Aircraft Propulsion
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Module No # 03
Lecture No # 14
Stagnation Conditions, Real Brayton Cycle with Stagnation Conditions

Welcome to the class we have seen that how does a Brayton cycle work and how to do calculation for various performance parameters for Brayton cycle. Then we moved on and considered the efficiency which are isentropic efficiencies for turbine and compressor. Then now we are moving towards next reality where we are going to considered turbine and compressor efficiencies along with the combustion efficiency and also we will account some more thing which we had not accounted.

So the thing which is we are going to account here is this stagnation condition now if we go back and see what did we consider what we did not consider.

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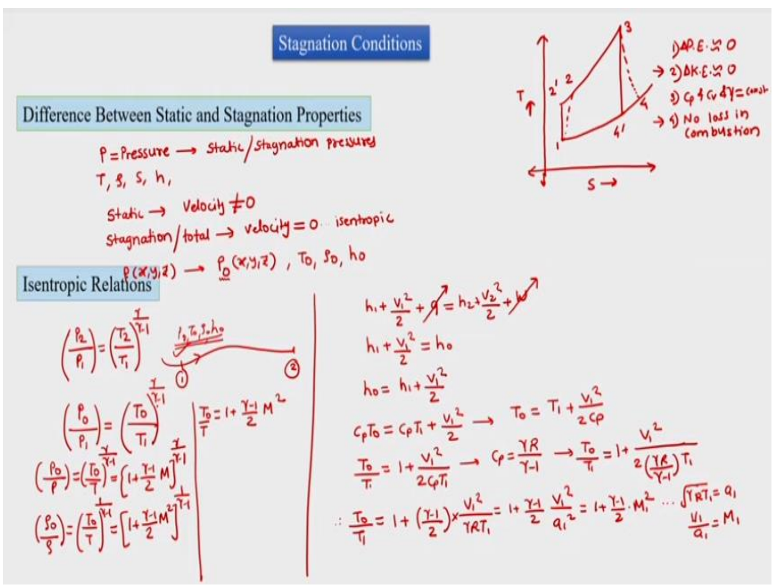

Now if we draw our Brayton cycle then in TS diagram we drew our Brayton cycle like this where 1, 2 dash, 3 and 4 dash was the cycle which is ideal cycle. And when we say that we want to consider real then we said that okay 1 to 2 is compressor and 3 to 4 is turbine. So this is what we had considered till time now here while working we made certain assumptions first assumption we made was we neglected potential energy between inlet and outlet of compressor or turbine.

So potential energy is neglected we neglected kinetic energy change in potential in turbine and compressor. So delta PE and delta KE we neglected then we also considered cp and cv and gamma they are constants we considered them as constant. Further we took into account that the losses in combustion are negligible no loss in combustion so with this prominent assumptions we derived few things for the Brayton cycle but now we have to move ahead and neglect some of the assumptions.

Prominently we will neglect the assumptions what we made for consideration of KE rather the assumptions which was considered as change in kinetic energy 0 and no loss in the combustion chamber we will now not consider this as assumptions and we will counter act for that assumption and take proper action for those assumptions. While considering that we have to first know what do we mean by stagnation condition.

So whenever we talk about few things in case of thermodynamic or fluid dynamic parameters. Suppose I consider P which is pressure then we say that there are two types of pressures one is static pressure or other is stagnation pressure. Dynamic pressure also exist but here we are talking about P dynamic pressure is $\frac{1}{2}\rho V^2$. So this P pressure is either static for stagnation similar thing exist for temperature, density, entropy, enthalpy and all other properties.

So all other properties are existing with two versions one is static or other is stagnation so what is the difference between static properties and stagnation properties. Now we mean here by saying something at static so let us say pressure at static pressure or temperature at static temperature then we mean that the quantity is measured in the presence of velocity. So if velocity is not brought to 0 and a quantity is measured then that quantity whatever we measure is called as static quantity.

