Aircraft Propulsion

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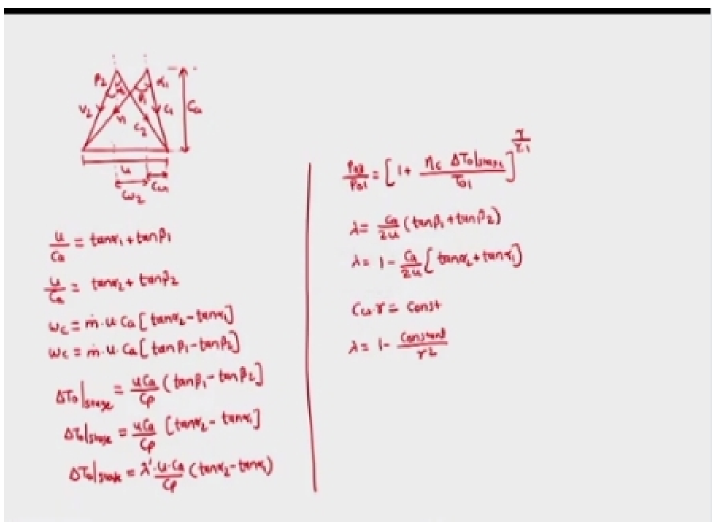
Lecture-31
Examples of Axial Flow Compressor

Welcome to the class we are going to see the examples on axial compressor, before moving to example let us see the salient features, what we learnt from the axial compressor which are essential to solve the example (refer time: 00:37).



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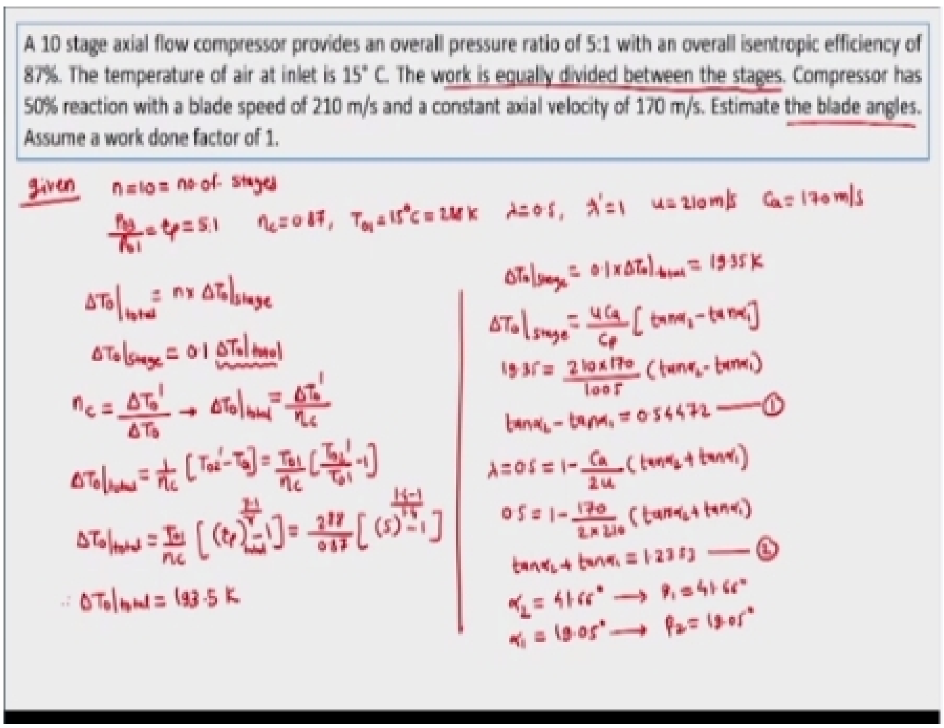
First is the velocity triangle, so this is the velocity triangle for axial compressor, where we have this as C_1, this as V_1 and then C_1 has an angle α_1, V_1 has an angle β_1. So this is C_2 which has angle α_2 and then we have V_2 which has angle β_2, then we have this as u, so further we would have this small will be C_{w1} and then this would be C_{w2} and then this height would be C_a.

