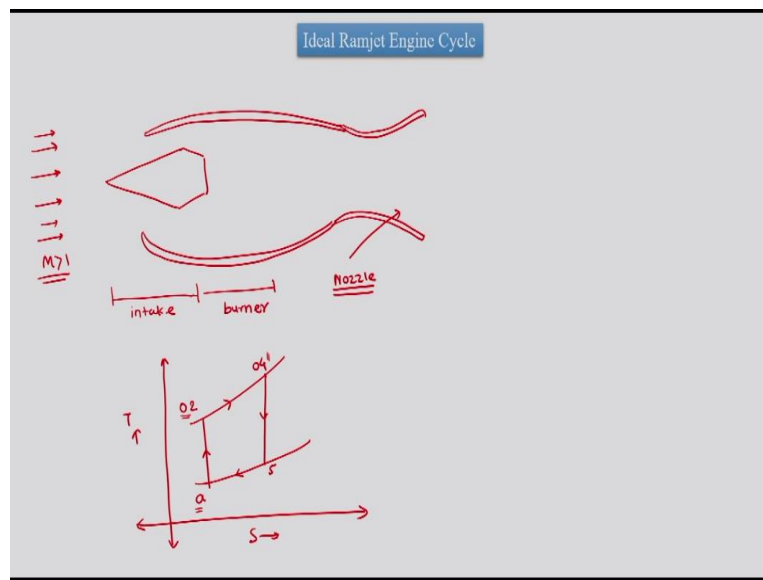


Aircraft Propulsion
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Lecture - 22
Ramjet Engine: Parameters and Losses

Welcome to the class. Today, we are going to see about the ramjet engine. We are going to see its configuration and then some derivations about ramjet engine, which would help us to find out the thrust or specific fuel consumption for a given ramjet configuration and for its given flying condition.

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So, let us first see what is the ideal ramjet engine cycle. A typical ramjet engine would look like this where we have attached here, which is getting converted into a nozzle later and then so this is a nozzle and before that we have a cone at the entry which has to do with generation of shock. So, this portion would act as intake. So, there will be Mach number more than 1, supersonic flow entering into this ramjet.

And this flow would get compressed due to the shock, which is present due to this cone and then this compression is in the absence of compressor which is a conventional compressor but instead of that, the flow is getting compressed due to shock and then that is why this portion is going to

act as intake plus compressor in comparison with conventional gas turbine. However, in the terminologies of ramjet, it will be called as intake.

Then, there will be fuel injection happening here onwards and then that fuel will get burnt. So, this is burner or our combustion chamber and then this is nozzle. So, these are the components of a typical ramjet engine, which does not have any rotating part. Now, if we plot TS diagram for ramjet engine, then it will be like this. We have atmospheric condition at the inlet and then that would lead to 02 after the intake.

This 02 number is taken from the numbers of the gas turbine where 02 is the state at the exit of the compressor. The same number is given here for the state, which is at the exit of the intake. Then, 02 to 03 is the heat addition process in the gas turbine and then 03 to 04 is turbine. So, the same thing here would be state as 04, which is the entry to the nozzle. Here, process of heat addition is 02 to 04 and then 4 to 5 is expansion.

So, this is how the ramjet cycle would work. Here, these numbers are purposely given such that this is inlet; this is exit of the compressor or exit of the intake. This is presently for ramjet exit of the combustion chamber or burner, but 04 is purposely given since 04 represents the condition at the outlet of the turbine or at the entry to the nozzle. So, 04 should be treated as entry to the nozzle and 5 is outlet of the nozzle.

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Expression for Thrust and Specific Fuel Consumption (Ideal Case)

$$\frac{P_{0a}}{P_a} = \left[1 + \frac{\gamma-1}{2} Ma^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{0s}}{P_s} = \left[1 + \frac{\gamma-1}{2} M_s^2\right]^{\frac{\gamma}{\gamma-1}}$$

$P_{0a} = P_{0s}$ $P_a = P_s$
 $\underline{Ma = M_s}$

$$M_s = \frac{U_s}{a_s} = M_a = \frac{U_a}{a_a}$$

$$\therefore U_s = U_a \cdot \frac{a_s}{a_a} = U_a \cdot \sqrt{\frac{\gamma R T_s}{\gamma R T_a}}$$

$$\therefore U_s = U_a \cdot \sqrt{\frac{T_s}{T_a}}$$

$$\therefore U_s = U_a \cdot \sqrt{\frac{T_{0s}}{T_{0a}}}$$

$$\downarrow \downarrow$$

$$C_j = C_a \cdot \sqrt{\frac{T_{0s}}{T_{0a}}}$$

Thrust = $(\dot{m}_a + \dot{m}_f) \cdot C_j - \dot{m}_a \cdot C_a = \dot{m}_a \left\{ \left(1 + \frac{\dot{m}_f}{\dot{m}_a}\right) C_j - C_a \right\}$

