Aircraft Propulsion
Vinayak N. Kulkarni
Department of Mechanical Engineering
Indian Institute of Technology- Guwahati

Lecture 02
Solved Examples for Flow process

Welcome to the class. We have seen that the overview of thermodynamics through different laws and then we have seen what are the different terminologies in the Thermodynamics specifically, we will deal in this class for the examples which we will deal with the review of thermodynamics practically in those examples we will focus on the control volume analysis or flow processes.

This is very much required since the components which are going to be studied for the gas turbine power plant each component would be a flow process based component hence. The analysis of that component can be well understood with the base what we would be constructing through these solved examples. So let us workout with examples.

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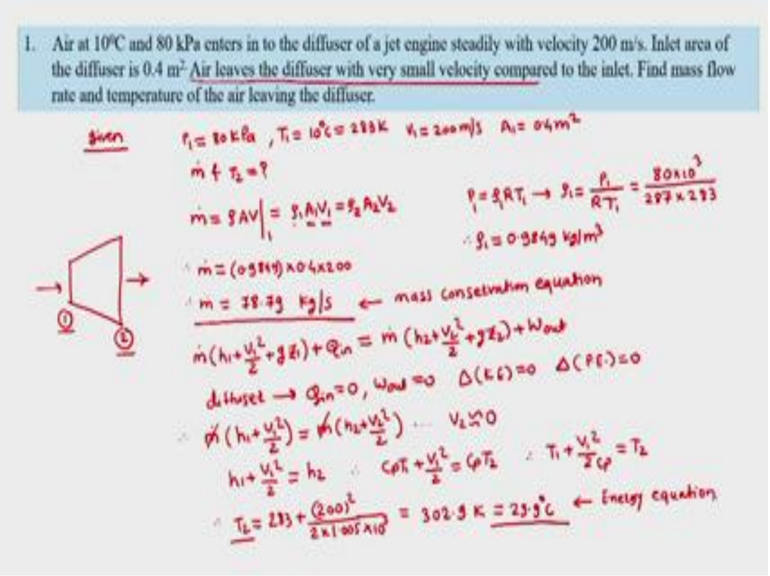

So, the first example says that air at 10 degree Celsius and 80 kilo Pascal enters into the diffuser of a jet engine steadily leave with velocity 200 metre per second inlet area of the diffusion is .4 metre square. Air leaves the diffuser with very small velocity compared to the inlet find out mass flow rate and temperature of the air leaving the diffuser. So, let us see what are the given things here?

So first given thing over here, so we are working with diffuser, so let us plot the diffuser, diffuser in general is a divergent area section for us. We are dealing with very low speeds, so for very low speeds the diffuser is a diverging section. So since this is the inlet of the diffuser station 1 is the outlet of the diffuser. Station 2 we are told that P1 is equal to 80 kilo Pascal and T1 is equal to 10 degree Celsius which is equal to 283 Kelvin. We are also told that V1 is equal to 200 metre per second and inlet area A1 is equal to .4 metre square.

Given:

$$P_1 = 80\ kPa\ ; T_1 = 10^0C = 283\ K\ ; V_1 = 200\frac{m}{s}; A_1 = 0.4\ m^2$$

We are told to find out the mass flow rate which is m dot and we are told to find out T2. So, these are the two things which we need to find out. So, we can know that mass flow rate is

$$\dot{m} = \rho AV|_1 = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

So, let us first find out Rho 1 from perfect gas equation we know

$$P_1 = \rho_1 RT_1 \rightarrow \rho_1 = \frac{P_1}{RT_1} = \frac{80 * 10^3}{287 * 283} = 0.9849\frac{kg}{m^3}$$

$$\dot{m} = 0.9849 * 0.4 * 200 = 78.79\frac{kg}{s}$$

So, this is what one of the expected answers and for this answer to arrive at we used mass conservation equation. We got this answer using mass conservation equation. For steady flow analysis always or for flow process we cannot just use energy equation. We also have to consider most of the times the mass conservation equation also.