Then we have derived certain relations which include $u/C_a = \tan\alpha_1 + \tan\beta_1$ further $u/C_a = \tan\alpha_2 + \tan\alpha_2$. We know compressor work, $W_c = \dot{m}\, u\, C_a (\tan\alpha_2 - \tan\alpha_1)$. So, $W_c = \dot{m}$ u

C_a (tan β_1 - tan α_2), there is a formula which we derived for $\Delta T_0|_s$ and then this is $\frac{uC_a}{C_p}$(tan β_1 - tan β_2).

So $\Delta T_0|_s = \frac{uC_a}{C_p}$(tan α_2 - tan α_1) further if we have the work done factor λ then $\Delta T_0|_s$ can be written as $\lambda' \frac{uC_a}{C_p}$(tan α_2 - tan α_1). Then we know that there is pressure rise $P_{03}/P_{01} = 1 + \eta_c$ $\Delta T_0|_s$, so this is stage efficiency divided by T_{01} bracket rise to $\frac{\gamma}{\gamma-1}$, so this is stage efficiency.

Further ° of reaction lambda = C_a upon twice u $\times$ tan β_1 + tan β_2, so it is equal to lambda = 1 - $\frac{C_a}{2u}$(tan α_2 + tan α_1). Free vortex theory says that C_wr = constant and as per that theory we have ° of reaction = 1 - $\frac{constant}{r^2}$. So these are the salient features which we learnt from the topic of axial compressor, so let us go ahead with the example.(refer time: 05:09)



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The example reads that a 10 stage axial flow compressor provides an overall pressure ratio of 5 : 1 with an overall isentropic efficiency of 87%. The temperature of air at inlet is 15 ° Celsius, work is equally divided between the stages, compressor has 50% reaction with a blade speed of to 210 m/s and constant axial velocity of 170 m/s, estimate blade angles. This is the fact which we have to calculate, where we have to assume work done factor to be 1.

So let us see what is given here, so we have few things which are given in this example n is 10 which is number of stages. Further pressure ratio P_{03}/P_{01} or what we say as r_p is 5:1. Then overall isentropic efficiency is told as 87%, inlet initial temperature is told as 15 $^\circ$ Celsius which is 288 Kelvin. λ is told as 0.5, work done factor $\lambda^{'}$ is said as 1. Blade speed u = 210 m/s and axial velocity C_a is 170 m/s.

We know that we are told that work is equally divided between all the stages, if work is equally divided between all the stages then total and total change in the total temperature is equal to number of stages $\times$ total temperature in a stage. So total temperature stage would be equal in all stages, since we know if we multiply both sides by C_p then we can get this C_p ΔT_0 as total compressor work and this is number of stages and this will become compressor work in a stage.

So this total work would get divided equally in all stages it implicitly means that the temperature rise is same in all the stages. So we can say that temperature rise in 1 stage = 0.1 ΔT_0 which is total, since there are 10 number of stages. We know the compressor efficiency, so we can find out the total temperature rise, so we know compressor efficiency, $\eta_c = \frac{\Delta T_0^{'}}{\Delta T_0}$.

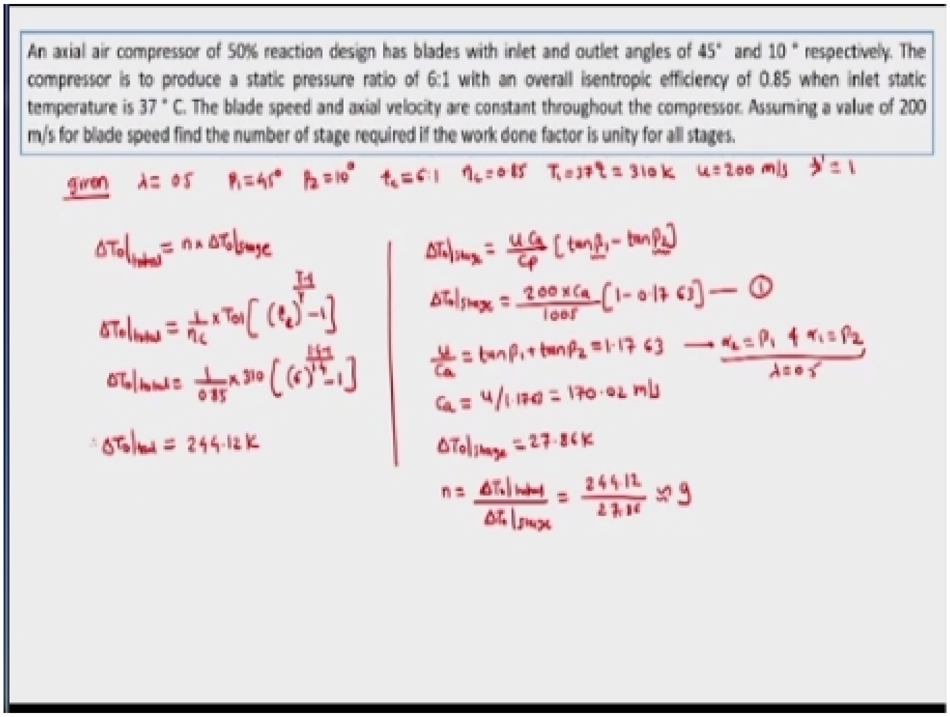
So Δ T_0 which is actually we need which is total = Δ $T_0^{'}$ divided by compressor efficiency. So $\Delta T_0|_{total} = \frac{1}{\eta_c}$ $(T_{02}^{'} - T_{01})$ we can take T_{01} common, so $\frac{T_{01}}{\eta_c}$. So $\frac{T_{02}^{'}}{T_{01}}$ - 1, we can use the relation which is isentropic relation for the total temperature ratio. So we can have $\frac{T_{01}}{\eta_c}$ $\times$ r_p pressure ratio bracket rise to $\frac{\gamma-1}{\gamma}$ - 1.