So if it is a pressure then it is a static pressure if it is a temperature then it is a static temperature. So here velocity is not 0 now if we consider stagnation which is also called as total while considering this velocity as to be brought to 0 but it as to be brought to 0 in isentropic way. So it has to be brought to 0 reversibly and adiabatically then the corresponding condition is called as stagnation condition.

So here as static condition P is dependent upon x, y, z as location so pressure is varying in all the location in the flow field. Similarly if we denote P_0 as the total pressure or stagnation pressure then that will also vary at all locations.

$$P(x, y, z) \rightarrow P_0(x, y, z, h)$$

So it is a special quantity which has to be evaluated at a point where we have to assume that if at that point the fluid particle is brought to 0 velocity isentropically then what would be the condition.

With this assumption if we measure the pressure then corresponding pressure is stagnation pressure or total pressure. So here we are bringing the velocity to 0 as P_0 is having 0 as subscript similarly T_0, ρ_0, h_0 all represents stagnation conditions at a given locations. So it is a imaginary quantity which one has to calculate by thinking or assuming flow to be brought to 0 velocity isentropically.

But there are certain realities where we can practically measure this and then that is where we say it has Pitot static tool where we have this is tube in which we have hole at the center and this center hole is connected from limb of manometer then fluid will go here and practically fluid is going to rest at this location. So if we measure pressure at this location then this pressure is called as this pressure would be called as total pressure or stagnation pressure.

So if I take a cylinder or sphere and flow is flowing over and then I measure pressure at suppose three locations 1, 2 and 3 at location 1 flow would directly come to rest since it is a stagnation point. So the pressure measured at point 1 which is stagnation pressure but at point 2 flow will be not stagnant since the normal direction is this at this and velocity is not in the direction of local normal.

So there is component of velocity which is not letting it go to 0 and then that is why velocity is not being 0 at point 2 and also at point 3 the pressure measured at point 2 and 3 is a static pressure. Similarly if we measure density or measure temperature at point 1 and point 2 and 3 and those will be static quantities and the quantity is whatever we measure at 1 will be total quantities.

So this is the basic difference between stagnation and static quantities so we are assuming that flow is bought to 0 velocity isentropically. So we have to consider isentropic relations and as isentropic relations we know that

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\left(\frac{P_0}{P_1}\right) = \left(\frac{T_0}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\left(\frac{\rho_0}{\rho}\right) = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}}$$

Suppose this is a stream line this is state 1 and this is state 2 and at state 2 we have brought the fluid to 0 velocity. Then we will go ahead and try to link it classical gas dynamic relation where let us apply the steady flow energy equation between station 1 and 2 then it says that

$$h_1 + \frac{V_1^2}{2} + q = h_2 + \frac{V_2^2}{2} + w$$

But we are saying that there is no heat addition between station 1 and 2 and then there is no work interaction.

$$h_1 + \frac{V_1^2}{2} = h_0$$

$$C_p T_0 = C_p T_1 + \frac{V_1^2}{2}$$

$$T_0 = T_1 + \frac{V_1^2}{2C_p}$$

$$C_p = \frac{\gamma R}{\gamma - 1}\ ; \sqrt{\gamma R T_1} = a_1 ; \frac{V_1}{a_1} = M_1$$

$$\frac{T_0}{T_1} = 1 + \frac{V_1^2}{2C_pT_1\left(\frac{\gamma R}{\gamma - 1}\right)} = 1 + \left(\frac{\gamma - 1}{2}\right)\frac{V_1^2}{\gamma RT_1} = 1 + \left(\frac{\gamma - 1}{2}\right)\frac{V_1^2}{a_1^2} = 1 + \left(\frac{\gamma - 1}{2}\right)M_1^2$$

$$\left(\frac{P_0}{P_1}\right) = \left(\frac{T_0}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\left(\frac{\rho_0}{\rho}\right) = \left(\frac{T_0}{T}\right)^{\frac{1}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{1}{\gamma-1}}$$

So this is what we call as stagnation condition and the stagnation condition relations so here all these relations are based upon the fact that we are assuming flow to be bought to 0 velocity with isentropic assumption. Now let us go back and see what do we mean by not accounting the velocity we have got we did the account the velocity and we have got certain relation but now we will start considering the velocity with the help of stagnation condition and try to find out the same relations for Brayton cycle.