However mass conservation has to be considered before considering the energy equation so, now let us apply energy equation between the station 1 and 2, so we know that steady flow energy equation is

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gZ_1\right) + Q_{in} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gZ_2\right) + W_{out}$$

So for the diffuser we have

$$Q_{in} = 0\ ; W_{out} = 0; \Delta(K.E) = 0; \Delta(P.E) = 0$$

So, we get

$$\left(h_1 + \frac{V_1^2}{2}\right) = \left(h_2 + \frac{V_2^2}{2}\right)$$

Further it is also told that they air leaves the diffuser with very small velocity as compared to inlet.

$$V_2 \approx 0$$

$$\left(h_1 + \frac{V_1^2}{2}\right) = h_2$$

$$C_pT_1 + \frac{V_1^2}{2} = C_pT_2$$

$$T_1 + \frac{V_1^2}{2C_p} = T_2$$

$$T_2 = 283 + \frac{(200)^2}{2 * 1.005 * 10^3} = 302.9\ K = 25.9^0C$$

This is other output of the example and this has been obtained using the energy equation.

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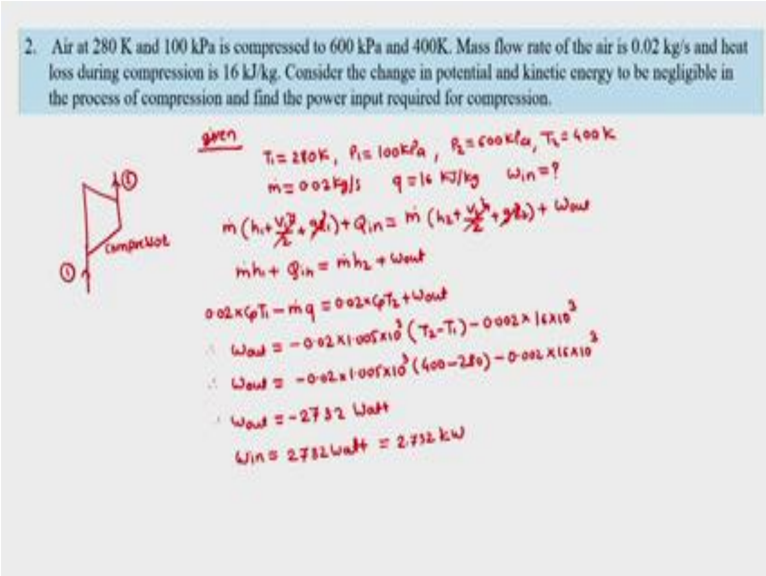

Now let us move to the next example, it states that air at 280 Kelvin and 100 kilo Pascal is compressed to 600 kilo Pascal and 400 Kelvin, mass flow rate of air is .02 kg per second and heat loss between the processes of compression is 16 kilo joule per kg. Consider the change in potential and kinetic energy to be negligible in the process of compression and find the power input required for a compression.

So here we are talking about a compressor so we will draw a compressor. Here it is station 1 here it is station 2 and this is a compressor. The given things in this example are:

$$T_1 = 280\ K\ ; P_1 = 100\ kPa; P_2 = 600\ kPa; T_2 = 400\ K; \dot{m} = 0.02\frac{kg}{s}; q = 16\frac{kJ}{kg}\ ; W_{in} =?$$

So we are supposed to find out what is the W in, In the process of compression so let us apply the energy equation, we will write down the same energy equation:

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gZ_1\right) + Q_{in} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gZ_2\right) + W_{out}$$

This is general steady flow energy equation. Now in this case $W_{out} = 0$, We are again not going to consider any change in Kinetic and potential energies, so we will have

$$\dot{m}h_1 + Q_{in} = \dot{m}h_2 + W_{out}$$

Sorry, we are supposed to find out W_{in} so we are working with compressor. So we need W only change in potential and kinetic energy is neglected so we know

$$0.02\ C_pT_1 - \dot{m}q = 0.02\ C_pT_2 + W_{out}$$
$$W_{out} = -0.02 * 1.005 * 10^3 * (400 - 280) - 0.002 * 16 * 10^3 = -2.732\ kW$$

Negative sign indicates that work output is negative. That means work in is positive and it is 2732 watt or it is equal to 2.732 kilo watt.

$$W_{in} = 2.732\ kW$$

So here in this example we just needed to find out the work output and then that needed only the energy equation.

