Aircraft Propulsion
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Module No # 03
Lecture No # 13
Examples of Gas Turbine Attachment

Welcome to the class now we will have some more solved examples or some more examples which we will solve for practice here onwards in some classes which we will across the chapters whatever we have learnt.

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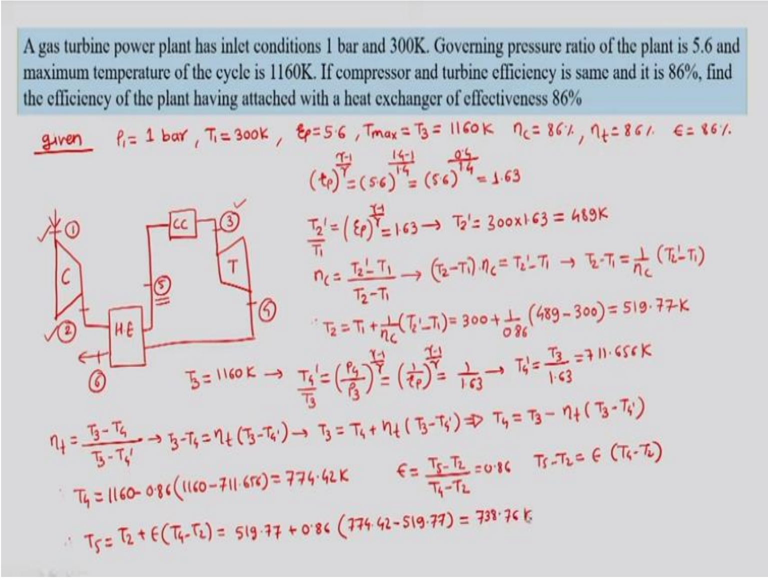

Now first example says that there is a gas turbine power plant which has inlet conditions as 1 bar and 300 Kelvin governing pressure ratio of the plant is 5.6 and maximum temperature of cycle is 1160 Kelvin. If the compressor and turbine efficiency is same and it is 86% find the efficiency of the plant having attached with the heat exchanger of the heat of effectiveness 86%. Let us solve this example so we have something which are given before that as the name suggests in the example it is the gas turbine power plant with heat exchanger.

So let us draw the schematic and this schematic would say that this is 1 this is compressor and after the compressor it will go into the heat exchanger after the heat exchanger we have combustion chamber after the combustion chamber we have turbine from the turbine it will go out into the heat exchanger. We will give our usual number 1 to the compressors, 2 to the compressor outlet, 5 to the heat exchanger outlet, then 3 to the combustion chamber outlet 4 to the turbine outlet and 6 to the heat exchanger outlet for the exhaust.

Then we know given things are

$$P_1 = 1\ bar\ ; T_1 = 300\ K; r_P = 5.6\ ; T_{max} = T_3 = 1160\ K\ ; \eta_c = 86\ \%; \eta_t = 86\ \%; \epsilon = 86\ \%$$

Before solving the example the most required quantity is

$$\left(r_p\right)^{\frac{\gamma-1}{\gamma}} = (5.6)^{\frac{\gamma-1}{\gamma}} = (5.6)^{\frac{1.4-1}{1.4}} = 1.63$$

Now we can calculate the temperature ideal temperature at the exit of the compressor which is

$$\frac{T_2'}{T_1} = \left(r_p\right)^{\frac{\gamma-1}{\gamma}} = 1.63 \rightarrow T_2' = 300 * 1.63 = 489\ K$$

Now we can use compressor efficiency formula which says that

$$\eta_c = \frac{T_2' - T_1}{T_2 - T_1}$$

$$T_2 = T_1 + \frac{1}{\eta_c}(T_2' - T_1) = 300 + \frac{1}{0.86}(489 - 300) = 519.77\ K$$

So we know now everything at the inlet and at the outlet of the compressor now we will move ahead and then we know what is temperature T3 at the inlet to the turbine.

