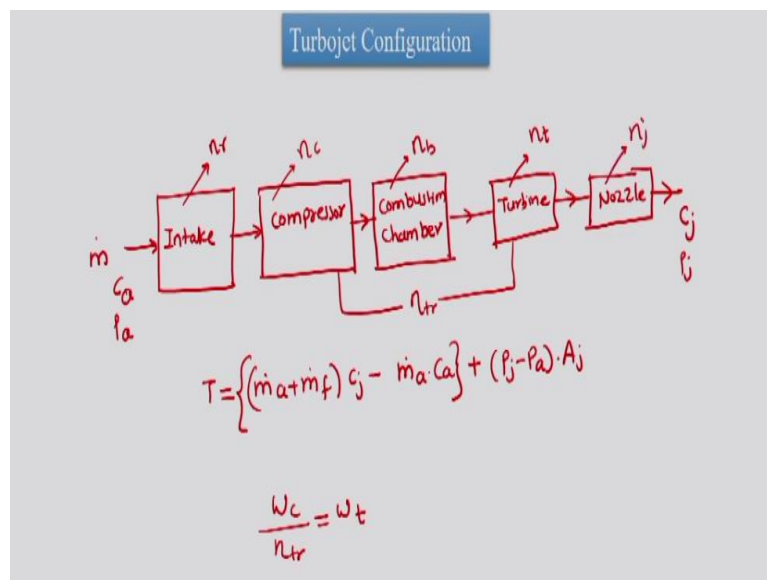


Aircraft Propulsion
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Lecture - 20
Turbojet Engine - Configuration and Examples

Welcome to the class. Today, we are going to talk about Turbojet engine its configuration and its examples how to deal with an example of Turbojet engine.

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Turbojet configuration as we know turbojet if we plot in schematic it would have first an intake there would be an intake after the intake there would be compressor. Air will first come into the intake then it will go to the compressor. And then after the compressor it will go into the combustion chamber or burner. After the burner it will go into the turbine which is basically the gas turbine for us then after the turbine it will go into the nozzle.

And then here it generates the necessary velocity which is C_j where entering velocity is C_a and then we generate the thrust basically which depends upon P_j , P_a and C_j , C_a and also \dot{m} . So we know thrust is $= (\dot{m}_a + \dot{m}_f) C_j - \dot{m}_a C_a + (P_j - P_a) A_j$. So this is the expression for thrust which we need to calculate for the turbojet engine. So we would know how to find out the properties in the intake.

We know that there is ram efficiency or isentropic efficiency of intake. It governs the compression which is achieved before the air it reaches to the compressor. Then there is

compressor efficiency which is telling us that if a compression in the compressor is not isentropic. There is combustion chamber efficiency or burner efficiency. It does not release the complete chemical energy of the fuel.

But some energy is lost then there is turbine efficiency which tells us that process is not completely isentropic in the turbine, but it has certain losses. There is one more efficiency that this turbine and compressor they are connected with each other and then that efficiency is called transmission efficiency where the power generated from the turbine is given to the compressor and some of the power is getting lost.

So we have to give extra power which is going to account for the transmission loss. If W_c is the necessary work input for the compressor. Then we have to produce the work which is more than the compressor work so this should be = turbine work. So turbine necessarily present in the turbojet to run the compressor. In ideal case turbine work = compressor work, but due to transmission losses we have to do more work in the turbine such that which we can produce necessary compression in the compressor by supplying the work.

Then we have η_j which is nozzle efficiency where the process is not reversible in the nozzle and it leads to the losses in the nozzle. So the whole idea in case of turbojet is different from the shaft power cycle what we had seen where the whole objective was to find out the W_{net} . Here objective is to find out thrust or specific fuel consumption and we have seen the formulas about these quantities.

So for a given condition of the turbojet we are supposed to find out the thrust or specific fuel consumption or maybe the necessary quantities. Most of the components like turbine compressor, combustion chamber would remain same in other turbofan engine also, but in principle this is the composition for a turbojet engine.

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A simple turbojet unit operates with a maximum turbine inlet temperature of 1200 K, a pressure ratio of 4.25:1 and a mass flow of 20 kg/s under design conditions, the following component efficiencies may be assumed.

Isentropic efficiency of compressor	:	87%
Isentropic efficiency of turbine	:	91.50%
Propelling nozzle efficiency	:	96.50%
Transmission efficiency	:	98.50 %
Combustion chamber pressure loss	:	0.21 bar

Assume $C_{pa} = 1.005 \text{ kJ/kg K}$ and Specific heat ratio of air = 1.4, $C_{pg} = 1.147 \text{ kJ/kg K}$ and Specific heat ratio of gas = 1.33

Calculate the total design thrust. Also calculate the total thrust and specific fuel consumption taking into consideration the nozzle choking condition. Assume that the unit is stationary and at sea level, where the ambient condition may be taken as 1 bar and 293K. Assume air fuel ratio of 50.