Let's $f = \frac{\dot{m}_f}{\dot{m}_a}$ = Fuel-Air ratio

\therefore Thrust = $T = \dot{m}_a [(1+f)C_j - C_a]$

Specific Thrust = $\frac{T}{\dot{m}_a} = (1+f)C_j - C_a$

Let's apply energy eqⁿ for burner

$$\dot{m}_a \cdot h_{02} + \dot{m}_f \cdot Q_{cv} = (\dot{m}_a + \dot{m}_f) \cdot h_{04}$$

$$h_{02} + f \cdot Q_{cv} = (1+f) \cdot h_{04}$$

$$\therefore T_{02} + f \frac{Q_{cv}}{C_p} = (1+f) T_{04}$$

$$\therefore T_{02} + f \frac{Q_{cv}}{C_p} = T_{04} + f T_{04}$$

$$\therefore T_{04} - T_{02} = f \left[\frac{Q_{cv}}{C_p} - T_{04} \right]$$

$$\therefore \frac{T_{04}}{T_{0a}} - \frac{T_{02}}{T_{0a}} = f \left[\frac{Q_{cv}}{C_p T_{0a}} - \frac{T_{04}}{T_{0a}} \right]$$

$$\therefore f = \frac{\frac{T_{04}}{T_{0a}} - \frac{T_{02}}{T_{0a}}}{\frac{Q_{cv}}{C_p T_{0a}} - \frac{T_{04}}{T_{0a}}} = \frac{\frac{T_{04}}{T_{0a}} - 1}{\frac{Q_{cv}}{C_p T_{0a}} - \frac{T_{04}}{T_{0a}}}$$

Having said this, we can work out with the derivation for finding out the expression for thrust and specific fuel consumption for this ideal ramjet. So, in this case, we have TS diagram known to us as a 02, 04 and 05. Knowing this for an ideal, we can write down

$$\frac{P_{0a}}{P_a} = \left(1 + \frac{\gamma-1}{2} * Ma^2\right)^{\frac{\gamma}{\gamma-1}}$$

So, if we know Mach number of the flow which is entering into the ramjet, then we can write down the total condition at a.

So, this is what P_{0a} we can find out. Similarly,

$$\frac{P_{05}}{P_5} = \left(1 + \frac{\gamma-1}{2} * M_5^2\right)^{\frac{\gamma}{\gamma-1}}$$

but we know that $P_{0a} = P_{05}$ and $P_a = P_5$. So, this gives us $Ma = M_5$, so Mach number at exit and at inlet of ramjet remains constant. So, this is what outcome of this simple derivation. This we can use for finding out thrust. We know $M_5 = U_5/a_5$ rather U_5 is velocity at the exit.

And then this is equal to Ma which is U_a/a_a which is entry velocity divided by entry acoustic speed. So, $U_5 = U_a * a_5/a_a$. We know that acoustic speed is $\sqrt{\gamma R T_5 / \gamma R T_a}$. So, $U_5 = U_a * (\frac{T_5}{T_a})^{0.5}$. We know that

$$\frac{T_5}{T_0} = 1 + \frac{\gamma-1}{2} * M_5^2$$

$T_a/T_{0a} = 1 + \gamma - 1/2 Ma^2$. So, we know that $T_5/T_{05} = T_a/T_{0a}$ and using that we can write U_a into T_{05}/T_{0a} .

So, this is useful for us since we need this and we always say that U_5 is C_j and U_a is C_a into T_{05}/T_{0a} . Now, this we can further use for finding out thrust where we know

$$\text{Thrust} = (\dot{m}_a + \dot{m}_f) * C_j - \dot{m}_a * C_a$$

So, we can take \dot{m}_a common and then we can write down $\dot{m}_a * (1 + \dot{m}_f/\dot{m}_a) * C_j - C_a$. Let us put $f = \dot{m}_f/\dot{m}_a$ which is fuel-air ratio.