We know T3 = 1160 Kelvin so we can find out T4 but before that we will use it find out ideal temperature at the outlet of the turbine which is

$$\frac{T_4'}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{r_p}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow T_4' = \frac{1160}{1.63} = 711.656\ K$$

Now in this temperature at the outlet of the turbine with ideal condition we can find out actual temperature using turbine efficiency formula.

$$\eta_t = \frac{T_3 - T_4}{T_3 - T_4'} \rightarrow T_4 = T_3 - \eta_t (T_3 - T_4')$$

$$T_4 = 1160 - 0.86 * (1160 - 711.656) = 774.42\ K$$

Now we have to evaluate the condition which are at the inlet to the combustion chamber using the effectiveness formula we know

$$\epsilon = \frac{T_5 - T_2}{T_4 - T_2} = 0.86$$

$$T_5 = T_2 + \epsilon(T_4 - T_2) = 519.77 + 0.86 * (774.42 - 519.77) = 738.76\ K$$

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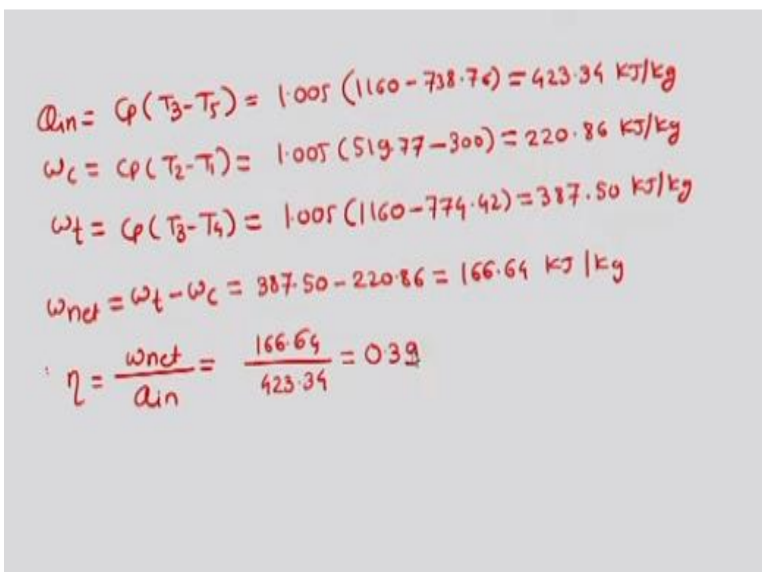

Now we know all temperatures we can find out

$$Q_{in} = C_p(T_3 - T_5) = 1.005 * (1160 - 738.76) = 423.34\ kJ/kg$$

$$W_c = C_p(T_2 - T_1) = 1.005 * (519.77 - 300) = 220.86\ kJ/kg$$

$$W_t = C_p(T_3 - T_4) = 1.005 * (1160 - 774.42) = 387.50\ kJ/kg$$

$$W_{net} = W_t - W_c = 387.50 - 220.86 = 166.64\ kJ/kg$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{166.64}{423.34} = 0.39$$

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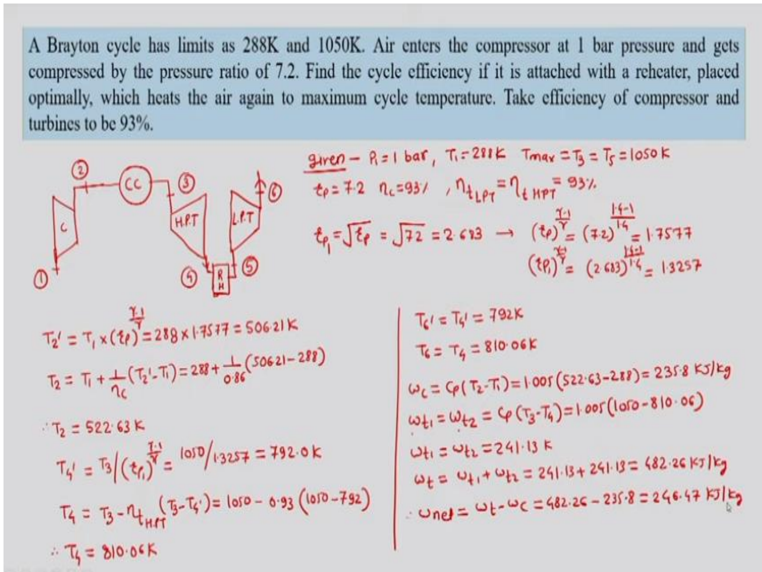

We will go ahead with next example we can start next example which tells that Brayton cycle has limits as 288 Kelvin and 1050 Kelvin. Air enters the compressor at 1 bar pressure and gets compressed by the pressure ratio of 7.2. Find the cycle efficiency if it is attached with a re-heater placed optimally which heats the air again to maximum cycle temperature. Take efficiency of compressor and turbine to be 93% we can first sketch the schematic of the cycle so that we can number the different component their entries and their outlets.