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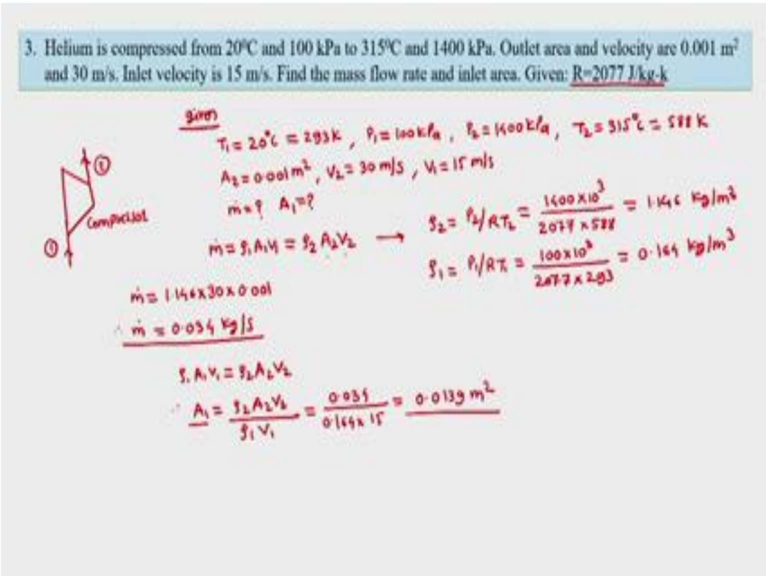

So, we will move next example and next example says that helium is compressed from 20 degree Celsius and 100 kPa to 315 degree Celsius and 1400 kilo Pascal outlet area and velocity are give which are .001 metre square and 30 metre per second respectively inlet velocity is 15 meter per second, find out mass flow rate and inlet area R for helium is given as 2077 joule per kg Kelvin.

Again this example deals with compressor. So we have one at the compressor inlet 2 at the compressor outlet and we will note down the given things for us. So given things include

$$T_1 = 20^0C = 293\ K\ ; P_1 = 100\ kPa, T_2 = 315^0C = 588\ K; A_2 = 0.001\ m^2; V_2 = 30\frac{m}{s}\ ;$$

$$V_1 = 15\frac{m}{s}$$

We are supposed to find out

$$\dot{m} = ?\ ; A_1 = ?$$

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{14000 * 10^3}{2077 * 588} = 1.146\frac{kg}{m^3}$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{100 * 10^3}{2077 * 293} = 0.164\frac{kg}{m^3}$$

$$\dot{m} = 1.146 * 30 * 0.001 = 0.034\frac{kg}{s}$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$A_1 = \frac{\rho_2 A_2 V_2}{\rho_1 V_1} = \frac{0.034}{0.164 * 15} = 0.0139\ m^2$$

This is also evaluated by us using only mass conservation equation. So we have to see which equations are important for solving the example in case of steady flow and then we might need both area and mass conservation. We might need energy and mass conservation or we might need either.

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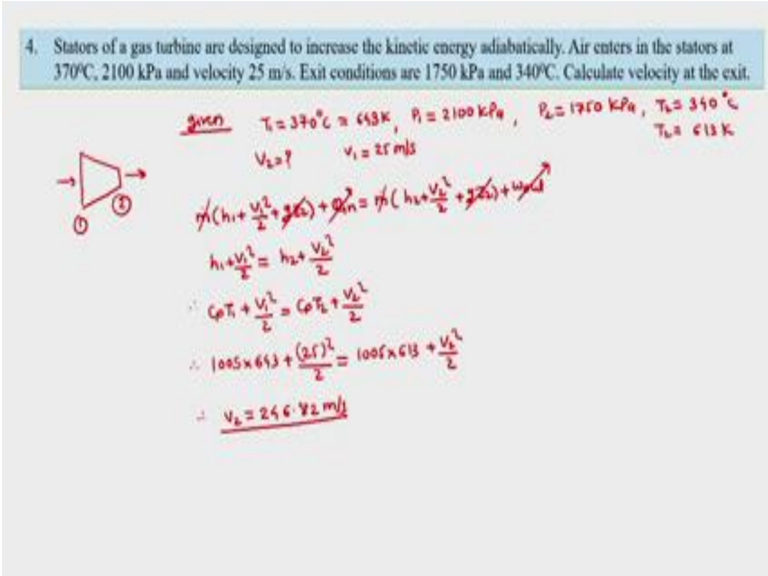

So next example state that stator of gas turbine are designed to increase the kinetic energy adiabatically in the example, it is stated that it is adiabatic. So, adiabatic means there is no heat interaction. So you it is given that q is zero. Air enters the stator at 370 degree Celsius 2100 kPa and velocity of 25 metre per second. Exit conditions are 1750 kPa and 340 degree Celsius calculate velocity at the exit.