So, thrust = $T = \dot{m}_a * (1 + f) * C_j - C_a$ Then, we can find out specific thrust which is

$$\text{Specific thrust} = T/\dot{m}_a = (1+f) * C_j - C_a$$

Here, we need to find out f , so for that let us apply energy equation for burner. This gives us \dot{m}_a into h_{02} is the entering enthalpy into the burner, then we add \dot{m}_f fuel which has calorific value Q_{cv} . This gives us $\dot{m}_a + \dot{m}_f$ into h_{04} as total enthalpy.

$$(\dot{m}_a * h_{02}) + \dot{m}_f * Q_{cv} = (\dot{m}_a + \dot{m}_f) * h_{04}$$

So, we can divide complete expression by \dot{m}_a and write down $h_{02} + f * Q_{cv} = (1 + f) * h_{04}$. Let us divide again this expression by C_p since $h = C_p * T$. So, we get $T_{02} + f * Q_{cv}/C_p = (1 + f) * T_{04}$. So, we have $T_{02} + f * Q_{cv}/C_p = T_{04} + f * T_{04}$. We can take the terms related to f together and then we can write down $T_{04} - T_{02} = f * Q_{cv}/C_p - 1 - T_{04}$. This gives us an expression to which we can divide completely by T_{0a} , so we can have

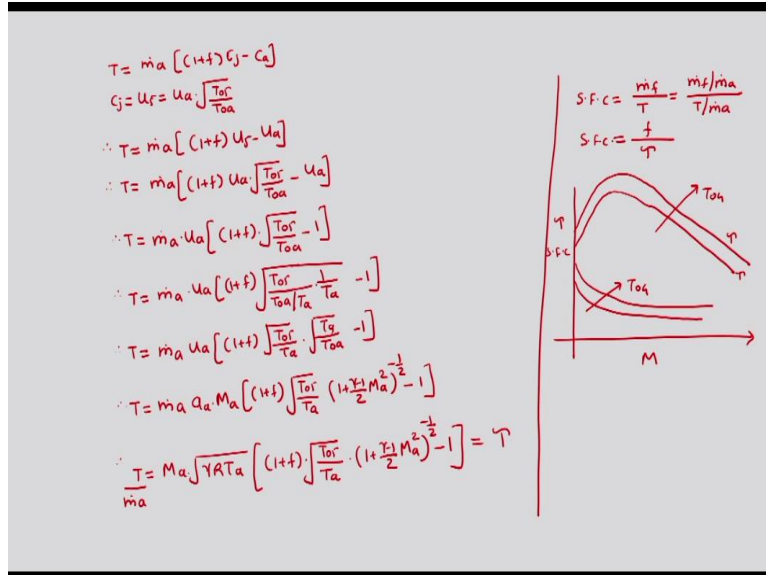
$$T_{04}/T_{0a} - T_{02}/T_{0a} = f * Q_{cv}/C_p * (T_{0a} - T_{04})/T_{0a}$$

So, we can get f which was required for us to find out thrust and then this f is so equal to $T_{04}/T_{0a} - T_{02}/T_{0a} / (Q_{cv}/C_p * (T_{0a} - T_{04})/T_{0a})$ where we know $T_{02} = T_{0a}$. So, we have

$$f = (T_{04}/T_{0a} - 1) / (Q_{cv}/C_p * (T_{0a} - T_{04})/T_{0a})$$

and then this we can use for thrust calculation.

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So, we can write down again the formula for thrust

$$T = \dot{m}_a * ((1+f) * C_j - C_a)$$

where we have seen that $C_j = U_5 = U_a * (T_{05}/T_{0a})^{0.5}$. So, we have thrust = $\dot{m}_a * ((1+f) * U_5 - U_a)$

So, *thrust* = $\dot{m}_a * ((1+f) * U_a * (T_{05}/T_{0a})^{0.5} - U_a)$. We can take U_a common, so we have $\dot{m}_a * U_a * ((1+f) * (T_{05}/T_{0a})^{0.5} - 1)$. We can divide and multiply so as to rearrange the term in the square root where we can say it as $T_{05}/T_{0a}/T_a$ into $1/T_a - 1$.

So, this is useful since this has relevance with Mach number and we can express this into two square roots which is T_{05}/T_a and $T_a/T_{0a} - 1$. So, we can write down now $\dot{m}_a U_a$ can be expressed in terms of Mach number, which is acoustic speed * Mach number * $1+f$, so T_{05}/T_{0a} will be as it is and then this T_a/T_{0a} can be written as $1 + \gamma - 1/2 Ma^2$ but this is in square root and 1 upon, so it will have $-1/2$.

