

Aircraft Propulsion
Vinayak N. Kulkarni
Department of Mechanical Engineering
Indian Institute of Technology - Guwahati

Lecture – 25
Understanding Turbomachines

Welcome to the class, in today's class we are going to talk about turbo machines, so the title of the talk is; understanding turbo machines, here our objective is in general, define the governing equation for the turbo machine. So, we know that we have come across the word turbo machines which are dealing with rotary machine and then they can be either were absorbing machines or work producing machines.

In case of work absorbing machines for us as per this course, we have compressor and for work producing machines, we have turbines, so before starting with the complete examples or specific topics on the compressor or turbine, we are going to deal in this class for the machine governing equation for the turbo machines.

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The image shows a handwritten derivation of the moment of momentum equation. On the left, a 3D coordinate system is shown with axes x, y, and z. A position vector \vec{r} is drawn from the origin to a point. A velocity vector \vec{v} is shown at that point. A small volume element δV is indicated, and a force vector $\delta \vec{F}$ is shown acting on it.

The derivation steps are as follows:

$$\frac{D}{Dt}(\vec{v} \delta V) = \delta \vec{F}$$

$$\vec{r} \times \frac{D}{Dt}(\vec{v} \delta V) = \vec{r} \times \delta \vec{F} \quad \text{--- (1)}$$

$$\frac{D}{Dt}[(\vec{r} \times \vec{v}) \delta V] = \frac{D\vec{r}}{Dt} \times \vec{v} \delta V + \vec{r} \times \frac{D}{Dt}(\vec{v} \delta V)$$

$$\because \frac{D\vec{r}}{Dt} = \vec{v}$$

$$\frac{D}{Dt}[(\vec{r} \times \vec{v}) \delta V] = \vec{v} \times \vec{v} \delta V + \vec{r} \times \frac{D}{Dt}(\vec{v} \delta V)$$

$$\frac{D}{Dt}[(\vec{r} \times \vec{v}) \delta V] = \vec{r} \times \frac{D}{Dt}(\vec{v} \delta V) \quad \text{--- (2)}$$

$$\therefore \frac{D}{Dt}[(\vec{r} \times \vec{v}) \delta V] = \vec{r} \times \delta \vec{F} \quad \text{--- (3)}$$

So, first let us consider the coordinate system as what we always deal with, this is x axis, y axis and z axis, consider that there is a fluid particle which is moving with velocity v and it is acted upon by a force δf , this play a particle as position vector r . Now, our objective from here is basically to find out if we are considering the mass motion into the turbo machine, then there will

be change in momentum and then hence, we need either some torque to be applied or we will get as an output.

So, we will have certain power required to be given or we will need certain power as an output, so this fluid particle is said to be getting entered into our turbo machine, so this fluid particle as well as velocity vector v , it is acted by force f , which is an external force, so for this if we write down the second law; Newton's second law that will state that

$$\left(\frac{D}{Dt}\right) (\vec{V} \rho d\mathcal{V}) = \delta\vec{F}$$

let $d\mathcal{V}$ be the volume, we will say this v to be differently such that it can be distinguished from the velocity v .

So, this is rate of change of momentum of the particle is equal to force applied, so this is our Newton's second law. Now, let us take moment of this equation, we will say

$$\vec{r} \times \left(\frac{D}{Dt}\right) (\vec{V} \rho d\mathcal{V}) = \vec{r} \times \delta\vec{F} \quad (1)$$

so this is the moment what we have taken for the momentum equation but let us know this equation as well but we can write down for a new term;

$$\left(\frac{D}{Dt}\right) ((\vec{r} \times \vec{V}) \rho d\mathcal{V}) = \left(\frac{D\vec{r}}{Dt}\right) \times \vec{V} (\rho d\mathcal{V}) + \vec{r} \times \left(\frac{D}{Dt}\right) (\vec{V} \rho d\mathcal{V})$$

so this is product rule what we are apply.

Here, we should note that but $D\vec{r}/Dt$ is velocity vector \vec{V} , so this gives us

$$\vec{V} \times \vec{V} (\rho d\mathcal{V}) + \vec{r} \times \left(\frac{D}{Dt}\right) (\vec{V} \rho d\mathcal{V})$$

but $\vec{V} \times \vec{V}$ is 0, so this ruled out, so we will get

$$\left(\frac{D}{Dt}\right) ((\vec{r} \times \vec{V}) \rho d\mathcal{V}) = \vec{r} \times \left(\frac{D}{Dt}\right) (\vec{V} \rho d\mathcal{V}) \quad (2)$$

we can see in equation 1, we have term which is $\vec{r} \times \left(\frac{D}{Dt}\right) (\vec{V} \rho d\mathcal{V})$, so putting equation 2 in equation 1, we can write down the

$$\left(\frac{D}{Dt}\right) ((\vec{r} \times \vec{V}) \rho d\mathcal{V}) = \vec{r} \times \delta\vec{F}$$

so this is what our new equation which is equation number 3.

