Aircraft Propulsion

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Lecture – 27

Centrifugal Compressor: Thermodynamic analysis, Stage Efficiency and Degree of Reaction

Welcome to the class, in last class, we were seeing about the centrifugal compressor, we are going to continue the same topic for discussion. We had seen that the centrifugal compressor, there is one impeller and then there is a diffusor and then there is a volute casing, the initially flow gets tangential velocity through the impeller which furthers gets diffused into the diffuser and casing to give the pressure rise.

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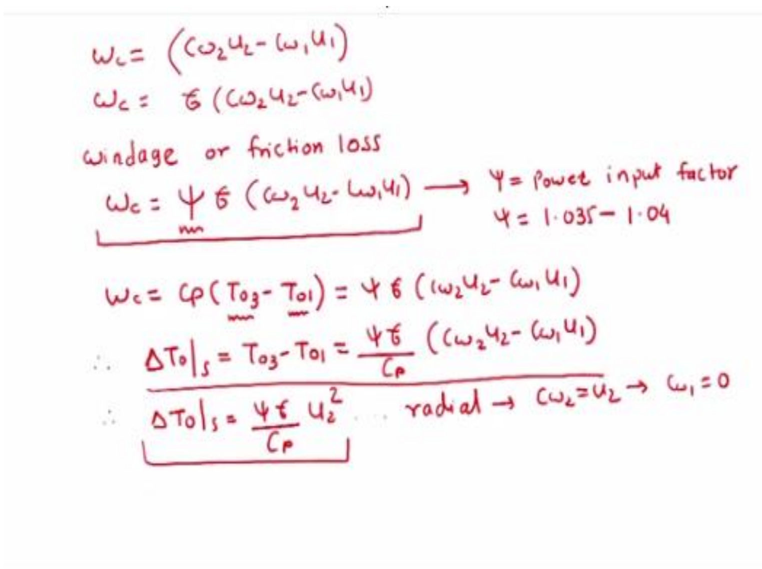

We had seen that the work input for the compressor was $C_{w2}u_2 - C_{w1}u_1$, this was an ideal work input required to give desired pressure rise, hence here we have also seen that due to the slip factor, this work input is altered and then we get a formula

$$W_c = (C_{w2}u_2 - C_{w1}u_1)\times\sigma$$

this slip factor will mainly due to the inertia of the air to follow the tangential velocity at the impeller sections.

Further, these work input would get altered due to the friction or also called as windage or friction loss, alter the work input and then we get work input as $\sigma \times \Psi$, so this is Ψ which is called as power input factor, so

$$Wc = \Psi \, \sigma(Cw_2u_2 - Cw_1u_1)$$

\where Ψ is called as power input factor, Ψ has a range of around 1.035 to 1.04, so this is the work input for a compressor which we need to evaluate from the velocity diagram.

But we know for more dynamically,

$$Wc = Cp\ (T03 - T01)$$

where T01 is total temperature at inlet to the impeller and T03 is the total temperature in the after the diffuser where we have pressure raise the desired value, so this becomes equal to psi sigma $Cw_2u_2 - Cw_1u_1$, so we can calculate $\Delta T0$ stage which is $T03 - T01 = \Psi\, \sigma/cp\ (Cw_2u_2 - Cw_1u_1)$, for a special case, of radial vanes we can have formula for this $\Delta T0s$ stage temperature rise as $u2^2 \times \Psi\ \sigma$ Cp, this is for only radial vanes where $Cw_2 = u_2$.

And basically, $Cw2 = \Psi$ u2 and then we have cw1 = 0, which is without any swirl at the inlet, so having said this we calculated the stage temperature rise, a general formula or a special formula, further we will proceed to find out the pressure rise.

