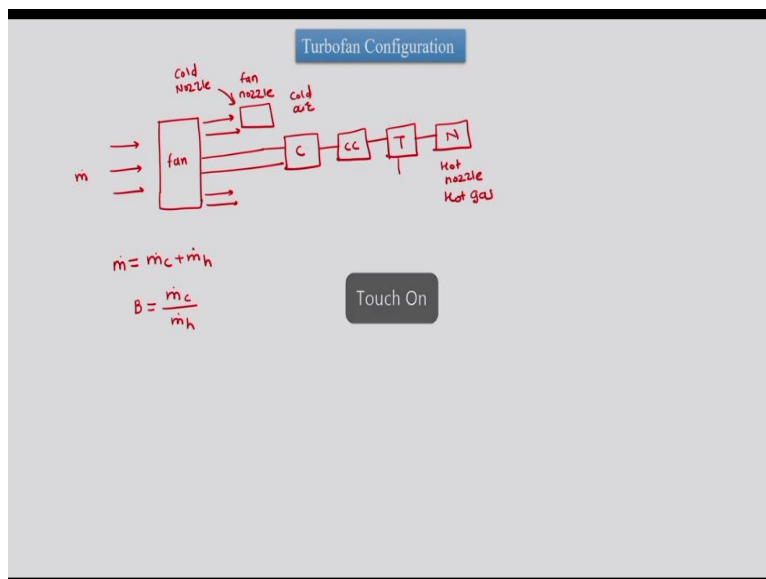


Aircraft Propulsion
Vinayak N. Kulkarni
Department of Mechanical Engineering
Indian Institute of Technology – Guwahati

Lecture - 21
Turbofan Engine: Configuration and Examples

Welcome to the class. We are going to see today about the examples for turbofan engine. First, we are going to see how is the composition of a given turbofan engine.

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In a turbofan engine, if I plot the schematic, then first there is a fan, there is a fan which is receiving complete mass flow rate of complete mass flow rate and then some air is going to pass directly to a fan nozzle. However, some air is going to pass from fan to a compressor, further it will go to a combustion chamber and then from there to a turbine and then from there to the nozzle.

So, this will have 2 nozzles, one is called as cold nozzle where air which was not having any combustion that is passing and other is air which is passing through the combustion chamber and then that is hot nozzle. So, we have basically 2 airs, from this nozzle we are having cold air and this is hot air or hot gas engine. So, if \dot{m} is total mass flow rate, then within that \dot{m}_c is cold mass flow rate plus \dot{m}_h is hot mass flow rate.

And then as per the specification of a turbofan engine, we have to define a bypass ratio which is percentage of cold air in comparison with hot air going from the fan. So, how much air

percentage of hot air is flowing over the fan? So, this is what the bypass ratio is and then we know that in case of turbojet engine, the jet velocity is high outlet. So, turbojet engine thrust is obtained from the combusted gas and then that combustion would lead to the lower efficiencies.

And then that is why to improve the efficiency, we have to find out a way and one of the ways is to implement fan. So, having fan we are partially generating some thrust based upon the cold nozzle and some thrust generated from hot nozzles, so this will improve the efficiency. Further, it also helps to reduce the noise. So, these are the details of turbofan configuration.

In general, the civil aircrafts which are seen by us, those civil aircrafts are generally having the turbofan engine where we can see under the wings, there are engines and then the frontal phase is comprised of fan and then downstream of it we have compressors, turbines, combustion chambers and the nozzle. The turbofan engine can have both nozzle as single where the both cold and hot gases will mix and then they will pass through a nozzle.

Or then there can be otherwise there can be 2 separate nozzles, which will have separate thrust generated. So, then there are 2 types of turbofan; one is mixed, another is unmixed. So, let us see some examples for turbofan.

(Refer Slide Time: 04:46)

The following data apply to a twin-spool turbofan engine, with the fan driven by the LP turbine and the compressor by the HP turbine. Separate cold and hot nozzles are used.