Now, this is the equation which we have arrived after taking moment of the momentum equation, let us keep term again and evaluate this term.

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$$\int_{sys} \frac{D}{Dt} [(\vec{r} \times \vec{v})] \rho dV = \int_{sys} \vec{r} \times \delta \vec{F} = \sum \vec{r} \times \vec{F} \leftarrow \text{Force acting on system}$$

$$\int_{sys} \frac{D}{Dt} [(\vec{r} \times \vec{v})] \rho dV = \sum \vec{r} \times \vec{F} \quad \text{--- (4)}$$

$$\frac{D}{Dt} (B_{sys}) = \frac{D}{Dt} \int_{sys} b \rho dV + \int_{sys} b \vec{v} \cdot \vec{n} ds \rightarrow \begin{matrix} b \rightarrow \text{specific property} \\ B \rightarrow \text{total property} \end{matrix}$$

$$\frac{D}{Dt} \int_{sys} b \rho dV = \frac{D}{Dt} \int_{sys} b \rho dV + \int_{sys} b \vec{v} \cdot \vec{n} ds \quad \text{--- (5)}$$

$$\frac{D}{Dt} \int_{sys} (\vec{r} \times \vec{v}) \rho dV + \int_{sys} (\vec{r} \times \vec{v}) (\vec{v} \cdot \vec{n}) ds = \sum (\vec{r} \times \vec{F}) \quad \text{--- (6)}$$

$$\int_{sys} (\vec{r} \times \vec{v}) (\vec{v} \cdot \vec{n}) ds = \sum (\vec{r} \times \vec{F}) \quad \text{--- (7)}$$

So, we will write down this equation again which states that

$$\left(\frac{D}{Dt} \right) ((\vec{r} \times \vec{v}) \rho dV) = \vec{r} \times \delta \vec{F}$$

but we will now integrate this equation over our complete system, so let us integrate this equation over our system, so we will have

$$\int_{sys} \left(\frac{D}{Dt} \right) ((\vec{r} \times \vec{v}) \rho dV) = \sum \vec{r} \times \delta \vec{F}$$

so system has many particles and every particle has certain position vector, so we are actually adding up external forces and their moments for all the particles but this should be say to be equivalent with the summation $\vec{r} \times \vec{f}$, where this \vec{f} is force acting on system.

This $\delta \vec{F}$ was force acting on a particle, so there will be many forces acting on many particles and these integration of force and its moment would get replaced by summation of finite number of forces which are acting on the system, hence this equation becomes D/Dt , system integral of D/Dt $\vec{r} \times \vec{v} \rho dV = \vec{r} \times \vec{f}$, so let us denote this equation again and we will work out with the left hand side term and find out what can be written. We know that in general, we can write down that

$$\frac{D}{Dt} [B_{sys}] = \frac{\partial}{\partial t} (\int_V \rho b dV) + \int_{sys} \rho b \vec{V} \cdot \vec{n} ds$$

And if vector integrate showing the direction of; n is the vector showing direction of area so, saying this, this is basically our Reynolds's transport theorem and as per this Reynolds's transport theorem, this b is specific property like if I take energy in the specific energy and if we take b, then capital B is total property, if I take b as mass, then capital B as will be total mass and small b will be 1, if I take B as energy; capital B as energy, then it will be density into energy into volume.

And in that case, specific b will be just specific energy hence these Reynolds's transport theorem is practically saying that this b is volume integral of $\rho b dV$, so this is what b system, b system is property of this system, so this is integrated over the complete volume or system and then that is

$$\frac{D}{Dt} [\int_V \rho b dV] = \frac{\partial}{\partial t} (\int_V \rho b dV) + \int_s \rho b \vec{V} \cdot \vec{n} ds \quad (4)$$

Now, let us compare this equation number 4 and equation number 5, specifically we are supposed to compare the left hand side of both equations.

And find out which is our b, so it tells out that our b is $\vec{r} \times \vec{v}$, so we can write down the equation number 4 in terms of equation number 5 and then we can get it as

$$\frac{\partial}{\partial t} (\int_V \rho (\vec{r} \times \vec{V}) dV) + \int_s \rho (\vec{r} \times \vec{V}) \vec{V} \cdot \vec{n} ds = \sum \vec{r} \times \vec{F}$$

so having said this now this becomes our equation which is obtained from taking moment of momentum equation.