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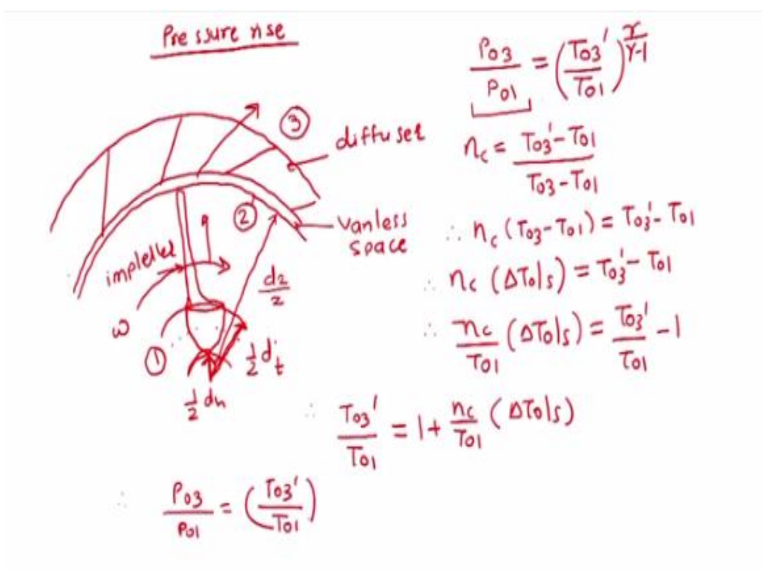

So, we are working now for pressure rise in case of a centrifugal compressor, here we know that our centrifugal compressor is in this way, where we have this as impeller and then this is a diffuser and flow goes in radial direction here and gets diffused in the diffuser, so this is diffuser, this is impeller, this diameter is 1/2 di, this is radius and this is 1/2 dh or this i is also at the hub, so this is called as hub of the impeller, so this is called as hub of the impeller.

So, diameter of the hub of the impeller is dh, so radius is 1/2 dh, this is called as tip of the impeller, so this is dt is the diameter, so dt/2 is the radius and then we have this which is exit of the impeller, so radius is d2/2 and this is when less space, we were saying 1 at the i of the impeller here it is 1 for us which is midway between hub and tip and then we have 2 here and we have 3 outside which is after the diffuser and in the volute casing.

So, after this the flow would go into the casing and these impeller rotates with omega angular velocity, now here we know that if suppose, we take isentropic compression, then

$$\frac{P03}{P01} = \left(\frac{\mathrm{T03}'}{T01}\right)^{\frac{\gamma}{\gamma-1}}$$

however, here we are interested to find out the pressure rise, to evaluate the pressure rise, we should know this temperature, where T03 is the isentropic temperature; total temperature in the casing our after the diffuser.

So, to evaluate that we need efficiency formula for the compressor, this is compressor efficiency, also stage efficiency which is

$$stage\ efficiency\ = \frac{(T03' - T01)}{(\ T03 - \ T01)}$$

where we know numerator represents the ideal work input, the denominator represents actual work input, so we have; we can take T01 common here and then we can have efficiency, we can have efficiency first, we will make use of the known things and write down the efficiency formula in the way that efficiency ×(T03 – T01) = (T03’ – T01)

This basically, is ΔT0 stage which we have already evaluated is equal to T03'– T01, we can take T01 common from right hand side and so we have

$$\frac{\eta_c}{T01}\,(\Delta T0 \text{ stage}) = T03'/T01 - 1$$

so ultimately, what we would have is

$$\frac{T03'}{T01} = 1 + \frac{\eta_c}{T01}\,(\Delta T0 \text{ stage})$$

so this remains our formula for the temperature rise.

We can use this formula which is related with isentropic temperature rise for the evaluation of the pressure ratio, so we have

$$\frac{P03}{P01} = \left(\frac{T03'}{T01}\right)^{\frac{\gamma}{\gamma-1}}$$

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$$\frac{P_{03}}{P_{01}} = \left\{1 + \frac{\eta_c}{T_{01}}\,\Delta T_0|_s\right\}^{\frac{\gamma}{\gamma-1}}$$

$$\therefore \underbrace{\frac{P_{03}}{P_{01}} = \left\{1 + \frac{\eta_c}{T_{01}}\,\frac{\psi\sigma U_2^2}{C_p}\right\}^{\frac{\gamma}{\gamma-1}}} \longrightarrow \underline{\text{Perfect gas}}$$

incompressible flow

$$T\,ds = dh - v\,dp = dh - dP/\rho$$

$$\therefore dh = dP/\rho \rightarrow dh_0 = dP_0/\rho$$

$$\underline{\underline{dP_0}} = \rho\, dh_0 = \rho\, C_p\, \Delta T_0|_s = \rho\, \cancel{C_p}\,\frac{\psi\sigma U_2^2}{\cancel{C_p}} = \underline{\underline{\rho\psi\sigma U_2^2}}$$

