Aircraft Propulsion

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Module No # 04
Lecture No # 17
Nozzle Flow

Welcome to the class. Today we are going to talk about nozzle flow. The reason for the discussion for nozzle flow lies in the fact that we are going to start about the engines, and we are going to discuss more about the details and components of the engines. And in case of main the turbo jet engine or in case of ramjet engine we know that nozzle is one of the parts. Initially air comes into the compressor its get compressed then it passes to the combustion chamber where it gets combusted and then it expands in the turbine.

This turbine produces the work which is required to run the compressor and then the rest of the enthalpy drop will take place in the nozzle. So rest of the expansion work in enthalpy drop so the convergent to kinetic energy will take place in the nozzle and hence it produces the thrust for a engine. So since the function of the nozzle is to have reduction in enthalpy and hence raise in kinetic energy and that is a major component in case of turbojet engine, turbofan engine and also in ramjet engine. So discussion about nozzle flow becomes important.

So let us start to discuss about nozzle flow and some basic about some nozzle flow. Here this part will actually deal mainly with the relations about nozzle flow which are derived from the basic gas dynamics. We are not going to go in the details of gas dynamics, but we are going to touch the concept of nozzle flow to the gas dynamic relations. Let us first understand the isentropic relations.
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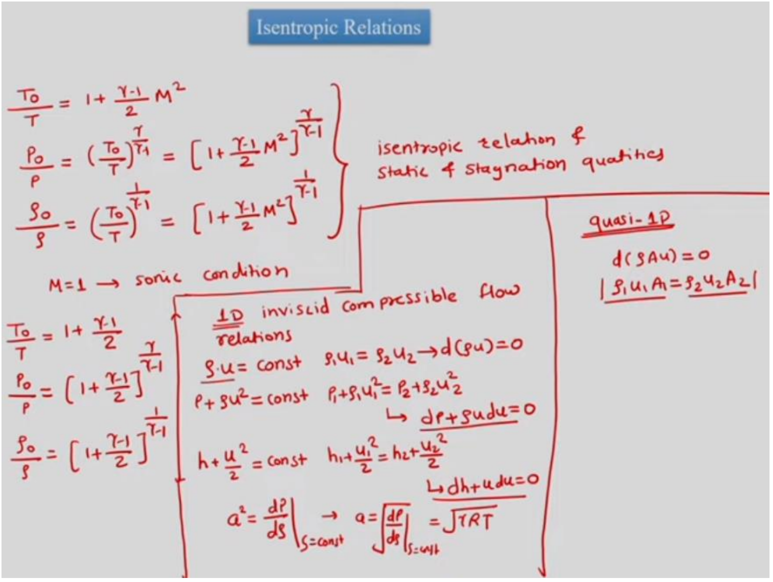

We have also derived partly those relations but we are going to revisit them and we know that the total temperature to static temperature relations says that the ratio

$$\frac{To}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

and hence

$$\frac{P0}{P} = \left(\frac{To}{T}\right)^{\frac{\gamma}{\gamma - 1}}$$

Similarly, stagnation to static density ratio,

$$\rho 0/\rho = \left(\frac{To}{T}\right)^{\frac{1}{\gamma - 1}}$$

and hence it is

$$= \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

So these relations are isentropic relations and they are between the static and stagnation quantities. So if we know a static quantity or total quantity then knowing the Mach number we can calculate the other unknown thing. Here we have made an assumption of calorically perfect gas and hence gamma is constant or here for this relation. These relations will be used by us in the further studies where we will vary the Mach number and we can get different ratios.

A special thing will be told or will be required is for the Mach number 1 or for sonic conditions and this relations will get transform to very simple relations which states that

$$\frac{To}{T} = 1 + \frac{\gamma - 1}{2}$$

M^2 square will become 1. Similarly,

$$\frac{P0}{P} = \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{\gamma}{\gamma - 1}}$$

and

$$\frac{\rho 0}{\rho} = \left(1 + \frac{\gamma - 1}{2}\right)^{\frac{1}{\gamma - 1}}$$

So these are the isentropic relations between static and stagnation quantity for Mach number 1.

