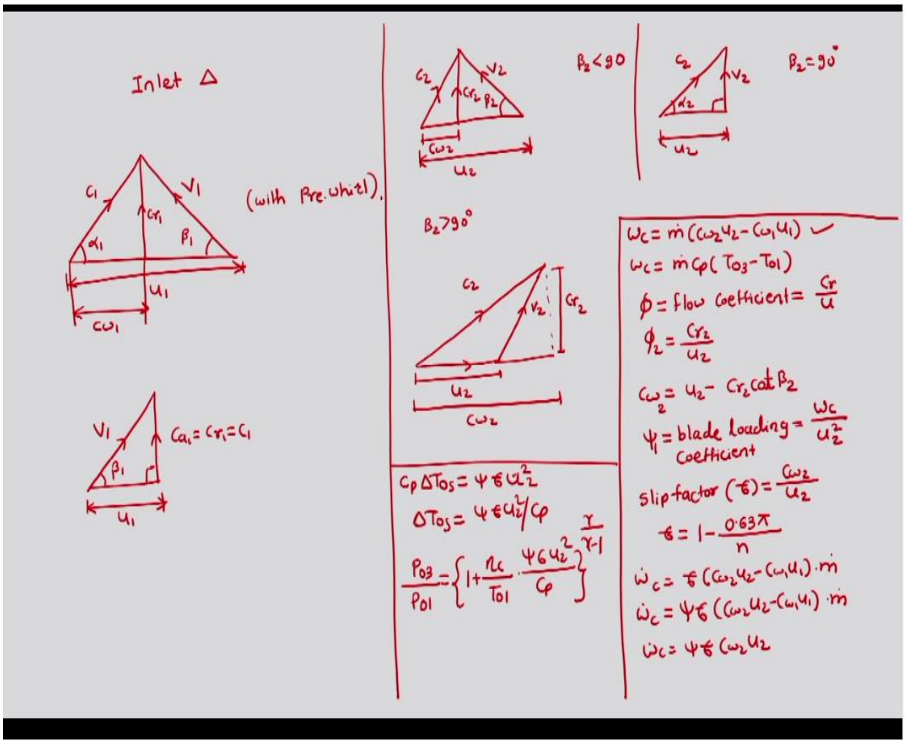
Aircraft Propulsion

Vinayak N. Kulkarni
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Lecture - 28
Examples of Centrifugal Compressor

Welcome to the class. We have till time seen about the centrifugal compressor and now we are going to go ahead with the examples on centrifugal compressor. So, before starting the examples, we will make recapitulation about the centrifugal compressor.

(Refer Slide Time: 00:46)



What we know is inlet velocity triangle and inlet velocity triangle is like this for centrifugal compressor where we have this is Cr1, this is C1, this is V1 and then big length is u1 and then this is Cw1. So, angle made by C1 is α1 and v1 is β1. So, this is inlet velocity triangle with prewhirl. If there is no prewhirl that is without guide when we know that the velocity triangle would be of this kind where we have $Ca_1 = Cr_1 = C1$. This is v1, α1 is 90 degree, this is β1 and then we have u1.

Then, we have outlet velocity triangles, which are of 3 types. Outlet velocity triangle of this kind is for backward sweep which is β2 less than 90, so this is v2, this is β2, this is u2, and this is C2 and cw2 where the central vertical line is Cr2. Then, for β equal to 90 which is for radial case. For radial outlet, we have velocity triangle of this kind where we have this as u2, this is v2 and this is c2.

This is α2, β2 is equal to 90 degree, then we have the velocity triangle which is for β2 greater than 90 degree, which is forward sweep and for forward sweep velocity triangle is of this kind where this is V2, this is C2, this is u2 and then we have this big as Cw2 and then this is Cr2. So, this is about the velocity triangle for the centrifugal compressor. Then, we have some formulas what we had seen.

The first formula what we know is about compressor power where we have

$$\dot{m}(Cw2\ u2 - Cw1\ u1)$$

. Similarly, compressor power input is also

$$\dot{m}Cp(T03 - T01)$$

We have seen that phi is flow coefficient and its definition we have made it as Cr/u. So, in general we needed φ2 which was Cr2/u2 and then from the velocity triangle we can in general write Cw2 = u2 - cr2 cotβ2.