Overall Pressure ratio	:	25.0
Fan pressure ratio	:	1.65
Isentropic efficiency of each propelling nozzle	:	0.95
Bypass ratio m_c / m_h	:	5.0
Turbine inlet temperature	:	1550K
Fan, compressor and turbine polytropic efficiency	:	0.90
Mechanical efficiency of each spool	:	0.99
Total air mass flow	:	215 kg/s
Combustion pressure loss	:	1.50 bar

It is required to find the thrust and *SFC* under sea-level static conditions where the ambient pressure and temperature are 1.0 bar and 288 K. Assume $C_{pa} = 1.005 \text{ kJ/kg K}$ $C_{pg} = 1.147 \text{ kJ/kg K}$, Specific heat ratio of air = 1.4, Specific heat ratio of gas = 1.33

In the example, it is given that the following data apply to a twin-spool turbofan engine in case of twin-spool, we should understand that there will be 2 turbines instead of 1 and one turbine

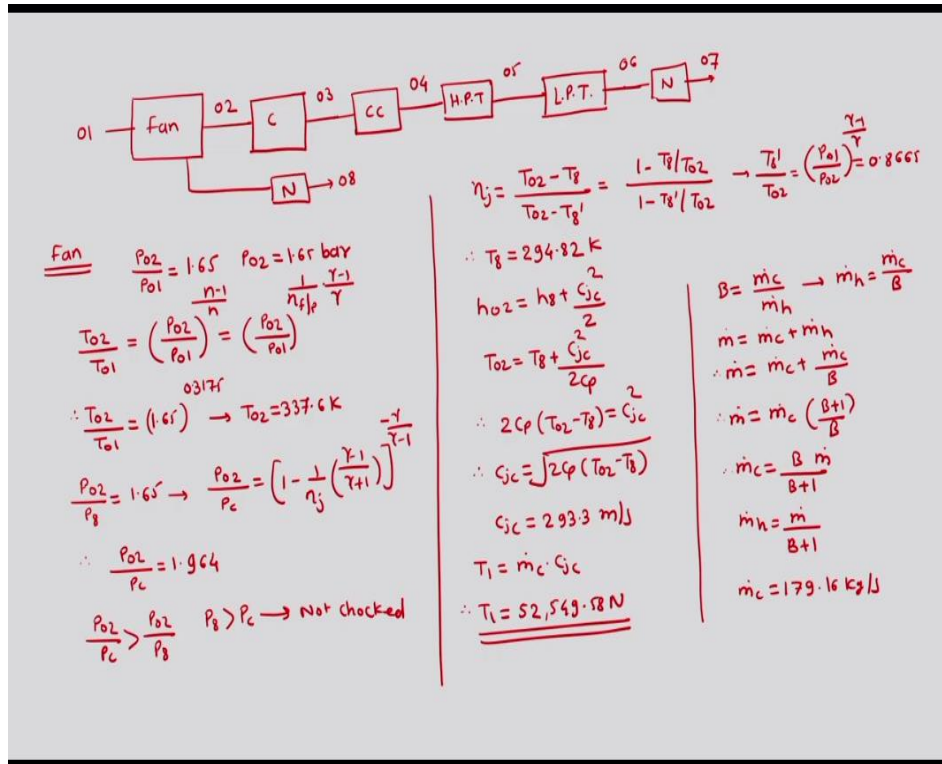
which is high pressure turbine will have its connection with compressor and low pressure turbine will have connection with fan, such that both would be driven by separate turbines.

We have seen such arrangement, multi-spool arrangement in case of the aircraft arrangements. Following data apply to a twin-spool turbofan engine, with a fan driven by low-pressure turbine and compressor by high pressure. Separate cold and hot nozzles are used. Overall pressure ratio is 25, this pressure ratio overall means from the atmospheric condition to the condition at the outlet of the compressor.