Now we will see something new and that is for the 1D inviscid compressible flow relations. We partially know those relations in case of incompressible flow we state that area into velocity at inlet of a duct is equal to area into velocity at the outlet of a duct. But that relation is for incompressible inviscid flow. But now we are going to write the expressions for mass momentum and energy related to 1D incompressible 1D inviscid compressible flows.

And as per this relation,

$$\rho u = \text{constant or } \rho_1 u_1 = \rho_2 u_2$$

and hence

$$d(\rho u) = 0$$

It is mass conservation equation for one dimensional case. So it states that density into velocity this product at station 1 if a duct is equal to the same product at station 2 in the duct. Other relation for momentum equation states that

$P + \rho u^2$ = constant or $P_1 + \rho_1 u_1^2$square = $P_2 + \rho_2 u_2^2$

and hence in the differential form this equation $dP + \rho u\ du = 0$.

This relation states that pressure plus density into velocity square at station 1 is constant and hence it is equal to the same summation pressure plus density into velocity square at station 2.

Similarly the energy equation will be

$$h + u^2/ 2 = \text{constant.}$$

here we are having the assumption that the flow is adiabatic there is no any heat addition. So this says that

$$h_1 + u_1 u_1^2 / 2 = h_2 + u_2^2/ 2$$

and this in differential form will become dh + u du = 0.

And hence this relation will be very much known to us. since we know that

$$h_1 + u_1^2 / 2 = h_0$$

which is total enthalpy in station 1 and that is equal to total enthalpy in station 2 and both the enthalpy will be the same since there is no heat addition between station 1 and station 2. There is one more relation which should be known to us in the compressible flow regime which says that acoustic speed square is equal to dp / dρ in the entropy constant state.

So we have

$$a = \sqrt{(\mathrm{dp}/\mathrm{d\rho})} \text{at constant entropy}$$

$$= \sqrt{\gamma RT}$$

So this is also a non- relation for 1D inviscid compressible flows. Further these relations as I said are for 1D but these relations would also be valid for quasi 1D. Where quasi 1D relation by meaning it is more than 1D but less than 2D complications. Where we are feeling that flow has variation in only one direction and there is minor variation in the second direction.

That is what we mean in the quasi 1D case and in case of 1D we mean that there is only variation in one direction or in the direction of flow. So in the quasi 1D the differential form of pressure and energy is same but the change is there in the mass conservation equation for the quasi 1D that is

$$d(\rho A u) = 0$$

So we have

$$\rho_1 A_1 u_1 = \rho_2 A_2 u_2$$

and this is mass flow rate into area. This is basically kg per second mass passing through one section is equal to kg per second mass passing from the other section mass fluxing to area is equal to mass fluxing to area where ρu is use as mass flux. So these are the isentropic relations having knowingly as isentropic relation we can further derive few relations related to the nozzle.

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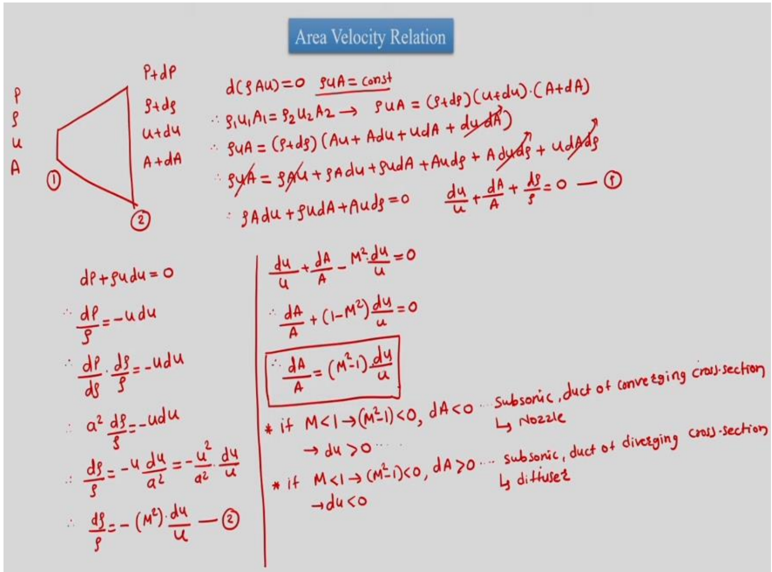

