Aircraft Propulsion

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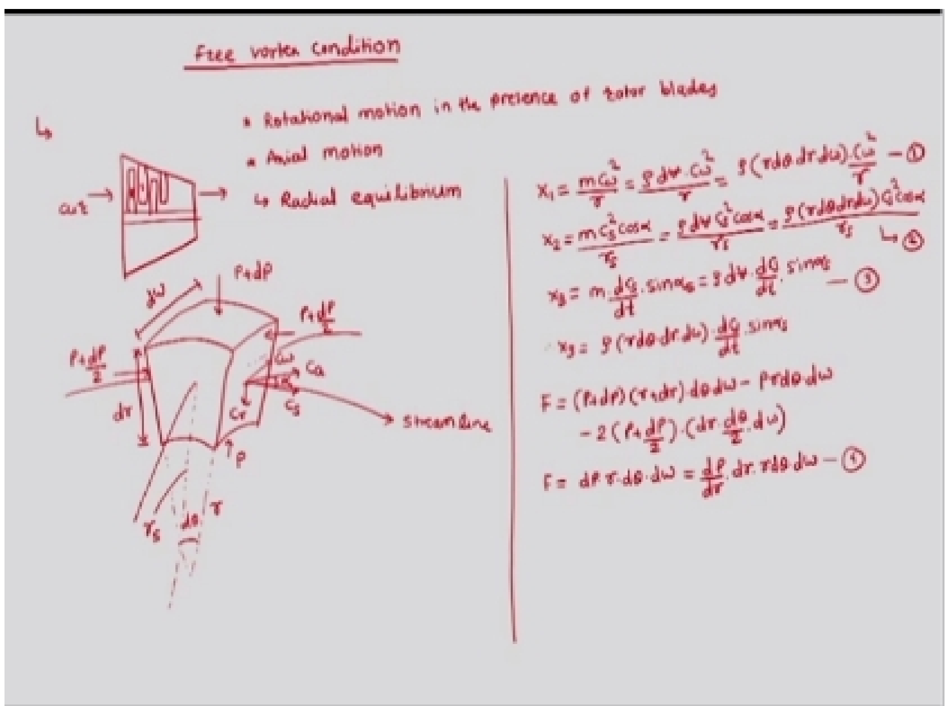
Lecture-30
Axial Flow Compressor Free vortex Condition

Welcome to the class, we are talking about axial compressor, today's discussion will be focused upon free vortex condition in the design of axial compressor. So let us see what does it mean by free vortex condition for the design of axial compressor (refer time: 00:44).



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Actually when a fluid element is passing from the compressor, we know that this is a schematic of axial compressor, this is the shaft. And fir$_s$t the fluid element will go from rotor and then it will go from the stator. So this is rotor, this is stator and then these rotor$_s$ and stator$_s$ are arranged in series. So in this case we are having flow in axial direction for the air and this air is experiencing 2 motions, 1 is the rotational motion in the presence of the rotor blades.

So there is rotational motion in the presence of rotor blades and then there is axial motion of the fluid particle. So these are 2 motions experienced by the fluid particle, let us consider radial equilibrium and condition for radial equilibrium. Here let us feel that there is a fluid particle of a particular shape which we have drawing now. This is the fluid particle and then this fluid particle is flowing, this fluid particle is passing from the compressor in this manner.

So this fluid particle is going along us stream line, so this is stream line in the steady flow. And then but this fluid particle is also rotating along with the rotor, so this is the rotational direction. So let us consider the geometrical dimensions let this be dr, since this is along the radius of the compressor. And then this is the r radius of the compressor and then there will be the stream line which is having curvature and that stream line has it's own radius.

And then that radius we are denoting by r_s stream lines radius, so this we are denoting by r_s. Now fluid is flowing with axial velocity and then it has radial velocity C_r and then this is the velocity which is tangent to this stream line. And this is C_s which is making angle α with axial velocity and then there will be fluid element which is moving this direction with velocity C_w. So for the pressure which are acting upon the fluid element let us consider that the pressure acting on this surface is dp pressure acting on this surface is acting on the bottom surface is p on top surface is p + dp.

