Aircraft Propulsion
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Module No # 04
Lecture No # 16
Examples of Real Cycle

Welcome to the class now we will see some examples on real cycle here we know by meaning real cycle we have we will consider the losses in the combustion chamber, losses in the heat exchanger and also we will have efficiencies of turbine and compressor. So this part would have the pressure ratios for turbine and compressor will be different so let us see how to solve some examples related to real cycle.

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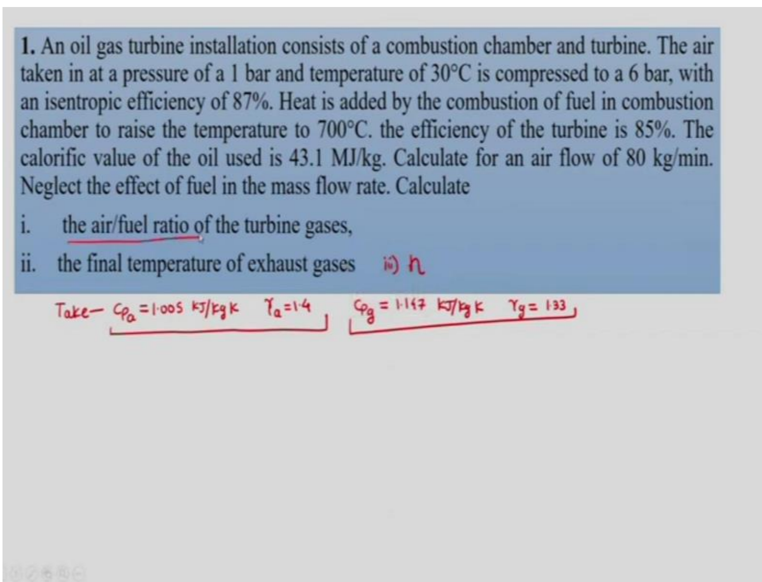

First example states that an oil gas turbine installation consist of a combustion chamber and turbine. The air taken in at pressure of 1 bar and temperature 30 degree Celsius is compressed to 6 bar with an isentropic efficiency of 87%. Heat is added by combustion of fuel in combustion chamber to raise the temperature to 700 degree Celsius, the efficiency of turbine which is isentropic efficiency is 85%, the calorific value of fuel is 43 mega joule per kg.

Calculate for an air flow of 80 kg per minute neglect the effect of fuel in mass flow rate. Calculate air fuel ratio or the gas turbine and also final temperature of exhaust gases. So these things we have to calculate this is a simple part of the example and here we have to also find efficiency of the cycle. So now we will start solving this example a point to be noted here that CP of air is 1.005 and gamma of air 1.4 and CP of gas is 1.147 kilo joule per kg Kelvin and gamma of gas is 1.33. So the specific heat and specific heat ratio for air and gas are different which we had not taken in case of perfect cycle.

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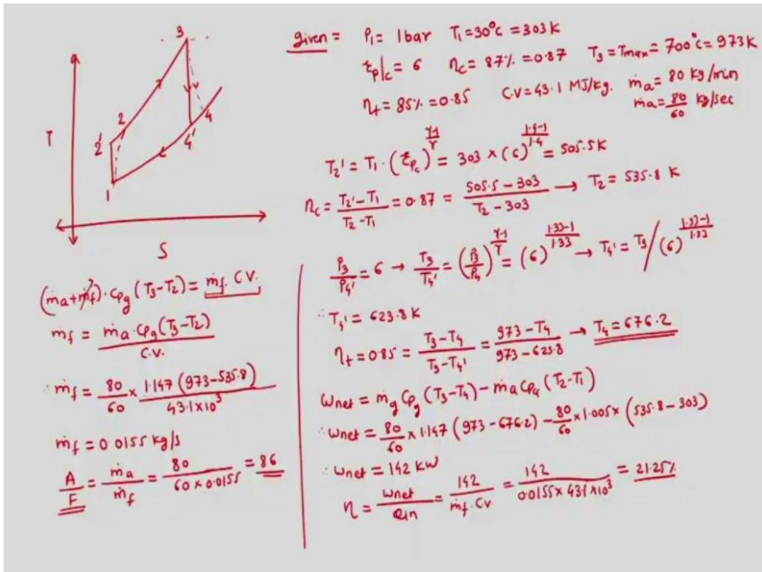

So having said this we will go ahead and solve the example as per the example first let us draw the TS diagram and in the TS diagram we will have 1 to 2’ dash as isentropic compression, 2 for real compression, 3 is the T max point or maximum temperature point and then we have 4’ as expansion and 4 as real point so having said this we know what are the given things to us in the example.