So, knowing this we can write down the formula for pressure rise as

$$\frac{P03}{P01} = \left(1 + \frac{\eta_c}{T01}\,(\Delta T0 \text{ stage})\right)^{\frac{\gamma}{\gamma-1}}$$

but we have already evaluated our stage temperature ratio or rise, this we can put over here and then our formula for a special case of radial vanes with no inlet swirl becomes

$$= \left(1 + \frac{\eta c}{T01}\left(\frac{\sigma \Psi u2^2}{Cp}\right)\right)^{\frac{\gamma}{\gamma - 1}}$$

If we would be interested to put the formula in general, then we can put the general formula for temperature rise and then obtain the formula for the pressure, okay, so this is the case where we can obtain the pressure rise. Here, we are using the formula for isentropic relation, so this become formula for perfect gas and which is hence the compressible fluid, so for any compressible flow, the formula can be obtained from thermodynamic relation which is

$$Tds = dh – vdp$$

which is dh – dp/ rho but process is isentropic.

So, dh = dp/rho which is in terms of total enthalpy dh0 = dp0/rho, so we have dp0 = rho dh0 and we know rho dh0 is ρ Cp×ΔT0s and then this formula becomes ρCp, so this formula becomes ρCp×ΔT0s and we have already found out ×ΔT0s as $\sigma\Psi$ $u2^2$upon /Cp, so this Cp, Cp gets cancelled and we get ρ $\sigma\Psi$ $u2^2$, so this is the pressure rise for the incompressible flow.

And this formula what we have derived is the pressure rise for compressible flow where we have used perfect gas relation for the adiabatic process. Having said this now, we have seen what is the configuration of centrifugal compressor, how to find out work input for this centrifugal compressor accounting different losses, how to use this work input to find out temperature rise and then we have seen how to find out depressor rise.

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stage efficiency.

$$\eta_c = \eta_{st} = \frac{W_c'}{W_c} = \frac{C_p(T_{03}' - T_{01})}{C_p(T_{03} - T_{01})} = \frac{C_p(T_{03}' - T_{01})}{\psi\sigma(C_{w_2}u_2 - C_{w_1}u_1)}$$

$$\therefore \eta_{st} = C_pT_{01}\left[\left(\frac{T_{03}'}{T_{01}}\right) - 1\right] \Big/ \left\{\psi\sigma(C_{w_2}u_2 - C_{w_1}u_1)\right\}$$

$$\therefore \eta_{st} = C_pT_0\left[(r_p)^{\frac{\gamma-1}{\gamma}} - 1\right] \Big/ \left\{\psi\sigma(C_{w_2}u_2 - C_{w_1}u_1)\right\}$$

Now, we will go ahead and then work for stage efficiency, this is what we have said, compressor efficiency or stage efficiency is ideal work input divided by actual work input. Ideal work input are

$$W_{ideal} = C_p \times (T_{03}' - T_{01})$$

$$W_{actual} = C_p \times (T_{03} - T_{01})$$

but actually, we have found out work input in terms of velocity triangle, so we can keep the ideal work input as it is and actual work input we can represent in terms of the one which we have obtained for velocity triangle which is $\Psi\sigma(C_{w2}u_2 - C_{w1}u_1)$.

So, stage efficiency or compressor efficiency we can find out as

$$\eta_c = \frac{C_p(T_{03}' - T_{01})}{\Psi\sigma\,(C_{w2}u_2 - c_{w1}u_1)}$$

then this we can further simplify and in terms of pressure rise or pressure ratio, we can get the stage efficiency as

$$\eta_{st} = \frac{C_pT_{01}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)}{\Psi\sigma\,(C_{w2}u_2 - c_{w1}u_1)}$$

this formula can be further made simplify for the special case of radial vanes without inlet swirl will become in the denominator $\Psi\sigma\, u_2^2$.