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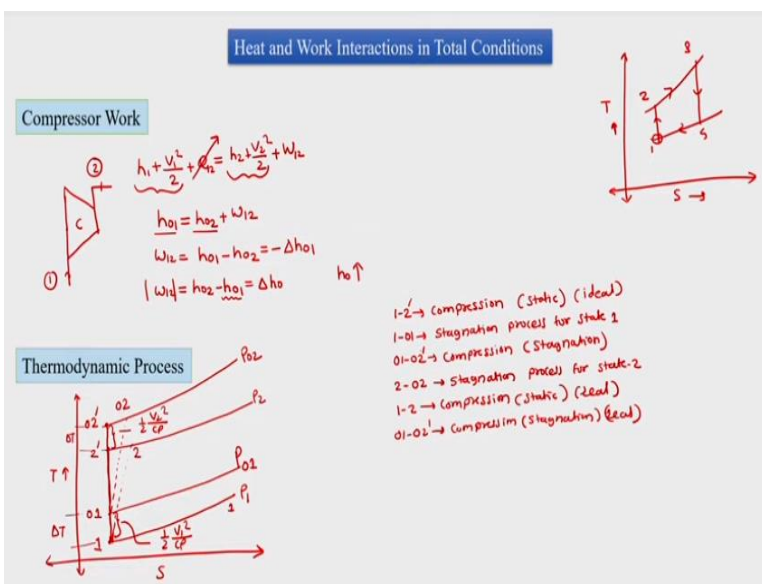

So we know that our Brayton cycle had a compressor which is the first entity and then for the compressor we have process 1 to 2 and the we have 3 to 4 as turbine. So for compressor 1 to 2 we will consider the steady flow energy equation. So we will draw the compressor in the Brayton cycle so this is station 1 this is station 2 this is compressor. So we will apply a steady flow energy equation which state

$$h_1 + \frac{V_1^2}{2} + Q_{12} = h_2 + \frac{V_2^2}{2} + W_{12}$$

So if we start considering total enthalpy at station 1 and total enthalpy at station 2 then the work done or work input whatever we get will actually account the velocities at station 1 and 2 so we are going towards more realistic estimates.

$$h_{01} = h_{02} + W_{12}$$

$$W_{12} = h_{01} - h_{02} = -\Delta h_{01}$$

$$W_{12} = h_{02} - h_{01} = \Delta h_0$$

$$|W_{12}| = h_{02} - h_{01} = \Delta h_0$$

So we can see that if we had work into the system total enthalpy increases. So for the compressor total enthalpy increases so we can now move and add the thermodynamic process for compressor.

So we will say that okay this is T this is S this is temperature and entropy and now we will plot particularly 4 lines where we are saying that this is station 1 as what we wrote here so this is the static conditions at 1 now at 1 we are knowing or needing h_{01}. So for that let us bring the flow isentropically to 0 velocity so if you bring it isentropically to 0 velocity then we will reach to the 01 conditions this is 01.

Now as what we were consider earlier we were considering compressor work to be input in the process 1 to 2 so 1 to 2 is compression using static conditions 1 to 01 is actually stagnation process for state 1 but now in the new ways we are saying that 01 to 02 is the actually work input in the process of compression. So 01 to 02 is again compression but it is from the perspective of stagnation conditions.

So practically we have 2, 02 is stagnation process for state 2 so practically we are having this as $\frac{\frac{1}{2}V_1^2}{C_p}$ and then this is equal to $\frac{\frac{1}{2}V_2^2}{C_p}$ since this is ΔT and this is also ΔT since Y is a temperature axis it is not energy so this is not $\frac{1}{2}V_1^2$ this is $\frac{\frac{1}{2}V_1^2}{C_p}$. So this is what the thermo dynamic process for compressor if we start accounting the stagnation condition but the process of our interest was here 2 dash so here process of which is ideal process actually from our perspective and 01 and then we have real process as 1 to 2.

