

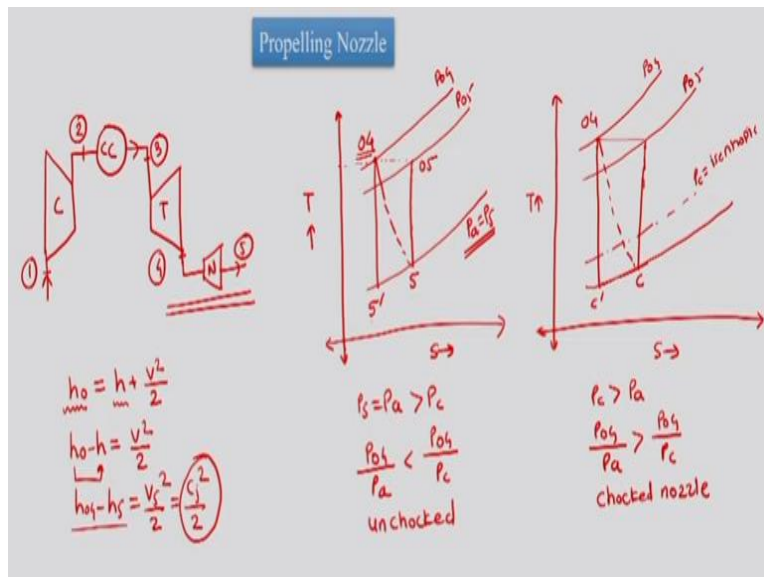
Aircraft Propulsion
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Module No # 04
Lecture No # 19
Propelling Nozzle, Nozzle Efficiency

Welcome we will start discussing about the remaining part from our lecture of our intake and nozzle we had covered for the intakes. Now we will discuss about the nozzle so the nozzle which we are going to talk about it is called as propelling nozzle. Actually there is nozzle in a gas turbine power plant and that nozzle is exclusively used for generating the jet then the nozzle is called propelling nozzle if the nozzle is used in the turbine as the component then that is also a nozzle.

So practically there are two nozzles in the gas turbine power plant about which rather from which we are going to talk about the propelling which is exclusively used for generating the thrust.

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So we know that a typical gas turbine power plant as compressor it as state 1 at the inlet it has state 2 at the inlet then we have combustion chamber after the combustion chamber we have gas turbine which as process 3 to 4 and from the 4 we have nozzle from 4 to 5. So we are talking about process 4 to 5 in the nozzle and objective of the nozzle in general is to act as a duct which would decrease the pressure and increase the kinetic energy.

So if we plot the T-S diagram for process 4 to 5 then this TS diagram would depend upon a factor which is related with choking and what does it mean by choking we had already seen in one of our classes where we came to know that mass flow rate in the nozzle does not present increase if at all we decrease the nozzle decrease pressure and in that choking case the minimum cross section or the throat as attain the Mach 1 sonic condition.

So in general in this cruise subsonic aircrafts the nozzle are convergent where maximum Mach number is expected to be sonic at the exit. Which is going to be attained after having choking state so this is suppose the entry to the nozzle which is 04 total conditions at the exit of the turbine and then from the 4 we generally come to static state which is 5. We are using total conditions at the entry and we are using static condition at exit this as a relevance we state that

$$h_0 = h + V^2/2$$

where h_0 is total enthalpy at the entry to the inlet of the nozzle and h is the static enthalpy at the outlet of the nozzle.

So $h_0 - h$ is the kinetic energy which is available at the outlet under the assumption that entry kinetic energies negligible. So we are moving from h_0 to h so practically h_0 to $h_5 = V^2/2$ and in our language it is $c_j^2/2$. So we are interested in finding out the jet velocity so we are putting the process from stagnation condition to static condition only which is unlike in the case of compressor or turbine where we have all the graphs between total to total conditions or stagnation to stagnation condition.

So this process is this way but in reality we will have a process which is 4 to 5 so this is 5 dash this is 5 but the nozzle is not going to operate isentropic duct there is loss in due to friction due to which we are having this dotted line which is real expansion in the nozzle. However the nozzle is adiabatic duct due to which the total temperature at the inlet and at the end outlet are same so this is 05 condition which is corresponding to the total condition at point 5.