Now, we will consider flow to be steady in our turbo machine and hence if flow is steady which is expected in case of a turbine or a compressor, then the time variation will be neglected or it will not be present, so we have

$$\int_s \rho (\vec{r} \times \vec{V}) \vec{V} \cdot \vec{n} ds = \sum \vec{r} \times \vec{F} \quad (7)$$

now we will continue with this equation but before that we plot the sketch so that it will be helpful for us to understand.

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$T_{shaft} = \int_S (\rho \vec{v} \cdot \vec{n}) dS (\vec{r} \times \vec{v})_i$
 $T_{shaft} = \sum_{i=1}^n \dot{m}_i (\vec{r} \times \vec{v})_i$
 $T_{shaft} = -\dot{m}_1 (r_1 \omega_1) + \dot{m}_2 (r_2 \omega_2)$
 $T_{shaft} = \dot{m}_2 (r_2 \omega_2) - \dot{m}_1 (r_1 \omega_1)$
 $\dot{m}_1 = \dot{m}_2 = \dot{m}$
 $T_{shaft} = \dot{m} [r_2 \omega_2 - r_1 \omega_1]$
 ↳ Euler Turbomachinery equation
 $Power = W_p = T_{shaft} \times \omega$
 $\therefore W_p = (r_2 \omega_2 - r_1 \omega_1) \cdot \dot{m} \omega$
 $\therefore W_p = \dot{m} [\omega r_2 \omega_2 - \omega r_1 \omega_1]$
 $\therefore W_p = \dot{m} [u_2 \omega_2 - u_1 \omega_1]$

Since, we are supposed to take the only one component into account, so this is the shaft of our turbo machine and then there will be flow which is going lime this turbo machine and this is centre line to the path and then this machine is rotating in this direction, so far the inlet, mean height for the inlet is suppose r, which is along the radius, mean height for the outlet is suppose r2, let area at inlet is A1, area at the outlet be A2.

And hence, there is a flow which is going from this turbo machine and we are supposed to find out the equation for the flow. Here, flow is coming with 3 velocities, so for inlet we have 3 velocities which are like this we have radial velocity Cr1 like this, we have axial velocity Ca1 and in this direction, there is tangential velocity Cw1, so total velocity is C1. Similarly, for outlet also, we will have Cr2, we will have Ca2 and then we will have Cw2 and this is C2.

So, we have 3 velocities which are entering and 3 velocities which are leaving, considering this, we are more interested in torque, so we are more interested in the tangential velocity, so we are going to have this equation written along only that direction, so if we write down this equation only in one direction, we can write down this for shaft, so which is basically force acted this force whatever it, $r \times f$ and we are talking about it for only one direction and then that force into its moment will give us torque, we are talking about torque acting on shaft is equal to this is surface integral for our $\rho \cdot v \cdot n \cdot ds$ of $r \times v$ for that direction component.

$$\int_S \rho \vec{V} \cdot \vec{n} \, ds \quad (\vec{r} \times \vec{V})$$

But τ shaft; we know that this is surface integral and now this surface integral can be said to be integrated for finite number of inlets and outlets, so it will become summation for all inlets and outlets, let there will be n number of inlets and outlets and this equation can be written as this way, so this term is mass flow rate, so \dot{m}_i dot into $\vec{r} \times \vec{v}$, for that inlet in that direction or that inlet in only one direction.

So, this would become here as

$$\tau_{\text{shaft}} = -\dot{m}_1 (\vec{r}_1 \times \vec{C}_{w1}) + \dot{m}_2 (\vec{r}_2 \times \vec{C}_{w2})$$

if we see here, then for inlet, the normal component for area is outward, here as well it will be outward, so the velocity vector for $C1$ is in opposite direction for this area vector but this $C2$ will be aligned, so dot product of velocity and area which is $\vec{v} \cdot \vec{n}$, this will turn out to be negative, so mass flow rate will have negative sign at the inlet and mass flow rate will have positive sign at the outlet.

$T_{\text{shaft}} = \int (\rho \vec{V} \cdot \vec{n}) ds \quad (\vec{r} \times \vec{V})_i$
 $T_{\text{shaft}} = \sum_{i=1}^n \dot{m}_i (\vec{r} \times \vec{v})_i$
 $T_{\text{shaft}} = -\dot{m}_1 (r_1 \omega_1) + \dot{m}_2 (r_2 \omega_2)$
 $T_{\text{shaft}} = \dot{m}_2 (r_2 \omega_2) - \dot{m}_1 (r_1 \omega_1)$
 $\dot{m}_1 = \dot{m}_2 = \dot{m}$
 $T_{\text{shaft}} = \dot{m} [r_2 \omega_2 - r_1 \omega_1]$
 \rightarrow Euler Turbomachinery Equation
 $\text{Power} = W_p = T_{\text{shaft}} \times \omega$
 $\therefore W_p = (r_2 \omega_2 - r_1 \omega_1) \cdot \dot{m} \omega$
 $\therefore W_p = \dot{m} [\omega r_2 \omega_2 - \omega r_1 \omega_1]$
 $\therefore W_p = \dot{m} [u_2 \omega_2 - u_1 \omega_1]$

