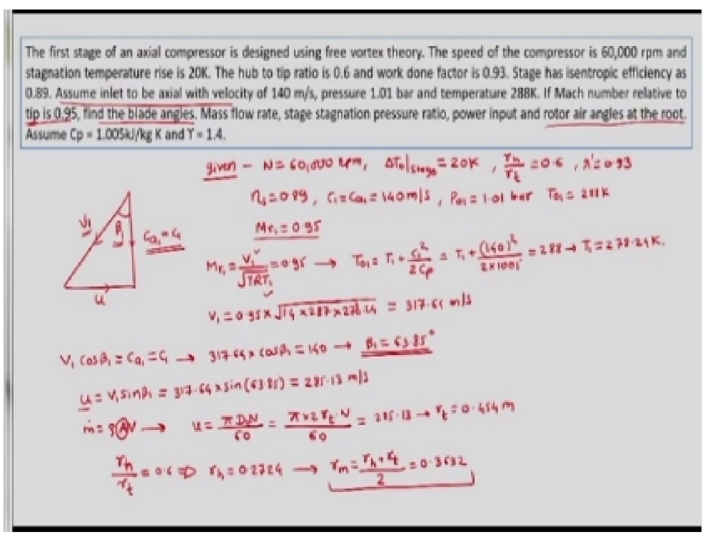
Aircraft Propulsion

Prof. Vinayak N. Kulkarni

Department of Mechanical Engineering
Indian Institute of Technology-Guwahati

Lecture-32
Examples of Axial Flow Compressor

Welcome to the class we will see some more examples on axial compressor the example reads that the first stage of an axial compressor is designed using free vortex theory. (refer time: 00:36).



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The speed of compressor is 60,000 rpm and stagnation temperature rise is 20 K. Hub to tip ratio is 0.6 and work done factor is 0.93, stage has isentropic efficiency of 0.89. Assume inlet to be axial with velocity of 140 m/s, pressure 1.01 bar and temperature 288 K. If Mach number relative to the tip is 0.95, find the blade angles, mass flow rate, stagnation pressure ratio, power input and rotor angle at the root.

Assume see C_p to be 1.005 and gamma to be 1.4, in this case we are told that inlet is having axial velocity. So we can draw the velocity triangle special for this inlet is we are having C_{a1}

= C_1 and this is V_1 and this is u, so this is β. For this case we are having $\alpha_1 = 0$, such that it is totally axial velocity then we will say what are the given things in this example, in this example N is said to be 60,000 rpm, then $\Delta T_0|_s = 20$ Kelvin, hub to tip pressure we can take it for the radius of hub and radius of tip as 0.6, work done factor as 0.93 then stage efficiency as 0.89.

Then we are told that $C_1 = C_{a1}$ is 140 m/s, P_{01}, 1.01 bar T_{01} 288 K, we are told that Mach number with relative velocity at the tip. It is important we are told to do everything at the tip then that tip Mach number with relative velocity is 0.95. Having said this we can find out the first the blade angles. So for that we have to find out beta, beta can be found out if we know any of the other velocities we know just C_1 at this moment.