So 1 to 2 is compression. In static sense which is real 1 to 2 is static sense which is real and we are saying this is a constant pressure line P1 this P 01 this is P2 this is P02 but really we are now accounting velocities so it will become naught so it will become 01, 02 dash so it is compression from stagnation perspective and then that is basically in real case. So as what our nomenclature says 1 to 2 is real and 1 to 2 dash is ideal or the isentropic process. So this is how we would draw a new thermodynamic process for compressor now we will see the isentropic efficiency of compressor.

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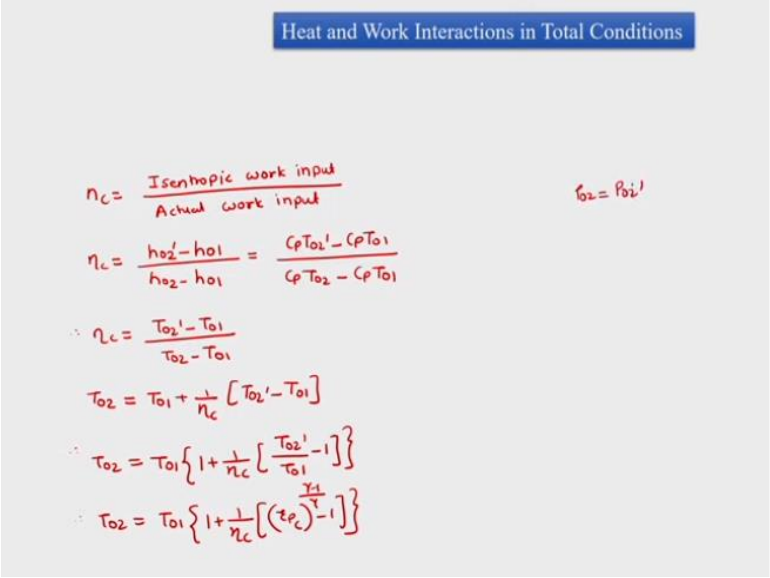

We have said that η_c is isentropic efficiency of compressor here we know that it is having isentropic work input divided by actual work input. So we have

$$\eta_c = \frac{Isentropic\ work\ input}{Actual\ Work\ input} = \frac{h'_{02} - h_{01}}{h_{02} - h_{01}} = \frac{C_p T'_{02} - C_p T_{01}}{C_p T_{02} - C_p T_{01}}$$

$$\eta_c = \frac{T'_{02} - T_{01}}{T_{02} - T_{01}}$$

$$\frac{T'_{02}}{T_{01}} = \left(r_{p_c}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{02} = T_{01} + \frac{1}{\eta_c[T'_{02} - T_{01}]}$$

$$T_{02} = T_{01}\left\{1 + \frac{1}{\eta_c}\left[\frac{T'_{02}}{T_{01}} - 1\right]\right\}$$

$$T_{02} = T_{01}\left\{1 + \frac{1}{\eta_c}\left[\left(r_{p_c}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\right\}$$

Here we should note that we are using not just r_p but we are using r_{p_c} which states that only for compressor and we will see why it is so in next few minutes when we come back for turbine.

We should also note here that we are considering that there is no loss in total pressure if we come from 01 to 02 or from 2 dash to 02 dash. So practically we are considering the $P_{02} = P'_{02}$ so these are the two assumptions which we are making.