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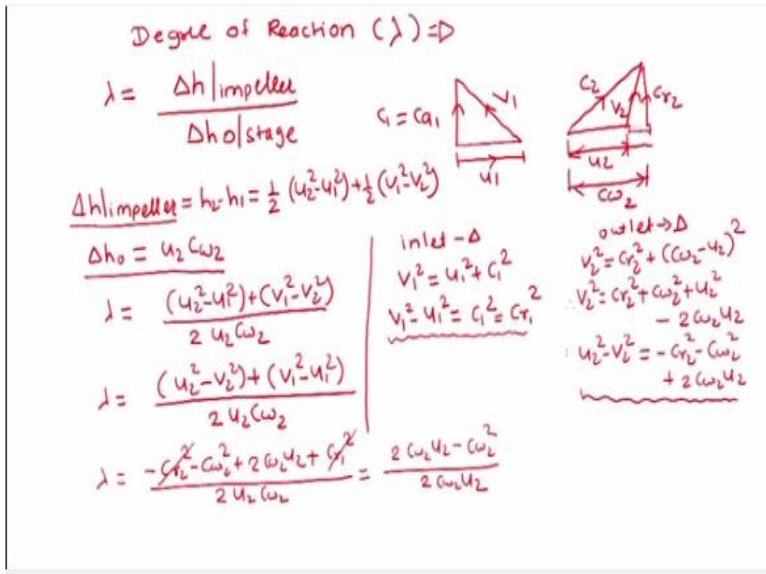

So, this is how we can find out the stage efficiency for the centrifugal compressor, so now we will move ahead for the next derivation which is for the degree of reaction which we are saying it as R or which we will say also as λ. So, λ in some books it is also represented as R, so λ by definition is Δh in the impeller divided by Δh0 in the stage, change in enthalpy in the rotor divided by total enthalpy change in the stage.

So, this is the case or derivation or expression for the degree of reaction when somebody; when some turbo machine is of impulse type, then degree of reaction is 0, when it is complete reaction based, then we will have complete enthalpy rise in the rotor, then degree of reaction would be 1 and for intermediate degrees, between 0 and 1 we have to evaluate using this expression where which says that enthalpy rise or enthalpy change in the impeller divided by total enthalpy change in the stage.

We should redraw the velocity triangles, velocity triangle at the inlet is this way for us where we have C1 = Ca1 and this was v1 and this is u1, for the outlet velocity triangle this is one representative and this is v2, this is C2 and this is u2 and if you drop a normal, then this becomes Cr2 and complete would become Cw2, we know that these are the 2 velocity triangles for one sample centrifugal compressor, we can make use of them.

First, we will have Δh impeller = h2 - h1 and then this we know from Euler turbine equation or turbo machinery equation is equal to $1/2\ u_2^2 - u_1^2 + 1/2\ v_1^2 - v_2^2$. Knowing this, we can make use to put in the expression for degree of reaction, further $\Delta h0$ is known to us as $Cw2 \times u2$ for particular case without inlet swirl, this is the basic work input what is given to the flow in the centrifugal compressor and that complete work input is used to rise the enthalpy.

So, $\Delta h0$ is $u2 \times Cw_2$, where enthalpy rise in the impeller is based upon the change in relative velocity and change in tangential velocity, so λ can be written as

$$\lambda = \frac{u1^2 + v1^2 - v2^2}{2\ u2Cw2}$$

we can rearrange the terms and we can write

$$\lambda = \frac{(u2^2 - v2^2) + (V1^2 - u1^2)}{2\ u2Cw2}$$

Now, we can go back from the to the velocity triangle.

And from inlet velocity triangle, we can write down

$$v1^2 = u1^2 + C1^2$$

which gives us $v1^2 - u1^2 = c1^2$which is practically Ca and which is further going to be $Cr1^2$, since we have constant radial velocity assumption, then from outlet velocity triangle we can write down $cr2^2$ which is this vertical line is equal to either plus; from the outlet velocity triangle, we can write down as $v2^2 = cr2^2 +$ this small length and this small length is u2, this small length is cw2 – u2 bracket square.