Let us proceed with an example which states that a simple turbojet unit operates with a maximum turbine inlet temperature of 1,200 kelvin and a pressure ratio of 4.25:1. And mass flow of 20 kg per second under design conditions, following component efficiencies maybe assumed. Isentropic efficiency of compressor is 87%, isentropic efficiency of turbine is 91.5%, propelling nozzle efficiency is 96.5%, transmission efficiency is 98.5%.

Combustion chamber pressure loss is 0.21 bar. Assume specific heat of air as 1.005 kilo joule per kg Kelvin, specific heat of air as 1.4. Specific heat of gas as 1.47 kilo joule per kg Kelvin and specific heat ratio for the gas as 1.33. Calculate the total design thrust also calculate the total design, total thrust and specific fuel consumption taking into account or taking into consideration nozzle choking condition.

Assume that the unit is stationary and at sea level where the ambient condition may be taken as 1 bar and 293 Kelvin. Assume air fuel ratio as 50. A notable point in this example that nothing is told about intake so we can exclude the intake where we can feel that the given conditions as 1 bar and 293 Kelvin are the inlet conditions to the compressor. So these are the compressor inlet conditions and we can proceed with the example.

Further it is notable over here that we are told to use different properties for air and gas which is specific heat and specific heat ratio.

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which is

$$= 1/\eta_c T_{01} (T_{02}'/T_{01} - 1)$$

And then this is related with isentropic relation so we can have

$$T_{02} - T_{01} = \frac{T_{01}}{\eta_c} \left(\left(\frac{P_{02}}{P_{01}} \right)^{\frac{\gamma}{\gamma+1}} - 1 \right)$$

$$= \frac{T_{01}}{\eta_c} \left((rp)^{\frac{\gamma}{\gamma+1}} - 1 \right)$$

T₀₁ is given as 293 Kelvin this is 293/compressor efficiency which is 0.87 and into the bracket 4.25 and bracket raise to 0.285 – 1 and this gives us T₀₂ = 465.6 Kelvin. So this is the value in reality at the exit of the compressor.

Now we know that compressor work/transmission efficiency = turbine work. In this example we needed the entry total temperature for the nozzle for which we need the turbine pressure ratio, but we do not know turbine pressure ratio so we have to calculate the turbine pressure ratio first for which we are going to use the relation which states that compressor work/transmission efficiency equal to turbine work.

So we have to be careful here since in the compressor we have

$$\frac{C_{p\text{air}}(T_{02} - T_{01})}{\eta_{tr}} = C_{p\text{gas}} (T_{03} - T_{04})$$

We know everything over here except T₀₄. So we can calculate T₀₄ so first

$$T_{03} - T_{04} = (C_{p\text{air}}/C_{p\text{gas}} \eta_{tr})(T_{02} - T_{01}).$$

Now we know T₀₃ = 1200 specific heat of air is 1005/same for gas is 985 it is given to us as 1147 into 0.985 which is transmission efficiency into T₀₂ which we have just calculated – T₀₁ which is 293 and this gives us T₀₄ and hence T₀₄ is 1046.4 Kelvin. So this is the total temperature at the exit of the turbine. Now here we can use this total temperature for finding out the pressure ratio for the turbine.

Since we know turbine efficiency we can use this formula as what we used T₀ turbine efficiency is actual work done/ideal work done. So this can be formulated using turbine pressure ratio as what we did in case of compressor we formulated it in terms of pressure ratio. Here same way we can formulate it for the pressure ratio using this where we have

$$T03 - T04 = \eta_t T03 \left(1 - \left(\frac{1}{r_{pt}} \right)^{\frac{\gamma_g - 1}{\gamma_g}} \right)$$

But here this should be noted that this is gamma for gas. Now we are just unknown with r_{pt} which is pressure ratio for turbine we know T02 we know T04. So from this we can calculate T03 is known which is 1200 T04 is 1046.4 turbine efficiency is given which is 0.915 into 1200 $1 - 1/r_{pt}$ turbine bracket raise to 1.33 $- 1/1.33$. We are told that specific heat ratio for gas is 1.33.

If we rearrange the terms we can get turbine pressure ratio as 1.835 so this is the turbine pressure ratio. Now knowing this we can find out P04 which is the exit pressure of the turbine we first should know what is P03. $P03 = P02 - \Delta P_{0cc}$. So it will be 4.25 we are told that exit entry to the compressor is 1 bar compressor pressure ratio is 4.25 so total pressure at the exit of the compressor is 4.25 and loss is 0.21.

So we can know here that 4.04 bar knowing this we can calculate P04 from the fact that turbine pressure ratio $r_{pt} = P03/P04$. So $P04 = P03/r_{pt}$ it gives us P04 as 2.2 bar. So now we know what is the exact value of the pressure at the exit to the turbine. In the example now it is asked to us to find out the total thrust and or other designed value of the thrust. Here we are not going to discuss about the choking condition.