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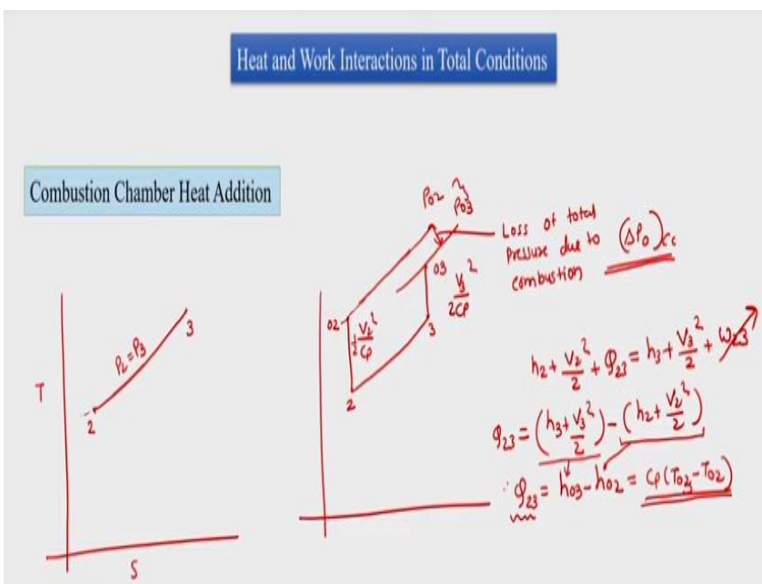

Now let us go and see that heat addition in this combustion process or combustor now first how did we represent the combustion process in case of just TS diagram we said that it is isobaric combustion so it is said 2 to 3 or it is 2 dash to 3 dash or 3 accordingly as per the ideal or real cycle. So we were considering that P2= P3 which was static pressure but now we will account it in terms of total pressure and so we will have suppose 2 here and 3 here and process 2 to 3 like this.

So first will go from 2 to 2 dash isentropically so this is 02 2 to 02 isentropically such that we will get stagnation condition and we will go isentropically from 3 to 03 where we are accounting this as $\frac{\frac{1}{2}V_2^2}{C_p}$and this is $\frac{\frac{1}{2}V_3^2}{C_p}$cp. And then if we try to connect them then we would see that this was P02 and this is P03 where P03 is less than P02 and then this difference in total pressure is loss of total pressure due to combustion and this is how we will start accounting the total pressure loss.

So if we start writing the steady flow energy equation then we can mention

$$h_2 + \frac{V_2^2}{2} + Q_{23} = h_3 + \frac{V_3^2}{2} + W_{23}$$

$$Q_{23} = \left(h_3 + \frac{V_3^2}{2}\right) - \left(h_2 + \frac{V_2^2}{2}\right) = h_{03} - h_{02} = C_p(T_{03} - T_{02})$$

So this is how we will account the heat addition in the process of combustion in this we will account the $(\Delta P_0)_{cc}$ means total pressure loss in the combustion chamber or in the process of combustion. So having said this we will move to our next entity in that next entity is turbine.

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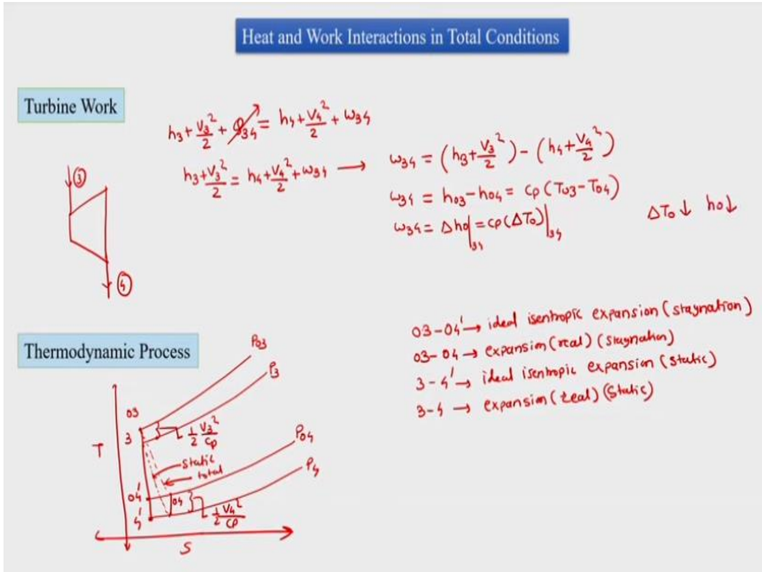

So in case of turbine let us find out the turbine work for that we will again consider the TS diagram in the thermo dynamic process we will consider and then we need to draw 4 lines corresponding to static and stagnation condition. So this is P3 this is P03 this is P04 and this is P4 so for us 3 to 4 dash was the process of isentropic expansion. But now 4 point is brought to 04 and then we get the total condition at 4 point and 3 point is brought to 03 to get the total conditions.