And on side surfaces the pressure which is acting is (P+dp)/2, let the angle between the 2 faces is $d\theta$, from the geometry dw is the width of the fluid element. Now we are drawn the free body diagram of the fluid element and now we will consider different accelerations and the forces which are acting upon the fluid element. So first we will consider acceleration which is suppose X_1 and that is due to the centripetal force and that acceleration is in the presence of velocity C_w.

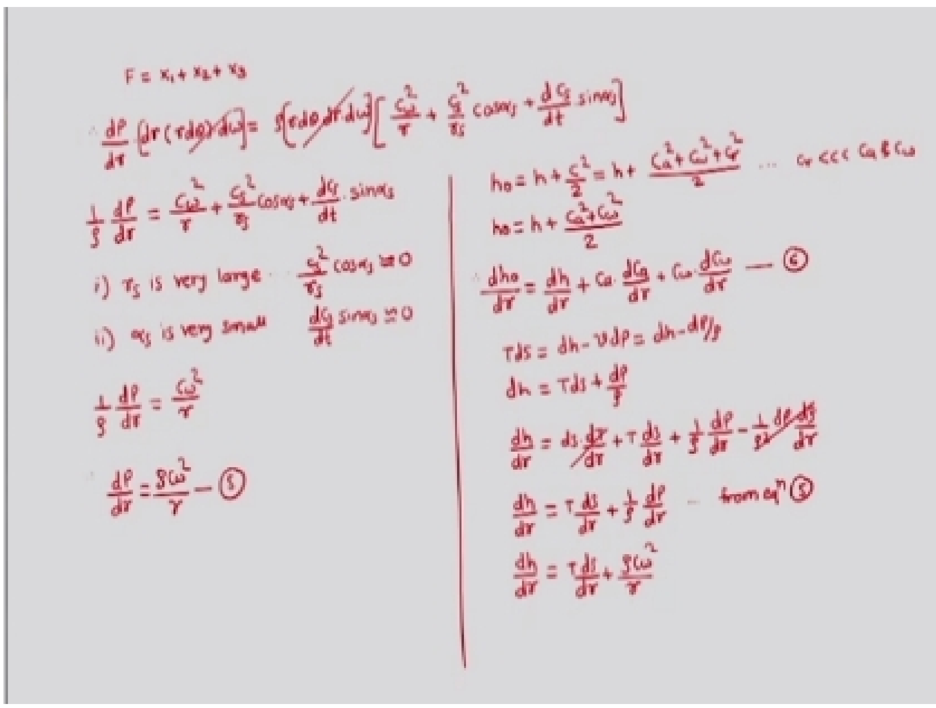
So this is $(mC_w^2)/r$, this is due to the rotational motion along the rotor. Now we can find out m which is mass, which is $(\rho dV\ C_w^2)/r$. But we can further write down volume as $((r\ d\theta\ dr\ dw)\rho\ C_w^2)/r$. so this is the first acceleration in the presence of rotational motion, second acceleration is due to the curvature of the stream line and then that acceleration $mC_s^2\cos\ \alpha$.

Since it is the direction of velocity $C_s^2\cos\ \alpha/r_s$ which is the radius of the curvature of the stream line. So this is again (rho dV C_s^2 cos α)/r_s and this gives us $[\rho(r\ d\theta\ dr\ dw)C_s^2\cos\alpha]/r_s$. now there is third acceleration for the fluid element and that acceleration is due to change in velocity with respect to time and that is $(dC_s/dt)\sin\ \alpha_s$.