The next part of the example deals with choking conditions. So let us first resolve the conditions we know that

$$\frac{T04}{T5} = \left(\frac{P04}{P5} \right)^{\frac{\gamma - 1}{\gamma}}$$

which is so we have just now calculated it has 2.2/1 and bracket raise $2 P04/P5$ dash or $P5 = \gamma - 1/\gamma$. This is gamma for gas considering this we will get this ratio putting P04 as 2.2 and P5 dash as 1 we will get it as 1.21 this ratio as 1.21.

$\epsilon_{pk} = 4.25:1$ $T_{03} = 1200 \text{ K}$ $\dot{m} = 25 \text{ kg/s}$ $\eta_c = 87\%$ $\eta_t = 91.5\%$
 $\eta_{tran} = 98.5\%$ $\eta_j = 96.5\%$ $\Delta P_{04} = 0.21 \text{ bar}$

$\eta_c = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}} \rightarrow T_{02} - T_{01} = \frac{1}{\eta_c} (T_{02}' - T_{01}) = \frac{1}{\eta_c} T_{01} \left[\left(\frac{P_{02}'}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$
 $T_{02} - T_{01} = \frac{T_{01}}{\eta_c} \left\{ \left(\frac{P_{02}}{P_{01}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right\} = \frac{T_{01}}{\eta_c} \left\{ (\epsilon_{pk})^{\frac{\gamma-1}{\gamma}} - 1 \right\} = \frac{293}{0.87} [(4.25)^{0.287} - 1]$
 $\therefore T_{02} = 465.6 \text{ K}$

$\eta_t = \frac{T_{03} - T_{04}}{T_{03} - T_{04}'}$
 $T_{03} - T_{04} = \eta_t T_{03} \left[1 - \left(\frac{1}{\epsilon_{pk}} \right)^{\frac{\gamma-1}{\gamma}} \right]$
 $1200 - 1046.4 = 0.915 \times 1200 \left[1 - \left(\frac{1}{\epsilon_{pk}} \right)^{\frac{\gamma-1}{\gamma}} \right]$
 $\therefore \epsilon_{pk} = 1.335$
 $P_{03} = P_{02} - \Delta P_{04} = 4.25 - 0.21 = 4.04 \text{ bar}$
 $\epsilon_{pk} = \frac{P_{03}}{P_{01}} \rightarrow P_{03} = \frac{P_{03}}{\epsilon_{pk}} = 2.2 \text{ bar}$

$\frac{T_{04}}{T_{04}'} = \left(\frac{P_{04}}{P_{04}'} \right)^{\frac{\gamma-1}{\gamma}} = 1.21$
 $\eta_j = \frac{T_{04} - T_5}{T_{04} - T_5'}$
 $T_5 = T_{04} - \eta_j (T_{04} - T_5')$
 $T_5 = 867 \text{ K}$
 $h_{04} = h_5 + \frac{C_j^2}{2}$
 $T_{04} = T_5 + \frac{C_j^2}{2c_p}$
 $\therefore C_j = 641.4 \text{ m/s}$

$T = (\dot{m}_a + \dot{m}_f) C_j - \dot{m}_a C_a$
 $T = \dot{m}_a (C_j - C_a) = \dot{m}_a C_j$
 $T = 16030 \text{ CN}$

$\frac{W_c}{N_{tr}} = \omega_t$
 $\frac{C_{pa}(T_{02} - T_{01})}{N_{tr}} = C_{pg}(T_{03} - T_{04})$
 $T_{03} - T_{04} = \frac{C_{pa}}{C_{pg} \eta_{tr}} (T_{02} - T_{01})$
 $T_{04} = T_{03} - \frac{C_{pa}}{C_{pg} \eta_{tr}} (T_{02} - T_{01})$
 $T_{04} = 1200 - \frac{1005}{1147 \times 0.985} (465.6 - 293)$
 $\therefore T_{04} = 1046.4 \text{ K}$

Here our objective is to find out T5 once T5 is known then we can find out what is the value for the exit jet velocity, but before that we should know what T5. From this we can know now T5 dash T5 can be found out from nozzle efficiency which says that

$$\eta_j = \frac{T_{04} - T_5}{T_{04} - T_5'}$$

So we have $T_5 = T_{04} - \eta_j(T_{04} - T_5')$ and then this gives us with all the known quantities the value of T5 as 867 Kelvin.

So now we know T5 we can write down the energy equation for the nozzle which says that h_{04} which is the h_{04} at the exit of the nozzle total enthalpy at the entry of the nozzle = $h_5 + C_j$ or C_5 square/2. So we have $T_{04} = T_5 + c_j$ square/2 c_p where this is c_p for gas where c_p for gas. So we can calculate C_j from here knowing the rest of the knowing the T_{04} and T_5 we can calculate C_j and it is 641.4 meter per second.