So, $\vec{r} \times \vec{V}$ for one direction which is for the torque will become $\vec{r} \times \vec{C}_w$, in this we will also $\vec{v} \cdot \vec{n}$ into \vec{C}_w , so we will have

$$\tau_{\text{shaft}} = \dot{m}_2 (\vec{r}_2 \times \vec{C}_{w2}) - \dot{m}_1 (\vec{r}_1 \times \vec{C}_{w1})$$

but we have $\dot{m}_1 = \dot{m}_2$, so $\tau_{\text{shaft}} = \dot{m}$ that we can say $\dot{m}_1 = \dot{m}_2 = \dot{m}$, so it will become $r_2 C_{w2} - r_1 C_{w1}$,

$$\tau_{\text{shaft}} = \dot{m}(r_2 C_{w2} - r_1 C_{w1})$$

this equation is called as Euler turbo machinery equation. So, as per this equation, now we can find out what is the power, so we know torque, power; so let us say W_p is power, so that is equal to torque shaft into angular velocity which is ω . So, this we will write as ω so

$$W_p = (r_2 c_{w2} - r_1 c_{w1}) \dot{m} \omega$$

so this will give us

$$W_p = \dot{m} \omega r_2 c_{w2} - \omega r_1 c_{w1},$$

what is ωr ; ωr is tangential velocity, so this

$$W_p = \dot{m} (u_2 c_{w2} - u_1 C_{w1}),$$

This u which is tangential velocity of the machine, C_{w2} is tangential velocity of the fluid at the inlet, so this becomes the expression for power.

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$$\begin{aligned} T &= \dot{m} (r_2 c_{w2} - r_1 c_{w1}) & W_p &= \dot{m} (u_2 c_{w2} - u_1 c_{w1}) \checkmark \\ u_2 c_{w2} > u_1 c_{w1} &\rightarrow \omega_p \rightarrow +ve \rightarrow \text{Compressor} \\ u_1 c_{w1} > u_2 c_{w2} &\rightarrow \omega_p \rightarrow -ve \rightarrow \text{Turbine} \\ \tau_c &= \dot{m} [r_2 c_{w2} - r_1 c_{w1}] & \tau_t &= \dot{m} [r_1 c_{w1} - r_2 c_{w2}] \\ W_c &= \dot{m} [u_2 c_{w2} - u_1 c_{w1}] & W_t &= \dot{m} [u_1 c_{w1} - u_2 c_{w2}] \\ \frac{W_c}{\dot{m}} &= u_2 c_{w2} - u_1 c_{w1} & \frac{W_t}{\dot{m}} &= u_1 c_{w1} - u_2 c_{w2} \\ dW_c &= d(u c_w) & dW_t &= -d(u c_w) \end{aligned}$$

So, let us analyse this equation along with torque equation where we said

$$\text{Torque, } \tau = \dot{m} (r_2 C_{w2} - r_1 C_{w1})$$

$$\text{power } W_p = \dot{m} (u_2 C_{w2} - u_1 C_{w1}).$$

So, now if we have u_2, C_{w2} to be greater than u_1, C_{w1} , then this will give us W_p as positive and here we are saying that angular momentum at the outlet is more than angular momentum at the inlet and it is would happen if we are supplying energy so hence, this will become the expression

for compressor. Similarly, if u_1, C_{w1} is more than u_2, C_{w2} then W_p will be negative, so angular momentum at inlet was more than the outlet.

So, power was produced here and this power was therefore for the turbine, so energy actually got transfer from the system to the surrounding, so τ shaft for the compressor is $\dot{m} (r_2 C_{w2} - r_1 C_{w1})$ and W_c for compressor is $\dot{m} (u_2 c_{w2} - u_1 c_{w1})$, so this is total, so if I say specific work output is

$$W_c / \dot{m} = u_2 c_{w2} - u_1 c_{w1}$$

so this is possible to be written like small work input required for compressor is equal to small change in momentum between inlet and outlet for the tangential case.