We are told that in the example P1 = 1 bar, T1 = 30 degree Celsius so it is 303 Kelvin and then we are told that pressure ratio which is r compression rpc = 6 and then compression efficiency isentropic efficiency of compressor is 87% so it is 0.87, T3= T max = 700 degree Celsius so it is +273 so it is 973 Kelvin. We are told that turbine efficiency isentropic turbine efficiency is 85% so it is 0.85 calorific value of fuel is 43 mega joule per kg.

Calorific value is amount of heat liberated when 1 kg of fuel is burnt and we are also told that mass flow rate of air = 80 kg per minute so it is 80/60 kg per second. So knowing this we can proceed and solve first we are told that compression pressure ratio is 6 so we can find out $T_{2'}$ so

$$T_{2'} = T_1 * (r_{p_c})^{(\gamma-1)/\gamma}.$$

Here we are told that gamma is 1.4 so we can take it as 1.4 and then accordingly we can calculate so it is 1.4 – 1 / 1.4 so $T_{2'}$ comes out to be 505.5 Kelvin. Now we have to find out T_2 so for T_2 we can use the efficiency of compressor so compressor efficiency is ideal work input divided by actual work input. Ideal is related with

$$\eta_c=(T_{2'}-T_1)/(T_2-T_1)$$

and it is 0.87 so this is equal to ($T_{2'}$ is 505.5) – (T_1 is 303) / T_2 which is unknown and T_1 is known and this gives us T_2 = 535.8 Kelvin.

Now we should know here that actual relation for heat addition is

$$(\dot{m}_a + \dot{m}_f)*Cpg*(T_3 - T_2) = \dot{m}_f * \text{Calorific value of fluid}.$$

So this is the total amount of heat liberated by burning the fuel and this heat is received by the mixture of air and fuel to raise the temperature from 2 to 3 but we can neglect the mass of fuel in the mixture just keep mass of fuel in calorific value to find out the fuel flow rate and it is stated in the statement of the example also.

So we can have

$$\dot{m}_f = (\dot{m}_a)*Cpg*(T_3 - T_2) / \text{calorific value}$$

so we have $\dot{m}_f = \dot{m}_a$ which is 80/60 kilogram per second into *Cpg* and *Cpg* is known to us given as 1.147 and T_3 is maximum temperature which is 973. T_2 is calculated 535.8 divided by calorific value which is 43.1 but 43.1 is in kilo mega joule per kg. So we have to convert it into kilo joule per kg and then this will match with the unit for the Cp and this gives us the mass flow rate of fuel as 0.0155 kg per second.

So we will get air fuel ratio as 80 air fuel ratio is practically mass of air divided by mass of fuel. We are neglecting the mass of fuel in the total mixture so also we have used total mixture wave mass is 80/60 for convenience but for calculation of air fuel ratio we have 80/60 / 0.0155 and this gives us air fuel ratio as 86 this is one of the answer since if we go back we are supposed to find out air fuel ratio.

Now we will proceed with next calculation which is exhaust gas we are not given with any losses for the turbine pressure ratio so pressure ratio of turbine and compressor is same as in the example. Since we are not told that there is loss in the heat exchanger side or there is loss in the combustion chamber for total pressure so for us $P_3 / P_{4'}$ is also 6. So from there we can find out

$$T_3 / T_{4'} = (P_3 / P_4)^{\wedge}(\gamma - 1)/\gamma$$

but here γ is for gas.