Let us consider that there is a divergent duct and this divergent duct has an inlet which is station 1 and an outlet which is station 2 and then we know P, ρ, u and A are the property at station 1. So similarly, P + dP, ρ + dρ, u + du and A + dA are the properties at station 2 where we have infinitesimal change in area A. As per mass conservation equation we know that

$$d(\rho A u) = 0$$

or

$$\rho\ u\ A = \text{constant.}$$

So we know that

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2.$$

We know that at station 1 we have ρ u A and at station 2 we have ρ + dρu +du and A + dA. This gives us ρu A = ρ + dρ when we multiply these two brackets then we will get Au + Adu + udA + du dA. But this higher order term. We will neglect and then we can multiply this and we can get ρA u = ρA u + ρA du + ρu dA + A u dρ and then we will have again some higher order term which are A du dρ + u dA dρ which we will again neglect.

Further this ρAu will also get cancelled and hence we will have ρA du = ρA du + ρu dA + A u dρ = 0. Since we know that ρA u is constant, we will divide both the sides by ρA u and then we can get $du/u + dA/A + d\rho/\rho = 0$. So let us say that this is equation number 1. Now we will use momentum equation in differential form which says that $dp + \rho u\, du = 0$. So we have $dP/\rho = -u\, du$. We can further write it like $dP/d\rho$ into $d\rho/\rho = -u\, du$.

Suppose we are considering again as isentropic case where entropy is constant then

$$a^2\, d\rho/\rho = -du.$$

We can divide this a square on other side and we can get - u du upon a square but we will divide and multiply by u so we can get - u square upon a square into du / u. As per the definition of Mach number we know that

$$\frac{d\rho}{\rho} = M^2 \left(\frac{du}{u}\right)$$

So this we will says as equation number 2.

So we will put equation number 2 into equation number 1 and we can get

$$du/u + dA/A - M^2\, du/u = 0.$$

So we have

$$dA/A + (1 - M^2)du/u = 0.$$

So we can write

$$dA/A = (M^2 - 1)du/u.$$

So this relation is called as area velocity relation.

Now let us understand or analyze this relation if we try to analyze this relation we can say that let us say that if Mach number less than 0 it is a minus. So if Mach number is sorry let us assume that Mach number is less than 1 which is subsonic.

So if Mach number is less than 1 that would give us $M^2 - 1$ as less than 0 so this will be a negative number. And since this is negative number we will further assume that let us say that dA is less than 0 where we mean that dA area is decreasing so dA is less than 0. So we are saying that it is subsonic duct subsonic flow in a duct of converging cross section. So for this case we can say that the A left hand side is negative right hand side since Mach number is subsonic is also negative.

So this gives us the fact that we will have du to be positive which is greater than 0 that means this duct is acting as nozzle. So we have subsonic duct with converging cross section acts as nozzle.

Now if Mach number is still less than 1 flow is still subsonic, so we still have M square - 1 less than 0 but now let us assume that dA greater than 0. So what we are talking about is subsonic flow in a duct of diverging cross section.

If we are talking about such then this dA is positive left hand side is positive. But right hand side says that it is negative since M square - 1 is negative. But right hand side also to be positive since dA is positive so du has to be negative. Then this M square - 1 is negative du is negative so then this product will become positive which state that it acts as diffuser.

By the definition nozzle is the entity which increases the kinetic energy at the expands of enthalpy drop, diffuser is the entity which decreases kinetic energy at the expands and raises the enthalpy. So we have diverging cross section acting as diffuser for subsonic flow.