Here we have completely considered expansion of the gas to the exit pressure where we have considered $P_5 = P_a$ which is 1 bar. So there is no pressure component existing in the thrust. So we have thrust

$$T = (\dot{m}_a + \dot{m}_f) C_j - \dot{m}_a C_a$$

but mass of fuel is negligible then thrust = $\dot{m}_a(C_j - C_a)$, but further C_a is at sea level there is

zero velocity so it is $\dot{m} \times C_j$ and C_j . we have calculate $\dot{m} \times C_j$ we have calculate \dot{m} is give which is 25 kg per second.

So we can get thrust equal to 16036.6 Newton so this is the value of the designed trust where we have considered complete expansion in the nozzle till we attend the exit pressure equal to atmospheric pressure. We did not consider any choking condition. Now let us move on to the next part of the example which says us that let us consider the choking conditions. We had seen that first we have to find out what is the choking pressure and then based upon that we can work up for the choking condition.

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Accounting Choking condition

$$\frac{P_{04}}{P_c} = \left[1 - \frac{1}{\eta_j} \left(\frac{\gamma-1}{\gamma+1} \right) \right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{04}}{P_c} = \left[1 - \frac{1}{0.95} \left(\frac{1.33-1}{1.33+1} \right) \right]^{\frac{1.33}{1.33-1}}$$

$$\frac{P_{04}}{P_c} = 1.895$$

$P_{04} = 2.2 \text{ bar}$
 $P_a = 1 \text{ bar}$

$$\frac{P_{04}}{P_a} = 2.2 \text{ bar}$$

$$\frac{P_{04}}{P_c} < \frac{P_{04}}{P_a}$$

$$\frac{P_a < P_c}{\text{choked}}$$

$$P_c = \frac{P_{04}}{1.895} = \frac{2.2}{1.895} = 1.16 \text{ bar}$$

$$T_c \rightarrow \eta_j = \frac{T_{04} - T_c}{T_{04} - T_c'} \rightarrow T_c' = \left[1 - \frac{1}{\eta_j} \left(1 - \frac{T_c}{T_{04}} \right) \right] \cdot T_{04}$$

$$\frac{T_c'}{T_{04}} = 1 - \frac{1}{\eta_j} \left(1 - \frac{T_c}{T_{04}} \right) = \left(\frac{P_c}{P_{04}} \right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{1.895} \right)^{\frac{1.33-1}{1.33}} = 0.8533$$

$$\frac{1}{\eta_j} \left(1 - \frac{T_c}{T_{04}} \right) = 1 - 0.8533 \Rightarrow \frac{1}{0.95} \left(1 - \frac{T_c}{1066.4} \right) = 1 - 0.8533$$

$$T_c = 898.26 \text{ K}$$

$$C_j = a = \sqrt{\gamma R T_c} = \sqrt{1.33 \times 284.59 \times 898.26}$$

$$C_j = 583.09 \text{ m/s}$$

$$\dot{m} = \rho_j A_j C_j \rightarrow A_j = \frac{\dot{m}}{\rho_j C_j} = \frac{25}{583.09 \times \frac{P_c}{R T_c}}$$

$$A_j = 0.094 \text{ m}^2$$

$$T = \dot{m} a (C_j - a) + (P_j - P_a) \cdot A_j$$

$$T = 25 \times 583.09 + (116 - 1) \times 10^5 \times 0.094 = 16081.25 \text{ N}$$

$$SFC = \frac{\dot{m}_f}{T} = \frac{\dot{m}_f / \dot{m}_a}{T / \dot{m}_a} = \frac{1/50}{16081.25/25}$$

$$SFC = 0.111 \text{ kg/Nhr}$$

$C_p = \frac{\gamma R}{\gamma - 1}$
 $R = (\gamma - 1) \cdot C_p$
 $R = (1.33 - 1) \cdot 1147$
 $R = 0.284594$
 $R = 284.594 \text{ J/kgK}$

Let us continue with the second half of the example where we need to see account, accounting choking conditions as per these this is the second part so we have to see whether the nozzle is choked so P_{04}/P_c this is theoretical relation which we have developed, which we have derived depends upon nozzle efficiency and gamma.

$$\frac{P_{04}}{P_c} = \left(1 - \left(\frac{1}{\eta_j} \right) \left(\frac{\gamma - 1}{\gamma + 1} \right) \right)^{\left(\frac{\gamma}{\gamma - 1} \right)}$$

So we have $P_0/P_c = 1 - 1/\text{point nozzle efficiency}$ is given as 96.5% to 965 into gamma g is 1.33 – 1 1.33 + 1 – 1.33/1.33 – 1. So if we put this we will get $P_0/P_3 = 1.895$. So this is the theoretical relation, but we know what is our P_0 . We have derived we have got P_0 as 2.2 bar P_0 is 2.2 bar and P_0 or sorry P_a atmospheric pressure, atmospheric pressure in this example is said to be 1 bar.