So here we are using total pressure ratio which is given as 5:1, so when we keep here T_{01} is 288 compressor efficiency is said as 0.87 then r_p is 5 and γ is $\frac{1.4-1}{1.4}$ - 1. So we get $\Delta T_0|_{total}$ as 193.5 Kelvin, so we can say that $\Delta T_0|_s = 0.1 \times \Delta T_0$ total. So it is equal to 19.35 Kelvin we can find out Δ T_0 we used the $\Delta T_0|_s$ to find out the blade angles.

Since we are suppose to estimate blade angles, so we have to find out β_1 and β_2 , so we need 2 expressions. So first expression we can make use of for $\Delta T_0|_s$ and that expression says that it is $\frac{uC_a}{C_p}(\tan\alpha_2 - \tan\alpha_1)$. So we can put $\Delta T_0|_s$ which is 19.35 = u which is 210 $\times$ C_a which is 170 divided by C_p which is 1005 $\times$ $(\tan\alpha_2 - \tan\alpha_1)$ and this gives us a relation which says that $\tan\alpha_2 - \tan\alpha_1 = 0.54472$ we name it as equation number 1.

Further we can make use of relation which is for $^\circ$ of reaction which is said to be $0.5 = 1 - \frac{C_a}{2u} \times (\tan\alpha_2 + \tan\alpha_1)$. So knowing this relation we can put all the numbers we can put 0.5 = 1 - C_a is 170 divided by 2 $\times$ 210 $\times$ $(\tan\alpha_2 + \tan\alpha_1)$. So we have $\tan\alpha_2 + \tan\alpha_1$ as 1.2353 we will name it as equation number 2, we can solve this equation simultaneously.

We will get $\alpha_2 = 41.66^\circ$ and α_1 has 19.05° since it is a 50% reaction $\alpha_2 = \beta_1$ and β_1 is 41.66° and $\alpha_1 = \beta_2$ which is 19.05°. So this is where we have solved the example which was related to finding out blade angles.(refer time: 13:31)
We will move to next example which reads that an axial flow compressor of 50% reaction de-



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sign has blades with inlet and outlet angles as 45° and 10° respectively. The compressor is to produce a static pressure ratio of 6:1 with an overall isentropic efficiency of 0.85 when inlet static temperature is 37° Celsius. The blade speed and axial velocity are constant throughout the compressor assuming a value of 200 m/s of blade speed find number of stages of compressor required if work done factor is unity for all stages.

So it is given in the example that we are having λ which is degree of reaction as 0.5, we are told that inlet and outlet blade angles β_1 is 45 ° and β_2 is 10°. We are told that the overall pressure ratio is r_p pressure ratio or r_c, r_p or r_c pressure ratio is 6:1 an isentropic efficiency is 0.85, T_1 is 37° Celsius which is equal to 310 K. We are told that u = 200 m/s and $\lambda^{'}$ is 1 which is work done factor.