And hence we can get an expression which says that thrust upon mass flow rate

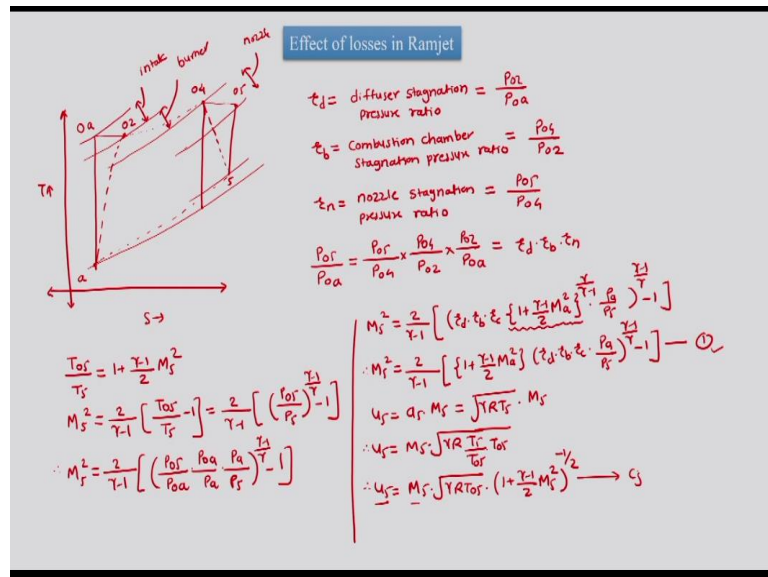
$$T/\dot{m}_a = Ma * (\gamma R * T_a)^{0.5} * [(1+f) * (T_{05}/T_a)^{0.5} * (1 + \gamma - 1/2 Ma^2)^{-0.5} - 1]$$

where we know that $T_{05} = T_{04}$ or which is the exit total temperature. So, this is the specific thrust which we will say as τ and hence using this we can write down the formula for specific fuel consumption which is $\dot{m}_f / \text{thrust}$.

So, amount of fuel required to generate you need thrust, so it is \dot{m}_f/thrust . Let us divide both numerator and denominator by \dot{m}_a , so we can get $\dot{m}_f/\dot{m}_a T/\dot{m}_a$. So, we can get f/τ , so this is the formula for specific fuel consumption knowing f and τ we can find out. Let us plot τ and SFC with respect to Mach number where we can get the τ will initially increase with Mach number for given maximum temperature and then decrease.

If we increase the maximum temperature, then it will increase at all Mach numbers, so this is the direction in which T_{04} will increase and this is for τ and then specific fuel consumption actually decreases with Mach number and it is basically increases with total temperature. This is the variation of specific thrust and specific fuel consumption in case of an ideal ramjet.

(Refer Slide Time: 18:30)



Let us work out if there are losses in the ramjet. Now, we are considered only ideal condition. Let us find out what will be the reality for the ramjet and then for that we are here in state a if we isentropically would go then we would have gone to $0a$, so this is T axis, this is S axis, but in reality we go to 02 , so we have this real process in the intake for compression which is 02 , then there is loss in the combustion chamber or burner and then we reach to 04 .

And if we would have expanded isentropically, then we would have come here till we expand the atmospheric condition but we do not expand till the atmospheric condition there is a choking, so choking leads to the expansion which is still 5 state and then if we are here, then this is 05 state.

So, this is the real cycle for ramjet. Now, let us define certain terminologies, let us say r_d is diffuser which is rather intake stagnation pressure ratio and that is defined as P_{02}/P_{0a} .

Then, we have combustion chamber will say r_b , it will not conflict with r_c which is compression ratio which is combustion chamber stagnation pressure ratio, which is P_{04}/P_{02} . Then, we have r_n which is nozzle stagnation pressure ratio and then that is P_{05}/P_{04} . So, there is loss in total pressure in all the processes. So, these losses are accounted through the ratios. In case of intake, this is the loss; in case of combustion chamber, this is the loss and in case of nozzle, this is the loss.

This is for intake, this is for burner and this is for nozzle. So, if I try to find out the ratio of P_{05}/P_{0a} and this ratio can be written as $P_{05}/P_{04} * P_{04}/P_{02} * P_{02}/P_{01}$ or P_{0a} which is atmospheric state. So, this is necessarily diffuser pressure ratio into burner pressure ratio into nozzle pressure ratio. So, the exit total pressure divided by inlet total pressure is the product of all the pressure ratios from diffuser, combustion chamber and nozzle.