So as per the example, we are working with a nozzle since it is a stator of the turbine this is 1 this is 2 and then we are given

$$T_1 = 370^0C = 643\ K; P_1 = 2100\ kPa; P_2 = 1750\ kPa; T_2 = 340^0C = 613\ K$$

$$V_2 = ?;\ \ V_1 = 25\frac{m}{s}$$

Here we are not needing anything related to mass conservation equation so let us apply the energy conservation equation between the station 1 and the station 2 which is the steady flow energy equation. Let us consider steady flow energy equation between station 1 and 2 this is the stator of the turbine so it has no work interaction. We are said that process is adiabatic in this case not that under root cancels and we are obviously having no potential energy changes.

So, we have

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gZ_1\right) + Q_{in} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gZ_2\right) + W_{out}$$

$$\left(h_1 + \frac{V_1^2}{2}\right) = \left(h_2 + \frac{V_2^2}{2}\right)$$

$$C_p T_1 + \frac{V_1^2}{2} = C_p T_2 + \frac{V_2^2}{2}$$

$$1005 * 643 + \frac{25^2}{2} = 1005 * 613 * \frac{V_2^2}{2}$$

$$V_2 = 246.82 \frac{m}{s}$$

So we just used only energy equation to solve this example.

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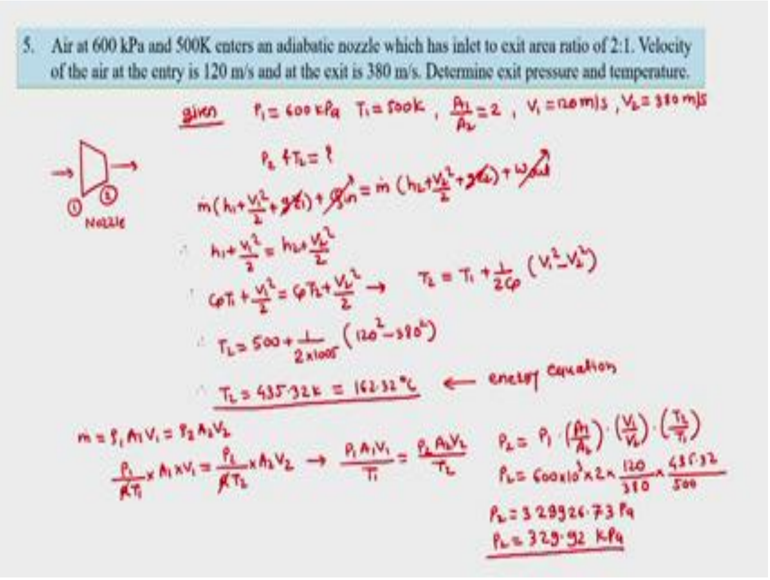

Now move on to the next example to state that air at 600 kPa and 500 Kelvin enters an adiabatic nozzle which has inlet to exit area of 2:1 and velocity of air at the entry is 120 metre per second and at the exit is 380 metre per second. Determine exit pressure and temperature since we are told that it is a nozzle. So for us low speed nozzle is a converging duct, so let us consider two stations 1 and 2 for the nozzle it is adiabatic given things in this example our gas is air entry is 600 kPa temperature T1 is 500 Kelvin we have told that A 1 by A 2 is equal to 2. And V 1 which is velocity at the inlet is 120 metre per second and velocity at the exit is 380 meter per second. We are supposed to find out P 2 and T 2 are the question.

$$P_1 = 600\ kPa; T_1 = 500\ K; \frac{A_1}{A_2} = 2; V_1 = 120 \frac{m}{s}; V_2 = 380 \frac{m}{s}; P_2 = ?, T_2 = ?$$

So, let us apply the first energy equation over here

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gZ_1\right) + Q_{in} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gZ_2\right) + W_{out}$$

This is the nozzle, it is an adiabatic nozzle. So No heat interaction. It is a nozzle. It has no work interaction. We further do not consider any potential energy change. So, we have $\dot{m}$ getting cancelled.