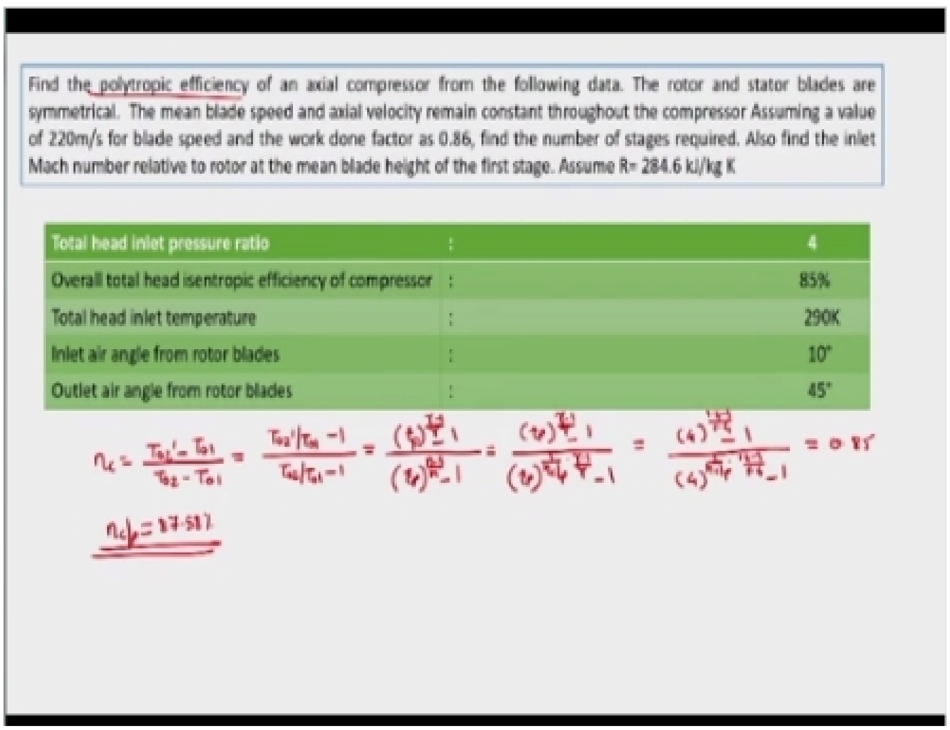
So we have to find out in this example the number of stages required, so here as what we did we can we know that total temperature change in the compressor is equal to number of stages total temperature change in a stage if we divide every stage is having same temperature rise. So for that we can find out the overall temperature change or total temperature change in the complete compressor as $\frac{1}{\eta_c}$ T_{01} r_c which is compression ratio bracket rise to $\frac{\gamma-1}{\gamma}$ - 1.

So knowing this we can put $\Delta T_0|_{total}$ = 1 upon compressor efficiency which is 0.85, total temperature is 310, compression ratio is 6, $\frac{1.4-1}{1.4}$ - 1, so we get total temperature change in the complete compressor is 244.12 K. Now we can find out the stage temperature rise $\Delta T_0|_s$ this is equal to $\frac{uC_a}{C_p} \times (\tan \beta_1 - \tan \beta_2)$, since we are given with β_1 and β_2.

So we can put this over here and we can get that $\Delta T_0|_s$ = u is told to us which is 200, so we are having u as 200 × C_a divided by C_p as 1005 tan β_1, β_1 is 45 so tan is 1 tan β_2 is 0.1763 we will put as equation number 1. We know that $u/C_a = \tan \beta_1 + \tan \beta_2$, so that is equal to 1.1763. So we know u = 200, so C_a = u divided by 1763 so C_a becomes 170.02 m/s.

Now we can put this C_a in this equation number 1 and we can get $\Delta T_0|_s$ as 27.86 Kelvin. So this is the temperature rise in the stage and this temperature rise in the stage is required for us for the calculation of total number of stages having said this. Here we are making use of the fact that $\alpha_2 = \beta_1$ and $\alpha_1 = \beta_2$. Since we are told that we are having degree of reaction as 0.5, so we can know that total number of stages is $\Delta T_0|_{total}/\Delta T_0|_s$.