So compressor let us say has entries 1 and 2 if there is a combustion chamber which will have entry 2 and 3 then we are working with a re-heater so there will two turbines and the first turbine will be high pressure turbine and then we will have a second low pressure turbine. So 3 to 4 and between this two turbines we will have a re-heater so 3 to 4, 4 to 5 is re-heater then 5 to 6 is low pressure turbine.

We will note down the given things in this example

$$P_1 = 1\ bar; T_1 = 288\ K; T_{max} = T_3 = T_5 = 1050\ K; r_p = 7.2\ ;\ \eta_c = 93\ \%\ ; \eta_{t_{LPT}} = \eta_{t_{HPT}} = 93\ \%$$

So having noted this we can solve first again it is told importantly to us that the re-heater is placed optimally with both the high pressure and low pressure turbine they should do same work and then we should have pressure ratio for the turbine any turbine

$$r_{p_1} = \sqrt{r_p} = \sqrt{72} = 2.683$$

$$(r_p)^{\frac{\gamma-1}{\gamma}} = (7.2)^{\frac{1.4-1}{1.4}} = 1.7577$$

$$(r_{p_1})^{\frac{\gamma-1}{\gamma}} = (2.683)^{\frac{1.4-1}{1.4}} = 1.3257$$

$$T_2' = T_1 * (r_p)^{\frac{\gamma-1}{\gamma}} = 288 * 1.7577 = 506.21\ K$$

$$T_2 = T_1 + \frac{1}{\eta_c}(T_2' - T_1) = 288 + \frac{1}{0.86}(506.21 - 288) = 522.63\ K$$

$$T_4' = \frac{T_3}{(r_{p_1})^{\frac{\gamma-1}{\gamma}}} = \frac{1050}{1.3257} = 792.0\ K$$

$$T_4 = T_3 - \eta_{t_{HPT}}(T_3 - T_4') = 1050 - 0.93 * (1050 - 792) = 810.06\ K$$

$$T_6' = T_4' = 792\ K$$

$$T_6 = T_4 = 810.06\ K$$

$$W_c = C_p(T_2 - T_1) = 1.005 * (522.63 - 288) = 235.8\ kJ/kg$$

$$W_{t_1} = W_{t_2} = C_p(T_3 - T_4) = 1.005 * (1050 - 810.06)$$

$$W_t = W_{t_1} + W_{t_2} = 241.13 + 241.13 = 482.26\ kJ/kg$$

$$W_{net} = W_t - W_c = 482.26 - 235.8 = 246.47\ kJ/kg$$

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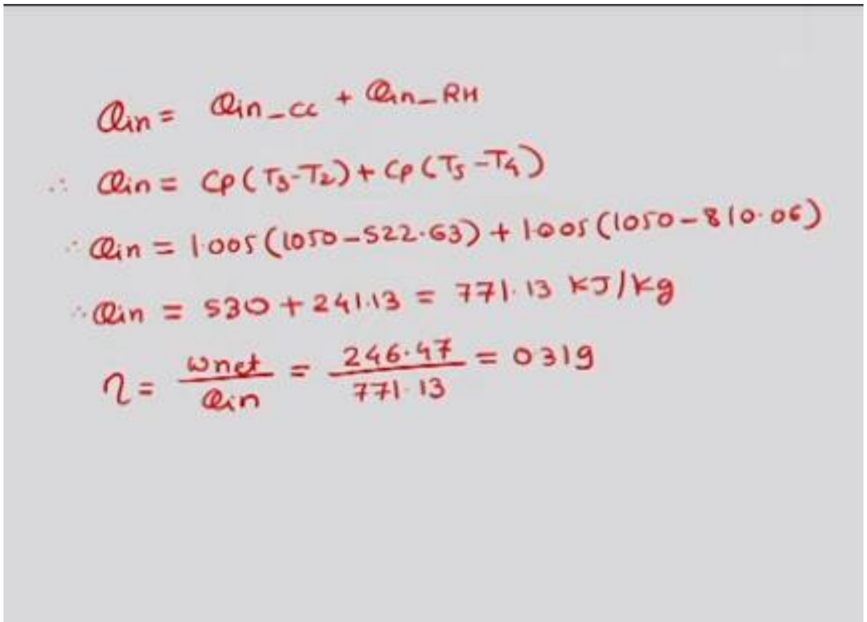

$$Q_{in} = Q_{in_{cc}} + Q_{in_{RH}}$$

$$Q_{in} = C_p(T_3 - T_2) + C_p(T_5 - T_4)$$

$$Q_{in} = 1.005 * (1050 - 522.63) + 1.005 * (1050 - 810.06)$$

$$Q_{in} = 530 + 241.13 = 771.13\ kJ/kg$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{246.47}{771.13} = 0.319$$