Since it is at sea level so we have $P_0/P_a = 2.2$ bar. Hence, we have P_0/P_c is $< P_0/P_a$ So P_a is $< P_c$ this says that nozzle is choked. So since is choked we have to find out the choking condition or rather exit condition. So exit condition is exit pressure which is P_c and $P_c = P_0/1.895$ and so we have 2.2/1.895 and thus gives us P_c to be 1.16 bar. Now let us find out T_c which is actual temperature at the exit of the nozzle.

But this should be depended upon nozzle frequency. We know nozzle efficiency is

$$\eta_j = \frac{T_0 - T_c}{T_0 - T_c'}$$

Here we can represent this as $T_c \text{ dash} = 1 - 1/\text{nozzle efficiency}$ into $1 - T_c/T_0$ into T_0 . So $T_c \text{ dash}/T_0$ is $1 - \eta_j$ $1 - T_c/T_0$, but we know this is $= P_c/P_0$ bracket raise to gamma – 1/gamma. This is for gases so this is 1.1895 bracket raise to 1.33 – 1/1.33 and this comes to be 0.8533.

And hence using this that is known efficiency of the nozzle we can find out okay. So we can calculate T_c from here where we will say that $1/\eta_j$ into $1 - T_c/T_0 = 1 - 0.8533$ in this number would be known to us and from here we can further go ahead put the value of η_j and then we will say $1/0.965$ into $1 - T_c/T_0$ we had calculated T_0 it is 1046.4 so it is $1045.4 = 1 - 0.8533$ this gives us $P_c = 898.26$ Kelvin.

Having known this T_c we can calculate c_j which = acoustic speed which is = under root gamma RT_j or T_c . Since, we have mach number 1 we know here we are not given with r so first we have to find out R . We know $C_p = \gamma R / \gamma - 1$. So $R = \gamma - 1 \times C_p / \gamma$. We are told that gamma = 1.33 and C_p is 1.147. So this gives us $R = 284$ point this is 10 to the power 3 since it is 1.147 is in kilo joule per kg Kelvin.

So R is kilo joule per kg Kelvin is 0.28459 so $R = 284.59$ joule per kg Kelvin. Now this R we can use here and we can see gamma is 1.33 R is 284.59 into T_c is 898.26. So this gives us $C_j = 583.09$ meter per second. Knowing this is we can calculate thrust, but before that we have to calculate the thrust based upon pressure since this is the thrust this value will give us thrust only from momentum.

So let us write down the formula to calculate which is not given to us we know

$$\dot{m} = \rho_j \times A_j \times C_j$$

so $A_j = \dot{m} / \rho_j C_j$ \dot{m} is given to us as 25 and we know now C_j we have calculated which is 583.09 and ρ_j is basically P_c / RT_c so we know P_c since we have calculated P_c as 1.16 bar, T_c is 98, R is 284. So this gives us A_j and that comes to be 0.094 meter square.

Knowing this A_j knowing this A_j now we can calculate

$$\text{Thrust} = (\dot{m}_a + \dot{m}_f) C_j - \dot{m}_a C_a + (P_j - P_a) A_j$$

so \dot{m} we have C_a is 0 so 25 into C_j is 583.09 P_j is known to us which is P_c 1.16 atmospheric pressure into 10 to the power 5 since these are in bar into 0.94 this gives us value of thrust as 16081.25 Newton. This is useful to calculate specific fuel consumption which is $\dot{m}_f / \text{thrust}$.

So let us find out since we are given with air fuel ratio let us use that. Let us divide numerator and denominator/ \dot{m}_a . So this is given air fuel ratio is given as 50 so this is 1/50 and this is 16081.25/25. So this gives us the value as point we have multiple this by 3600 to get the number in the units of hours so we will get 111 kg per Newton hour. So this is how we can solve the example for the Turbojet where we were supposed initially to find out the thrust if gas is completely expanded or if there is chocking of the nozzle.

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Following is the data for a turbojet flying at 805 km/hr speed at an altitude having pressure of 0.458 bar and temperature 248K.

Combustion Chamber pressure loss	:	0.21 bar
Compression total head pressure ratio	:	4:1
Calorific value of the fuel	:	43000 kJ/kg
Ram efficiency of intake	:	95%
Turbine inlet temperature	:	1100K
Efficiencies for compressor, turbine and transmission	:	0.85, 0.9, 0.99 respectively
Nozzle Efficiency	:	0.95

assume nozzle to be convergent with exit area 0.0935 m² the mass flow of fuel is small compared to the mass flow of air
 assume $C_{p,air} = 1.005 \text{ kJ/kg K}$, $C_{p,gas} = 1.147 \text{ kJ/kg K}$, Specific heat ratio of air = 1.4, Specific heat ratio of gas = 1.33

So let us see for the next example and next example states that following is the data for turbojet flying at 805 kilometer per hour speed at an altitude having pressure of 0.458 bar and temperature 248 Kelvin, combustion chamber pressure loss is given as 0.21 bar compression total pressure head or compressor head is given as 4:1, calorific value of fuel is given as 43000 kilo joule per kg which is 43 mega joule per kg.