So we have 244.12 divided by 27.86 and hence the answer comes around 9, so we need 9 stages for doing this compression work.(refer time: 20:03)



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Let us go to the next example which says that find the polytrophic efficiency of an axial compressor from the following data the rotor and stator blades are symmetrical. So the rotor and stator blades are symmetrical this statement itself means that we are given with degree of reaction 0.5. So we can take $\alpha_2 = \beta_1$ and $\beta_2 = \alpha_1$, mean blade speed and axial velocity remains constant throughout the compressor.

Assuming a value of 220 m/s of blade speed and the work done factor to be 0.86, find the number of stages required, also find the inlet Mach number relative to the mean blade height of the stage. Assume R to be 284.6, given data is total head inlet pressure ratio is 4 overall head isentropic efficiency is 85%, total inlet total head inlet temperature is 290 Kelvin, inlet air angle for the rotor is 10°, outlet air angle for the rotor is 45°.

So first we have to find out polytrophic efficiency, so we will we do not have to write the given things for finding out polytrophic efficiency. Since we can use the formula what we already know compressor efficiency is ($T_{02}^{'}$ - T_{01}) divided by (T_{02} - T_{01}). So we can take T_{01} common, so $T_{02}^{'}$ divided by T_{01} - 1 divided by T_{02} upon T_{01} - 1, so here we can make it in terms of compression ratio or pressure ratio.

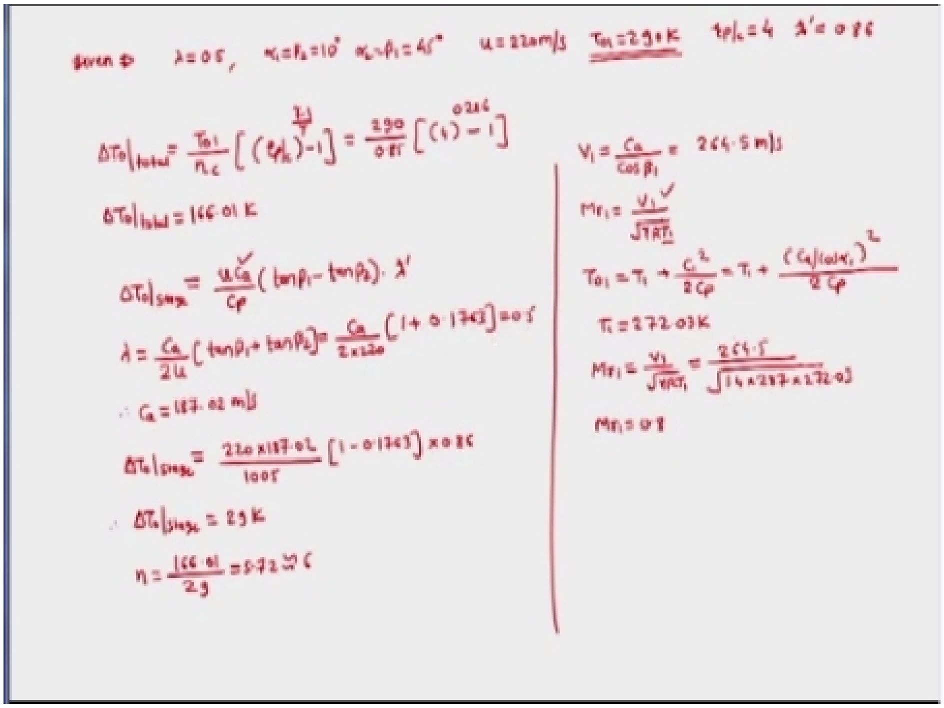
So we can write r_c bracket rise to $\frac{\gamma-1}{\gamma}$ - 1 we will mention it as pressure ratio which are same, so pressure ratio of the compressor. So this is bracket rise to $\frac{\gamma-1}{\gamma}$, since it is isentropic and in the bottom we have polytrophic process it is $\frac{n-1}{n}$ - 1 but we know that we can represent the denominator in terms of polytrophic efficiency. So we can have r_p bracket rise to $\frac{\gamma-1}{\gamma}$ - 1.

We can say in the denominator it is 1 upon compressors polytrophic efficiency × $\frac{\gamma-1}{\gamma}$ - 1. Now we can put whatever it is known to us pressure ratio is 4 bracket rise to 1.4 - 1 divided by 1.4 - 1 and here also it is 4 rise to compressors polytrophic efficiency into 1.4 - 1 divided by 1.4 - 1 = 0.85. Here in this case we just do not know what is compressors polytrophic efficiency and this comes out to be 87.58%.