$$\left(h_1 + \frac{V_1^2}{2}\right) = \left(h_2 + \frac{V_2^2}{2}\right)$$

$$C_p T_1 + \frac{V_1^2}{2} = C_p T_2 + \frac{V_2^2}{2}$$

$$T_2 = T_1 + \frac{1}{2C_p}(V_1^2 - V_2^2)$$

$$T_2 = 500 + \frac{1}{2 * 1005}(120^2 - 380^2)$$

$$T_2 = 435.32\ K = 162.32^0 C$$

So now I need to find out pressure for that will apply mass conservation equation

$$\dot{m} = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\frac{P_1}{RT_1} * A_1 * V_1 = \frac{P_2}{RT_2} * A_2 * V_2$$

$$\frac{P_1 A_1 V_1}{T_1} = \frac{P_2 A_2 V_2}{T_2}$$

$$P_2 = P_1\left(\frac{A_1}{A_2}\right)\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right)$$

$$P_2 = 600 * 10^3 * 2 * \frac{120}{380} * \frac{435.32}{500}$$

$$P_2 = 329.92\ kPa$$

So this is how we are coming to know that we need. Mass and energy both conservation equations to be applied for solving a particular equation for particular example of steady flow process.

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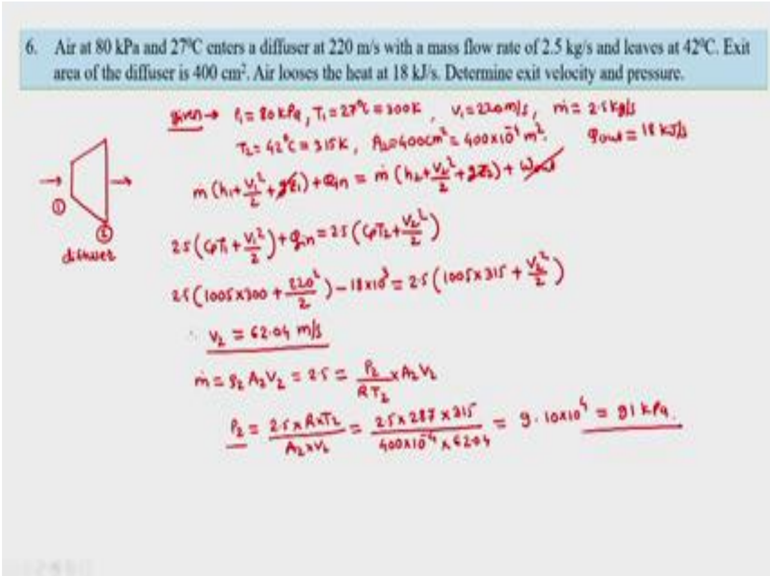

Let us move on to the next example it states that air at 80 kPa and 27 degree Celsius enters a diffuser at 220 metre per second. With mass flow rate of 2.5 kg per second and leaves at 42 degree Celsius exit area of the diffuser is 400 cm square, air loses heat at 18 kilo, joule per second determine the exit velocity and pressure. So, for this example, again we are working with diffuser for low velocity, diffuser is a divergent section and we have 1 at for inlet and 2 for the outlet in case of the diffuser

$$P_1 = 80\ kPa; T_1 = 27^0C = 300\ K; V_1 = 220\frac{m}{s}; \dot{m} = 2.5\frac{kg}{s}; T_2 = 42^0C = 315\ K$$

$$A_2 = 400\ cm^2 = 400 * 10^{-4}\ m^2; Q_{out} = 18\frac{kJ}{s}$$

So, let us apply again energy equation steady flow energy equation

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gZ_1\right) + Q_{in} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gZ_2\right) + W_{out}$$

$$2.5\left(C_pT_1 + \frac{V_1^2}{2}\right) + Q_{in} = 2.5\left(C_pT_2 + \frac{V_2^2}{2}\right)$$

$$2.5\left(1005 * 300 + \frac{220^2}{2}\right) - 18 * 10^3 = 2.5\left(1005 * 315 * \frac{V_2^2}{2}\right)$$

$$V_2 = 62.04\ m/s$$

So, we got this answer from energy conservation equation. We want pressure to be found out at the exit. We know that

$$\dot{m} = \rho_2 A_2 V_2 = 2.5 = \frac{P_2}{RT_2} A_2 V_2$$

$$P_2 = \frac{2.5 * R * T_2}{A_2 V_2} = \frac{2.5 * 287 * 315}{400 * 10^{-4} * 62.04} = 9.10 * 10^4 = 91\ kPa$$

So we got second answer of this question based upon the mass conservation equation. So we have to remember here that how to apply the mass and Energy Conservation equations for the different components like compressor diffuser or nozzle stators, rotors.

And these are the physical components for gas turbine power plant. We need this understanding while deriving different performance parameters for that gas turbine components or gas turbine power plant in general. I hope this will help us for getting those desired equations or the values. Thank you.