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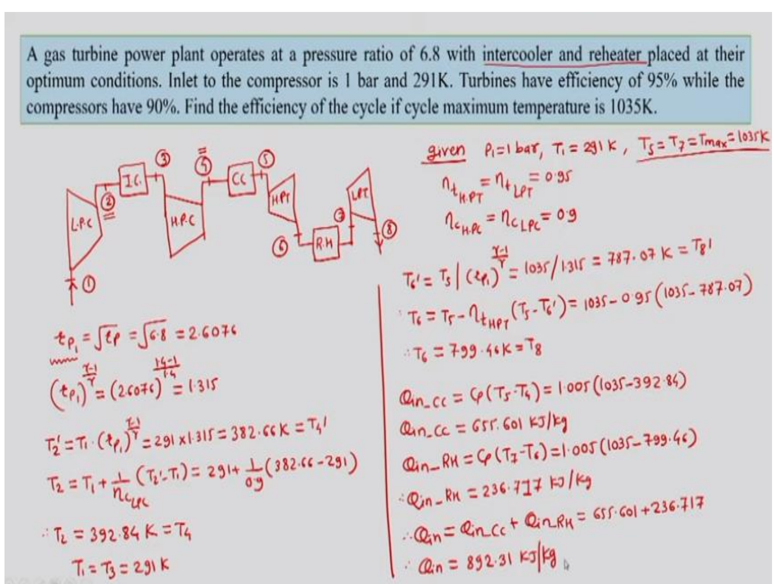

We will move ahead with next example it says that a gas turbine power plant operates at a pressure ratio of 6.8 with intercooler and re-heater placed at their optimum conditions. Inlet to the compressor is 1 bar and 291 Kelvins turbines have efficiency of 95% while the compressor have 90% find the efficiency of the cycle if maximum temperature is 1035 Kelvin. This example has both intercooler and re-heater and they are placed optimally.

So let us sketch the diagram and this is low compressor and then after that and we have intercooler then we have high pressure compressor then we have combustion chamber we have higher pressure turbine then we have re-heater then we have low pressure turbine. So this is state 1, state 2, state 3 , state 4, state 5, state 6, state 7 and state 8 so we have to evaluate 8 states. But the conditions are given to be optimum, so given things for us in this example are

$$P_1 = 1\ bar; T_1 = 291\ K; T_5 = T_7 = T_{max} = 1035\ K$$

$$\eta_{t_{HPT}} = \eta_{t_{LPT}} = 0.95$$

$$\eta_{c_{HPC}} = \eta_{c_{LPC}} = 0.9$$

We can start we can know that for optimum conditions we can find

$$r_{p_1} = \sqrt{r_p} = \sqrt{6.8} = 2.6076$$

$$\left(r_{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (2.6076)^{\frac{1.4-1}{1.4}} = 1.315$$

$$T_2' = T_1 * \left(r_{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 291 * 1.315 = 382.66\ K = T_4'$$

$$T_2 = T_1 + \frac{1}{\eta_{c_{LPC}}(T_2' - T_1)} = 291 + \frac{1}{0.9}(382.66 - 291) = 392.84\ K = T_4$$

$$T_1 = T_3 = 291\ K$$

$$T_6' = \frac{T_5}{\left(r_{p_1}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{1035}{1.315} = 787.07\ K = T_8'$$

$$T_6 = T_5 - \eta_{t_{HPT}}(T_5 - T_6') = 1035 - 0.95(1035 - 787.07) = 799.46\ K = T_8$$