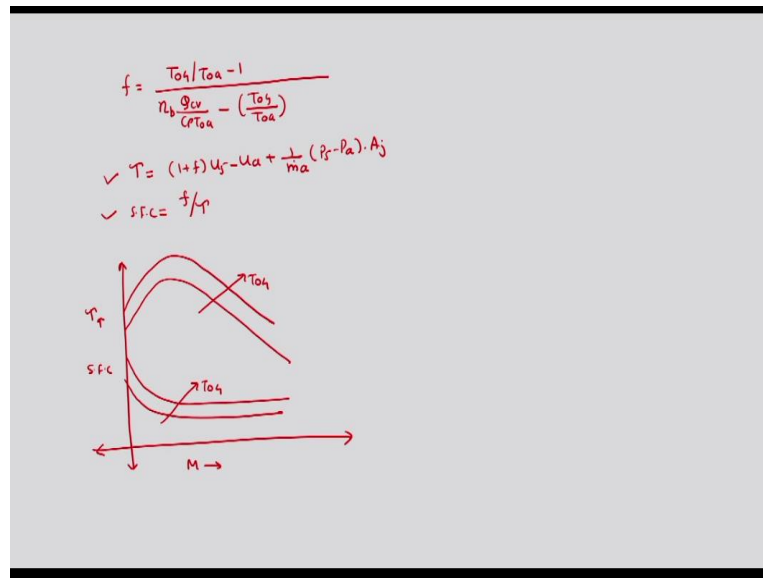
Let us feel that $T_{05}/T_5 = 1 + \gamma - 1/2 * M_5^2$. Now, we are trying to find out the derivation again for specific thrust and specific fuel consumption. So, we should know the temperature at 5 such that we can calculate the velocity. So, knowing this we can use this expression and say $M_5^2 = 2 / (\gamma - 1) * T_{05}/T_5 - 1$, so which can be written as $2 / (\gamma - 1) * T_{05}/T_5$ is $(P_{05}/P_5)^{(\gamma - 1)/\gamma} - 1$.

So, $M_5^2 = 2 / (\gamma - 1)$ where we will write down P_{05}/P_5 as $(P_{05}/P_{0a} * P_{0a}/P_a * P_a/P_5)^{(\gamma - 1)/\gamma}$. So, this gives us $M_5^2 = 2 / \gamma - 1$ into P_5/P_a , P_{0a} is seen to be product of 3 stagnation pressure ratios and we have P_{0a}/P_a which is $[1 + \gamma - 1/2 Ma^2]^{\gamma/(\gamma - 1)} * P_a/P_5$ then whole bracket complete bracket raise to $(\gamma - 1)/\gamma - 1$.

So, if we consider only this bracket then it has power $\gamma / (\gamma - 1)$ but this whole bracket has power $(\gamma - 1)/\gamma$. So, it gets canceled, so we have $M_5^2 = 2 / (\gamma - 1) * [1 + \gamma - 1/2 * Ma^2 * r_d * r_b * r_c * P_a/P_5]^{\gamma/(\gamma - 1) - 1}$. So, we know that we will number this as equation 1. We know that U_5 is equal to a_5 into M_5 , so it is equal to $\gamma RT_5 * M_5$.

So, we can write down $U_5 = M_5 \cdot \gamma R T_5 / T_{05} \cdot T_{05}$. So, $M_5 \cdot \gamma R \cdot T_{05}$. and T_5/T_{05} is $(1 + \gamma - 1/2 M_5^2)^{-1/2}$. So, this we can find out U_5 from finding out M_5 . M_5 would be found out from expression 1 knowing all the governing parameters. This M_5 will be put over here to find out U_5 , which is necessarily the jet speed.

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So, further we know what is the formula for f and the same formula is useful here also which is

$$f = (T_{04}/T_{0a} - 1) / (Q_{cv} \cdot C_p \cdot T_{0a} - T_{04}/T_{0a})$$

This formula was for ideal case where combustion was taking place completely and then heat chemical energy was released completely, but now we have burner efficiency, which is η_b . So, this is for real case. So, now we know these parameters, we can find out f and then we can use this to find out the thrust, which is

$$T = (1+f) \cdot U_5 - U_a + 1/\dot{m}_a \cdot (P_5 - P_a) \cdot A_j.$$

So, this is how we can find out thrust and then also specific fuel consumption, which is f/τ . So, this is basically τ since this is specific thrust. The variation of τ , specific thrust and specific fuel consumption is similar where specific thrust initially increases and then decreases with Mach number for a given T_0 and then with increase in T_0 , it increases. That is necessarily T_{04} .

Specific fuel consumption decreases with Mach number and then it has again increment with T_{04} where these numbers would get reduced where thrust would get reduced in the case of losses. So, here we end our discussion about ramjet and we will see some more examples for ramjet in one of the classes. Thank you.