Ram efficiency of intake is given as 95%, turbine inlet temperature is given as 1100 Kelvin efficiencies for compressor turbine and transmission are respectively given as 0.85.9 and 0.99. Nozzle efficiency is given as 0.95 assume the nozzle to be convergent with exit area of 0.0935 meter square and then we are saying that Cp of air, Cp of gas and specific heat ratios of either are given.

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$\eta_r = 0.95 = \frac{P_{01} - P_a}{P_{0a} - P_a}$
 $C_a = 805 \text{ km/hr} = \frac{805 \times 1000}{3600} = 223.6 \text{ m/s}$
 $a_a = \sqrt{\gamma R T_a} = 315.66 \text{ m/s}$
 $M_{a1} = \frac{C_a}{a_a} = 0.708$
 $\frac{P_{0a}}{P_a} = \left[1 + \frac{\gamma-1}{2} M_{a1}^2\right]^{\frac{\gamma}{\gamma-1}}$
 $\frac{P_{0a}}{P_a} = \left[1 + 0.2 \times (0.708)^2\right]^{\frac{1.4}{0.4}}$
 $\frac{P_{0a}}{P_a} = 1.3977$
 $P_{0a} = 0.64 \text{ bar}$

$0.95 = \frac{P_{01} - 0.458}{0.64 - 0.458}$
 $P_{01} = 0.63 \text{ bar}$
 $r_{p,c} = 4 \rightarrow \frac{P_{02}}{P_{01}} = 4 \rightarrow P_{02} = 2.52 \text{ bar}$
 $T_{02} = T_{01} + \frac{T_{01}}{\eta_c} \left[(r_{p,c})^{\frac{\gamma-1}{\gamma}} - 1 \right]$
 $\frac{T_{01}}{T_a} = 1 + \frac{\gamma-1}{2} M_{a1}^2 \rightarrow T_{01} = 272.86 \text{ K}$
 $T_{02} = 429.05 \text{ K}$
 $\frac{W_c}{\eta_{tr}} = W_t$
 $c_{p,a} (T_{02} - T_{01}) = \eta_{tr} c_{p,g} (T_{03} - T_{04})$
 $T_{03} - T_{04} = \frac{C_{p,a}}{\eta_{tr} C_{p,g}} (T_{02} - T_{01})$
 $T_{03} - T_{04} = 138.23 \text{ K}$
 $T_{04} = 961.76 \text{ K}$

$\eta_t = \frac{T_{03} - T_{04}}{T_{03} - T_{04}'} = \frac{138.23}{1100 - T_{04}'} = 0.99$
 $T_{04}' = 966.4 = \text{K}$
 $P_{03} = P_{02} - \Delta P_{l,cc} = 2.31 \text{ bar}$
 $\frac{P_{03}}{P_{04}} = \left(\frac{T_{03}}{T_{04}'}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1100}{966.4}\right)^{\frac{1.33}{0.33}}$
 $\frac{P_{03}}{P_{04}} = 1.833$
 $P_{04} = 1.26 \text{ bar}$
 $\frac{P_{04}}{P_a} = \frac{1.26}{0.458} = 2.751 = \frac{\gamma}{\gamma-1}$
 $\frac{P_{04}}{P_c} = \left[1 - \frac{1}{\eta_j} \left(\frac{\gamma-1}{\gamma+1}\right)\right]^{\frac{\gamma}{\gamma-1}}$
 $\frac{P_{04}}{P_c} = 1.91$
 $\frac{P_{04}}{P_a} > \frac{P_{03}}{P_c} \rightarrow P_c > P_a \rightarrow \text{checked}$
 $P_{04} = 1.91$
 $T_{04} = 961.76 \text{ K}$

So let us solve this example in this example as we know we are having first intake then we are having compressor then we are having combustion chamber then we are having turbine then we are having nozzle. So this is atmospheric condition and then this will be P01 this will be 01, 02, 03, 04 and then this depends upon the choking or it practically in static this is 5. So we are told that ram efficiency is 95%.

So we will use this and we know how to find out ram efficiency using the formula $P01 - Pa/P0a - Pa$ actual maximum pressure rise divided by ideal maximum pressure rise. Here we need to know P0a for that we have to first find out mach number. So before that we are given that Ca = 805 kilometer per hour and this if we convert into meter per second then we will get it as this is for kilometer this is for second.