We are talking about radial equilibrium so this acceleration is in the radial direction, so we again can have $\rho\ dV(dC_s/dt)\ \sin\ \alpha_s$. So we have $X_3 = \rho(r\ d\theta\ dr\ dw)(dC_s/dt)\sin\ \alpha_s$. Now these are the accelerations let us consider the force which is F that force is prominently due to pressure, so consider pressure forces acting in acting on the fluid element.

So for the top surface we have force which is (P + dp)(r + dr)dθ dw while for the bottom P rdθ dw and for the side forces the pressure force will be 2(P + dp/2) (dr.$\frac{d\theta}{2}$.dw). If we expand this and neglect the higher order terms we can write F = dp.r.dθ.dw only one term would remain which would be this term hence this term can be written as $\frac{dp}{dr}$.dr.r.dθ. dw.

So we have 1 acceleration X_1 due to the tangential velocity, acceleration 2 due to the curvature of stream line, acceleration 3 due to the unsteadiness in the flow or the time varying C_s velocity and then we have acceleration, we have force which is due to the pressure.



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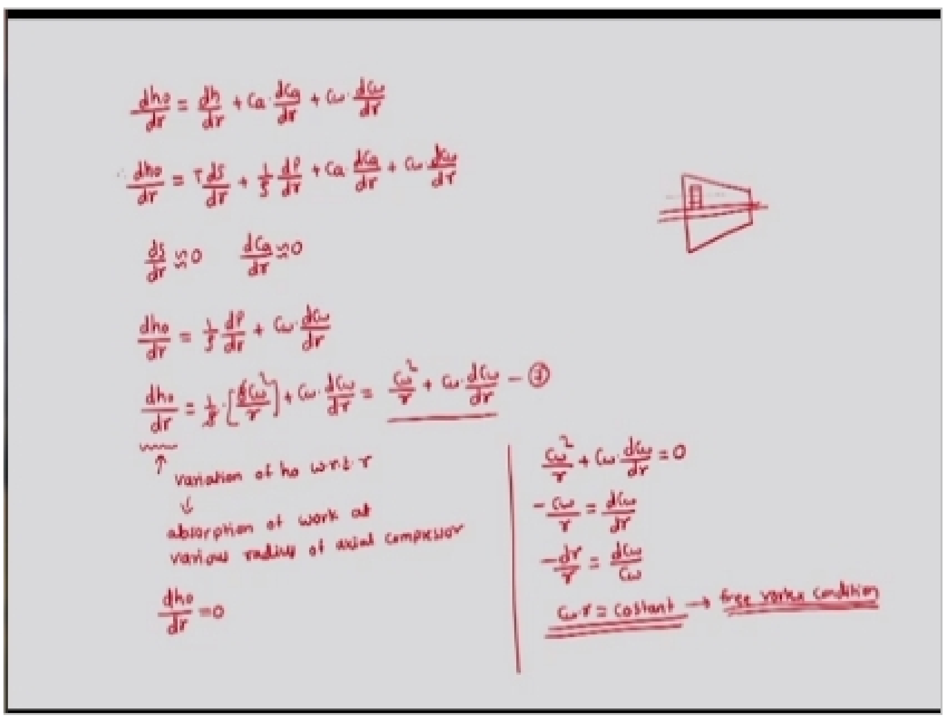
(refer time: 10:52). Now let us equate this and we can see that F = X_1 + X_2 + X_3, so we have $\frac{dp}{dr}$.dr.r.dθ. dw is equal to we can take mass which is common which is ρ(r.dθ.dr.dw) and all the term related to velocity will be inside the bracket. So first will be for the tangent velocity of the fluid element, second is curvature term for the stream line which is C_s^2 cos α_s and third term is (dC_s/dt)sin α_s.

Now this term gets cancelled, so we will have dp and the ρ would remain, so that will be $\frac{1}{\rho}$ (dp/dr) = C_w^2/r + (C_s^2/r_s) cos α_s + (dC_s/dt)sin α_s . Now let us make certain assumptions. First assumption here is that r_s is very large, so (C_s^2/r_s)cos $\alpha_s \simeq 0$. Second assumption is α_s is very small and this makes $(dC_s/dt)\sin \alpha_s$ which is very small and then that would go to 0.