So we will have a process like this where we will have this as 04 point and this would be the ideal 04 point which is 04 dash. So this line represents $\frac{\frac{1}{2}V_3^2}{C_p}$ and then this difference represents $\frac{\frac{1}{2}V_4^2}{C_p}$ so this is how we can get the total condition at different stations and then this is the representation for the case where we have this internal is for this static and this external is for total.

So let us say 03 to 04 dash is expansion isentropic expansion ideal isentropic expansion in terms of stagnation quantities then we have 03 to 04 so this is expansion which is real in terms of stagnation quantities and then we have 03 just 3 to 4 dash is ideal isentropic expansion in terms of static quantities and we have 3 to 4 as expansion which is real and that is with static quantities. So this is what we had also seen in case of compressor so these are the different processors what we had considered for compressor.

So 01 to 02 dash is ideal 1 to 2 1 to 2 dash is compression which is with static and ideal 01 to 02 dash was compression with stagnation and that is ideal. Similarly 1 to 2 was compression which was with static and real and 1 to 01 to 02 dash was compression with stagnation condition was ideal. So the same thing we are looking with the turbine and then we have plotted the process for turbine.

Now we can use the steady flow energy equation for turbine so let us plot this is the turbine so we have station 3 here we have station 4 here

$$h_3 + \frac{V_3^2}{2} + Q_{34} = h_4 + \frac{V_4^2}{2} + W_{34}$$

$$h_3 + \frac{V_3^2}{2} = h_4 + \frac{V_4^2}{2} + W_{34}$$

$$W_{34} = \left(h_3 + \frac{V_3^2}{2}\right) - \left(h_4 + \frac{V_4^2}{2}\right) = h_{03} - h_{04} = C_p(T_{03} - T_{04})$$

$$W_{34} = \Delta h_0|_{34} = C_p(\Delta T_0)|_{34}\,; \Delta T_0(\downarrow) \rightarrow h_0(\downarrow)$$

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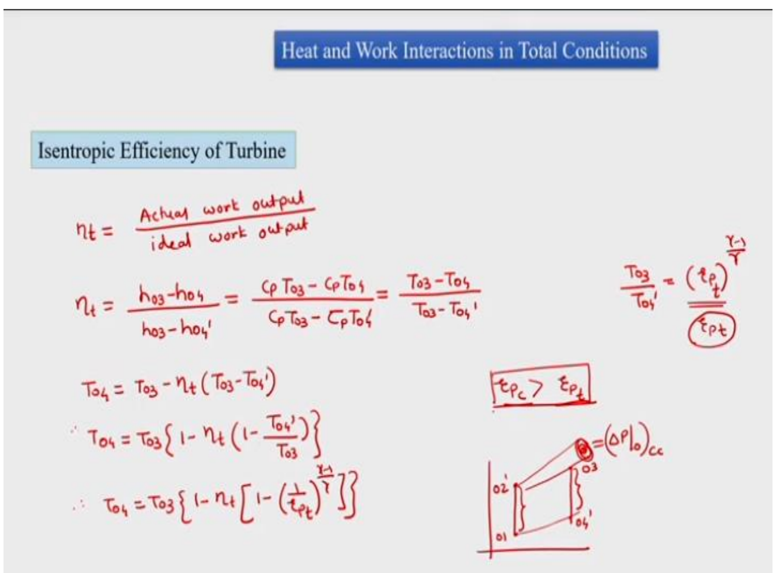