Here this TS diagram such as that we are working with nozzle where $P_4 = P_5$ and nozzle as completely expanded the gas with that is subsonic or supersonic expansion but there is complete expansion and exit till fluid as attain the pressure which is equal to the pressure at the ambience.

However in the choking condition this graph would get altered and then we need to bring in the choking pressure also into the figure.

So we are here it is not 4 state so this is P04 this is P05 so gas expands from 04 till 5 dash isentropically. However it has crossed Pc which is isentropic choking straight okay so since it has crossed the choking straight so the pressure is further lower and then it is for us it is C dash now atmospheric condition are basically there and then nozzle exit as not attained the atmospheric pressure. Nozzle should again expand but it is insufficient to expand the gas hence the nozzle exit pressure is more than the atmospheric pressure.

So it as cross the choking condition but it has not attained the pressure which is required at the exit. However still there is the process which is in real which is after the choking in reality it would have seduce state it would have state C. So this is T this is S so this figure as relevance when P5 = Pa which is greater than Pc however isentropic expansion if it would have continued and then the nozzle exit would have attained C pressure with C pressure it would have choked but atmospheric pressure is more than the choking pressure.

So P04 upon Pa is less than P04 upon Pc so this nozzle is un-choked however in this case Pc is less than Pa Pc is greater than Pa and we have P04 upon Pa is greater than P04 upon Pc so this is choked nozzle. Having said this we will work out with the efficiency of this nozzle.

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Nozzle Efficiency

$\eta_j = \text{isentropic efficiency of propelling nozzle} = \frac{T_{04} - T_5}{T_{04} - T_5'}$

$T_{04} - T_5 = \eta_j (T_{04} - T_5')$

$T_{04} - T_5 = \eta_j T_{04} \left(1 - \frac{T_5'}{T_{04}}\right)$

$\frac{T_5'}{T_{04}} = \left(\frac{P_5'}{P_{04}}\right)^{\frac{\gamma-1}{\gamma}}$

$T_{04} - T_5 = \eta_j T_{04} \left(1 - \left[\frac{P_5'}{P_{04}}\right]^{\frac{\gamma-1}{\gamma}}\right) = \frac{1}{2} S^2$

choking

$\eta_j = \frac{T_{04} - T_c}{T_{04} - T_c'}$

$T_{04} - T_c' = \frac{1}{\eta_j} (T_{04} - T_c)$

$T_c' = T_{04} - \frac{1}{\eta_j} (T_{04} - T_c) \quad \text{--- (1)}$

$\frac{P_{c'}}{P_{04}} = \left(\frac{T_c'}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}}$

$\frac{P_{c'}}{P_{04}} = \frac{P_c}{P_{04}} = \left[\left(T_{04} - \frac{1}{\eta_j} (T_{04} - T_c) \right) \frac{1}{T_{04}} \right]^{\frac{\gamma}{\gamma-1}}$

$P_c = P_{c'} = P_{04} \left[1 - \frac{1}{\eta_j} \left(1 - \frac{T_c}{T_{04}}\right) \right]^{\frac{\gamma}{\gamma-1}}$

$P_c = P_{c'} = P_{04} \left[1 - \frac{1}{\eta_j} \left(1 - \frac{T_c}{T_{04}}\right) \right]^{\frac{\gamma}{\gamma-1}}$

$\frac{T_{04}}{T_c} = 1 + \frac{\gamma-1}{2} M_c^2 = 1 + \frac{\gamma-1}{2} = \frac{\gamma+1}{2}$

$P_c = P_{c'} = P_{04} \left\{ 1 - \frac{1}{\eta_j} \left(1 - \frac{2}{\gamma+1}\right) \right\} = P_{04} \left\{ 1 - \frac{1}{\eta_j} \left(\frac{\gamma-1}{\gamma+1}\right) \right\}^{\frac{\gamma}{\gamma-1}}$

$\frac{P_{04}}{P_c} = \left\{ 1 - \frac{1}{\eta_j} \left(\frac{\gamma-1}{\gamma+1}\right) \right\}^{-\frac{\gamma}{\gamma-1}}$

$\frac{P_{04}}{P_c} = \left\{ 1 - \frac{\gamma-1}{\eta_j(\gamma+1)} \right\}^{-\frac{\gamma}{\gamma-1}} = \left\{ \frac{2}{\gamma+1} \right\}^{-\frac{\gamma}{\gamma-1}}$

So let us define η_j as nozzle efficiency which is also called as isentropic efficiency of propelling nozzle or just nozzle this is going to be defined from our reference figure here if we take this figure we are expanding from state 04 to 5 instead of 04 to 5'. So this efficiency as formula

$$\eta_j = \frac{T_{04} - T_5}{T_{04} - T_{5'}}$$

So this relevance with actual kinetic energy gained in the nozzle expansion and this as relevance with ideal kinetic energy gained in the nozzle.