Fan pressure ratio is 1.65, so within that basically we are having fan pressure ratio which is 1.65, isentropic efficiency of each propelling nozzle is 0.95, bypass ratio which is \dot{m}_c/\dot{m}_{hot} as 5, turbine inlet temperature as 1500 Kelvin. Fan, compressor and turbine they have propulsive efficiency as 0.9, mechanical efficiency of each spool is 0.99. Here, it is a twin-spool arrangement so low-pressure turbine is connected with the fan, so there is one transmission.

High-pressure turbine is connected with compressor, there is one another transmission. So, both have mechanical efficiency of 0.99, total air flow rate is 215 kg per second, combustion pressure loss is 1.5 bar, it is required to find out thrust and specific fuel consumption under sea-level where pressure and temperature are 1 bar and 288 Kelvin. Again, we are given with specific heat of air and gas separately and we are given with their gammas.

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So, let us start solving this example where we need first we will number each corner here where we will have first a fan, then fan has 01 condition, at the exit we have 01, then compressor we have 02, then 03 at the end of the compressor, then we have combustion chamber, it has 04, then we have high-pressure turbine it has exit as 05, we have low-pressure turbine it has 06 and then we have 1 nozzle, it has 07.

But there is one more nozzle here, which is cold nozzle, its exit is 08. So, let us work out for fan to start with. We are told that fan pressure ratio is 1.65, so P_{02}/P_{01} is 1.65 but we are told that P_{01} is 1 bar, so we have P_{02} as 1.65 bar. In this example, we are given with propulsive efficiencies, practically we need to know T_{02}/T_{01} from P_{02}/P_{01} and then that if it would have been isentropic flow, we would have written as γ upon $\gamma - 1/\gamma$.

But since the flow is not isentropic, some polytropic efficiency is given, so we have to refrain it as $n - 1/n$ with a polytropic index and we know this polytropic index has relevance with isentropic index γ through polytropic efficiency of fan. So, this is $1/\text{fan's polytropic efficiency}$ into $\gamma - 1/\gamma$. So, we have T_{02}/T_{01} is 1.65 bracket raise to 0.3175 and this gives us T_{02} as 337.6 Kelvin where T_{01} is given as 288 Kelvin.

Knowing T01 and T02, we can move ahead and then we can work out for the nozzle since we know P02/P8, which is atmospheric and that ratio is again 1.65, but what is known to us that this nozzle has efficiency and that nozzle efficiency is leading to the fact that we have to see whether this nozzle having certain efficiency choked or not. So, for that we have to find out choking condition and theoretical choking condition is

$$\frac{P_{02}}{P_c} = \left(1 - \left(\frac{1}{\eta_j} \right) \left(\frac{\gamma - 1}{\gamma + 1} \right) \right)^{\left(\frac{\gamma}{\gamma - 1} \right)}$$

Here, this γ is for air, so it is 1.4 and efficiency is given as 95%. So, P02/Pc comes out to be 1.964. So, we know now P02/Pc is greater than P02/P8, so P8 is greater than Pc. This means that nozzle is not choked. This nozzle is not choked. So, we have to find out the conditions at the exit of the nozzle for this. So, we know exit pressure is P8, which is atmospheric but we do not know what is the velocity.

So, to find out velocity, we have to use the relations based upon the known quantities. So, before that let us find out temperature. So, using the nozzle efficiency, which is

$$\eta_n = \frac{T_{02} - T_8}{T_{02} - T_8'}$$

So, we can take T02 common, so we will have T02 is taken common, so $1 - T_8/T_{02} = 1 - T_8'/T_{02}$. Here, we know that T_8'/T_{02} is isentropic relation, which is P_{01}/P_{02} bracket raise to $(\gamma - 1)/\gamma$ and this ratio comes out to be 0.8665.