Similarly, τ shaft or turbine can be written as

$$\tau \text{ shaft} = \dot{m} (r_1 C_{w1} - r_2 C_{w2})$$

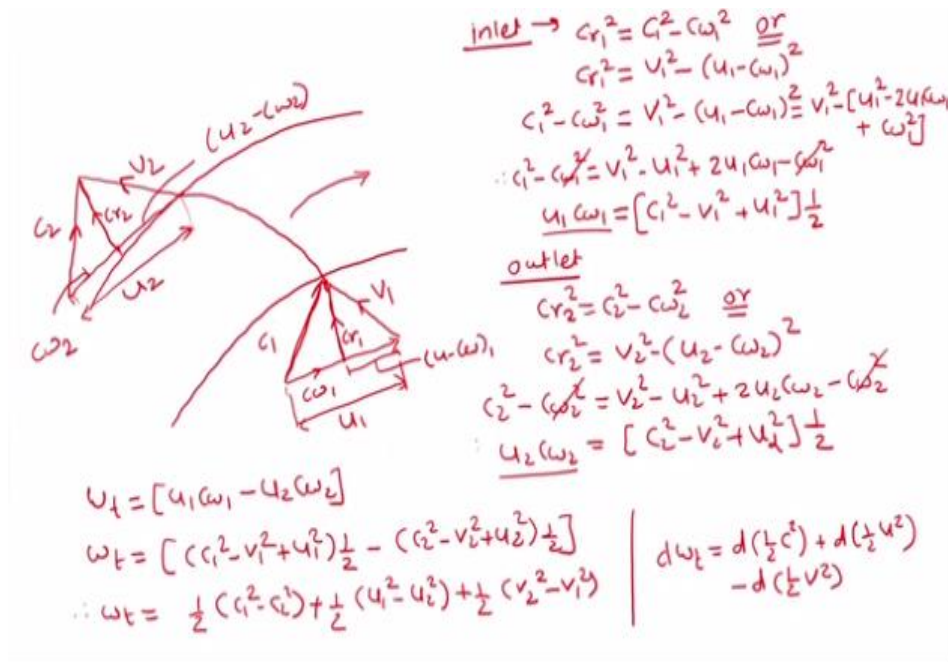
so we have turbine power as $\dot{m} u_1 c_{w1} - u_2 c_{w2}$, so specific power output will be

$$W_t / \dot{m} = u_1 C_{w1} - u_2 C_{w2},$$

so this again small work output of turbine is equal to change in angular velocity; angular momentum but that would have negative signs that changes negative. Hence, let us find out, let us take basic Euler turbine pump equation and understand further.

Here, we can see that we have term u into c_w for the equation; for the power, now what is this we have to replace with, so for that let us plot the velocity triangle for the turbo machine and work out.

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So, we will have suppose, if this turbo machine, these are the; we know that for any turbo machine, there is a blade or moving part, so we are trying to; that is where we are talking about this work transfer or energy transfer. So, energy transfer we are mainly focussing upon the moving part or rotor, so we are talking about inlet and outlet of a rotor. So, here we are trying to draw the velocity triangle for the rotor, so now this is turbo machine in general may be a compressor or turbine.

Then, there is fluid which is entering with absolute velocity which has said as C_1 and then there will be tangential velocity for the machine as u_1 but this flow is entering with radial velocity with C_{r1} , so this fluid has tangential velocity $C\omega_1$, therefore it will entering with relative velocity V_1 , relative velocity is always tangential to the blade, so this is at the outlet, so outlet velocity triangle, so this is u_2 , tangential velocity of the outlet, this is the velocity absolute at the outlet, so this is C_2 and this is v_2 which is relative velocity at the outlet.

And this section is $C\omega_2$, so now considering these 2 velocity triangles, we can write down for inlet as C_{r1} square, this vertical line C_{r1} ; this vertical line

$$C_{r1}^2 = C_1^2 - C\omega_1^2$$

but this is from this triangle but we can also write down from right velocity triangle as r C_{r1} square = v_1 square - $u_1 - C\omega_1$ bracket square, so this is $u - C\omega_1$, so both equations can be equated and then we can write

$$C_1^2 - C_{w1}^2 = V_1^2 - (u_1 - C_{w1})^2.$$

So, right hand side term will become $V_1^2 - u_1^2 - 2 u_1 c_{w1} + C_{w1}^2$, so $C_1^2 - C_{w1}^2$ will become v_1 square - u_1 square + twice $u_1 c_{w1} - c_{w1}$ square, so c_{w1} square would get cancelled and then we can get

$$u_1 C_{w1} = (C_1^2 - V_1^2 + u_1^2) / 2,$$

so similarly we will work out for outlet velocity triangle where as well we can write down

$$C_{r2}^2 = C_2^2 - C_w^2$$

this is for the absolute velocity.