So having said this we can write down

$$T_{04} - T_5 = \eta_j (T_{04} - T_{5'})$$

we can take T_{04} common and then $1 - T_{5'}$ upon T_{04} . Then this is as isentropic relation which is having relevance with

$$\frac{T_{5'}}{T_{04}} = \left(\frac{P_5}{P_{04}} \right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{04} - T_5 = \eta_j T_{04} \left(1 - \left(\frac{p_5}{P_{04}} \right)^{\frac{\gamma-1}{\gamma}} \right)$$

Thus as it is said it is equal to half C_j^2 so using this nozzle efficiency turbine exit total temperature and the conditions where we have $P_5 = P_a$ which is ambience condition in total pressure at exit of the turbine we can calculate the jet exit velocity which is required further for calculation of the thrust. Where we know thrust is equal to mass flow rate into $C_j - C_a$ in the case of complete expansion of the gas.

However in case of chocking we would have little difference in case of chocking we have

$$\eta_j = T_{04} - T_c / T_{04} - T_{c'}$$

So let us write down $T_{04} - T_{c'} = 1$ upon $\eta_j T_{04} - T_c$. So we can write down the T_c dash = $T_{04} - \eta_j$ into $T_{04} - T_c$ here since nozzle as got chocked we are unable to know what is this temperature where this is a P_c pressure. $P_c = P_{c'}$ we would not know what is the exit pressure and we also would not know what is the temperature which otherwise would have attain by the gas.

So we are writing down as expression for $T_{c'}$ which is equal to

$(T_{04} - 1) / \eta_j (T_{04} - T_c)$. Here we can use suppose we will re-plot the Ts diagram for nozzle where this is 04 this is C dash c and 05. Here we are interested to find out the choking condition so we should know what is the choking pressure in which nozzle would get choked. So we are interested in finding out will number this equation as number 1 Pc dash upon P04 for given P04 in which Pc or Pc nozzle would get choked.

So this is P_c/P_{04} and this can be found from Pc' upon T04 bracket raise $\frac{\gamma-1}{\gamma}$. Here we can put Tc' from expression 1 so we can write down

$$\left(\frac{P_{c'}}{P_{04}}\right) = \left(\frac{P_c}{P_{04}}\right) = (T_{04} - \left(\frac{1}{\eta_j}\right) (T_{04} - T_c) \left(\frac{1}{T_{04}}\right))^{\frac{\gamma}{\gamma-1}}$$

So we can get Pc is equal to Pc dash for given P04 we can further take T04 common which would get cancelled with exist in T04 from the denominator and lead to

$$P_c = P_{c'} = P_{04} \left(\left(1 - \left(\frac{1}{\eta_j}\right) (1 - T_c/T_{04})\right) \right)^{\frac{\gamma}{\gamma-1}}$$

Here we should also note that since the nozzle process or process in the nozzle is the adiabatic we have $T_{04} = T_{05}$ so

$$P_c = P_{c'} = P_{04} \left(\left(1 - \left(\frac{1}{\eta_j}\right) (1 - T_c/T_{05})\right) \right)^{\frac{\gamma}{\gamma-1}}$$

Here if we see closely the process c n05 are 05 to c if we isentropically compress or if we isentropically expand from 05 to c then we have got the Mach number 1 existing at state 1. So we can utilize this from knowing Mach number in reality equal to 1 for choking we have

$$T_{05}/T_c = 1 + \frac{\gamma-1}{2} M_c^2$$

Nozzle Efficiency

$\eta_j = \text{isentropic efficiency of propelling nozzle} = \frac{T_{04} - T_f}{T_{04} - T_{s'}} \checkmark$