Further we know Ψ which is a blading coefficient, blade loading coefficient which is also called as pressure coefficient and this is $Wc/u2^2$. Then, we have slip factor which is sigma and for that we are knowing it is tangential velocity ratio divided by u2 and then this sigma has a correlation as 1 - 0.63 φ/n where n is number of blades. Ideal work input whatever we have written was this.

But this work input with slip factor $\sigma C_{w_2} u_2 - Cw1\ u1 \times \dot{m}$. if there is frictional loss or windage then we have formula $\Psi \sigma(C_{w_2} u_2 - cw1)\ u1 \times \dot{m}$. So, in a particular case, we can write down the formula without inlet prewhirl as

$$\dot{w} = \Psi\ \sigma\ C_{w_2} u_2.$$

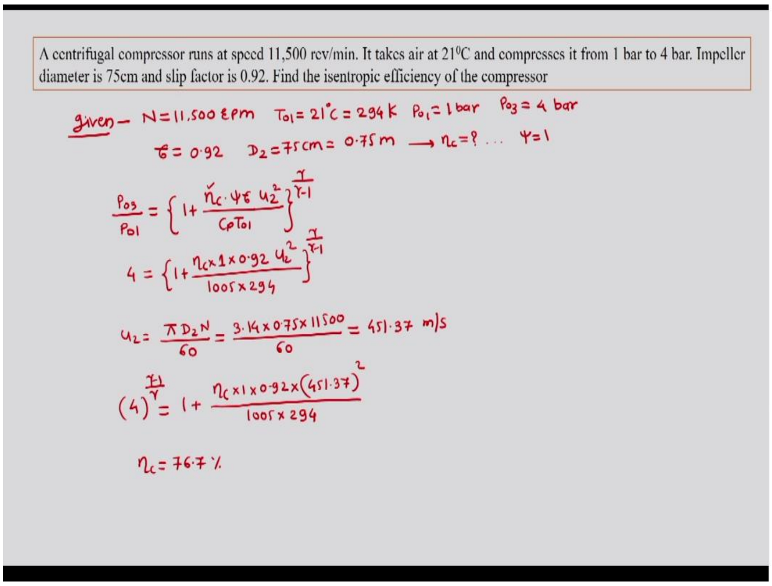
Further we have formula for stage temperature rise which is delta T0s and then that we can write down for the radial exit as $\Psi\ \sigma\ u_2^2$.

So, ΔT0s was $\Psi \sigma u2^2/Cp$. Similarly, we have pressure ratio

$$\frac{P03}{P01} = \left(1 + \left(\frac{\eta}{T01}\right)\left(\frac{\Psi \sigma u2^2}{Cp}\right)\right)^{\frac{\gamma}{\gamma-1}}$$

So, this is recapitulation of centrifugal compressor for velocity triangle on various diagrams.

(Refer Slide Time: 07:39)



Now, will go ahead and solve the example. First example reads that a centrifugal compressor runs at a speed of 11,500 revolutions per minute. It takes air at 21°C and compresses it from 1 bar to 4 bar. Impeller diameter is 75 centimeter, slip factor is 0.92. Find isentropic efficiency of the compressor. So, we will see what are the given things. Given things in the example is N which is 11,500 rpm.