Knowing this ratio and efficiency, we can find out basically T8 and this T8 would come out to be 294.82 Kelvin. So, this is the temperature at the exit. Now, we know that for the nozzle

$$h_{02} = h_8 + \frac{C_j^2}{2}$$

We are telling this jet as 1 or we can say it as cold, no problem. So, this will, this gives us

$$T_{02} = T_8 + \frac{C_j^2}{2C_p}$$

So, this gives us twice $C_p \times T_{02} - T_8 = C_j^2$ cold square.

So, C_j^2 cold = square root of twice C_p into $T_{02} - T_8$ and this gives us C_j cold as 293.3 meter per second. So, this is the velocity of the air at the exit of the jet. Now, we need to find out the

thrust from the nozzle 1. So, thrust 1 is equal to $m \cdot c_j$ but we should know what is m_{cold} and m_{cold} can be found out from bypass ratio since bypass ratio is \dot{m}_c / \dot{m}_h and we know \dot{m}_h is also equal to $\dot{m}_c + \dot{m}_h$.

So, let us represent \dot{m}_h in terms of \dot{m}_c , so $\dot{m}_h = \dot{m}_c / B$. So, $\dot{m} = \dot{m}_c + \dot{m}_c / B$ and so we have $\dot{m}_h = \dot{m}_c \times B + 1 / B$. So, $\dot{m}_c = B / B + 1 \times \dot{m}$ and then parallelly we can also find out \dot{m}_h and $\dot{m}_h = \dot{m}_c / B$, so it is $\dot{m} / B + 1$ and now we have \dot{m}_c from the known bypass ratio, which is 5 and known mass flow rate we can know \dot{m}_c as 179.16 kg per second.

So, thrust from the cold nozzle is 179.16 into C_j since the speed which is entering air has that is zero value, so basically we are having $m \cdot c_j - C$, but C is 0, so this thrust turns out to be 52549.58 Newton. So, this is the value of thrust, which is obtained from the nozzle fan.

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The slide contains a schematic diagram of a gas turbine cycle and handwritten calculations. The diagram shows the flow from state 01 through a fan (02), compressor (03), combustion chamber (04), high-pressure turbine (05), low-pressure turbine (06), and nozzle (07). A bypass stream goes from 02 through a nozzle (08) to the nozzle (07). The main flow is labeled \dot{m}_h and the bypass flow is \dot{m}_c .

Calculations:

- Pressure ratios: $\frac{P_{03}}{P_{02}} = \frac{P_{03}}{P_{01}} \times \frac{P_{01}}{P_{02}}$
- Temperature ratios: $\frac{T_{03}}{T_{02}} = \left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}}$
- Temperature values: $T_{03} = 2.37 \times 337.63 = 800.25 \text{ K}$, $T_{04} = 1550 \text{ K}$
- Work and heat: $\frac{W_c}{\dot{m}_r} = W_{1HP}$, $\dot{m}_h c_{p_a} (T_{03} - T_{02}) = \dot{m}_h c_{p_g} (T_{04} - T_{03})$, $T_{04} - T_{05} = \frac{c_{p_a}}{\dot{m}_r c_{p_g}} (T_{03} - T_{02}) = 409.09 \text{ K}$, $T_{05} = 1140.90 \text{ K}$
- Temperature difference: $T_{05} - T_{06} = 263.29 \text{ K} \rightarrow T_{06} = 877.60 \text{ K}$
- Pressure ratios: $P_{01} = 1 \text{ bar}$, $P_{02} = 1.65 \text{ bar}$, $P_{03} = 25 \text{ bar}$, $T_{04} = P_{03} - \Delta P_{03} = 23.5 \text{ bar}$, $P_{04} = \frac{T_{04}}{T_{03}} = \frac{P_{03}}{P_{02}} = \frac{P_{01}}{P_{02}} = \frac{P_{03}}{P_{02}}$, $P_{04} = \frac{T_{04}}{T_{03}} = 3.944 \rightarrow \frac{P_{05}}{P_{06}} = \frac{T_{05}}{T_{06}} = 3.245$, $P_{04} = \frac{P_{04}}{P_{03}} = \frac{23.5}{3.944} = 5.958 \text{ bar}$, $P_{06} = \frac{P_{06}}{P_{05}} = \frac{5.958}{3.245} = 1.835 \text{ bar}$