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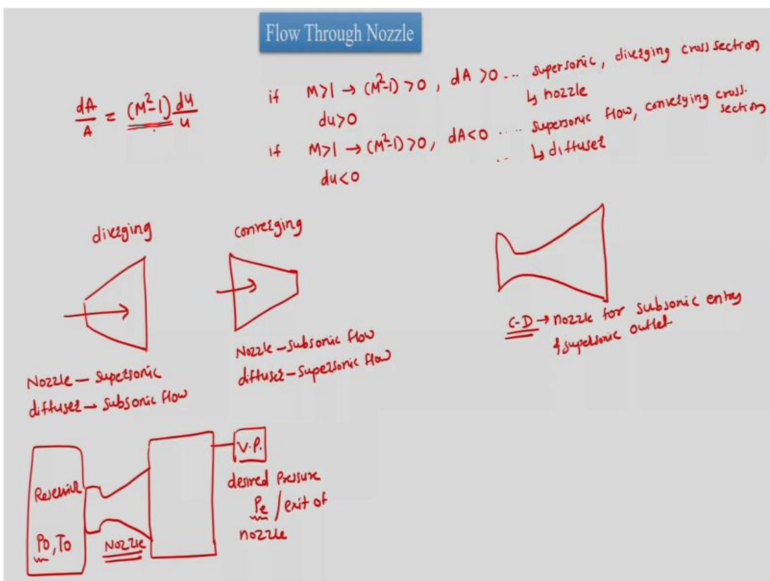

We will write down this relation again which states that dA / A = M^2 - 1 into du / u. Now if we have Mach number greater than 1 which gives as M square - 1 greater than 0 so it is positive and let us assume that dA is again greater than 0. Now we are talking about supersonic flow in a duct with diverging cross section. So here right hand side is left hand side is positive right hand side is actually positive so we will get du also to be positive and this means that supersonic flow in a diverging duct acts as nozzle.

Now let us consider fourth case where we have again have M more than 1 so M square - 1 will again be more than 0 so we have assign one more assumption that less than dA less than 0 so we are considering supersonic flow in a converging cross section. Here since left hand side less than 0 it is negative right hand side bracket term is positive. But since left hand side is negative right hand side also has to be negative. So for that du has to be less than 0 which states that supersonic flow in a converging duct would be diffused since supersonic duct is going to act as diffuser.

So we have basically two ducts one is diverging duct in the direction of flow it is diverging and then we have one more duct which is converging in the direction of flow. So this diverging duct will be acting as nozzle for supersonic flow and it is going to act as diffuser for subsonic flow. This converging section will act as nozzle in subsonic flow and diffuser in supersonic flow. So this is the judgement about the duct flow.

So we are talking about the nozzle so we have to consider both the sections but depending upon the flow Mach number. However, further we can have a sub convergent divergent duct which is also called as CD duct convergent divergent duct acting as nozzle for subsonic entry and supersonic outlet. So this is what we are calling it as the basic soft flow through nozzle where we are deriving the relation for area and Mach number.

Now flow through nozzle would have basically many kinds of complexity and those complexity we are going to discuss here consider now that we are having a convergent divergent duct placed in a chamber which is attached with a vacuum tank or which is attached with a vacuum pump and then on one side which is attached to a reservoir which has pressure P0 and temperature T0.

The vacuum pump we can connect outlet of the nozzle which is a convergent divergent duct we are connecting to a chamber which is connected to vacuum pump which can maintain desired pressure Pe or exit of nozzle. Now we are saying this pressure as Pe so there are many kinds of flow situation which will be governed by the pressure difference between P0 and Pe. We are going to discuss those situations.

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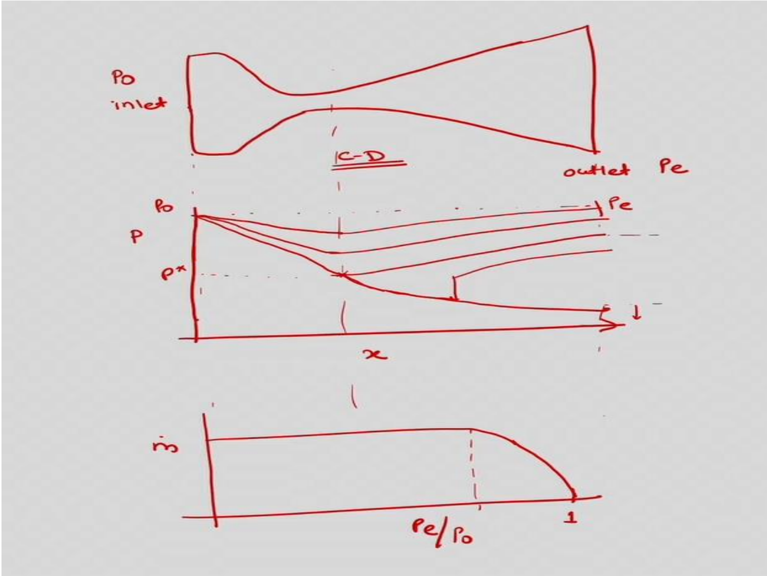