So we have to take this point into account so we have this also as 6 but gamma for gas is 1.33 so 1.33- 1 / 1.33 so this gives us $T_{4'} = T_3$ / 6 bracket raised to (1.33 – 1)/ 1.33 so $T_{4'}$ can be calculated and it is 623.8 Kelvin. So we will get T_4 if since we know turbine efficiency and turbine isentropic efficiencies given as 85% and

$$\eta_t = (T_3 - T_4)/(T_3 - T_{4'})$$

So we will have this is equal to T_3 and T_3 is known to us which is (973 – T_4) /(973 – $T_{4'}$) is 623.8 and this gives us T_4 which is exhaust gas temperature as 676.2 and so using the turbine and compressor efficiencies we can find out the exhaust gas temperature here. This example is different from the example which we would have solved for the case where we had non-ideal components.

In case of non-ideal components we had solved similar example but there we did not consider gamma and CP to be different so we had turbine and compressor both were handling same fluid. Since it has same specific heat ratios and same specific heats but in this example they are different. So we can find out w net since we are supposed to find out efficiency so for

$$w\ net = m_g * Cpg * (T_3 - T_4) - m_a * Cpa * (T_2 - T_1).$$

So we have w net is equal to mass of gas is again mass of air which is 80/60 *(CP of gas which is 1.147)*(T_3 is 973) – T4 and (T_4 is 676.2) 80/60 *(CP of air which is 1.005)* T_2 and (T_2 is 535.8) – (T1 is 303). So we can get w net is equal to 142 kilowatt so overall efficiency is

$$\eta = w\ net/Qin$$

we know it is 142/(mass of fuel * calorific value)and so it is 142 mass of fuel is 1.0115 into 43.01 into 10 to the power 3 and this gives us the efficiency as 21.25%. So this is the takeaway from the present example where we have to consider the CP and CV to be different for the compressor and turbine and then accordingly we have to calculate.

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2. The following data refer to a gas turbine set employing a regenerator:

Isentropic efficiency of the compressor	:	82%
Mechanical Transmission efficiency	:	99%
Pressure Ratio	:	7:1
Maximum cycle temperature	:	1000K
Combustion efficiency	:	97%
Calorific value of the fuel	:	43.1MJ/kg
Air mass flow	:	20kg/s
Regenerator effectiveness	:	75%
Regenerator gas side pressure loss	:	0.1bar
Ambient temperature and pressure	:	327K; 1 bar
Isentropic efficiency of the turbine	:	85%

Calculate the output, specific fuel consumption and overall thermal efficiency. Assume that the pressure losses in the air side of the regenerator and consumption are accounted for in the compressor efficiency. Compare these results with those obtained for the same plant without the regenerator, and with regenerator but without pressure loss and also comment on the results. Neglect the effect of fuel mass addition in the heat balance in the combustion chamber but include in the turbine calculation.

So we will move on to the next example and next example states that following data refers to the to a gas turbine set employing a regenerator isentropic efficiency of compressor is 82% mechanical efficiency is of transmission is 99% pressure ratio is 7 as to 1 maximum cycle temperature is 1000 Kelvin combustion efficiency is 97% calorific value of fuel is 43.1 mega joule per kg.

Air mass flow rate is 20 kg per second regenerator effectiveness regenerator is heat exchanger and its effectiveness is 75% regenerator gas side pressure loss gas side means at exit of the turbine there will be gas which is giving heat to the air which is going to the combustion chamber. So there the pressure loss is 0.1 bar ambient conditions in temperature and pressure as 327 Kelvin and 1 bar isentropic efficiency of turbine is 87% ok 85%.

Calculate the output specific fuel consumption and overall thermal efficiency so these three things we have to calculate what we have to assume? That assume that the pressure losses in the air side of the regenerator and consumption are accounted for in the compressor efficiency. So compressor efficiency accounts for the losses which are in the pressure losses compare these results with those obtained with the same plant without regenerator and with regenerator but without pressure loss and also comment on the results.

Neglect the effect of fuel in mass addition and in heat balance in combustion chamber but include it in the turbine calculation ok.

