2)}{C_p} \\ \lambda &= \frac{C_a}{2u} (\tan \beta_1 + \tan \beta_2) \rightarrow \tan \beta_1 + \tan \beta_2 = \frac{2\lambda u}{C_a} \\ C_a &= \frac{2\lambda u}{\tan \beta_1 + \tan \beta_2} = \frac{2 \times 0.5 \times 220}{1.0723 + 0.212} = 171.22\text{ m/s} \\ \Delta T_{0|_{\text{stage}}} &= \frac{1 \times 220 \times 171.22}{1005} (1.0723 - 0.212) \\ \Delta T_{0|_{\text{stage}}} &= 32.24\text{ K} \end{aligned}$$

$$\begin{aligned} n &= \frac{\Delta T_{0|_{\text{overall}}}}{\Delta T_{0|_{\text{stage}}}} \\ n &= \frac{218.55}{32.24} = 6.77 \end{aligned}$$

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So, these are the things which are given, now we first find out overall temperature change from the efficiency formula. So,  $\frac{T_{01}}{\eta_c} \left( (r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right)$ . So  $T_{01}$  is 303, isentropic efficiency 0.87, so c, so we can find out overall temperature change in the compressor and that turns out to be 218.55 K. Now, we are supposed to find out the stage temperature change.

And then we can divide them and find out number of stages, the formula for stage temperature change is  $\frac{\lambda u C_a}{C_p} (\tan \beta_1 - \tan \beta_2)$ . But in this formula, we are not aware about  $C_{a1}$ . So, to find out  $C_{a1}$  we can make use of degree of reaction, which says that  $\lambda = \frac{C_a}{2u} (\tan \beta_1 + \tan \beta_2)$ . So, we will have  $\tan \beta_1 + \tan \beta_2 = u/C_a$ .

So,  $C_a = u/(\tan \beta_1 + \tan \beta_2)$ , we know u it is 220 m/s,  $\tan \beta_1$  which is tan of  $47^\circ$  and it is 1.0723 + tan of  $\beta_2$  which is tan  $12^\circ$  which is 0.212 and then it gives us  $C_a$  as 171.22 m/s, this helps us find out  $\Delta T_{0|_s}$ . So, we know work done factor is unity u is 220,  $C_a$  is just now found out as 171.22 divided by  $C_p$  which is 1005 into  $\tan \beta_1$  is 1.0723 - 0.212 which is  $\tan \beta_2$ .

$\Delta T_{0|_s} = 32.24\text{ K}$ , thus we can find out number of stages as  $\Delta T_{0|_{\text{overall}}} / \Delta T_{0|_s}$ . So,  $n = 218.55/32.24$  and this is 6.77. So, we have to use 7 stages to achieve this required pressure rise of 5.5 total pressure ratio.(refer time: 13:31)

Find the polytropic efficiency of an axial compressor from the following data. The rotor and stator blades are symmetrical. The mean blade speed and axial velocity remain constant throughout the compressor. Assuming a value of 200m/s for blade speed and the work done factor as 0.84, find the number of stages required. Also find the inlet Mach number relative to rotor at the mean blade height of the first stage. Assume  $R = 284.6 \text{ kJ/kg K}$

Total head inlet pressure ratio	:	6.5
Overall total head isentropic efficiency of compressor	:	90%
Total head inlet temperature	:	292K
Inlet air angle from rotor blades	:	12°
Outlet air angle from rotor blades	:	43°

Given  $\epsilon_p = 0.9$ ,  $\eta_c = 0.9$ ,  $T_{01} = 292$ ,  $\alpha_1 = 12^\circ$ ,  $\alpha_2 = 43^\circ$ ,  $\lambda = 0.5$ ,  $u = 200 \text{ m/s}$   $\Rightarrow \lambda = 0.84$   
 $\beta_2 = 12^\circ$ ,  $\beta_1 = 43^\circ$

$$\eta_c = \frac{\frac{T_1}{T_2} - 1}{\frac{T_1}{T_2} - 1} = 0.9 = \frac{(6.5)^{\frac{0.286}{0.9}} - 1}{(6.5)^{\frac{0.286}{\eta_c}} - 1} \Rightarrow (6.5)^{\frac{0.286}{\eta_c} - 1} = 0.286 \rightarrow \eta_c = 92.2\%$$