As per those we will first draw the nozzle again suppose this is the CD convergent divergent duct this is inlet this is outlet and we are going to plot few things for this special case and those plots we will discuss here. So this is pressure versus x and then we will plot later on mass flow rate. Suppose the exit pressure is Pe and the inlet pressure in P0 and to start with in our experiment Pe = P0 and then that slot it leads to so let us say mass flow rate and then exit pressure by P0 ratio and this is suppose 1.

So Pe = P0 so for this particular case there will not be any flow through the nozzle or the duct. But now we will decrease the value of the exit pressure and that decrement in the value of the exit pressure would be this is the value of P0 and we were talking about the value of Pe = P0. So this would be a straight line constant pressure in the complete duct so there is no mass flow rate in the duct. Now it decreases the exit pressure.

Since we are decreasing the exit pressure there will be fluid which is coming out from the reservoir side it will decrease the pressure since flow rate is very small the velocities are very small and we know that subsonic flow will be getting enthalpy drop and increment in velocity in the convergent duct. So convergent duct act as nozzle for the subsonic flow but it will get minimum pressure and maximum Mach number at the minimum cross section then this is the divergent duct.

So divergent duct or subsonic flow will act as diffuser and it increases the pressure. This will continue for few exit pressures. So there are many solutions possible as what increase the mass flow rate by decreasing the exit pressures. So for every exit pressure value there is a solutions. However for a particular exit pressure the flow will become Mach number 1 at the minimum cross section. So flow reaching Mach number 1 is the attainment of sonic state at the minimum cross section.

And hence the minimum cross section here on what we should consider as the throat since it has Mach number 1. There onwards the for the first instance this pressure will increase but for later any decrement in exit pressure below this particular value which was leading to P star which is the Mach 1 pressure. The any pressure would not alter the pressure profile in the convergent portion. Since for any exit pressure value there is unique variation for the convergent duct.

And then it will reach subsonic state it will be in subsonic state throughout the convergent duct read the sonic state and then it will try to further expand. But if it continues to expand then it will reach a state in the exit which is not equal to the exit pressure which is much below so for that all sake there will be a shock and the shock will increase the pressure and then will make the flow to be subsonic at this location of appearance of shock. So for further onwards the subsonic flow encounters the divergent duct and acts as diffuser.

But for particular exit pressure value the flow continues to expand and attains the exit pressure isentropically. So that matching is what the matched exit pressure corresponding to the outlet pressure. In this before this and the case where we had attained sonic condition at the throat this region is called as over expanded region.

Further here onwards if we decrease the pressure then it is called as under expanded since the nozzle does not give sufficient expansion to the flow and at the exit of the nozzle we still have flow expansion possible. Since the pressure at the exit is higher than the pressure of the chamber which is attain from the vacuum pump. In this case if we try to prolog mass flow rate then mass flow rate initially increasing as we decrease the exit pressure.

And for the exit pressure in which we have attain Mach one for the minimum cross section there mass flow rate becomes maximum but it does not change for any further decrement in mass flow exit pressure and then that is what we are calling it as chocking condition. So for a particular value of exit pressure we will attain Mach number 1 in the minimum cross section and for that case we will attain the maximum mass flow rate beyond which when we decrease the pressure at a exit there will not be any decrement in mass flow rate and mass flow rate becomes constant.

And this constants in Mach number should also be co related with no change in the subsonic pressure plot subsonic region pressure plot or Mach number plot. Since there is unique solution for the exit pressure of any value at the exit for the given P0 and given T0. So this is what we refer as the nozzle flow through a convergent divergent duct. Now we will try to find out what is the chocked mass flow rate.