So, now let us see and work out for the hot gas or practically for the gas turbine. Now, for hot gas first we have to find out what is the compressor's pressure ratio since given things are total pressure ratio and fan pressure ratio. So, we are knowing that P_{03} , we will just number everything again here as well for the reference. We have fan first, which has 01 and exit 02, compressor nozzle 08.

Then, we have combustion chamber, after the combustion chamber it is 04, then we have turbine 1 which is HP this has 05, we have LP this has 06 and then there is a nozzle, which has 07. Having said this what we are given is P_{03}/P_{01} , what we are interested is P_{03}/P_{02} , this can be written as P_{03}/P_{01} into P_{01}/P_{02} and this is total pressure ratio, which is given as number

which is 25 and this is for fan which is 1.65. So, $P_{03}/P_{02} = 25/1.65$, so we have 15.15 as the total pressure ratio for compressor. This is for r_{pc} .

Now, knowing this, we can find out T_{03}/T_{02} and this has to be equal to P_{03}/P_{02} since this is a real condition, so it will be $n^{-1/n}$. This is a compressor, which has polytropic efficiency and due to which we are not having isentropic index, we have polytropic index, but we know that these two indices are having relevance using polytropic efficiency. So, P_{03}/P_{02} bracket raise to 1 upon compressor efficiency propulsive into $\gamma - 1$ upon γ .

So, we have $T_{03}/T_{02} = P_{03}/P_{02}$, polytropic efficiency is given, γ is given. From those given numbers, it becomes 3175, which was same for fan as well and this ratio comes out to be 2.37. So, we can know now T_{03} is equal to 2.37 into 337.63 and this comes out to be 800.26 Kelvin. Now, we are given with T_{04} and T_{04} is given to us as 1550 Kelvin. Now, we are knowing that this compressor is connected with high pressure turbine.

So, compressor's work divided by transmission efficiency is equal to turbine's work high pressure. So, we have C_p of but here we have remember that we have to find out using mass flow rate and then this is mass flow rate hot into C_p of air into $T_{02} - T_{01}$ sorry $T_{03} - T_{02}/\text{transmission efficiency} = \dot{m} \text{ hot}$. So, here we are having $\dot{m} \text{ hot}$, here we are having $\dot{m} \text{ cold}$ and here we are having \dot{m} into C_p of gas into $T_{04} - T_{05}$.

Knowing this, we can calculate $T_{04} - T_{05}$, mass flow rate would cancel out and this would be $C_p \text{ air}/\text{transmission efficiency} C_p \text{ gas } T_{03} - T_{02}$ and from known quantities, which is T_{03} is known, T_{02} is known, which is 337. So, from this C_p of air, C_p of gas and transmission efficiencies are known and from there this difference comes to be 1140.90 Kelvin. Having known this number, we know now what is the temperature at temperature difference between 04 and 05 sorry this number comes to be 409.09 and this gives us T_{05} as 1140.90 Kelvin.

Then, we can go ahead and we can work out for 06 and this is LP turbine and we know that it is connected with fan with the other spool, so we have $\dot{m} C_p$ of air into $T_{02} - T_{01}$ is equal to this is work done for the fan, work input for the fan that is equal to $\dot{m} h \text{ hot}$ into C_p of gas into $T_{05} - T_{06}$ and then this into transmission efficiency okay. So, we can divide it over here and so we have $C_p \text{ of air}/C_p \text{ of gas into transmission efficiency into } T_{02} - T_{01} = \dot{m} \text{ hot}/\dot{m} \text{ into } T_{05} - T_{06}$.