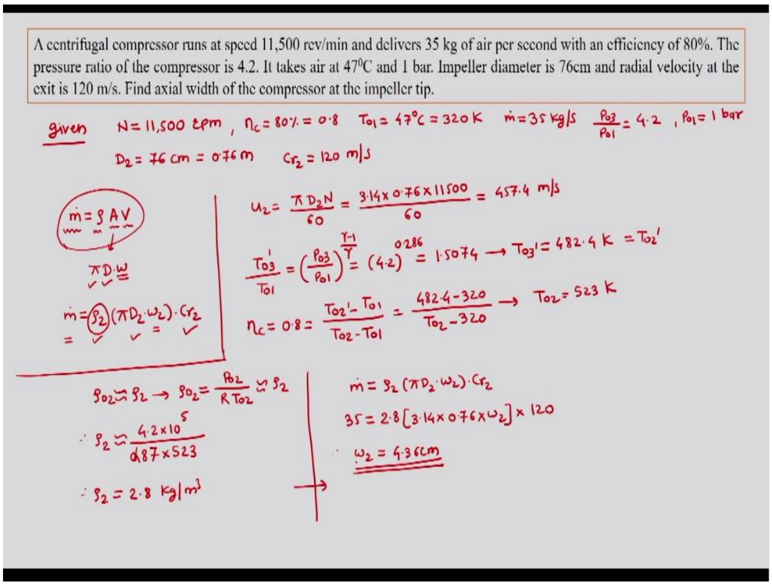
Then, we are given with T01 which is 21 degree Celsius. So, this is equal to 294 Kelvin, P01 is equal to 1 bar, P03 is equal to 4 bar. Then, we are given with slip factor σ as 0.92. We are told that diameter D2 is 75 centimeter, so it is 0.75 meter. We are supposed to find out what is compressor efficiency. If we look back, we are practically given everything from the formula which is

$$\frac{\text{P03}}{\text{P01}} = \left(1 + \frac{\eta c}{T01}\left(\frac{\sigma \Psi \text{u2}^2}{\text{Cp}}\right)\right)^{\frac{\gamma}{\gamma-1}}$$

So, we are just not knowing ηc, we are known with everything. For this example, we will take the losses due to friction as negligible so size equal to 1. So, left hand side P03 is 4, P01 is 1, so it is given so we have 1 plus eta c into size equal to 1 into sigma is equal to 0.92, we need to find u2 square divided by 1005 T01 is 294. Let us find out u2 where u2 is pi D2 N by 60 where we can take pi is 3.14, D2 is 0.75, N is 11,500 divided by 60.

This will give u2 as 451.37 meter per second. Having said this, we can write down 4 raise to $\frac{\gamma-1}{\gamma}$ = 1 + ηc×1 × 0.92 × $(451.37)^2$ /(1005×294). So, this gives us ηc = 76.7%. So, this is the isentropic efficiency of the compressor. So, this is how we would approach the example.

(Refer Slide Time: 11:39)



We will move on to the next example where it states that a centrifugal compressor runs at speed of 11,500 revolutions per minute and delivers 35 kg of air per second with an efficiency of 80%. The pressure ratio of the compressor is 4.2. It takes air at 47 degree Celsius and 1 bar. Impeller diameter is 76 centimeter; radial velocity at the exit is 120 meter per second. Find the axial width of the compressor at the impeller tip.

Here, we are given with few things, let us mention those. N is equal to 11,500 rpm, eta c is 80% which is equal to 0.8, T01 is equal to 47 degree Celsius which is 320 Kelvin, m dot is given as 35 kg per second, P03 upon P01 as 4.2 where P01 is given as 1 bar. D2 is given as 76 centimeter which is 0.76 meter, cr2 is told to be 120 meter per second. So, we are supposed to find out width of the impeller exit.

So, we know that ṁ is equal to ρ A×velocity. So, here we are given with m dot, we are supposed to find out area but within area we know for the annular passage we can write down area as φD × width, so diameter is given, φ is given, we need to find out ρ and V at the outlet so which is basically

$$\dot{m} = \rho_2 \, \pi \, D_2 \times w_2 \times cr_2$$

where Cr2 is given, ρ2 can be calculated and D2 is given.

We need to find out w2 for given m dot. So, let us see first we need to find out u2 and u2 is pi D2 N/60 and then this 3.14 into 0.76 into 11,500 divided by 60 which is 457.4 meter per second. So, this is the tangential velocity at the outlet. Now, we can find out, we need to find out density at outlet. For that, we need to know temperature at the outlet, so will proceed to find out temperature.