So, we have $v_2^2 = Cr_2^2 + C_{w2}^2 + u_2^2 - 2\ C_{w2}u_2$, so this over here is required as $u_2^2 - v_2^2$ would become $-\ cr_2^2 - Cw_2^2 + 2C_{w2}\ u_2$. Now, we have $v_1^2 - u_1^2$, we have $u_2^2 - v_2^2$ and this we can put in the formula for degree of reaction and then we can get $(u_2 - v_2)^2$ as $-\ Cr_2^2 - C_{w2}^2 + 2\ C_{w2}u_2 + v1^2 - u1^2$is cr1 square divided by $2\ u_2Cw_2$.

Now, this Cr; we have Cr1 = Cr2, so this is Cr1 square would get cancelled with $Cr2^2$ and then we have twice $Cw_2\ u_2 - Cw_2$ square divided by $2\ C_{w2}u_2$.

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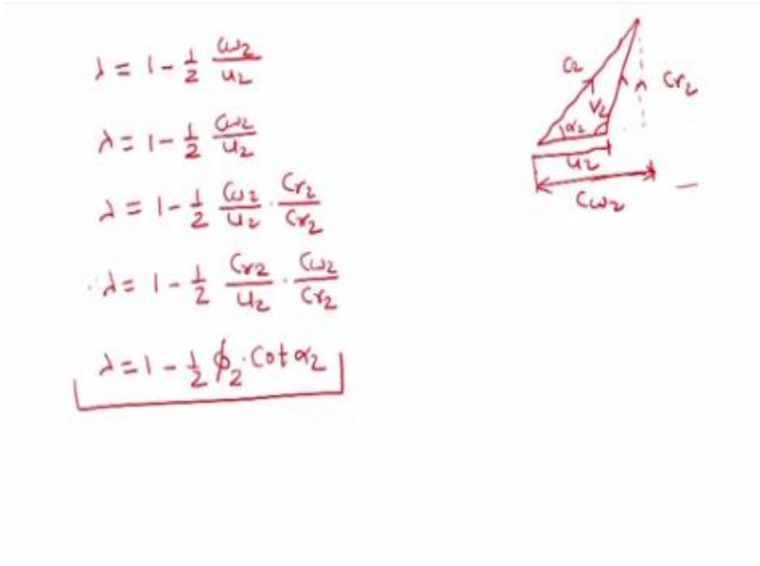

Hence, degree of reaction lambda would become equal to

$$\Lambda = 1 - (\,½)\left(\frac{Cw2}{u2}\right)$$

so this we can further reorganise using the inlet velocity triangle where we and outlet velocity triangle; we need to use the outlet velocity triangle here, since we have both the terms, Cw_2u_2, so outlet velocity triangle was like this where we had this as C2, so this complete was Cw2, this was u_2, this is v_2, this is Cr_2, so this was one velocity triangle and then we can have this further simplified as $\lambda = 1 - 1/2$.

So, we can write down Cw2 as u2, so degree of reaction lambda as become 1 - 1/2 (Cw2 /u2) where we can write down, this as 1 – 1/2 Cw2 ×u2 × Cr_2 / Cr2, so degree of reaction $\lambda = 1 - 1/2$, we can rearrange and then get Cr2 divided by u2 into Cw2 divided by Cr2, so this can give us 1 – 1/2, we have defined a flow coefficient which is Cr2 / u2, so that is φ2, Cw2 is this length, Cr2 is vertical.

$$\Lambda = 1 - (\,½)\varphi 2(\cot\alpha 2)$$

And this is basically, α2 so, we have this is equal to cotα2, so now this we can find out degree of reaction for the centrifugal compressor from the velocity triangles. So, we have seen what are the different components of the centrifugal compressor, how the flow takes place in the centrifugal compressor with different kinds of vanes, what will be the velocity triangle for the backward, radial or forward vanes and then we have seen how much is the pressure rise, we have seen how much is the temperature rise.

Also, I have derived the formula for stage efficiency and then we have found out what is the degree of reaction, so this is how we complete the necessary things for the centrifugal compressor, thank you.