$$Q_{in_{cc}} = C_p(T_5 - T_4) = 1.005(1035 - 392.84) = 655.601\ kJ/kg$$

$$Q_{in_{RH}} = C_p(T_7 - T_6) = 1.005(1035 - 799.46) = 236.717\ kJ/kg$$

$$Q_{in} = Q_{in_{cc}} + Q_{in_{RH}} = 655.601 + 236.717 = 892.31\ kJ/kg$$

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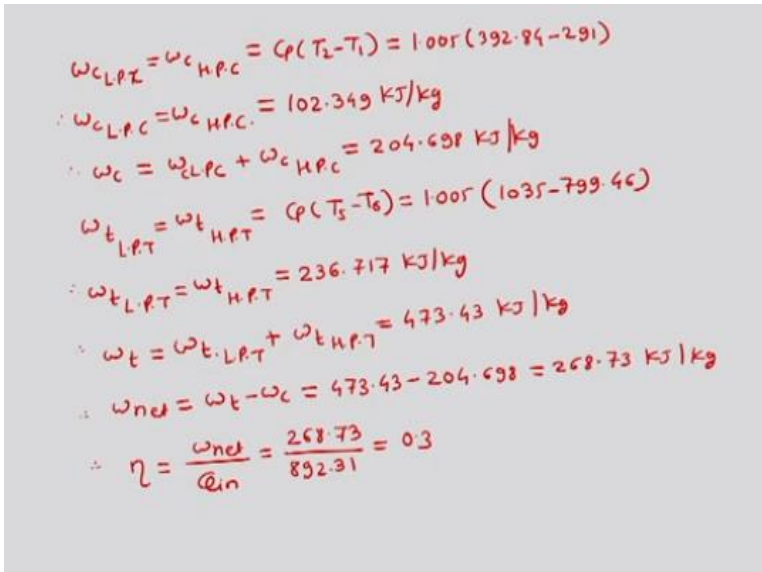

$$W_{c_{LPC}} = W_{c_{HPC}} = C_p(T_2 - T_1) = 1.005(392.84 - 291) = 102.349\ kJ/kg$$
$$W_c = W_{c_{LPC}} + W_{c_{HPC}} = 204.698\ kJ/kg$$

$$W_{t_{LPT}} = W_{t_{HPT}} = C_p(T_5 - T_6) = 1.005(1035 - 799.46) = 236.717\ kJ/kg$$
$$W_t = W_{t_{LPT}} + W_{t_{HPT}} = 473.43\ kJ/kg$$
$$\eta = \frac{W_{net}}{Q_{in}} = \frac{268.73}{892.31} = 0.3$$

So this is how we can solve the examples and elaborate in the exam we will have one example where the data will be given for many objectives and then we have find out all the intermediate steps.

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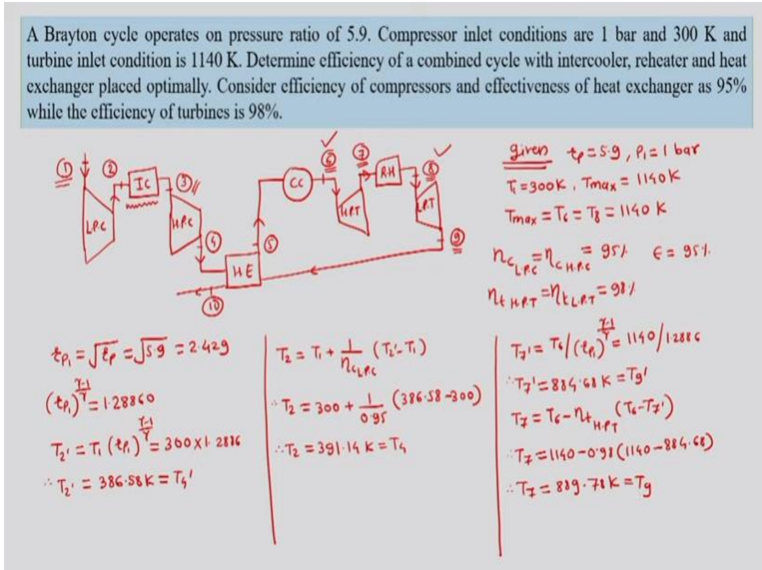

So the example reads as that there is a Brayton cycle and in the Brayton cycle it operates on pressure ratio of 5.9 compressor inlet condition are 1 bar and 300 Kelvin and turbine inlet condition is 1140 determine efficiency of combined cycle which intercooler, re-heater, heat exchanger placed optimally consider efficiency of compressor and turbine compressor and effectiveness of heat exchanger as 95% and efficiency of turbines is 98%.