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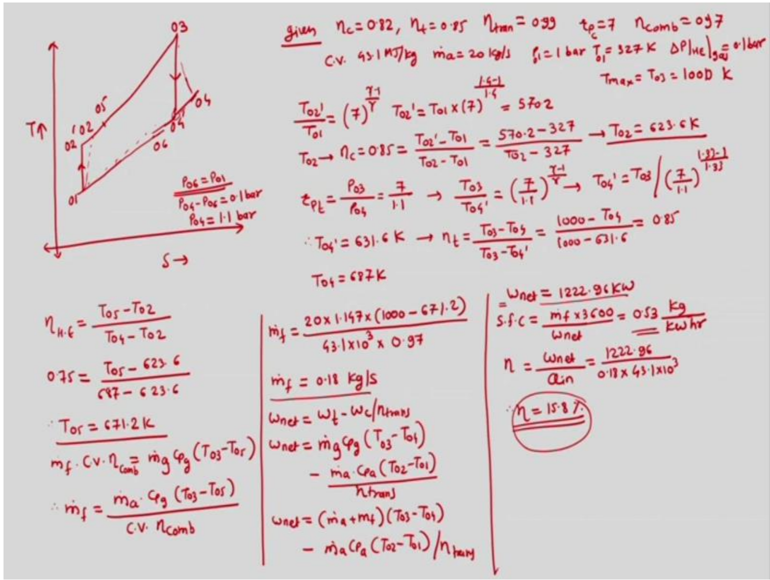

So we will go ahead and plot first the TS diagram or this example and again note down the given things. So TS diagram says that suppose we have 1 to 2' and 2 and then we have 3, 4. Now we have to remember one thing since we have to account losses we can solve this example whether we need to solve this example in terms of total pressure so it is 01, 02', 02, 03 and 04.

If losses are not given then the static condition can be used to solve the example but when losses are given we cannot solve the example in the static conditions. So it is 02', 02, 03 and then we have 04' and then it is 04 and then we have 05 in this side of the heat exchanger. So given things in the example include compressor efficiency as 82% we have turbine efficiency as 85% so it is 0.85.

We have transmission efficiency as 99% turbine is connected with compressor so compressor work we will calculate but that much work is just required for the process of compression. But we need to supply more work to get that work from the turbine so we practically would have compressor work more due to the transmission efficiency. So transmission efficiency is related with net work input and we have pressure ratio as 7 combustion efficiency as 97%.

And then we have calorific value as 43.1 mega joule per kg mass flow rate of air as 20 kg per second. P1 is 0.1 P1 is 1 bar T1 is 327 Kelvin and delta P in heat exchanger in gas side is 0.1 bar. So practically given to us fact is that P4 P04 is not same as P01 and so they are on different pressures so we will not connect them or rather if at all we can connect them we have to keep some gap and then that gap would be related to the pressure loss.

So we will solve with regenerator first and then we can work out without regenerator so we will find out first $T_{02'}$ since we are told the pressure ratio and that pressure ratio. So practically it is for compressor so pressure ratio for compressor is known so $T_{02'}$ / T_{01} is equal to so T1 is basically T01 and P1 is basically P01 is

$$T_{02'} / T_{01} = 7 ^ {(\gamma - 1)/\gamma}$$

and here we will take gamma as 1.4 for air and we have to take gamma 1.33 for gas.

So this gives us T2 $T_{02'}$ so $T_{02'} = T_{01}$ into 7 bracket raised to gamma which is 1.4 – 1 / 1.4 and this gives us $T_{02'}$ as 570.2 and we can calculate T_{02} from here using the compressor efficiency and it is 0.85 that is

$$\eta_c = (T_{02'} - T_{01}) / (T_{02} - T_{01}).$$

So $T_{02'}$ is 570.2 – T_{01} is 327 divided by we have T_{02} – 327 and this gives us T_{02} as 623.6 Kelvin.

Now we can calculate the temperature at the exit of the turbine since we are known with the turbine efficiency and we are also knowing the maximum temperature and that maximum temperature T max = T03 and that is 1000 Kelvin. So for that we can write down first we have to find out what is the pressure ratio for turbine? Pressure ratio for turbine is P_{03}/ P_{04} but this gas goes out at 04 and then it reaches the state which is 06 and 06 = P_{01}.

And then we are told that $P_{04} - P_{06}$ = 0.1 bar so we have P_{04} = 1.1 bar so we have P_{06} P0 pressure ratio for turbine = P_{03} / P_{04} so it is 7/1.1. So we can take it for

$$T_{03} / T_{04'} = (7/1.1)^{(\gamma - 1)/\gamma}$$

but here gamma will be 1.33. So we can calculate $T_{04'} = T_{03}$ / 7 /1.1 bracket raise to 1.33 – 1/ 1.33. This gives us $T_{04'}$ and $T_{04'}$ is 631.6 Kelvin now we can make use of it and turbine efficiency to find out T_{04}.