Now, same thing for relative velocity will be

$$C_{r2}^2 = V_2^2 - (u_2 - C_{w2})^2,$$

so this is $u_2 - C_{w2}$, so equating both terms, we will get $C_2^2 - C_w^2 = V_2^2$, sorry this is minus $v_1 v_2$ square minus, so v_2 square - u_2 square + twice $u_2 c_{w2} - c_{w2}$ square, so c_{w2} square would also get cancelled, so we will get

$$u_2 C_{w2} = (C_2^2 - V_2^2 + u_2^2) / 2.$$

So, this C_w terms are there in the equation for the work, so we know that suppose we are writing the work for the turbine, then you wrote that W_t ; specific work was $u_1 C_{w1} - u_2 C_{w2}$, so we have turbine work $= (C_1^2 - V_1^2 + u_1^2) / 2 - (C_2^2 - V_2^2 + u_2^2) / 2$, so we will have turbine work has one half $C_1^2 - C_2^2$, you can write down plus, first u term + $1/2 u_1^2 - u_2^2 + 1/2 V_1^2$ actually here it is $V_2^2 - V_1^2$, so we can write down for small w_t which is small work input.

This is basically, change in kinetic energy; absolute kinetic energy plus change in kinetic energy based on tangential velocity minus change in relative kinetic energy, so there are 3 terms.

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$$w_c = (w_2 u_2 - w_1 u_1)$$

$$w_c = \frac{1}{2} (c_2^2 - c_1^2) + \frac{1}{2} (u_2^2 - u_1^2) - \frac{1}{2} (V_1^2 - V_2^2)$$

$$d(w_c) = \underbrace{d\left(\frac{1}{2} c^2\right)}_{\text{impulse effect}} + \underbrace{d\left(\frac{1}{2} u^2\right)}_{\text{centrifugal effect}} - \underbrace{d\left(\frac{1}{2} V^2\right)}_{\text{reaction effect}}$$

$$d(w_t) = -\underbrace{d\left(\frac{1}{2} c^2\right)}_{\text{impulse effect}} - \underbrace{d\left(\frac{1}{2} u^2\right)}_{\text{centrifugal effect}} + \underbrace{d\left(\frac{1}{2} V^2\right)}_{\text{reaction effect}}$$

Similarly, if we see for compressor, then we have compressor work = $Cw_2 u_2 - Cw_1 u_1$, so compressor work was $\frac{1}{2} C_2^2 - C_1^2 + \frac{1}{2}$ into; one second; so this was having negative signs and this was having positive sign since absolute kinetic energy, we will write down it again for turbine, so for turbine, equation had become

$$d(w_t) = -d(1/2 C^2) - d(1/2 u^2) + d(1/2 V^2)$$

, so for this is for turbine.

So, for compressor,

$$W_c = \frac{1}{2} (C_2^2 - C_1^2) + \frac{1}{2} (u_2^2 - u_1^2) - \frac{1}{2} (V_1^2 - V_2^2)$$

this is for compressor, so for compressor, we can write down

$$w_c = +d(1/2 C^2) + d(1/2 u^2) - d(1/2 V^2)$$

so we can see that there are 3 terms for small work interactions and they are related with changes.

So, first term is $1/2 C^2$ change, so this is change in absolute kinetic energy.

So, change in absolute kinetic energy is positive for the rotor of compressor and it is negative for turbine, so this change in absolute energy; kinetic energy is called as due to is expected to be due to impulse effect and now, the second is due to centrifugal force, so this is tangential velocity which is $1/2 Mu$ square which is positive for compressor and negative for turbine, so second term is due to centrifugal effect. First term is impulse effect; second term is centrifugal effect.

Now, third term is basically, change in relative velocity and relative velocity is negative for compressor and it is positive for turbine, so relative velocity based kinetic energy changes negative for compressor and it is positive for turbine and this effect is called as reaction effect. So, there are 3 effects which are going to give us the work related to compressor or turbine.

So, we when deal with some specific machines something like radiator bind or centrifugal compressor, then the major effect is due to the term which is $1/2 u^2$ or minus $1/2 u^2$ but if we try to do with impulse turbine, then we will have only first term dominant, so this is what we will have to understand from this basic theme. So, now let us found out the enthalpy change, what would be there if we have flow through the turbo machine?

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$$c^2 = c_r^2 + \omega^2 + c_a^2$$

$$v^2 = v_r^2 + v_w^2 + v_a^2$$

$$\underline{v_r = c_r} \quad \underline{v_a = c_a}$$

$$\underline{\omega = u - v\omega}$$

$$W = c\omega_2 u_2 - c\omega_1 u_1 = h_{02} - h_{01}$$

$$(h_2 + \frac{c_2^2}{2}) - (h_1 + \frac{c_1^2}{2}) = [u_2 \omega_2 - u_1 \omega_1]$$

$$\therefore h_2 - h_1 = [u_2 \omega_2 - u_1 \omega_1] - \frac{c_2^2}{2} + \frac{c_1^2}{2} = [u_2 \omega_2 - u_1 \omega_1] - \left(\frac{c_2^2}{2} - \frac{c_1^2}{2} \right)$$

$$\frac{c_2^2}{2} - \frac{c_1^2}{2} = \frac{c_{r2}^2 + c_{a2}^2 + \omega_2^2}{2} - \frac{c_{r1}^2 + c_{a1}^2 + \omega_1^2}{2}$$

$$\therefore \frac{c_2^2}{2} - \frac{c_1^2}{2} = \frac{v_{r2}^2 + v_{a2}^2 + (u_2 - v\omega_2)^2}{2} - \frac{v_{r1}^2 + v_{a1}^2 + (u_1 - v\omega_1)^2}{2}$$