So let us consider the isentropic efficiency of turbine and there we have isentropic efficiency of turbine is equal to actual work output divided by ideal work output now we are trying to represent this in terms of total conditions. So we have

$$\eta_t = \frac{Actual\ work\ output}{Ideal\ work\ output} = \frac{h_{03} - h_{04}}{h_{03} - h'_{04}} = \frac{C_p T_{03} - C_p T_{04}}{C_p T_{03} - C_p T'_{04}} = \frac{T_{03} - T_{04}}{T_{03} - T'_{04}}$$

$$\frac{T_{03}}{T'_{04}} = \left(r_{p_t}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{04} = T_{03} - \eta_t(T_{03} - T'_{04}) = T_{03}\left\{1 - \eta_t\left(1 - \frac{T'_{04}}{T_{03}}\right)\right\} = T_{03}\left\{1 - \eta_t\left(1 - \left(r_{p_t}\right)^{\frac{\gamma-1}{\gamma}}\right)\right\}$$

Here it should be noted that we have used not r_p but r_{p_t} so the point to be noted here that we are using turbine pressure ratio. So there are two pressure ratios one is r_{p_c} which is compressor pressure ratio and then r_{p_t} which is turbine pressure ratio and for the cycle we will generally be using r_{p_c} but why this total pressures are different.

Since at the end of the compression we have reached certain total pressure from 01 to 02 dash or 02 but in the process of combustion this pressure got lost so 03. So there is loss in total pressure and then at the end of expansion we have come back to the same total pressure ratio maybe. But then the total pressure ratio between the inlets and outlets of the compression and turbine are not the same due to loss in total pressure in the combustion chamber.

$\Delta P_0|_{cc}$ so there are two total pressure ratio and within that compressor pressure ratio is always greater than the turbine pressure ratios.

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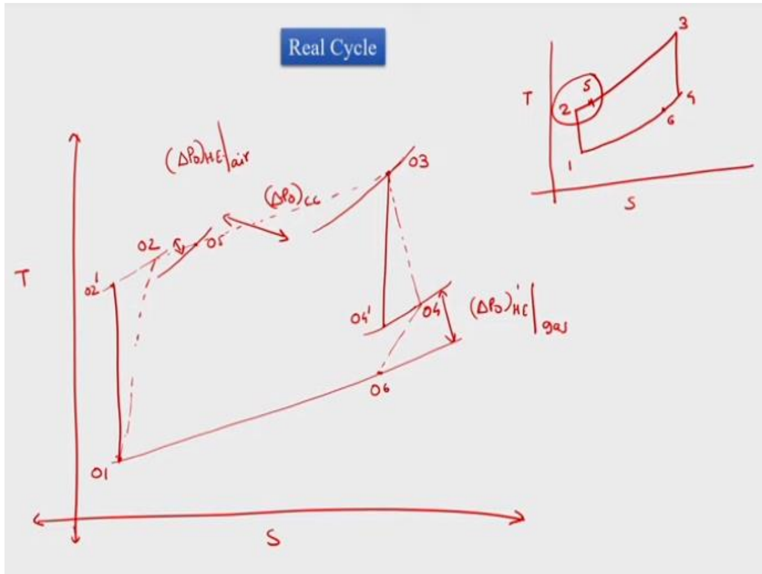

Having said this now we will draw the actual diagram which is the TS diagram for the cycle how would that dual cycle would look like. So let us plot actual TS diagram we know that we are here at stage 01 if were isentropically compressed then we will go state which is 02 dash but in reality we would go to a state which is 02. If there is a heat exchanger then there is certain loss in total pressure in heat exchanger and then we know that our earlier TS diagram with heat exchanger was like this which was 1 to 2, 2 to 3, 3 to 4.

So 2 to 5 was heat exchanger heat addition at the end of the compressor or before the combustion chamber and 4 to 6 was the loss of the heat in the heat exchanger from the gas side. So now if we see the process 2 to 5 it is isobaric heated addition but due to the flow there is loss in total pressure so we will have lower total pressure and then that is 05 and then there is heat addition in the combustion chamber and then we will end up in the point which is here and we will say it as 03 from 03 we would come to 04 dash.