This example has increased complexity since it has everything in it as intercooler heat exchanger and reheat. So we have first as low pressure compressor after low pressure compressor we have intercooler after intercooler we have the high pressure compressor after that we have heat exchanger then after the heat exchanger there is combustion chamber after the combustion chamber we have low high pressure turbine then we have a re-heater after the re-heater we have low pressure turbine and the exhaust of low pressure turbine goes into the heat exchanger.

So we will name the states as 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 so here we are given

$$r_p = 5.9; P_1 = 1\ bar; T - 1 = 300\ K; T_{max} = T_6 = T_8 = 1140\ K;$$
$$\eta_{t_{HPT}} = \eta_{t_{LPT}} = 98\ \%$$
$$\eta_{c_{HPC}} = \eta_{c_{LPC}} = 95\ \%$$
$$\epsilon = 95\ \%$$

$$r_{p_1} = \sqrt{r_p} = \sqrt{2.429}$$

$$\left(r_{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 1.28860$$

$$T_2' = T_1\left(r_{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 300 * 1.2836 = 386.58\,K = T_4'$$

$$T_2 = T_1 + \frac{1}{\eta_{c_{LPC}}}\left(T_2' - T_1\right) = 300 + \frac{1}{0.95}(386.58 - 300) = 391.14\,K = T_4$$

$$T_7' = \frac{T_6}{\left(r_{p_1}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{1140}{1.2886} = 884.68 = T_9'$$

$$T_7 = T_6 - \eta_{t_{HPT}}\left(T_6 - T_7'\right) = 1140 - 0.98 * (1140 - 884.68) = 889.78\,K = T_9$$

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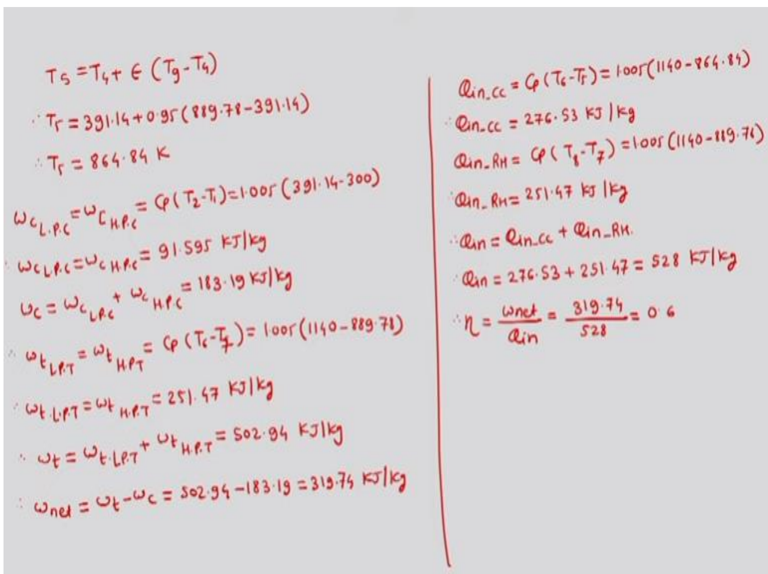

$$T_5 = T_4 + \epsilon(T_9 - T_4) = 391.14 + 0.95(889.78 - 391.14) = 864.84\,K$$

$$W_{c_{HPC}} = W_{c_{LPC}} = C_p(T_2 - T_1) = 1.005(391.14 - 300) = 91.595\,kJ/kg$$

$$W_c = W_{c_{HPC}} + W_{c_{LPC}} = 183.19\,kJ/kg$$

$$W_{t_{HPT}} = W_{t_{LPT}} = C_p(T_6 - T_7) = 1.005 * (1140 - 889.78) = 251.47\,kJ/kg$$

$$W_t = W_{t_{HPT}} + W_{t_{LPT}} = 502.94\,kJ/kg$$

$$W_{net} = W_t - W_c = 502.94 - 183.19 = 319.74\ kJ/kg$$

$$Q_{in_{cc}} = C_p(T_6 - T_5) = 1.005 * (1140 - 864.84) = 276.53\ kJ/kg$$

$$Q_{in_{RH}} = C_p(T_8 - T_7) = 1.005(1140 - 889.76) - 251.47\ kJ/kg$$

$$Q_{in} = Q_{in_{cc}} + Q_{in_{RH}} = 276.53 + 251.47 = 528\ kJ/kg$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{319.74}{528} = 0.6$$

This is how we will solve the examples related to the cycle we will continue with the example practice for the other sectors other chapters in the next class thank you.