So this is what we can get from the given data which is just the pressure ratio overall pressure ratio and the compressors isentropic efficiency.(refer time: 24:02)
We can make use of the given quantities for rest of the things, so given is here we are told that blades are symmetrical, so we are told that λ = 0.5. We are told that α_1 air angle at the inlet is 10°, so $\alpha_1 = \beta_2$ = 10° and $\alpha_2 = \beta_1$ = 45°, u is told as 220 m/s, T_{01} is 290 K. Then as it is known r_p compressor is told as 4 and work done factor $\lambda^{'}$ or λ_1 is given as 0.86.

So we are supposed to find out number of stages and relative Mach number at the inlet. So again for number of stages we can find out $\Delta T_0|_s$ using the formula for isentropic efficiency which says that the $T_{01}|_{total} = T_{01}/\eta_c \times r_p$ compressor bracket rise to $\frac{\gamma-1}{\gamma}$ - 1. So we have T_{01} which is told as 290 divided by 0.85 bracket rise to 4 sorry into 4 rise to 0.286 which is known to us - 1.

So this gives us total temperature change in the complete compressor as 166.01 Kelvin, for stage total temperature change we can use the formula of *eta* which says that $\frac{uC_a}{C_p}$ × (tan β_1 - tan β_2) × work done factor. But before that we have to find out C_a, since C_a is required in



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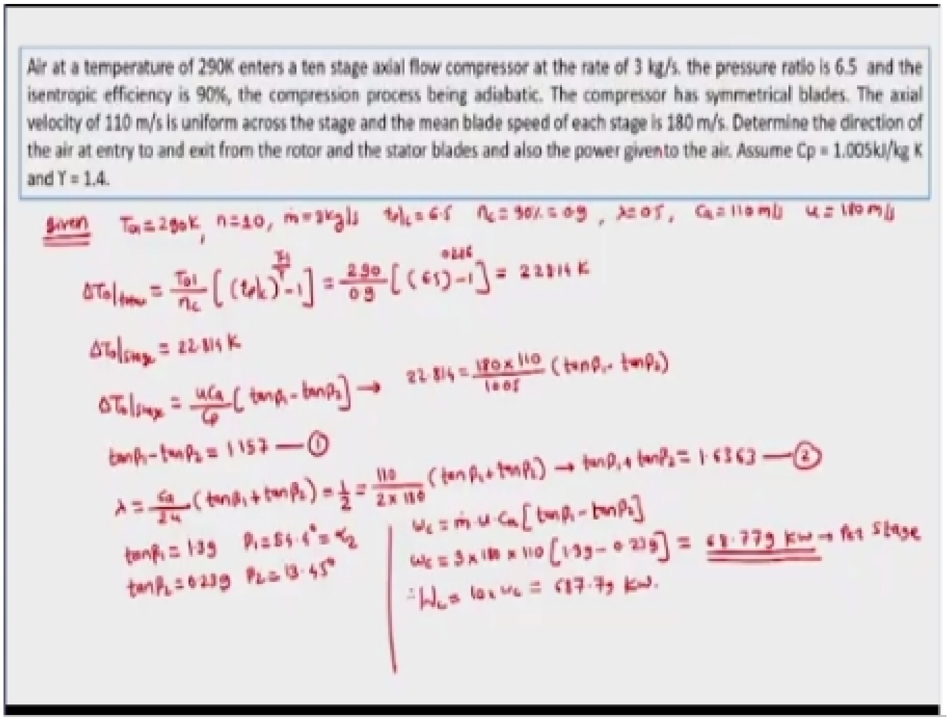
this formula, so we can (26:40) only u is given. So C_a can be find out found out from degree of reaction which is given as $\lambda = 2 = \frac{C_a}{2u} \times (\tan\beta_1 + \tan\beta_2)$.

So we can get it as (C_a/ 2 × 220) ×(tan β_1 is 0.1763 + β_1 is 1, tan β_1 is 1 and tan β_2 is 0.1763) this is equal to 0.5 and this gives us C_a which is 187.02 m/s. Now we know C_a we can find out $\Delta T_0|_s$ and $\Delta T_0|_s$ turns out to be 220 into 187.02 divided by 1005 into tan β_1 which is 1 - 0.1763 into 0.86. So we have $\Delta T_0|_s$ = 29 K.