So having these assumptions made we can write down $(dp/dr).(1/\rho) = C_w/r$ or $(dp/dr) = \rho C_w/r$. We will mention it as equation number 5, having said this now this was from the momentum equation. Let us get another expression from the thermodynamic which states that h_0 is the total enthalpy of the fluid element and that is equal to $h + C^2/2$ and then this is $h + C^2$ can be further decomposed into $(C_a^2 + C_w^2 + C_r^2)/2$.

Now here C_r is very very low in comparison with C_a and C_w so we can write $h_0 = h + (C_a^2 + C_w^2)/2$. So variation of h_0 with respect to r is dh_0/dr that can be said as $dh/dr + C_a(dC_a/dr) + C_w(d\,C_w/dr)$. Further we know that second law of thermodynamics and first law mix together tells that $TdS = dh - v\,dp$ or $dh - dp/\rho$, so dh, we can represent this as equation number 6, so $dh = T\,dS + dp/\rho$. dh/dr can be written as $ds(dT/dr) + T(dS/dr) + (1/\rho)\,(dp/dr) - (1/\rho^2)dp\,(d\rho/dr)$.

So we can neglect the higher order terms and then this term and this term would get cancelled and we will have $dh/dr = T\,(ds/dr) + (1/\rho)\,(dp/dr)$. However from equation number 5 from equation 5 we can write down $dh/dr = T\,(dS/dr) + \rho\, C_w^2/r$. Hence we can put this into equation number 6.



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(refer time: 17:14). And we can write down equation number 6 as $dh_0/dr = dh/dr + C_a\,(dC_a/dr) + C_w\,(d\,C_w/dr)$. So $dh_0/dr = T\,(dS/dr) + (1/\rho)\,(dp/dr) + C_a\,(dC_a/dr) + C_w\,(dC_w/dr)$.

Here we will consider (dS/dr) is close to 0 which means that entropy is constant along the radius this term would become prominent in case of compressible flows where shock is going to appear prominently, so dS/dr is going to be 0.

Otherwise, further we will also say that $dC_a/dr = 0$, since we have assumption that C_a is constant at various heights of the blade. So we have $dh_0/dr = (1 /\rho)$ (dp/dr) + $C_w(dC_w/dr)$, further we can write down for the dp/dr as $dh_0/dr = (1/\rho)\ \rho\ C_w^2/r + C_w\ (dC_w/dr)$. So we have $C_w^2/r + C_w\ dC_w/dr$, now this is our equation. We will number this as equation number 7.

In this equation number 7 we have left hand side which says that dh_0/dr and we have right hand side which has terms related to velocity. This left hand side term says that variation of h_0 with respect to r that means this term is related with absorption of work at various radius of axial compressor. Now we assume that we have designed axial compressor such that absorption is uniform, so $dh_0/dr = 0$.