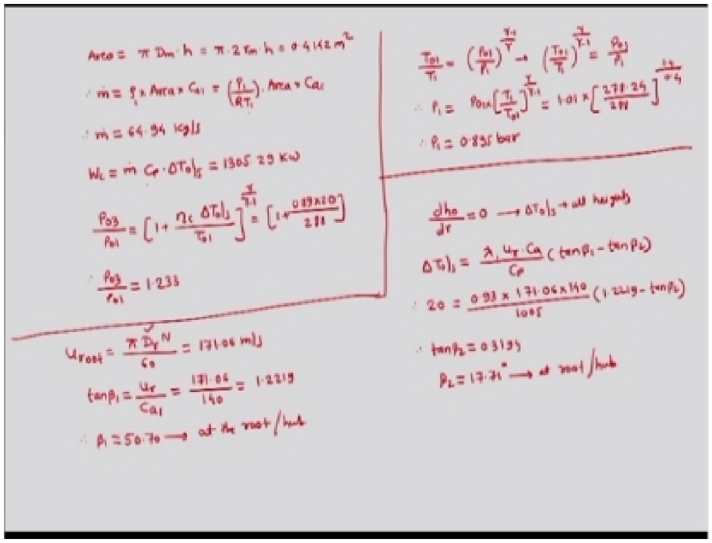
But we are given M_{r1}, so we have hint that we can find out V_1 since $M_{r1} = \frac{V_1}{\sqrt{\gamma R T_1}}$, here we are knowing it to be 0.95. So if we know T_1 then we can find out V_1, so let us find out T_1, we know total temperature is given it is equal to $T_1 + \frac{C_1^2}{2C_p}$. So $T_1 + C_1^2$ is $140^2/(2 \times 1005) =$ 288 K, so this gives us $T_1 = 278.24$ K. Having said this we can get now V_1.

Since $V_1 = 0.95$ into gamma 1.4, R 287 and T_1 is 278.24, so we know V_1, from here as 317.64 meters per second. Now we know V_1, we know C_1 we can find out beta, so which we know that $V_1 \cos \beta_1 = C_{a1}$ or C_1, so we know that 317.64 into $\cos \beta_1 = 140$, so it gives us β_1 as 63.85°. So this is one of the answers, having said this we have to next find out u we can find out u, since $u = V_1 \sin \beta_1$, V_1 is known $317.64 \times \sin 63.85$ and this gives us u as 285.13 m/s.

Now we are told to find out mass flow rate, stagnation pressure rise all these quantities, but for that we have to find out for mass flow rate we know $\dot{m} = \rho AV$. So, we know we need to find out area for that we need to find out radii and height. So let us work for this aspect to find out area, so we can use u which is $\pi DN/60$ we have to remember that we use tip Mach number, so we have this u at the tip, so we will get diameter at the tip.

So $\pi \times 2 \times$ radius at the tip $\times$ N divided by 60 and that is equal to 285.13, so from here we get radius at the tip as 0.454 m, having said this we are given that radius at the root or hub divided by radius at the tip = 0.6. So we can have a radius at the hub as 0.2724, so we can make use to find out mean radius and mean radius is radius at hub, less radius at tip divided by 2. And then that is 0.3632, we are interested in mean radius since we know that everything needs to be evaluated at mean radius.(refer time: 08:24)

So we can move ahead and find out area and area = π mean diameter $\times$ h and we know that it is equal to $\pi \times 2 \times$ mean radius $\times$ h. So for h height of the blade is known to us from radius at the root and radius at the hub and radius at the tip. So height is known from that we can find out area it turns out to be 0.4142 m^2. So we can now find out mass flow rate which is density $\times$ area $\times$ velocity and that velocity will be C_1 or C it is always C_{a1} for axial.



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But in this particular case $C_{a1} = C_1$, but density is not known at 1, so we will use $\frac{P_1}{RT_1}$ $\times$ area $\times$ C_{a1} but again next problem is T_1 is known, but we do not know P_1. So for that we can use isentropic relation which says that $\frac{T_{01}}{T_1} = \left(\frac{P_{01}}{P_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)}$. So we have $\left(\frac{T_{01}}{T_1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)} = \frac{P_{01}}{P_1}$. So P_1 = $\frac{P_{01}}{\left(\frac{T_{01}}{T_1}\right)^{\left(\frac{\gamma}{\gamma-1}\right)}}$, having said this we know P_{01}, we know T_1 and we know T_{01}.

So we can make use $1.01 \times T_1$ $(278.24/288)^{\frac{1.4}{0.4}}$, so we get P_1 as 0.895 bar. So we can make use of P_1 in this mass flow rate formula and we can get ρ_1 or equivalently we have written $\frac{P_1}{RT_1}$. So everything if we put we will get mass flow rate 64.94 kg/s, so we can find out then the compressor work which is $\dot{m} \times C_p \times \Delta\, T_0|_s$, which is given to us as 20 K $\times$ we have found out C_p is known.