So we have

$$\eta_t = T_{03} - T_{04} / T_{03} - T_{04'}.$$

T_{03} is 1000 – 631.6 / 1000 –T_{04} is not known $T_{04'}$ is 631.6 and this is equal to turbine efficiency which is 0.85 and then we can get T_{04} and that is 687 Kelvin. Now we have to work with heat exchanger and now our heat exchanger is non ideal so we know that effectiveness of heat exchanger is equal to actually heat transfer divided by maximum possible heat transfer .

$$\eta_e = T_{05} - T_{02} / T_{04} - T_{02}.$$

Now we know everything so we can calculate T_{05} here effectiveness of heat exchanger is given to us and that is basically told as regenerator effectiveness as 0.75. So T_{05} – T_{02} and T_{02} = 623.6 / T_{04} which is 687- 623.6 and this gives us T_{05} as 671.2 Kelvin. Now this is useful to us for finding out what is the mass flow rate of fuel we can use the same formula which is

$$\dot{m}_f = (\dot{m}_a)*Cpg * (T_{03} - T_{05}) / \text{calorific value}$$

But this is if combustion efficiency is 100% but combustion efficiency is not 100% so not all heat you will raise the temperature so there will be one multiplication which is combustion efficiency. So we have mass flow rate of fuel is equal to mass flow rate of gas but we are going to take it as air for the sake of simplicity

$$\dot{m}_f = (\dot{m}_a)*Cpg * (T_{03} - T_{05}) / (\text{calorific value} * \eta)$$

And then that will give us m dot f is equal to mass flow rate of air is given as 20 calorific value of gas is 1.147 kilojoule per kg Kelvin into 1000 – T_{05} we have just found out 671.2 divided by calorific value is 43.1 but it should be multiplied by 2 3 10 to the power of 3. Since it is mega joule per kg and combustion efficiency is given as 97% so mass flow rate of fuel is 0.18 kg per second. Now we have to find out what is the net work so net work is always turbine work minus compressor work so net work

$$w\ net = m_g * Cpg * (T_{03} - T_{04}) - m_a * Cpa * (T_{02} - T_{01}).$$

But this compressor work and turbine work we can consider for network if there is 100% transmission efficiency otherwise we need to supply more power for doing the compressor work.

And then that is why we have to divide this compressor work by transmission efficiency if transmission efficiency is not 100%. So w net is equal to

$$w\ net = (m_a + m_f) * Cpg * (T_{03} - T_{04}) - m_a * Cpa * (T_{02} - T_{01})/(transmission\ efficiency)$$

So we will have network = 1222.96 kilowatt since all the quantities are known to us over here knowing this we can find out specific fuel consumption and specific fuel consumption is mass flow rate of fuel divided by w net.

But generally it is specified as fuel flow rate per in per hour so it is m dot f into 3600 / w net and we can calculate get it as 0.53 into kg per kilowatt hour. So we can get thermal efficiency

$$\eta = w\ net/Qin$$

and we have 1222.96 kilowatt divided by mass flow rate of fuel is 0.18 into 43.1 into 10 to the power 3. So we have cycle efficiency or thermal efficiency of cycle as 15.8 cycle efficiency as 15.8%.

So this is the solution for cycle efficiency specific fuel consumption and network output if we have this is the solution for the example for network output specific fuel consumption and cycle efficiency if we have a regenerator. But now we are told to work out without regenerator so let us find out.