So we can find out number of stages as 166.01 divided by 29 and we get around 5.72, so it is around 6 stages required. Then we are suppose to find the relative Mach number and relative Mach number can be found out using relative velocity. But for that we have to first find out relative velocity from velocity triangle $V_1 = \frac{C_a}{cos\beta_1}$, so which is equal to 264.5 m/s then we need to know the formula which is relative M_1 is $V_1/\sqrt{(\gamma R T_1)}$.

So now V_1 is known we just do not know T_1 since we are known with T_{01}, so we can find out T_1 from T_{01} using the formula which says that $T_{01} = T_1 + \frac{C_1^2}{2C_p}$. But we again do not know C_1, so C_1 can be replaced again by C_a by saying that $C_1 = \frac{(\frac{Ca}{cos\alpha_1})^2}{2C_p}$. So this gives C_1 and then we can find out T_1 from here from known T_{01}, so we get T_1 as 272.03 Kelvin.
So we can know Mach number relative at inlet = $V_1/\sqrt{(\gamma R T_1)}$ and then this is equal to 264.5

divided by 1.4 into 287 into 272.03 and this gives us M_{r1} as 0.8. So this is how we would have solved the example for the axial compressor where we were asked to first find out the polytrophic efficiency and then the Mach number and also number of stages.(refer time: 31:19)



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We will move on to next example which states that air at a temperature of 290 Kelvin enters a 10 stage axial compressor at the rate of 3 kg/s, the pressure ratio is 6.5 and isentropic efficiency is 90%, the compression process being adiabatic. The compressor has symmetrical blades, axial velocity is 110 m/s and it is uniform across the stage and main blade speed of each stage is 180 m/s.

Determine the direction of air at entry to and exit from the rotor and the rotor blades and also power given to air given that assumes C_p as 1.005 and gamma as 1.4. So let us see what is given in this example given first is T_{01} which is 290 K, number of stages 10 then $\dot{m}$ is 3 kg/s. Then r_p compressor as 6.5, then compressors isentropic efficiency as 90% or 0.9, blades are symmetric.

So λ = 0.5 then axial velocity C_a = 110 m/s and then mean blade speed u = 180 m/s and then we are supposed to find out the exit and entry rotor and stator angles and power input given. So we can find out first total temperature rise which is equal to T_{01} upon compressor efficiency into pressure ratio of compressor bracket rise to $\frac{\gamma-1}{\gamma}$ - 1.

So we can know the overall compression, now overall total temperature rise as 290 divided by 0.9 which is compressor efficiency into 6.5 bracket rise to 0.286 - 1 and this gives us total temperature change in the complete compressor is 228.14 K. So since there are 10 stages, so Δ $T_0|_s$ = 22.814 K but we know that Δ $T_0|_s = \frac{uC_a}{C_p} \times$ (tan β_1 - tan β_2).

So here we know that 22.814 = 180 into 180 is u, C_a is 110 divided by C_p 1005 $\times$ tan β_1 - tan β_2. So we can get tan β_1 - tan β_2 = 1.157 and we will see it as equation number 1, we can know other equation from degree of reaction which is $\frac{C_a}{2u} \times$ (tan β_1 + tan β_2). But degree of reaction is 1/2 and 1/2 = C_a 110 divided by 2 into 180 into tan β_1 + tan β_2.

So we get other expression as tan β_1 + tan β_2 = 1.6363, so this we will say it has equation number 2 we can solve the equation simultaneously. We will get tan β_1 = 1.39 which gives us $\beta_1 = 54.4^\circ$ which is also is equal to α_2 and tan β_2 = 0.239 which gives β_2 = 13.45 degrees. Now we can find out compressor work it is told to us $\dot{m}$ $\times$ u $\times$ C_a $\times$ (tan β_1 - tan β_2).

So we have tan β_1, we have now tan β_2 we can put everything, so $\dot{m}$ is also given 3 u is 180, C_a is 110, tan β_1 is 1.39, tan β_2 is 0.239 and this gives us compressor work as 68.779 kW and this is obtained from the velocities triangle. So this is per stage. This is for 1 stage of the compressor, since there are no total 10 stages. So it is 10 into stage work, so it is 687.79 kW.

So this is how we have solved the other example given on the compressor having said this we will see the other examples and other concepts in the other classes, thank you.