$\dot{m} h / \dot{m}$ we have calculated as this gives us $\dot{m} h / \dot{m}$ as $1/B + 1$. So, this is going to give us $T_{05} - T_{06} = C_p$ of air / C_p of gas into transmission efficiency $T_{02} - T_{01}$ into $\dot{m} \dot{h} / \dot{m} \dot{h}$ and then this is basically known to us as $B + 1$. So, we can replace it with $B + 1$ and bypass ratio is given, T_{02} is known 337, T_{01} is given. All these numbers are given, from here we calculate $T_{05} - T_{06}$ and this number comes out to be 263.29 Kelvin and this gives us T_{06} to be 877.60 Kelvin.

Now, we are interested in finding out the turbine's pressure ratio, operating pressure ratio for the turbine. So, let us find out the operating pressure ratio for the turbine. We have basically, we are interested in finding out pressure P_{06} , since P_{06} is known, then we can check whether this nozzle is choked or not. So, for that we should know P_{06} . What we know is only P_{01} which is 1 bar, what we know latter P_{02} is 1.65 bar.

Then, we know P_{03} also and that was 25 bar as overall pressure ratio, then we could calculate P_{04} is P_{03} minus ΔP_0 combustion chamber and then this comes out to be 23.5 bar. Now, we do not know what is the value of P_{05} and what is the value of P_{06} , so for that let us find out P_{04}/P_{05} and this we know as relevance as $n - 1/n = T_{04}/T_{05}$ or rather P_{04}/P_{05} is equal to this n is due to polytropic efficiency of the turbine, otherwise it would have been isentropic index γ .

So, this can be written as P_{04}/P_{05} including the turbine's polytropic efficiency as $\gamma - 1/\gamma$. So, this number comes out to be P_{04}/P_{05} comes out to be 0.223. So, we know $P_{04}/P_{05} = T_{04}/T_{05}$ bracket raise to $1/0.223$ and then this ratio comes out to be 3.944 and similarly $P_{05}/P_{06} = T_{05}/T_{06}$ bracket raise to 0.223 and this gives us ratio which is T_{05} is known, we have calculated it here.

T_{06} is also known, we have calculated it here and this gives us the ratio and this ratio is 3.2456. So, knowing both ratios, we can calculate now P_{05} , P_{05} is equal to this ratio which is $P_{04}/P_{04}/P_{05}$. So, this is $23.5/3.944$ and then this gives us P_{05} as 5.958 bar. Then, $P_{06} = P_{05}/P_{05}/P_{06}$. So, it is $5.958/3.2456$ and this gives us P_{06} as 1.836 bar. Now, we know P_{06} which is the total pressure and entry to the nozzle.

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$$\begin{aligned}
 P_{06} &= 1.836 \text{ bar} \\
 \frac{P_{06}}{P_{0a}} &= 1.836 \checkmark \\
 \frac{P_{06}}{P_c} &= \left[1 - \frac{1}{\eta_j} \left(\frac{\gamma_g - 1}{\gamma_g + 1} \right) \right]^{\frac{-\gamma_g}{\gamma_g - 1}} \\
 \therefore \frac{P_{06}}{P_c} &= 1.916 \\
 \therefore \frac{P_{06}}{P_c} &> \frac{P_{06}}{P_a} \\
 P_a > P_c &\rightarrow \text{Not choked.} \\
 h_{06} &= h_7 + \frac{C_{j^2}}{2} \\
 \therefore T_{06} &= T_7 + \frac{C_{j^2}}{2C_p} \\
 \therefore \eta_j &= \frac{T_{06} - T_7}{T_{06} - T_7'} = \frac{1 - T_7/T_{06}}{1 - T_7'/T_{06}} \\
 \frac{T_7'}{T_{06}} &= \left(\frac{P_7}{P_{06}} \right)^{\frac{\gamma_g}{\gamma_g - 1}} = \left(\frac{1}{1.836} \right)^{\frac{1.33 - 1}{1.33}}
 \end{aligned}$$