But in reality we will come to 04 due to loss in the turbine but then we will have a heat exchanger in the process and then that heat exchanger would bring us to 06 due to further loss. So this is the real cycle what we can imagine for the gas turbine here this is loss which is ΔP_0 into the heat exchanger from air side. Similar complimentary loss is ΔP_0 into the heat exchanger for the gas side.

So this is the total pressure loss in the combustion chamber so these are all the losses which we have accounted when we started considering the total conditions where we are accounted the losses and so we are accounted the velocities.

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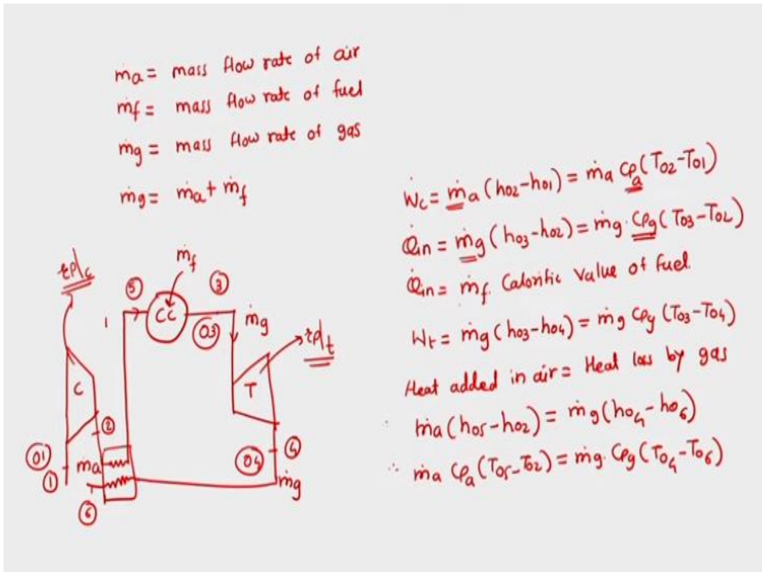

So now we will consider some more facts and those facts include that suppose $\dot{m}_a$ is mass flow rate of air $\dot{m}_f$ is mass flow rate of fuel $\dot{m}_g$ is after combustion which is mass flow rate of gas which is after combustion. So

$$\dot{m}_g = \dot{m}_a + \dot{m}_f$$

So if I write real really then we can have to do like this so we have state 1 or state 01 then we have state 2 or state 02 then we have combustion chamber and in the after the combustion chamber we have state 3 or 03 and then we have turbine and in the turbine we have at end we have 4 or 04.

In the compressor we have air which is going after the entry of the at the outlet of the compressor we have air coming. In the combustion chamber we are $\dot{m}_f$ adding which is fuel flow rate so to the turbine we are passing $\dot{m}_g$ and from the turbine we have $\dot{m}_g$ coming out. So if we write for the compressor then

$$W_c = \dot{m}_a(h_{02} - h_{01}) = \dot{m}_a {C_p}_a(T_{02} - T_{01})$$

$$Q_{in} = \dot{m}_g(h_{03} - h_{02}) = \dot{m}_g C_{p_g}(T_{03} - T_{02})$$

$$Q_{in} = \dot{m}_f * Calorific\ value\ of\ fuel$$

$$\mathrm{W_t} = \dot{m}_g(h_{03} - h_{04}) = \dot{m}_g C_{p_g}(T_{03} - T_{04})$$

$$Heat\ added\ in\ air = Heat\ loss\ by\ gas$$

$$\dot{m}_a(h_{05} - h_{02}) = \dot{m}_g(h_{04} - h_{06})$$

$$\dot{m}_a C_{p_a}(T_{05} - T_{02}) = \dot{m}_g(T_{04} - T_{06})$$

So this is what we can write down for heat exchanger based heat interaction so thus few things to be remembered here that we have to always use differential $C_p's$ for air and gas differential mass flow rates for turbine and compressor and then we have to also consider different pressure ratios which is r_p compressor and r_p turbine.

Since there are losses in the real cycle with this we conclude for the real cycle and its calculations thank you.