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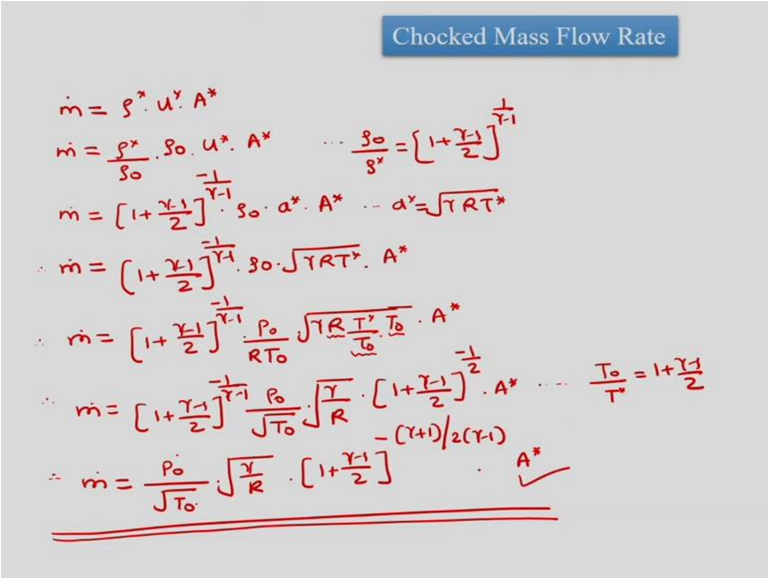

Chocked mass flow rate is suppose

$$\dot{m} = \rho * u * A *$$

where star quantities refer to the sonic condition which has been attained at the minimum cross section. We also know that from our area Mach number relation we also know that for Mach number 1 the term in the bracket will be 0. So this right hand side will become 0 so we mean that dA = 0. So it means that Mach number 1 will be always attained at the minimum cross section.

So if you are considering convergent duct then the convergent duct will reach sonic condition at the exit for a particular pressure difference. Similarly, if there is convergent divergent duct and this duct will have sonic condition in the minimum cross section. Having said this, we can write down ṁ = (ρ */ ρ0) ρ0 u*A*. But we know ρ */ ρ0 from the Mach number isentropic Relations which is $\left(1 + \frac{\gamma-1}{2}\right)^{\frac{-1}{\gamma-1}}$

The relation was stating that

$$\rho0/\rho^* = \left(1 + \frac{\gamma-1}{2}\right)^{\frac{1}{\gamma-1}}.$$

So the same relation is put over here is equal to ρ0 we have u*. u*is the velocity at sonic condition so but at sonic condition we have Mach number 1 so velocity is equal to speed of sound which is a*, a*A*. But we know

$$a *= \sqrt{(\gamma \mathrm{RT}} *)$$

So we can write down

$$\dot{m} = \left(1 + \frac{\gamma-1}{2}\right)^{\frac{-1}{\gamma-1}} \rho0\sqrt{\gamma \mathrm{RT}} * A *$$

So we have

$$\dot{m} = \left(1 + \frac{\gamma-1}{2}\right)^{\frac{-1}{\gamma-1}} \left(\frac{\mathrm{P0}}{\mathrm{RT0}}\right)\sqrt{(\gamma \mathrm{R}(\mathrm{T} */T0)T0)} A *$$

So which we can write down

$$\dot{m} = \left(1 + \frac{\gamma-1}{2}\right)^{\frac{-1}{\gamma-1}} \left(\frac{\mathrm{P0}}{\sqrt{To}}\right)\sqrt{\frac{\gamma}{R}} \left(1 + \frac{\gamma-1}{2}\right)^{\frac{-1}{2}} A *$$

Here we are using the relation which is

T0/T* =$1 + \frac{\gamma-1}{2}$.

So we have

$$\dot{m} = \left(\frac{\mathrm{P0}}{\sqrt{To}}\right)\sqrt{\frac{\gamma}{R}} \left(1 + \frac{\gamma-1}{2}\right)^{\frac{-1(\gamma+1)}{2(\gamma-1)}} A *$$

So when we have chocking condition then we can find out the mass flow rate using this relation where A star is the area at the minimum cross section or the throat.