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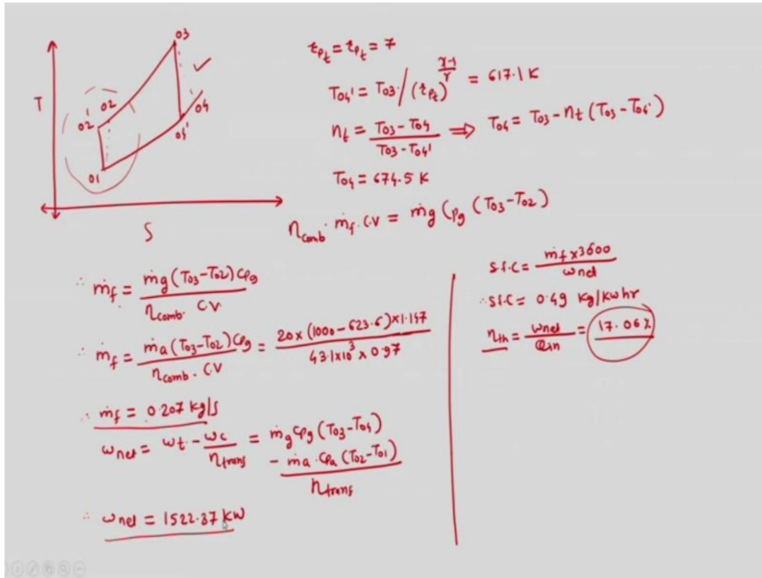

And in case when we do not have regenerator then that means that we do not any have pressure loss so since we do not have pressure loss this is a simple calculation since we will have turbine and compressor both will have same pressure ratio since there is no pressure loss in the regenerator so we have pressure ratio turbine is equal to pressure ratio compressor and then that is 7 it does not alter the calculation on compressor side but it alters the calculation on turbine side.

So we have to calculate freshly $T_{04'}$ from T_{03} it is

$$T_{04'} = T_{03} / (r_{pt})\wedge\ (\gamma - 1)/\gamma$$

where γ is 1.33. So this gives us $T_{04'}$ a new value without regenerator and no so no loss as 617.1 Kelvin. So we can find out T_{04} from this using the turbine efficiency since turbine efficiency is

$$\eta_t = T_{03} - T_{04} / T_{03} - T_{04'}.$$

And we will have here $T_{04} = T_{03} - \eta_t *(T_{03} - T_{04'})$ and so we get T_{04} as 674.5 Kelvin. So we can calculate mass flow rate of fuel a new way and that will be since now we do not have regenerator we have

$$\dot{m}_f = (\dot{m}_a)*Cpg * (T_{03} - T_{02}) / (calorific\ value * \eta)$$

But here for sake of convenience we will take it as mass of air mass flow rate of air. So we have calculated it we had given with 20 we have to multiply with CP of gas. So this is 20 into T02 T03 which is 1000 – T02 which is calculated as 623.6 divided by into CP which is 1.147 kilo joule per kg Kelvin / 43.1 into 10 to the power 3 into combustion efficiency as 0.97 and these uses fuel flow rate as 0.207 kg per second.

So we can calculate w net which is

$$w\ net = (m_{a} + m_{f}) * Cpg * (T_{03} - T_{04}) - m_{a} * Cpa * (T_{02} - T_{01}) / (transmission\ efficiency)$$

knowing all the quantities we can calculate w net and then that would turn out to be 1522.37 kilowatt. So we can calculate specific fuel consumption as mass flow rate of fuel into 3600 divided by w net and specific fuel consumption will turn out to be 0.49 kg per kilowatt hour.

We can calculate efficiency thermal efficiency of cycle or efficiency of cycle as

$$\eta = w\ net/Qin$$

W net is calculated Q in is mass flow rate into calorific value and this will turn out to be 17.06%. This example tells us few things we have solved with regenerator so there we had implemented heat exchanger and we know that heat exchanger is generally implemented for increasing efficiency.

Since it reduces the external heat addition but we can see that efficiency with regenerator was coming out to be 15.8 and without regenerator was coming out to be 17.06 which is more so with regenerator we are getting lower efficiency and without regenerator we have higher efficiency the difference was mainly due to the pressure loss and it was only 0.1 bar. So 0.1 bar has led to the decrement in efficiency of the cycle.

So this has also lead to increase in fuel flow rate with regenerator we have lower fuel flow rate without regenerator we have higher fuel flow rate with regen without regenerator we have network as 51522 and here network is 1222. So network as also got decreased with regenerator efficiency has also got decreased with regenerator and both are due to the pressure loss in the regenerator.

So we have to consider these pressure losses while considering regenerator this is how we can solve the examples for the real cycle where we have to account losses and when we are accounting losses we have to consider total conditions while solving the example thank you.