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We will move on to the next example and this example says that find the polytropic efficiency of an axial compressor from the following data. The rotor and stator are symmetrical, we know that when it is said the rotor and stator blades are symmetrical it means that degree of reaction is 50%. The mean blade speed and axial velocity remain constant throughout the compressor. Assume a value 100 m/s blade speed and work done factor of 0.84, find number of stages required, also find the inlet Mach number related to the rotor at mean blade height.

So, for this data given is total pressure inlet head is 6.5, overall total head inlet pressure ratio is 6.5, overall head isentropic efficiency of compressor 90%, total head inlet temperature is 292 K, inlet air angle from the rotor blades is 12°, outlet air angle from the rotor blades is 43°. So, we can write down given things pressure ratio is 6.5, overall isentropic efficiency is 0.9,  $T_{01}$  is 292 K.

We are told that air angles as  $\alpha_1$  as 12° and  $\alpha_2$  as 43°, blades are symmetrical. So,  $\lambda = 0.5$ . Hence, we can mention the same thing as  $\beta_2 = 12^\circ$  and  $\beta_1 = 43^\circ$ . We are So 292 divided by 0.9, 6.5 bracket raise to 0.286 - 1. So, overall temperature rise is 229.71 Kelvin, this helps us to find out number of stages which is  $\Delta T_0$  overall divided by  $\Delta T_0$  stage 229.71 divided by 174.66 divided by 21.02 and this gives us 10.92 which is 11 stages are required. (refer time: 21:45) e told that  $u$  is = 200 m/s and work done factor  $\lambda$  is 0.85, with this we will proceed we know isentropic efficiency of an axial compressor can be written

as  $\frac{r_p^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma-1}{\gamma} \cdot \frac{1}{\eta_{c|p}} - 1}$ . We know isentropic efficiency is 0.9, pressure ratio is  $6.5^{0.286} - 1$  divided by  $6.5^{\frac{0.286}{\eta_{c|p}} - 1}$ .

So, we will get from here  $6.5^{\frac{0.286}{\eta_{c|p}} - 1} = 0.7866$  from here we can find out compressors polytropic efficiency and it turns out to be 92.2%. So, we first found out polytropic efficiency, now we have to find out number of stages. Again, we have to find out stage temperature rise and overall temperature rise. So, we know for stage temperature rise we know the formula. (refer time: 17:55)

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And that formula is  $\Delta T_{0|s} = \frac{\lambda u C_a}{C_p} (\tan \beta_1 - \tan \beta_2)$ , we know  $\alpha_1 = \beta_2$  and  $\alpha_2 = \beta_1$ , but here we are not aware about  $C_a$ . So, we have to first find out  $C_a$ , we will make use of the formula for degree of reaction which is 0.5 and this gives us  $u/C_a = \tan \beta_1 + \tan \beta_2$  which is also  $\tan \alpha_2 = \tan \alpha_1$  and then this we know this gives us  $0.932125 + 0.9325$  and it gives us 1.1450.

So, we can know  $C_a = u/1.450$  and it comes out to be 174.66 m/s. So,  $\Delta T_{0|s}$  can be found out now and it is  $\frac{0.84 \times 174.66 \times 200}{1005} (\tan \beta_2 \tan \beta_1 - \tan \beta_2 \tan \beta_1)$  is  $0.9325 - 0.2125$  and this gives us stage temperature rise as 21.02 K. We can as well find out overall temperature rise and that is  $\frac{T_{01}}{\eta_c} \left( (r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right)$ .