What we can know quickly is

$$\frac{T03}{T01} = \left(\frac{P03}{P01}\right)^{\frac{\gamma-1}{\gamma}}$$

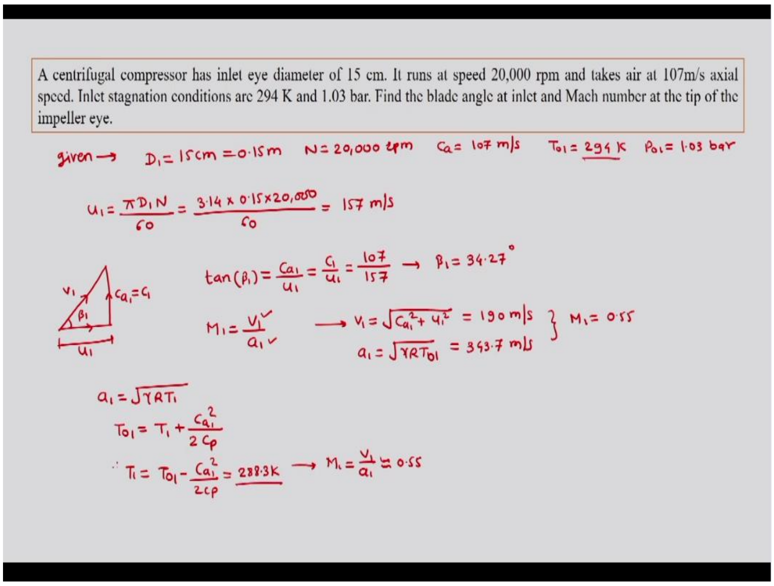
This is 4.2 bracket raise to 0.286, this will be 1.5074. So, this gives us T03 dash and then that is basically 482.4 Kelvin. So, this is the temperature at exit. Basically, this is also equal to T02 dash since in the diffuser we have no work input, so total temperature would remain constant. So, having said this now we can use formula for efficiency, which is given as 0.28.

So, this will become $(T_{02}' - T_{01}) / (T_{02} - T_{01})$ or this is as well $T_{03}' - T_{01}/T_{03} - T_{01}$. So, we have $(T_{02}' - T_{01})/(T_{02} - T_{01})$. We know T_{02}', it is 482.4 -320 divided by T_{02} – 320. So, this gives us T02 is equal to 523 Kelvin. Now, velocities are low, so we can neglect the kinetic energy and we can feel that rho 02 is roughly is equal to ρ_2 where $\rho 0_2$ is equal to P_{02} upon RT_{02} and this is roughly is equal to ρ_2.

So, rho 2 is equal to 4.2 into 10 to the power 5 divided by 287 into 523. So, this gives us rho 2 is equal to 2.8 kg per meter cube. Having known with rho 2, now we can go ahead with the formula for $\dot{m}$which is $\rho_2 \pi D_2 \times w_2 \times Cr_2$. Here, $\dot{m}$ is equal to 35, ρ2 is 2.8, π is 3.14, D2 is given as 0.76 into width at the exit into radial velocity which is given as 120. So, this gives us width at the exit for the impeller as 4.36 centimeter.

This is the way we have to make use of different known factors. We learnt few things from here that we have to use the formula for mass flow rate at the exit if we need to calculate the width at the outlet of the impeller.

(Refer Slide Time: 19:07)



Let us go with the next example. This example reads that centrifugal compressor has inlet eye diameter of 15 centimeter. It runs at 20,000 rpm and takes air at 107 meter per second axial speed. Inlet stagnation conditions are 294 Kelvin and 1.03 bar. Find the blade angle at the inlet and Mach number at the tip of the impeller eye. Here, everything told to us to calculate is at the inlet itself.

So, we will write down the given things first. In the given things, we are told diameter at the inlet of the eye that is D1 is equal to 15 centimeter, so it is 0.15 meter and is told to be 20,000 rpm, axial velocity is given as 107 meter per second. T_{01} is 294 Kelvin and P_{01} is equal to 1.03 bar. So, knowing this we can find out u1, which is pi D1 N/60, so we have 3.14, D1 is 0.15, N is 20,000 divided by 60 and this gives us u1 as 157 meter per second.