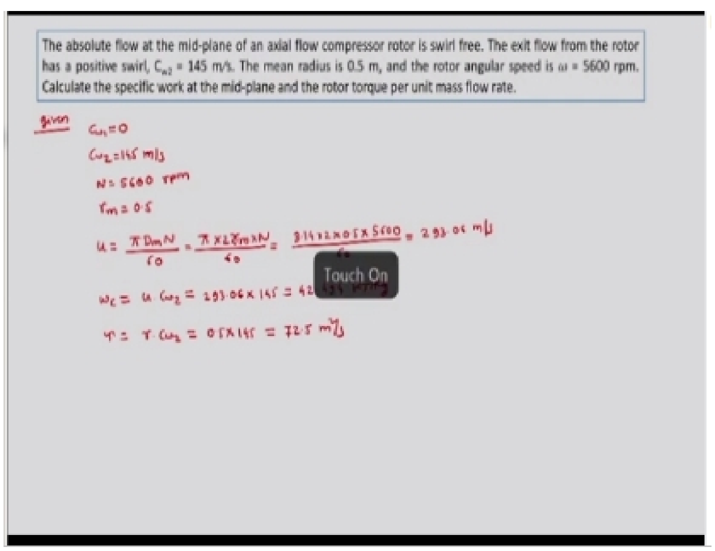
So it gives us work input as 1305.29 kW, so then we can find out pressure ratio which is $P_{03}/P_{01} = \left(1 + \frac{\eta_c \Delta T_0|_s}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$. So $T_0|_s$ is known compressor efficiency is 0.09, 0.89 $\times$ 20 divided by 288, so P_{03}/P_{01} which is stage pressure ratio it turns to be 1.233. Now we are told to find out the angles we are told to find out rotor air angles at the root.

So everything what we work for the angles was at the tip, so now we have to find out at the root. So to find out at the root, we have to go ahead and then first find out u velocity at the root, so let us find out u velocity at the root which is π D at the root or hub $\times$ N divided by 60. So it turns out to be 171.06 m/s, we have found out radius at the hub or radius at the root. And then we know N we are using this formula we will get u at the root.

But whether it is hub or root, we have everywhere axial velocity same, so from the velocity triangle and β_1 = u at the root divided by C_{a1}. So we can get 171.06 divided by 140, so it gives 1.2219, so we have β_1 = 50.70 at the root, inlet blade angle at the root is this. Now we have to find out outlet blade angle, so for outlet blade angle we can assume we can we are rather told that it is free vortex design.

And in the free vortex design we know assumption is dh_0 by dr is 0 that is $\Delta h_0|_s$ or $\Delta T_0|_s$ is same at all the heights at all heights we can make use of it and then we can write $\Delta T_0|_s$ for the root or for the tip is power work done factor $\times$ u at the root in or hub $\times$ C_a divided by C_p $\times$ (tan β_1 - tan β_2). So we have this as 20 is factor work done factor as 0.93 u as 171.06, this is 140 divided by 1005.

So we know tan β_1 which is 1.2219 - tan β_2, so this gives us tan β_2 = 0.3194, so β_2 = 17.71° at root or hub. So this is how we would solve the example which is giving different requirements for root and hub. Let us work out with the next example, which states that absolute velocity at the mid plane of an axial flow compressor rotor, is swirl free.(refer time: 16:11)