So, let us plot again the velocity triangle in workout, so this is our turbo machine and we had before that, we will just plot the velocity triangles what we had plotted, so this was c_{r2} , and this was C_2 , this was V_2 , this was u_2 and this was Cw_2 and for inlet velocity triangle, this is for outlet. For inlet, we have again similar triangle in general, for a turbo machine and this is c_1 , cr_1 , v_1 , we will have this as u_1 and then this is u_1 and this is cw_1 .

So, let us try to find out what is the enthalpy change when flow goes from inlet to outlet in a turbo machine, so we know that

$$C^2 = C_r^2 + C_w^2 + C_a^2$$

so z is basically, axial direction, so C_a^2 . So similarly, relative velocity we will also have 3 terms where we have $V_r^2 + V_w^2 + V_a^2$, in the present case of our velocity triangles, V_r in general is equal to C_r and $V_a = C_a$, further in all the velocity triangles, we have V_w , we have $C_w = u - V_w$.

So, this is this term which is $u_2 - C_w^2$ and then this is $u_1 - C_w$, so we should note this and we should work out, so while working out, we know that if we take the general work expression in case of compressor suppose, it is $w = C_w^2 u_2 - C_w^1 u_1$ and this is basically $= h_{02} - h_{01}$, so we are adding the work, we know that we had seen in case of Brayton cycle with stagnation condition, what is the work input for the compressor, it was $C_p(t_2 - t_1)$.

So, basically that is $h_{02} - h_{01}$ that was the work which we are considering from thermodynamic point of view which we have obtained now from the velocity triangles, so having said this we have this as an expression, here this total enthalpy we can release, decompose into static, enthalpy plus kinetic energy, so we have $h_2 + c_2^2 / 2 - h_1 + c_1^2 / 2$ and this is $= u_2 c_w^2 - u_1 c_w^1$.

So, enthalpy change which is

$$h_2 - h_1 = u_2 C_w^2 - u_1 C_w^1 - C_2^2 / 2 + C_1^2 / 2,$$

so this minus sign for c_1 square would become plus on the other side, so let us work out for this term which is equal to suppose, $u_2 c_w^2 - u_1 c_w^1 - C_2^2 / 2 + C_1^2 / 2$, so let us work out with this term which is $C_2^2 / 2 - C_1^2 / 2$ and this is $(C_r^2 + C_a^2 + C_w^2) / 2 - (C_r^1 + C_a^1 + C_w^1) / 2$ but we know now the equivalence and according to that we can write down this

$$\frac{C_2^2}{2} - \frac{C_1^2}{2} = \frac{V_r^2 + V_a^2 + (u_2 - V_w^2)^2}{2} - \frac{V_r^1 + V_a^1 + (u_1 - V_w^1)^2}{2}$$

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$$\frac{C_2^2}{2} - \frac{C_1^2}{2} = \frac{V_2^2 + V_{a2}^2 + (V_{w2}^2 - 2u_2V_{w2} + u_2^2)}{2} - \frac{V_1^2 + V_{a1}^2 + (V_{w1}^2 - 2u_1V_{w1} + u_1^2)}{2}$$

$$\therefore \frac{C_2^2}{2} - \frac{C_1^2}{2} = \frac{V_2^2 - 2u_2V_{w2} + u_2^2}{2} - \frac{V_1^2 - 2u_1V_{w1} + u_1^2}{2} = \left[\frac{V_2^2 - V_1^2}{2} \right] + \left[\frac{u_2^2 - u_1^2}{2} \right] - (u_2V_{w2} - u_1V_{w1})$$

$$h_2 - h_1 = [u_2C_{w2} - u_1C_{w1}] - \left\{ \left(\frac{V_2^2 - V_1^2}{2} \right) + \left(\frac{u_2^2 - u_1^2}{2} \right) - (u_2V_{w2} - u_1V_{w1}) \right\}$$

$$h_2 - h_1 = (u_2C_{w2} - u_1C_{w1}) - \left\{ \left(\frac{V_2^2 - V_1^2}{2} \right) + \left(\frac{u_2^2 - u_1^2}{2} \right) - (u_2V_{w2} - u_1V_{w1}) \right\}$$