So  $292/0.9$ ,  $(6.5)^{0.286} - 1$ . So, overall temperature rise is 229.71 K, this helps us to find out number of stages which is  $\Delta T_{0|overall}$  divided by  $\Delta T_{0|s}$ , 229.71 divided by 174.66 divided by 21.02 and this gives us 10.92 which is 11 stages are required. (refer time: 21:45)

Air at a temperature of 288K enters a eight stage axial flow compressor at the rate of 5 kg/s, the pressure ratio is 5.5, and the isentropic efficiency is 80%, the compression process being adiabatic. The compressor has symmetrical blades. The axial velocity of 120 m/s is uniform across the stage and the mean blade speed of each stage is 185 m/s. Determine the direction of the air at entry to and exit from the rotor and the stator blades and also the power given to the air. Assume  $C_p = 1.005 \text{ kJ/kg K}$  and  $\gamma = 1.4$ .

given  $r_p = 5.5$ ,  $T_{01} = 288 \text{ K}$ ,  $\eta_c = 0.8$ ,  $C_a = 120 \text{ m/s}$ ,  $u = 185 \text{ m/s}$ ,  $n = 8$ ,  $m = 5$ ,  $\lambda = 0.5$

$$\Delta T_{0|overall} = \frac{T_{01}}{\eta_c} \left[ (r_p)^{\frac{\gamma}{\gamma-1}} - 1 \right] = \frac{288}{0.8} \left[ (5.5)^{0.46} - 1 \right]$$

$$\Delta T_{0|overall} = 226.2 \text{ K}$$

$$P_{\text{work}} = \dot{m} C_p \Delta T_{0|overall} = 5 \times 1005 \times 226.2 = 1134.615 \text{ kW}$$

$$\Delta T_{0|stage} = \frac{u \times u \times \lambda}{C_p} (\tan \beta_1 - \tan \beta_2)$$

$$\frac{\Delta T_{0|overall}}{n} = \frac{1.2 \times 185 \times 170}{1005} (\tan \beta_1 - \tan \beta_2) = \frac{226.2}{8}$$

$$\tan \beta_1 - \tan \beta_2 = 1.28 \quad \text{--- (1)}$$

$$\frac{u}{C_a} = \tan \beta_1 + \tan \beta_2 = \frac{185}{120}$$

$$\tan \beta_1 + \tan \beta_2 = 1.5416 \quad \text{--- (2)}$$

$$\tan \beta_1 = 1.51 \rightarrow \beta_1 = 54.65^\circ$$

$$\tan \beta_2 = 0.1316 \rightarrow \beta_2 = 7.45^\circ$$

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We will move to next example, it states that air at a temperature of 288 K enters an 8 stage axial flow compressor at the rate of 5 kg/s, pressure ratio is 5.5 and isentropic efficiency is 80%, compression is adiabatic, the compressor has symmetrical blades, again degree of reaction is 50%. The axial velocity is 120 m/s which is uniform across the stage and the mean blade speed of each stage is 185 m/s.

Determine the direction of air at entry and at entry to an exit from the rotor and stator blades and also power given to the air. Assume  $C_p$  as 1.005,  $\gamma$  as 1.4. Let us mention what are the given things, given things are pressure ratio is 5.5,  $T_{01}$  is 288 Kelvin, compressor's isentropic efficiency is 80%, it is told that  $C_a$  is 120 m/s and  $u$  is 185 m/s, number of stages are 8, mass flow rate is 5 and degree of reaction is 0.5.

We can find out  $\Delta T_{0|overall}$  as  $\frac{T_{01}}{\eta_c} \left( (r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right)$ , 288 divided by 0.8 into  $(5.5)^{0.286} - 1$ . So, now we have found out overall temperature rise as 226.2 K. So power required =  $\dot{m} C_p \times \Delta T_{0|overall}$ ,  $\dot{m}$  is said as 5 kg per second  $C_p$  is 1005  $\times$  226.2 and this gives us a number

1136.635 kW.

Now we can find out  $\Delta T_{0|s}$  and delta T0 stage is  $\frac{\lambda u C_a}{C_p} (\tan \beta_1 - \tan \beta_2)$ . So,  $\Delta T_{0|s}$  we can find out from  $\Delta T_{0|overall}/n =$  this is 1 work done factor u is 185,  $C_a$  is 120 divided by 1005  $\times \tan \beta_1 - \tan \beta_2$ , what we know overall is 226.2 divided by number of stages is 8.