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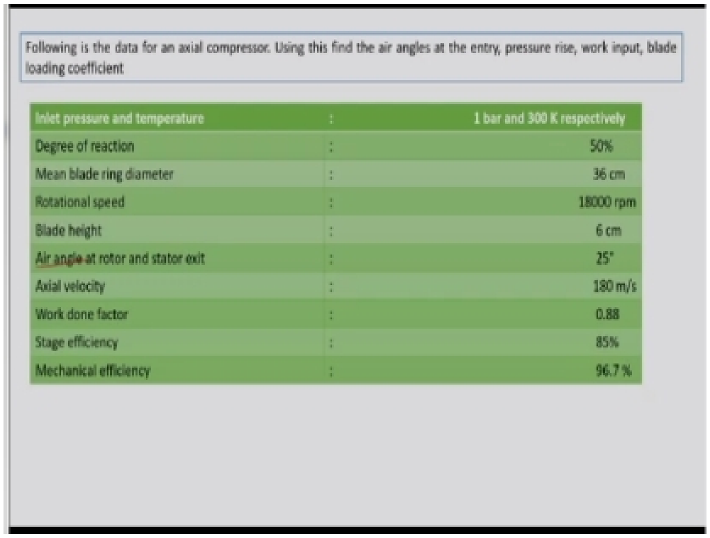
So it is told that we have axial entry, so we have C_{w1} as 0 the exit flow from the rotor as positive swirl and we are to given with C_{w2} at 145 m/s. mean radius is 0.5 and rotor angular speed ω is 5600 rpm. Calculate specific work at the mid plane and rotor torque per unit mass rate this is a simple example where everything is given. We are having N as 5600 rpm and then we have mean radius r_m as 0.5, we can find out u first which is $\frac{\pi D_m N}{60}$, so it is $\pi \times 2 \times r_m \times N/60$.

So we know 3.14 × 2 × 0.5 × 5600/60, so this gives us velocity 293.06 m/s. We are supposed to find out compressor work or stage work which for as is u $\times$ C_{w2} and C_{w1}, but C_{w1} is 0. So we have u $\times$ C_{w2} and u is 293.06, C_{w2} is 145. So it is basically 42.494 kJ/kg. We can also find out torque and torque is r $\times$ C_{w2}, for this case which is 0.5 $\times$ 145. So we have torque 72.5 m/s, for the you need mass flow rate assumption this this is a small example, where we found out all the things which are required.(refer time: 18:47)



Following is the data for an axial compressor. Using this find the air angles at the entry, pressure rise, work input, blade loading coefficient

Inlet pressure and temperature	:	1 bar and 300 K respectively
Degree of reaction	:	50%
Mean blade ring diameter	:	36 cm
Rotational speed	:	18000 rpm
Blade height	:	6 cm
Air angle at rotor and stator exit	:	25°
Axial velocity	:	180 m/s
Work done factor	:	0.88
Stage efficiency	:	85%
Mechanical efficiency	:	96.7 %

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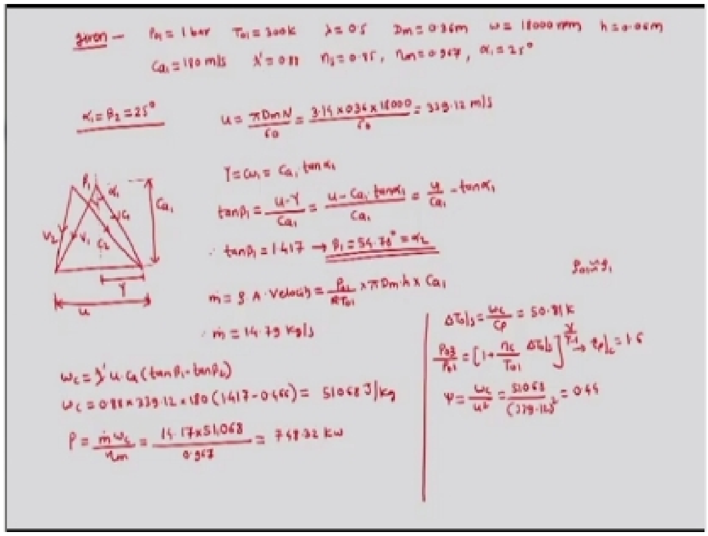
We can go ahead with the next example, which states that following is the data for an axial compressor using this find out air angle at entry pressure rise, work input, blade loading coefficient and the given things are inlet pressure and temperature, 1 bar, 300 Kelvin, degree of reaction 50%, mean blade ring diameter 36 cm, rotational speed 18,000 rpm, blade height 6 cm, air angle at root and stator exit is 25°, axial velocity 180 m/s, work done factor 0.88, stage efficiency 85% and mechanical efficiency 96.7%.(refer time: 19:38)