And P_0 and T_0 are the total conditions corresponding to reservoir where we need the values of gamma which is specific heat ratio and R which is specific gas constant. So knowing this we can find out mass flow rate at the exit of the nozzle. Now we will go ahead and find out the relation which is area Mach number relation.

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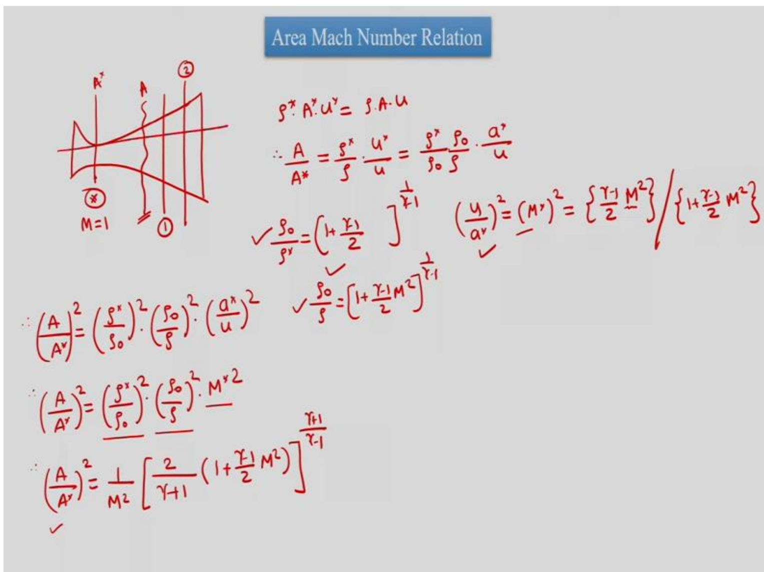

Here we let us consider we are having convergent divergent duct and we are interested to find out what is the Mach number in this section since we know the Mach number in this section. So in this star condition we know that it is Mach number 1 and then we are interested in finding out the Mach number in section 2, in section 1, in section 3 or in any section. We know that mass flow rate is constant where working with steady nozzle so mass flow rate through the minimum cross section which is $\rho^* A^* u^*$ equal to mass flow rate in any section other than star which is supposed $\rho A u$.

So we will have A which is area at any section divided by A* which is area at the sonic state is equal to $\rho^*/\rho \times u^*/u$. We can further write this by stating

$(\rho^*/\rho_0)\ \rho_0\ u^*\ \ a^*/u$

We know that

$\rho0/\rho* = \left(1 \ + \frac{\gamma-1}{2}\right)^{\frac{1}{\gamma-1}}$

And there is a gas dynamic relation which says that

$$\left(\frac{u}{a*}\right)^2 = \mathrm{M}*^2 = (\frac{\gamma-1}{2}\mathrm{M}^2)/(1+\frac{\gamma-1}{2}\mathrm{M}^2$$

So this Mach number at the section for which area is A and we are interested to find out this Mach number only. So we can write

$$\left(\frac{A}{A*}\right)^2 = \left(\frac{\rho*}{\rho0}\right)^2 \left(\frac{\rho0}{\rho}\right)^2 \left(\frac{a*}{u}\right)^2$$

So we have

$$\left(\frac{A}{A*}\right)^2 = \left(\frac{\rho*}{\rho0}\right)^2 \left(\frac{\rho0}{\rho}\right)^2 (M*)^2$$

we have this relation for M star.

$$\left(\frac{A}{A*}\right)^2 = \left(\frac{1}{\mathrm{M}}\right)^2 \left((\frac{2}{\gamma+1})(1 \ + \frac{\gamma-1}{2}\mathrm{M}^2)\right)^{\frac{\gamma+1}{\gamma-1}}$$

So we know area A in this section we know A star in this section so we know left hand side so we will it relatively find out the Mach number which would patch or match the left hand side value and hence we can know the Mach number for any cross section if flow is having supersonic velocity in a convergent divergent duct. This relation is also valid in the convergent section provided we have chocking condition in the throat.

For this relation we will have complete isentropic flow in the complete duct so there has to be no shock. Having said this, we end up our discussion about the nozzle flow and now we can go ahead and solve or work with the different other components with the engine. Thank you