So this we will get it as 223.6 meter per second. Knowing this we know acoustic speed and ambient condition is γRT where this is atmospheric we have atmospheric air so gamma is 1.4 and R is 287 Ta is given and that Ta value is 248 putting these values we will get acoustic speed as 315.66 Kelvin or meter per second. So mach number is Ca/aa and we get mach number as 0.708.

Knowing this mach number we know

$$\frac{P0a}{Pa} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\left(\frac{\gamma}{\gamma - 1}\right)}$$

So here we know P0a/Pa is $1 + 1.4 - 1$ is 0.4, $0.4/2$ is 0.2 into 0.708 bracket square $1.4/0.4$. So P0a/Pa gives us the ratio as 1.3975 and this gives us P0a as 0.64 bar. We are given with Pa as 0.458 bar. So this gives us P0a. So we can put this P0a and find out for the P01 so we are 0.95 from the ram efficiency $P01 - 0.458/64 - 0.458$ and this gives us P01 as 0.63 bar.

So we get P01 as 0.63 bar having said this now we can move ahead and calculate the other corner quantities for this cycle. So we are told that compression pressure ratio for the compressor is 4. So we know $P02/P01$ is = 4 so $P02 = 4$ into 0.36 and that is 2.52 bar. Now we can calculate T02 since $T02 = T01 + T01$ into compressor efficiency bracket raise to pc bracket raise to $\gamma - 1/\gamma - 1$.

We know how to derive this formula from the fact that compressor efficiency

$$\eta_c = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}}$$

Then if we take T_{01} common then T_{02}'/T_{01} becomes P_{02}'/P_{01} which is γ raised to $1/\gamma$. Having known this we can find out T_{02}' , but for that again we should know T_{01} and T_{01} can be found out being the process as isentropic being the process as adiabatic in the intake we can know it as $1 + \gamma - 1/2 Ma^2$.

And so this gives us T_{01} as 272.86 Kelvin. So we have T_{02} putting this all numbers here we can get $T_{02} = 429.05$ Kelvin. Now we have calculated this and we have calculated this. So let us move and calculate for the conditions for 3 and 4, but we are given with the turbine inlet temperature as 1100 Kelvin. So we are knowing T_{03} , but we do not know the condition at T_{04} , but we know that there is a transmission efficiency related with compressor and turbine.

So we have compressor work/transmission efficiency equal to turbine work. So we have C_p of air into $T_{02} - T_{01} =$ transmission efficiency into C_p of gas into $T_{03} - T_{04}$. So we have $T_{03} - T_{04} = C_p$ of air/transmission efficiency into C_p of gas into $T_{02} - T_{01}$. So this gives us we know C_p of air which is 1.001 we know C_p of gas 1.147, transmission efficiency is given as 99%.

And then T_{02} is found out 429 T_{01} is known which is 272 so this gives us $T_{03} - T_{04}$ as 138.23 Kelvin. So we get T_{04} which is getting subtracted from 1100 and then we have 961.76 Kelvin. Now what we are not knowing at this moment is the turbine pressure ratio. So we can know turbine pressure ratio from turbine efficiency formula where

$$\eta_t = \frac{T_{03} - T_{04}}{T_{03} - T_{04}'}$$

Here we know now $T_{03} - T_{04}$ is evaluated as 138.23 and divided T_{03} is 1100 - T_{04} dash, but this efficiency is given to us as 0.9. So knowing this we can calculate the ideal temperature at the exit of the turbine as T_{04}' and that is = 946.40 Kelvin. So knowing this we can proceed, but then first let us find out what is P_{03} , but $P_{03} = P_{02} - \Delta P_0$ combustion chamber which is given as 0.21 bar.

And P_{02} is 2.52 bar since we have evaluated P_{02} we have got it as P_{02} we have got it as 2.52 bar - 0.21 bar so it comes to be 2.31 bar. So we know now

$$\frac{P03}{P04} = \left(\frac{T03}{T04'} \right)^{\frac{\gamma_g}{\gamma_g - 1}}$$

So this would be 1100/946.40 bracket raise to 1.33/0.33 and we get here P03/P04 = 1.833 so this gives us P04 and P04 comes out to be 1.26 bar. Since we know P03 is 2.31/1.833 this gives us P04 as 1.26 bar.

So we can find out which is the pressure known total pressure at the entry to the nozzle is known atmospheric condition is known. So driving pressure ratio for the existing nozzle is P04/Pa which is 1.26/0.458 this ratio comes to be 2.751. So knowing this pressure ratio we can compare with the choking condition and choking condition pressure ratio says that

$$\frac{P04}{Pc} = \left(1 - \left(\frac{1}{\eta_j} \right) \left(\frac{\gamma_g - 1}{\gamma_g + 1} \right) \right)^{\left(\frac{\gamma_g}{\gamma_g - 1} \right)}$$

Now nozzle efficiency is given to us as 95% gamma gas and all the other things are known which is 1.33 putting those all numbers we will get P04/Pc as 1.91. So here P04/Pa which is 2.751 is greater than P04/Pc this is that Pc is greater than Pa so we have nozzle choked. So since nozzle is choked we have to find out the temperature at the exit of the nozzle for the choking condition, but we know the process is adiabatic from 4 to 5 and then there is mach number 1 mach number 1 in the exit, mach number 1 in the exit of the nozzle. Nozzle is adiabatic.