So having said this we can write down what are the given things in this example, where we are told that P_{01} = 1 bar, T_{01} = 300 K, 50% reaction λ 0.5, mean diameter as 0.36 m, then we are told that ω is 18,000 rpm, then height of the blade is 0.06 m. Then we are told axial velocity is 180 m/s, work done factor λ' is 0.88, stage efficiency isentropic is 0.85 and mechanical efficiency is 0.967.

We are told α_1, absolute velocity angle as 25°, since it is told that the angle is air angle when it is air angle, it is the absolute velocity angle. So we know now, we are told to first find out blade angles but for 50% reaction we know $\alpha_1 = \beta_2$ but α_1 is given, so β_2 is also known, which is 25°. So now we have to find out β_1, before that we will first plot the velocity triangle here this is C_1, this is V_1, this is C_2, this is V_2, this is α_1, this is β.



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So and then this is C_{a1} knowing this, we can go ahead and first we can find out u which is $\frac{\pi D_m N}{60}$, we know π is 3.14, D_m is 0.36 into N and N is told to be 18,000/60, so we got u as 339.12 m/s. Now we are given with this C_{a1} which is 180 m/s, we are given with α_1, so we can find out this distance which is suppose y. So y are for us $C_{w1} = C_{a1} \times \tan \alpha_1$, so this is y.

Further we can make use of this y since this is complete u, so we will have V_1 we know that tan β_1 is equal to in terms relative velocity we can say that it is equal to $\frac{u-y}{C_{a1}}$. So it is $\frac{u-C_{a1} tan\beta_1}{C_{a1}}$, so it is $\frac{u}{C_{a1}}$ - tan β_1, so sorry it is tan α_1. Having said this we can find out we know α_1, we know u, we know C_{a1}.

So we know tan β_1, from this relation and it turns out to be 1.417 and this gives us β_1 = 54.78° and then this itself is α_2. Since we are given with 50% reaction rate 50% degree of reaction, now we can go ahead and find out mass flow rate. So mass flow rate is $\rho \times$ area $\times$ velocity and so we have we do not know ρ, so we can find out rho which is $\frac{P_1}{RT_1}$ for all sake we will assume total density is same as stagnation density.

So roughly our assumption is ρ_{01} is same as ρ_1 , so this is $\frac{P_{01}}{RT_{01}} \times \pi \, D_m \times$ h $\times$ C_{a1}. So we know now, this everything we can find out mass flow rate, from here which turns out to be 14.79 kg/s knowing this we can find out compressor work as λ' work done factor $\times$ u $\times$ C_a $\times$ (tan β_1 - tan β_2). So compressor work input is 0.88 $\times$ 339.12 $\times$ C_a is 180, tan β_1 is 1.417 - 0.466

This gives us compressor work as 51068 J/kg, but compressor work is different from power input, since we are told that there is certain mechanical efficiency. So power input is ($\dot{m} \times W_c$)/η_m , so $\dot{m}$ is 14.17 power input is uhhh to be calculated. So 51068 divided by mechanical efficiency 0.967 it gives us 748.32 kW.

Now we can find out total temperature rise in the stage, then that stage temperature rise we know total temperature rise in the stage is work done of the compressor divided by C_p, so it turns out to be 50.81 K. Now we can find out pressure rise $P_{03}/P_{01} = \left(1+\frac{\eta_c \Delta T_0|_s}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$. From here, we get compressor pressure ratio which is P_{03}/P_{01} as 1.6 for the stage.

And then we can find out the factor which is the blade loading coefficient which is ψ and ψ is W_c/u^2. So W_c is 51068 u^2 is $(339.12)^2$ it comes out to be 0.44. So this is the way we will solve the examples of axial compressor, thank you.