$$h_2 - h_1 = [u_2^2 - u_1^2 - (u_2V_{w2} - u_1V_{w1})] - \left\{ \left(\frac{V_2^2 - V_1^2}{2} \right) + \left(\frac{u_2^2 - u_1^2}{2} \right) - (u_2V_{w2} - u_1V_{w1}) \right\}$$

$$h_2 - h_1 = \left[\frac{u_2^2}{2} - \frac{u_1^2}{2} \right] - \left[\frac{V_2^2 - V_1^2}{2} \right] \quad \left. \begin{array}{l} T ds = dh - dP/\rho \\ dh = T ds + dP/\rho \end{array} \right\}$$

$$dh = d\left(\frac{u^2}{2}\right) - d\left(\frac{V^2}{2}\right)$$

$$T ds + \frac{dP}{\rho} = d\left(\frac{u^2}{2}\right) - d\left(\frac{V^2}{2}\right)$$

So, let us expand then, we can write $C_2^2/2 - C_1^2/2 = V_2^2 + v_2^2 +$ now, the term will become $V_2^2 - 2u_2V_{w2} + u_2^2/2 - v_2^2 + v_2^2 + v_2^2 - 2u_1V_{w1} + u_1^2/2$, so we have $C_2^2/2 - C_1^2/2 =$ we can clubbed this 3 terms on the numerator of both and then that we can write as v_2^2 which is total relative velocity square + either $- 2u_2v_2 + u_2^2/2 -$ total relative velocity square $- 2u_1v_1 + u_1^2/2$.

Now, this we will put in our equation which had c square term and then we can write $h_2 - h_1 =$ this actually we can further simplify and we can write down here as $v_2^2/2 - v_1^2/2 + u_2^2/2 - u_1^2/2 - u_2C_{w2} - u_1V_{w1}$, so keeping this in the enthalpy expression, we can get the first term as it is which is $u_2^2 - u_1^2$ - first term was $u_2C_{w2} - u_1C_{w1}$, this was this - change in kinetic energy and this term is minus for all $v_2^2/2 - v_1^2/2 + u_2^2/2 - u_1^2/2 - u_2V_{w2} - u_1V_{w1}$, then curly braces complete.

Now, here we will replace our $C_{w2}/u - V_w$ and then $u_2u_2 - u_2v_2 - u_1u_2v_1 - v_1v_1 -$ curly braces $v_2^2/2 - v_1^2/2 + u_2^2/2 - u_1^2/2 - u_2v_2 - u_1v_1$ curly braces complete, so we will have

$$h_2 - h_1 = u_2^2 - u_1^2$$

from here $- u_2v_2 + u_1v_1 -$ curly braces, we have $v_2^2/2 - v_1^2/2 + u_2^2/2 - u_1^2/2 - u_2v_2 - u_1v_1$ curly braces complete, so we will have a term which can be written over here as minus sign will be taken out and then this term can be bracketed with minus sign here.

And then we have a term $u_2 v_{w2} - u_1 v_{w1}$ here and with minus sign again, this has minus sign so, with plus sign here, this minus and this minus will make it plus and this is minus, so we can cancel this, so we will have $h_2 - h_1 =$ we have u_2^2 and this u_2^2 will have minus sign with $1/2 u_2^2$, so this will become $u_2^2/2 - u_1^2/2 - v_2^2/2 - v_1^2/2$, so we can write it as dh , which is

$$\text{change in enthalpy, } dh = d(u^2/2) - d(v^2/2)$$

so there is change in enthalpy for the flow which is flowing through the turbo machine which will take place in the rotor only due to the change in the tangential velocity or due to change in relative velocity.

Now, we know from thermo dynamics that

$$Tds = dh - dp/\rho$$

so vdp is dp/ρ ,

so we can write down

$$dh = Tds + dp/\rho,$$

so we can put this over here and write down

$$Tds + \frac{dp}{\rho} = d\left(\frac{u^2}{2}\right) - d\left(\frac{v^2}{2}\right)$$

We assume the flow to be isentropic in case of compressors or turbines, so this term is neglected, so it is little Ts , it is negligible or 0, in reality, so

$$\frac{dp}{\rho} = d\left(\frac{u^2}{2}\right) - d\left(\frac{v^2}{2}\right)$$

so pressure change in the compressor or turbine rotor will take place due to change in tangential velocity and change in relative velocity.

So, this was the discussion related to the governing equation or the turbo machines which are either compressor or turbine, basic understanding of the velocity triangles and why there will be torque, power output and why there is pressure raise and enthalpy change while flow is taking place over the rotor of a turbo machine. Thank you.