(Refer Slide Time: 51:54)

Handwritten calculations on a slide:

- $T_{04} = T_{05} = 961.76 \text{ K}$
- $\frac{T_{04}}{T_5} = 1 + \frac{\gamma_g - 1}{2} = \frac{\gamma_g + 1}{2} = 1.67$
- $T_5 = 825.54 \text{ K} = T_c$
- $C_j = \sqrt{\gamma_g R_g T_5}$
- $C_j = \sqrt{1.33 \times 287 \times 825.54}$
- $C_j = 558.99 \text{ m/s}$
- $\dot{m} = \rho_j A_j C_j = \frac{P_j}{R T_j} A_j C_j$
- $\dot{m} = \left(\frac{P_{03}}{1.9147} \right) \times \frac{1}{287 \times 825} \times A_j \times 558.99$
- $\dot{m} = 14.69 \text{ kg/s}$
- $S_j = \beta_c = \frac{P_c}{R T_c}$
- $T_{thrust} = T_e \dot{m} (C_j - a_e) + (P_e - P_a) A_j$
- $T = 14.69 (558.99 - 223.6) + (0.657 - 0.458) A_j$
- $T = 6790.75 \text{ N}$
- S.F.C. \Rightarrow
- $\dot{m} \cdot q_{cv} = \dot{m} a C_j (T_{02} - T_{04})$
- $\dot{m}_f = \dot{m} a C_j (T_{02} - T_{04}) / q_{cv}$
- $\dot{m}_f = 0.261 \text{ kg/s}$
- $S.F.C. = \frac{\dot{m}_f}{T} = 0.138 \text{ kg/Nhr}$

So knowing this we know T04 is = T05 which is from the adiabaticness and this is 961.76 Kelvin so we can find out

$$\frac{T_0}{T_5} = 1 + \frac{\gamma_g - 1}{2} M^2$$

but M is 1. So this is just gamma gas + 1/2. So this is 1.65 knowing this we can calculate till 5 and till 5 comes out to be 825.54 Kelvin. Knowing T5 we calculate cj which is again speed at the exit of the jet, nozzle jet speed at the exit of the nozzle which is gamma gas R gas into T5 or Tc.

This is same as Tc then this is equal to Cj gamma gas is 1.33 we have found out R and then R was 284 in earlier example. In the same way we can find out R here as well and then Tc is 825.54. Knowing this we get Cj = 558.99 meter per second. Now what we need to evaluate here is mass flow rate since we are given with area of the nozzle. So for thrust calculation we know need to know the mass flow rate.

So it is rho j Aj Cj, but we do not know rho j which is equal to, here rho j = pc = Pc/RTc so it has Pj/RTj x Aj x cj. So m dot we can use here Pc further needs to be calculated. So Pc which is Pj that is P04/choking condition criteria which had come to be 1.9167 into 1/284 into Tj which is 825 into area of the jet which is given and now cj which is 558.99 and all these numbers give us mass flow rate as 14.64 kg per second.

We have to remember here that P04 should be taken in pascal. So having known this we can find out thrust which is T which is (m_dot_a + m_dot_f) Cj - m_dot_a Ca + (Pj - Pa) Aj. So thrust we know m dot now 14.64 cj is known 558.99 ca is given 223.6 + Pj is Pc which is 0.657 which has been obtained as this which is P/RT which is this P/RT - atmospheric pressure is 0.458 into Aj which is given as 0.09.

So this gives us thrust value as 6770.75 Newton. We also need to calculate specific fuel consumption and then that specific fuel consumption can be found out before that we have to found out mass flow rate of fuel. So for that we can apply energy equation for the burner will have

$$\dot{m}_f \times Q_{cv} = \dot{m}_a C_p \text{ gas} \times (T_03 - T_02).$$

So

$$\dot{m}_f = \dot{m}_a \times C_p \text{ gas} (T_03 - T_02) / Q_{cv}.$$

We have calculate T03, where T03 is given as 1100 we know T02 cp of gas is given as 1.147

mass flow rate of air is calculated as 14.64 and Q_{cv} which is the calorific value is given as 43 mega joule. So \dot{m}_f turns out to be 0.261 kg per second so we can know now what is specific fuel consumption that is \dot{m}_f/F or upon T and this gives us specific fuel consumption in terms of unit of hour as 0.138 kg per Newton hour where we have multiplied \dot{m}_f/T into 3600.

Having said this, this is the method to calculate the examples or to solve the examples for the turbojet engine. Thank you.