So, we get  $\tan \beta_1 - \tan \beta_2 = 1.28$ , we will number this equation as equation number 1, from degree of reaction 5,  $u/C_a = \tan \beta_1 + \tan \beta_2$ . So u is 185 and  $C_a$  is 120. So we have  $\tan \beta_1 + \tan \beta_2 = 1.5416$ . So we can solve equation number 1 and equation number 2 simultaneously. So, that we can get  $\tan \beta_1 = 1.41$  and this gives us  $\beta_1 = 54.65^\circ$  and  $\tan \beta_2$  is 1. 0.1316. So,  $\beta_2 = 7.49^\circ$ . So, this is how we would solve this example.(refer time: 27:36)

Determine the stage efficiency and work done factor for an axial flow compressor, if actual pressure ratio developed is 1.25 and actual temperature rise is 35K. Blade inlet and outlet angles are  $46^\circ$  and  $15^\circ$ . Peripheral and axial velocities are 280 m/s and 195 m/s respectively. Assume  $C_p = 1.005 \text{ kJ/kg K}$  and  $\gamma = 1.4$  and total temperature at the inlet is 295K.

given  $r_p = 1.25$ ,  $T_{01} = 295$ ,  $\beta_1 = 46^\circ$ ,  $\beta_2 = 15^\circ$ ,  $u = 280 \text{ m/s}$ ,  $C_a = 195 \text{ m/s}$ ,  $\Delta T_{0s} = 35 \text{ K}$

$$\Delta T_{0s} = \frac{T_{01}}{n_s} \left[ (r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$\eta_s = \frac{T_{01}}{\Delta T_{0s}} \left[ (r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$\eta_s = \frac{295}{35} \left[ (1.25)^{\frac{1.4-1}{1.4}} - 1 \right]$$

$$\eta_s = 85.49\%$$

$$\Delta T_{0s|stage} = \frac{\lambda u C_a}{C_p} (\tan \beta_1 - \tan \beta_2)$$

$$35 = \frac{\lambda \times 280 \times 195}{1005} (\tan 46^\circ - \tan 15^\circ)$$

$$\lambda = 0.8395$$

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We will move on to the next example, it reads that determine the stage efficiency and work done factor for an axial flow compressor if actual pressure ratio developed is 1.25 and actual temperature rises 35 K, blade inlet and outlet angles are  $46^\circ$  and  $15^\circ$ . Peripheral and axial velocities are 280 m/s and 195 m/s respectively.  $C_p$  is 1.005 kJ/kg.K,  $\gamma$  is 1.4 and total temperature at inlet is 295 K.

So let us say what is given here and given here is actually it's a stage pressure rise, it is 1.25  $T_{01}$  is 295,  $\beta_1$  is  $46^\circ$ ,  $\beta_2$  is  $15^\circ$ , u peripheral speed is 280 m/s and the axial speed  $C_a$  is 195 m/s. Having said this we can find out the stage temperature rise basically using the formula

which is the  $\frac{T_{01}}{\eta_c} \left( (r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right)$ .

Here we are given the stage temperature rise as 35 K, that is also given. So, we can say stage efficiency is  $\frac{T_{01}}{\eta_c} \left( (r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right)$ , so stage efficiency turns out to be  $295/35 \times (1.25)^{\frac{\gamma-1}{\gamma}} - 1$  and directly we get stage efficiency as 55.43%. Then we are supposed to find out work done factor for that we can make use of the formula  $\Delta T_{0|s} = \frac{\lambda u C_a}{C_p} (\tan \beta_1 - \tan \beta_2)$

So, we have  $35 = \text{work done factor} \times u$ ,  $u$  is 280 into  $C_a$  is 195 divided by 1005 and  $\tan \beta_1$  is  $\tan$  of 46 -  $\tan$  of 15. So we have work done factor as 0.8393. This is a simple example where we had to use straight formulas to find out the quantity thank you.