$T_{04} - T_s = \eta_j (T_{04} - T_{s'})$

$\therefore T_{04} - T_f = \eta_j T_{04} \left(1 - \frac{T_{s'}}{T_{04}}\right)$

$\frac{T_{s'}}{T_{04}} = \left(\frac{P_{s'}}{P_{04}}\right)^{\frac{\gamma-1}{\gamma}}$

$\therefore T_{04} - T_f = \eta_j T_{04} \left(1 - \left(\frac{P_{s'}}{P_{04}}\right)^{\frac{\gamma-1}{\gamma}}\right) = \frac{1}{2} S^2$

choking

$\eta_j = \frac{T_{04} - T_c}{T_{04} - T_{c'}} \checkmark$

$T_{04} - T_{c'} = \frac{1}{\eta_j} (T_{04} - T_c)$

$\therefore T_{c'} = T_{04} - \frac{1}{\eta_j} (T_{04} - T_c) \text{--- (1)}$

$\frac{P_{c'}}{P_{04}} = \frac{P_c}{P_{04}} = \left(\frac{T_{c'}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}}$

$\frac{P_{c'}}{P_{04}} = \frac{P_c}{P_{04}} = \left[\left(T_{04} - \frac{1}{\eta_j} (T_{04} - T_c)\right) \frac{1}{T_{04}} \right]^{\frac{\gamma}{\gamma-1}}$

$\therefore P_c = P_{c'} = P_{04} \left[1 - \frac{1}{\eta_j} \left(1 - \frac{T_c}{T_{04}}\right) \right]^{\frac{\gamma}{\gamma-1}}$

$\therefore P_c = P_{c'} = P_{04} \left[1 - \frac{1}{\eta_j} \left(1 - \frac{T_c}{T_{04}}\right) \right]^{\frac{\gamma}{\gamma-1}}$

$\frac{T_{04}}{T_c} = 1 + \frac{\gamma-1}{2} M_c^2 = 1 + \frac{\gamma-1}{2} = \frac{\gamma+1}{2}$

$\therefore P_c = P_{c'} = P_{04} \left\{ 1 - \frac{1}{\eta_j} \left(1 - \frac{2}{\gamma+1}\right) \right\}^{\frac{\gamma}{\gamma-1}} = P_{04} \left\{ 1 - \frac{1}{\eta_j} \left(\frac{\gamma-1}{\gamma+1}\right) \right\}^{\frac{\gamma}{\gamma-1}}$

$\therefore \frac{P_{04}}{P_c} = \left\{ 1 - \frac{1}{\eta_j} \left(\frac{\gamma-1}{\gamma+1}\right) \right\}^{\frac{-\gamma}{\gamma-1}}$

$\frac{P_{04}}{P_c} = \left\{ 1 - \frac{\gamma-1}{\eta_j(\gamma+1)} \right\}^{\frac{-\gamma}{\gamma-1}} = \left\{ \frac{2}{\gamma+1} \right\}^{\frac{-\gamma}{\gamma-1}}$

M_c is choking Mach number but choking Mach number is 1.

so we can write down

$$P_c = P_{c'} = P_{04} \left(1 - \left(\frac{1}{\eta_j}\right) \left(1 - \frac{2}{\gamma+1}\right)\right)^{\frac{\gamma}{\gamma-1}}$$

$$= P_{04} \left(1 - \left(\frac{1}{\eta_j}\right) \frac{\gamma-1}{\gamma+1}\right)^{\frac{-\gamma}{\gamma-1}}$$

So this formula should be used to find out the choking condition for given P_{04} for nozzle once we attain if we have pressure P_c which is lower than the exit pressure then it is not choked if P_c is higher than the exit pressure then it is choked. So for that we actually would can also understand that if nozzle efficiency is 100% then

$$\frac{P_{04}}{P_c} = \left(1 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{-\gamma}{\gamma-1}}$$

so which would lead to

$$= \left(\frac{2}{\gamma+1}\right)^{\frac{-\gamma}{\gamma-1}}$$

gamma +1 to upon gamma + 1 bracket raise to - gamma upon gamma -1 and this is the isentropic relation between the static and total pressure for Mach number 1.

So here is here was the discussing for nozzle and a associated thing here how this is how we end the topic about discussion for the propelling nozzle thank you.