$$\begin{aligned}
 T_7 &= 760.98 \text{ K} \\
 C_{j^2} &= \sqrt{2C_p (T_{06} - T_7)} \\
 \therefore C_{j^2} &= 517.45 \text{ m/s} \\
 T_h &= \dot{m} h C_{j^2} = \frac{\dot{m}}{B} C_{j^2} = \frac{217}{6} \times 517.45 = 18542.12 \text{ N} \\
 \therefore T &= T_h + T_c \\
 \therefore T &= 71091.7 \text{ N}
 \end{aligned}$$

Now, this is how will we proceed to see whether the nozzle is choked or not. What we have calculated till time is P06 and that was 1.836 bar but P01 is 1 bar, so P06/P0a is 1.836. Now, let us see if this nozzle is choked. By seeing P06/Pc as $1 - \eta_j \gamma_{\text{gas}} - 1 / \gamma_{\text{gas}} + 1$ bracket raise to $-\gamma_{\text{gas}} / \gamma_{\text{gas}} - 1$. γ_{gas} is 1.33, nozzle efficiency is 95%, so we can get P06/Pc = 1.916.

So, we have P06/Pc is greater than P06/Pa, so we have Pa greater than Pc, so nozzle is not choked okay. So, we have to go ahead and find out what is the velocity of the second jet. So, for that again we will use h06 is equal to h7 plus $C_j^2 / 2$, so we have $T_{06} = T_7 + C_j^2 / 2C_p$ gas but here we need to know what is the T7. So, we can find out T7 from nozzle efficiency, which says that $T_{06} - T_7 / T_{06} - T_7'$.

We can take T06 common, so $1 - T_7 / T_{06} / 1 - T_7' / T_{06}$ but T_7' / T_{06} is related with P7/P06 bracket raise to $\gamma_{\text{gas}} - 1 / \gamma_{\text{gas}}$ and we have seen that this ratio P7 which is Pc which is P7/Pa is 1.836, so this can be evaluated from here and T_7' / T_{06} can be found out from Pc/P06 which is 1.836 bracket raise to $1.33 - 1 / 1.33$. So, we can get T_7' / T_{06} which is required over here.

We know nozzle efficiency, we know T06, knowing all this, we can calculate T7 and T7 comes out to be 760.98 Kelvin. Knowing this, we can calculate Cj. C_j^2 square hot = square root of twice Cp gas into T06 - T7. So, knowing this we get Cj hot as 517.45 meter per second. So, we can calculate second thrust or hot thrust as $\dot{m} \cdot C_j$ hot but \dot{m} is again $\dot{m} / B + 1$ into Cj hot.

So, this is $215/6 \times 517.45$, so this thrust comes out to be 18542.12 N. So, total thrust is

$$T = T_{\text{hot}} + T_{\text{cold}}$$

, so we have total thrust as 71091.7 N. So, the major care which we have to take in the example which we need to solve for the fan is this that we should take care for the mass flow rate. The nozzle has only cold mass flow rate and other gas turbine part like compressor, combustion chamber has hot mass flow rate.

So, this is coming into 2 ways; first thing it will come for calculation of thrust of the first nozzle where we have to use only \dot{m}_c , but while equating the networks that means the work of high-pressure turbine with compressor, we should use hot gas and while equating the work for fan and low-pressure turbine, we have to use both the mass flow rates, one is total mass flow rate for fan and the hot mass flow rate for low-pressure turbine.

And that is how this example becomes little complicated. Further, we have to always check whether the nozzle is choked or not in either cases by looking at the critical conditions and then if nozzle is not choked, then we have to use the corresponding formula or if nozzle is choked, then we have to use associated formula. This is how we can solve the example for turbofan engines. Thank you.