From the velocity triangle, we know that this is alpha 1, this is u1, this is β1, this is v1 and this is Ca1 is equal to C1. So, we know that tanβ1 is equal to Ca1/ u1 which is C1/u1, so it is 107 divided by 157 and this gives us beta 1 is equal to 34.27 degrees. So, this is the value of β1. We are supposed to find out Mach number at the tip of the impeller eye. So, we know Mach number M_1 is equal to V_1/a_1.

We are supposed to find out relative Mach number, so this relative Mach number can be calculated if we know V_1 and a_1, we can find out V1 from this velocity triangle which says that it is equal to square root of $(Ca_1^2+u1^2)$ and then this turns out to be 190 meters per second. Now, we are supposed to find out a1, a1 is equal to square root of γRT1. Now, instead of taking this as static temperature, if we take it as total temperature, we can get a1 as 343.7 meter per second.

And then this combination gives us M1 as 0.55 but if we do not take a1 using total temperature and if we calculate static temperature required for calculation of a1, we need to use the formula for calculation of total temperature as

$$T01 = \mathrm{T1} + \frac{\mathrm{Ca}^2}{2Cp}$$

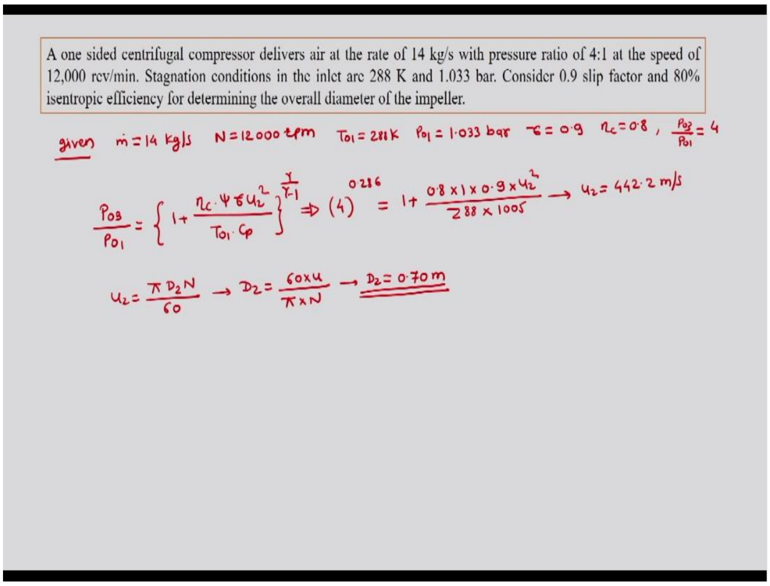
So, T1 = T01 – ca1 square divided by 2 Cp. So, this gives us T1 as 288.3 Kelvin. So, this is the static temperature at the inlet.

Now, we can calculate

M1 = v1/a1

and this also becomes close to 0.55 since total temperature and static temperature are very close by.

(Refer Slide Time: 24:15)



Having known this, we can move ahead with the next example and this example reads that a one sided centrifugal compressor delivers air at the rate of 14 kg per second with pressure ratio of 4 is to 1 at the speed of 12,000 revolutions per minute. Stagnation conditions at the inlet at 288 Kelvin and 1.033 bar. Consider 0.9 as slip factor and efficiency as 80% for determining overall diameter of the impeller.

So, we are given in this example as m dot is 14 kg per second and then revolutions N as 12,000 rpm, T01 is 288 Kelvin, P01 is 1.033 bar, slip factor sigma is 0.9, efficiency is 0.8. Having known this, we can use formula P03 upon P01 since we are given as P03 upon P01 as 4. So, this is equal

$$= \left(1 + \frac{\eta c}{T01}\left(\frac{\sigma \Psi u2^2}{Cp}\right)\right)^{\frac{\gamma}{\gamma-1}}$$

In this formula, we can take Ψ is equal to 1, so we will have P03/ P01 is 4 raise to 0.286 = 1 + 0.8 × 1×0.9 into $u2^2$ / (288 ×1005). This gives us u_2 = 442.2 meter per second. We know that u2 = πD_2 N/60, so D2 = 60 ×u / πN. So, we get D_2 from the know quantities as 0.70 meter. So, here in this example, we were expected to find out the diameter.