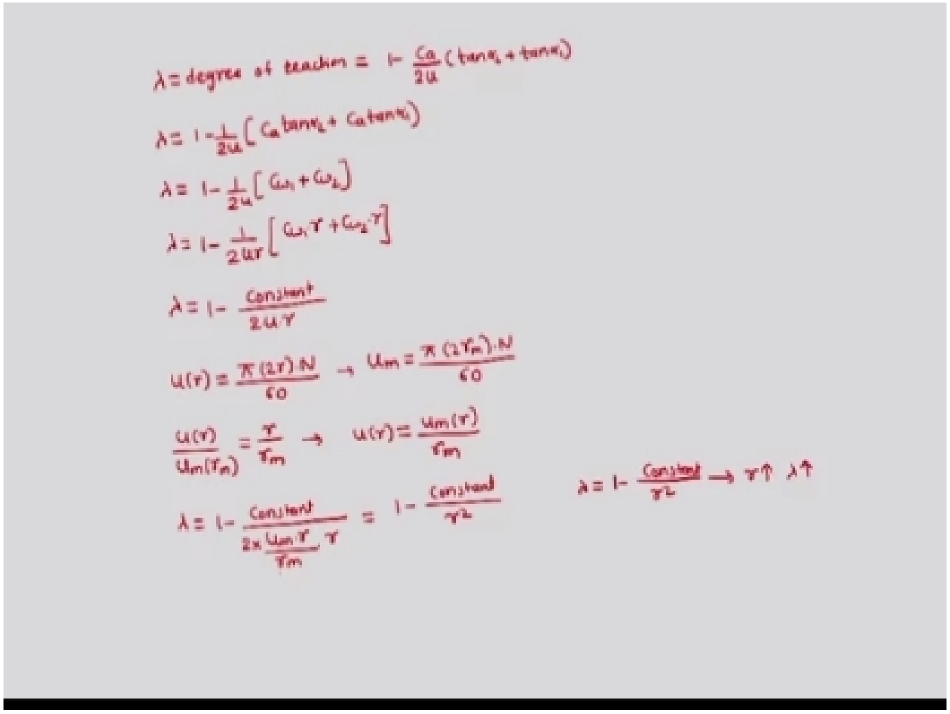
So with this assumption we can write down $C_w^2/r + C_w\ (dC_w/dr) = 0$, so we have C_w/r it is minus = dC_w/dr which is $-dr/r = dC_w/C_w$ and which says that $C_w r$ = constant. Upon integration we would get this $C_w r$ as a constant and this is called as free vortex condition. If in the axial compressor $C_w r$ is constant at different radii then it is called as free vortex condition.

We should remember again that in the axial compressor if this is the compressor, this is the shaft and this is the rotor. So for the rotor blade we have always consider that we are working at the mean height, so we can work at any height it corresponds to any radius. But our all derivations till time for the pressure rise for stage temperature rise where related to the velocity triangle drawn at mean height. But if we try to plot velocity triangles at different heights with the constraint of free vortex condition then we will have $C_w r$ as constant(refer time: 22:50).

So we will see it is repercussion, it says that $C_w r$ as constant, so let us see λ as degree of reaction and we have derived a relation which says that 1 - $(C_a/2u)(\tan\alpha_2 + \tan\alpha_1)$. So λ = 1 - $(1/2u)(C_a \tan\alpha_2 + C_a \tan\alpha_1)$, this from the velocity triangle we can write down as C_{w1} + C_{w2} for the terms in the bracket. Let us multiply and divide by r such that we will have 2 u r and then we will have $C_{w1}r + C_{w2}r$.

But $C_w r$ is constant, so we have 1 - (constant/(2 u r)), so this is u velocity at some radius r. So this u is at some r and we know it is equal to $(\pi\ (2r)\ N)/60$. And this we can represent if we know u mean which is the mean blade speed at mean radius which is $(\pi\ (2r_m)\ N)/60$. Let us divide both the terms then we have $u(r)/u_m(r_m) = r/r_m$.

So we can have $u(r) = u_m$ into r/r_m, we will put it in the expression of λ and we can get $\frac{constanst}{2\frac{u_m.r}{r_m}.r}$. So this here mean radius and mean blade speed would be constant, so 1 - that constant will get further strengthened and we will have only r^2. So this expression says that λ =



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1 - constant/r^2, so if r increases that means if the location from the blade root increases.

Then degree of reaction increases for the axial compressor which is designed with free vortex principle. So this is how we would work for the design of axial compressor and then associated derivations. So here we have seen in all that how to find out the velocity triangles what is the variation of different thermodynamic and fluid dynamic parameters along the axial compressor what are different components of an axial compressor.

We derive for stage temperature rise we derived for pressure ratio, we derived for degree of reaction and then for the constraint of the free vortex condition. We will see next topics in the next class, thank you.